Research Article

\(H_\infty\) Fuzzy Control for Nonlinear Singular Markovian Jump Systems with Time Delay

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This paper investigates the problem of \(H_\infty\) fuzzy control for a class of nonlinear singular Markovian jump systems with time delay. This class of systems under consideration is described by Takagi-Sugeno (T-S) fuzzy models. Firstly, sufficient condition of the stochastic stabilization by the method of the augmented matrix is obtained by the state feedback. And a designed algorithm for the state feedback controller is provided to guarantee that the closed-loop system not only is regular, impulse-free, and stochastically stable but also satisfies a prescribed \(H_\infty\) performance for all delays not larger than a given upper bound in terms of linear matrix inequalities. Then \(H_\infty\) fuzzy control for this kind of systems is also discussed by the static output feedback. Finally, numerical examples are given to illustrate the validity of the developed methodology.

1. Introduction

Singular systems, also known as descriptor systems, have been widely studied in the past several decades. They have broad applications and can be found in many practical systems, such as electrical circuits, power systems, network, economics, and other systems [1, 2]. Due to their extensive applications, many research topics on singular systems have been extensively investigated such as the stability and stabilization [3, 4] and \(H_\infty\) control problem [5, 6]. A lot of attention has been paid to the investigation of Markovian jump systems (MJSs) over the past decades. Applications of such class of systems can be found representing many physical systems with random changes in their structures and parameters. Many important issues have been studied for this kind of physical systems, such as the stability analysis, stabilization, and \(H_\infty\) control [7–10]. When singular systems experience abrupt changes in their structures, it is natural to model them as singular Markovian jump systems (SMJSs) [11–13]. Time delay is one of the instability sources for dynamical systems and is a common phenomenon in many industrial and engineering systems such as those in communication networks, manufacturing, and biology [14]. So the study of SMJSs with time delay is of theoretical and practical importance [15, 16].

The fuzzy control has been proved to be a powerful method for the control problem of complex nonlinear systems. Specially, the Takagi-Sugeno (T-S) fuzzy model has attracted much attention due to the fact that it provides an efficient approach to take full advantage of the linear control theory to the nonlinear control. In recent years, this fuzzy-model-based technique has been used to deal with nonlinear time delay systems [17, 18] and nonlinear MJSs [19, 20]. But singular Markovian jump fuzzy systems (SMJFSs) are not fully studied [21, 22], which motivates the main purpose of our study. In this paper, a new method using the augmented matrix will be given to the control of SMJFSs. By this method the number of LMIs will be decreased, so the complexity of the calculation will be greatly reduced when the number of fuzzy rulers is relatively large. And, at the same time, some new relaxation matrices added will reduce the conservation of control conditions compared with
previous literatures. And when using the augmented matrix to design the static output feedback control, there are not any crossing terms between system matrices and controller gains, so assumptions for the output matrix [23], the equality constraint for the output matrix [24], and the bounding technique for crossing terms are not required; therefore, the conservatism brought by them will not exist.

In this paper, the $H_{\infty}$ fuzzy control problem for a class of nonlinear SMJFSs with time delay which can be represented by T-S fuzzy models is considered. Our purpose is to design fuzzy state feedback controllers and static output feedback controllers for SMJFSs with time delay, such that closed-loop systems are stochastically admissible (regular, impulse-free, and stochastically stable) with a prescribed $H_{\infty}$ performance $y$. Sufficient criteria are presented in forms of LMIs which are simple and easy to implement compared with previous literatures. Finally, numerical examples are given to illustrate the merit and usability of the approach proposed in this paper.

*Notations.* Throughout this paper, notations used are fairly standard; for real symmetric matrices $A$ and $B$, the notation $A \geq B$ ($A > B$) means that the matrix $A - B$ is positive semidefinite (positive definite). $A^T$ represents the transpose of the matrix $A$, and $A^{-1}$ represents the inverse of the matrix $A$. $\lambda_{\max}(B)$ $(\lambda_{\min}(B))$ is the maximal (minimal) eigenvalue of the matrix $B$. diag($\cdot$) stands for a block-diagonal matrix. $I$ is the unit matrix with appropriate dimensions, and, in a matrix, the term of symmetry is stated by the asterisk “$*$.” Let $\mathbb{R}^n$ stand for the $n$-dimensional Euclidean space, $\mathbb{R}_+^{nm}$ is the set of all $n \times m$ real matrices, and $\| \cdot \|$ denotes the Euclidean norm of vectors. $\mathcal{B}(\cdot)$ denotes the mathematics expectation of the stochastic process or vector. $L^p_2([0,\infty))$ stands for the space of $n$-dimensional square integrable functions on $[0,\infty)$. $C_{nd} = C([-d,0],\mathbb{R}^n)$ denotes Banach space of continuous vector functions mapping the interval $[-d,0]$ into $\mathbb{R}^n$ with the norm $\|\phi_{nd}\| = \sup_{-d \leq s \leq 0} \|\phi(s)\|$.

### 2. Basic Definitions and Lemmas

Consider a SMJFS; its $i$th fuzzy rule is given by

\[ R_i: \text{if } \xi^1(t) \text{ is } M_{i1}, \xi^2(t) \text{ is } M_{i2}, \ldots, \text{ and } \xi^k(t) \text{ is } M_{ik}, \text{ then} \]

\[
\begin{align*}
\dot{x}(t) &= A_i(r_i)x(t) + A_{d,i}(r_i)x(t-d) + B_i(r_i)u(t) + B_{wij}(r_i)w(t), \\
z(t) &= C_i(r_i)x(t) + C_{d,i}(r_i)x(t-d) + D_i(r_i)u(t), \\
x(t) &= \phi(t),
\end{align*}
\]

\[ \forall t \in [-d,0], i \in \mathcal{T} \equiv \{1,2,\ldots,k\}, \]

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input, $w(t) \in \mathbb{R}^p$ is the exogenous disturbance which belongs to $L^2_2([0,\infty))$, and $z(t) \in \mathbb{R}^p$ is the controlled output. $\phi(t) \in C_{nd}$ is a compatible vector-valued initial function, and $d$ is an unknown but constant delay satisfying $d \in [0,\bar{d}]$. The scalar $k$ is the number of If-Then rules. $M_{ij}$ ($i, j = 1,2,\ldots,l$) are fuzzy sets. $\xi^i(t) - \xi^i(t)$ are premise variables. $E \in \mathbb{R}^{n \times n}$ may be a singular matrix with rank $E = r \leq n$. $A_i(r_i), A_{d,i}(r_i), B_i(r_i), B_{wij}(r_i), C_i(r_i), C_{d,i}(r_i), D_i(r_i),$ and $C_{wij}(r_i)$ are known constant matrices with appropriate dimensions. $\{r_i, t \geq 0\}$ is a continuous-time Markovian process with right continuous trajectories taking values in a finite set given by $\mathcal{S} = \{1,2,\ldots,N\}$ with the transition rate matrix $\Pi \equiv \{\pi_{pq}\}$ satisfying

\[
\Pr \{r_{i+h} = q | r_i = p\} = \begin{cases} 
\pi_{pq} h + o(h) & p \neq q \\
1 + \pi_{pp} h + o(h) & p = q,
\end{cases}
\]

where $h > 0$, $\lim_{h \to 0} o(h)/h = 0$, and $\pi_{pq} \geq 0$, for $q \neq p$, is the transition rate from mode $p$ at time $t$ to $q$ at time $t+h$ and $\pi_{pp} = N - \sum_{q=1,qs} \pi_{pq}$.

By fuzzy blending, the overall fuzzy model is inferred as follows:

\[
\begin{align*}
\dot{E}x(t) &= \sum_{i=1}^{k} \lambda_i(\xi(t))(A_i(r_i)x(t) + A_{d,i}(r_i)x(t-d)) + B_i(r_i)u(t) + B_{wij}(r_i)w(t), \\
z(t) &= \sum_{i=1}^{k} \lambda_i(\xi(t))(C_i(r_i)x(t) + C_{d,i}(r_i)x(t-d)) + D_i(r_i)u(t) + C_{wij}(r_i)w(t), \\
x(t) &= \phi(t),
\end{align*}
\]

\[ \forall t \in [-\bar{d},0], i \in \mathcal{T} \equiv \{1,2,\ldots,k\}, \]

where $\xi(t) = [\xi^1(t) \xi^2(t) \cdots \xi^k(t)]^T$. $\beta_i(\xi(t)) = \Pi_{j=1}^{k} M_{ij}(\xi_j(t))$. Letting $\lambda_i(\xi(t)) = \beta_i(\xi(t))/\sum_{j=1}^{k} \beta_j(\xi(t))$, it follows that $\lambda_i(\xi(t)) \geq 0, \sum_{i=1}^{k} \lambda_i(\xi(t)) = 1$.

By the notational simplicity, in the sequel, for each possible $r_i = p \in \mathcal{S}, A_i(r_i) \equiv A_p, B_{d,i}(r_i) \equiv B_{d,p}, C_{d,i}(r_i) \equiv C_{d,p}, \lambda_i(\xi(t)) \equiv \lambda_p,$ and so on.

**Definition 1** (see [15, 25]). (i) For a given scalar $\bar{d} > 0$, the SMJS with time delay

\[
\begin{align*}
\dot{x}(t) &= A(r_i)x(t) + A_d(r_i)x(t-d), \\
x(t) &= \phi(t),
\end{align*}
\]

\[ t \in [-\bar{d},0] \]

is said to be regular and impulse-free for any constant time delay satisfying $d \in [0,\bar{d}]$, if pairs $(E, A(r_i))$ and $(E, A_d(r_i) + A_d(r_i))$ are regular and impulse-free.

(ii) System (4) is said to be stochastically stable if there exists a finite number $M(\phi(t), r_0)$ such that the following inequality holds:

\[
\lim_{t \to \infty} \mathbb{E} \left\{ \int_{0}^{t} \|x(s)\|^2 ds \mid r_0, x(s) = \phi(s), s \in [-\bar{d},0] \right\} < M(\phi(t), r_0).
\]
(iii) System (4) is said to be stochastically admissible if it is regular, impulse-free, and stochastically stable.

Lemma 2 (see [26]). Given matrices $E, X > 0, Y$, if $E^T X + Y A^T$ is nonsingular, there exist matrices $S > 0, L$, such that $ES + LQ = (E^T X + Y A^T)^{-1}$, where $A, Q \in \mathbb{R}^{n \times (n-r)}$, such that $E^T A = 0, E^T 0 = 0$, rank $A = \text{rank } \Theta = n - r$, $X, S \in \mathbb{R}^{n \times n}$, and $Y, L \in \mathbb{R}^{n \times (n-r)}$.

Lemma 3 (see [27]). For matrices $Q > 0, P$, and $R$ with appropriate dimensions, the following inequality holds:

$$PR^T + RB^T \leq R Q R^T + PQ^{-1} P^T.$$  \hspace{1cm} (6)

Lemma 4 (see [28]). For any constant matrix $X \in \mathbb{R}^{n \times n}$, $X = X^T > 0$, scalar $r > 0$, and vector function $\bar{x} : [-r, 0] \rightarrow \mathbb{R}^n$ such that the following integration is well defined; then

$$\int_{-r}^{0} \dot{x}^T(t + s) X \dot{x}(t + s) ds \leq \dot{x}^T(t) X \dot{x}(t) - \int_{-r}^{0} \dot{x}^T(t - r) X \dot{x}(t - r) ds.$$  \hspace{1cm} (7)

Lemma 5 (see [29]). Suppose there are piecewise continuous real square matrices $A(t), X$, and $Q > 0$ satisfying $A^T(t) X + X^T A(t) < 0$ for all $t$. Then the following conditions hold:

(i) $A(t)$ and $X$ are nonsingular.
(ii) $\| A^{-1}(t) \| \leq \delta$ for some $\delta > 0$.

Lemma 6 (see [30]). If the following conditions hold:

$$M_{ij} < 0, \quad 1 \leq i \leq r;$$

$$\frac{1}{r - 1} M_{ij} + \frac{1}{2} \left( M_{ij} + M_{ji} \right) < 0, \quad 1 \leq i \neq j \leq r,$$

then the following parameterized matrix inequality holds:

$$\sum_{i=1}^{k} \sum_{j=1}^{r} \alpha_i(t) \alpha_j(t) M_{ij} < 0,$$  \hspace{1cm} (9)

where $\alpha_i(t) \geq 0$ and $\sum_{i=1}^{r} \alpha_i(t) = 1$.

Based on the parallel distributed compensation, the following state feedback controller will be considered here:

$$u_p(t) = \sum_{i=1}^{k} \lambda_i K_{pi} x(t),$$  \hspace{1cm} (10)

where $K_{pi} (p \in \mathcal{D}, i \in \mathcal{T})$ are local controller gains, such that the closed-loop system

$$\dot{E} \bar{x}(t) = \sum_{i=1}^{k} \sum_{j=1}^{r} \lambda_i \lambda_j \left( (A_{pi} + B_{pi} K_{pi}) x(t) + A_{d,pi} x(t - d) + B_{w,pi} w(t) \right),$$

$$z(t) = \sum_{i=1}^{k} \sum_{j=1}^{r} \lambda_i \lambda_j \left( (C_{pi} D_{pi} K_{pi}) x(t) + C_{d,pi} x(t - d) + C_{w,pi} w(t) \right),$$

$$\bar{x}(t) = \sum_{i=1}^{k} \lambda_i \phi(t),$$

is stochastically admissible.

3. The Design of the State Feedback $H_\infty$ Controller

Firstly, the sufficient condition will be given such that system (11) is stochastically admissible. Combining (4) and (10), fuzzy closed-loop system (11) can be rewritten in the following form:

$$\dot{E} \bar{x}(t) = \sum_{i=1}^{k} \lambda_i \left( \overline{A}_{pi} \bar{x}(t) + \overline{A}_{d,pi} \bar{x}(t - d) + \overline{B}_{w,pi} w(t) \right),$$

$$z(t) = \sum_{i=1}^{k} \lambda_i \left( \overline{C}_{pi} \bar{x}(t) + \overline{C}_{d,pi} \bar{x}(t - d) + \overline{C}_{w,pi} w(t) \right),$$

$$\bar{x}(t) = \phi(t),$$

$$t \in [-\overline{d}, 0]$$  \hspace{1cm} (11)
Remark 7. For systems (11) and (12), it can be seen that
\[
\det \left( sE - \sum_{i=1}^{k} \sum_{j=1}^{k} \lambda_i \lambda_j \begin{bmatrix} A_{pi} + B_{pi}K_{pj} \end{bmatrix} \right)
= \det \left( s \begin{bmatrix} E & 0 \\ 0 & 0 \end{bmatrix} - \sum_{i=1}^{k} \sum_{j=1}^{k} \lambda_i \lambda_j \begin{bmatrix} A_{pi} + B_{pi}K_{pj} - I_m \end{bmatrix} \right)
= \det \left( sE - \sum_{i=1}^{k} \lambda_i \begin{bmatrix} A_{pi} + B_{pi}K_{pj} - I_m \end{bmatrix} \right)
= \det \left( sE - \sum_{i=1}^{k} \lambda_i \begin{bmatrix} A_{pi} + B_{pi}K_{pj} - I_m \end{bmatrix} \right)
= \det \left( sE - \sum_{i=1}^{k} \lambda_i \begin{bmatrix} A_{pi} + B_{pi}K_{pj} - A_{d,pi} \end{bmatrix} \right)
\]

By rank $E = \text{rank} \bar{E}$ and Definition 1, it can be obtained that the regularity and nonimpulse of system (11) are equal to the regularity and nonimpulse of system (12). So the stochastic admissibility of system (11) can be studied by system (12).

Theorem 8. For a prescribed scalar $\bar{d} > 0$, there exists a state feedback controller (10) with $u_p(t) = \sum_{i=1}^{k} A_i L_p Y^{-1}_p x(t)$ such that system (11) when $w(t) = 0$ is stochastically admissible for any constant time delay $d$ satisfying $d \in [0, \bar{d})$, if there exist matrices $\overline{P}_p > 0$, $\overline{Q}_p > 0$, $\overline{Z} > 0$, $L_{pi}$, $\overline{S}_p$, $Y_{p2}$, and $Y_{p3}$, $i \in \mathcal{I}$, $p \in \mathcal{S}$, such that

\[
\begin{bmatrix}
\Gamma_{1pi} & * & * & * & * \\
L_p + Y_p^T A_{p2} Y_{p2} Y_p & -Y_{p3} & -Y_{p3} & -Y_{p3} & -Y_{p3} \\
(\gamma_{p2}^T \gamma_{p2})/p & 0 & -\gamma_{p2} & -\gamma_{p2} & -\gamma_{p2} \\
\overline{d} A_{p1} Y_p - \overline{d} B_{pi} Y_{p2} & \overline{d} B_{pi} Y_{p3} & \overline{d} B_{pi} Y_{p3} & \overline{d} B_{pi} Y_{p3} & \overline{d} B_{pi} Y_{p3} \\
Y_p & 0 & 0 & 0 & -\overline{Q}_p & * \\
\overline{d} Y_p & 0 & 0 & 0 & 0 & -\overline{Q}_p & * \\
[I_r, 0] H^{-1} M_p^T & 0 & 0 & 0 & 0 & 0 & -I_p \\
\end{bmatrix}
\]

where $\Gamma_{1pi} = \pi_{pp} Y_p^T \gamma_{pi} + A_{p2} Y_p + Y_p^T A_{pi} - Y_p^T B_{pi} - B_{pi} Y_{p2} - Y_p^T \overline{d} Y_{p2} - E Y_p + \overline{Z} Y_p = (\overline{E} Y_p + \overline{S}_p \overline{R}^T)^T$, $L_p = K_{p} Y_{p2}$, $Y_{p2} = Y_{p3} \overline{v}_p Y_{p3}$, $M_p = \begin{bmatrix} \sqrt{\pi_{p1}} Y_{p1}^T & \sqrt{\pi_{p2}} Y_{p1}^T & \sqrt{\pi_{p1}} Y_{p1}^T & \sqrt{\pi_{p1}} Y_{p1}^T & \sqrt{\pi_{p1}} Y_{p1}^T \\
\sqrt{\pi_{p2}} Y_{p1}^T & \sqrt{\pi_{p2}} Y_{p1}^T & \sqrt{\pi_{p2}} Y_{p1}^T & \sqrt{\pi_{p2}} Y_{p1}^T & \sqrt{\pi_{p2}} Y_{p1}^T \\
\sqrt{\pi_{p1}} Y_{p1}^T & \sqrt{\pi_{p2}} Y_{p1}^T & \sqrt{\pi_{p2}} Y_{p1}^T & \sqrt{\pi_{p2}} Y_{p1}^T & \sqrt{\pi_{p2}} Y_{p1}^T \\
\sqrt{\pi_{p1}} Y_{p1}^T & \sqrt{\pi_{p2}} Y_{p1}^T & \sqrt{\pi_{p2}} Y_{p1}^T & \sqrt{\pi_{p2}} Y_{p1}^T & \sqrt{\pi_{p2}} Y_{p1}^T \\
\sqrt{\pi_{p1}} Y_{p1}^T & \sqrt{\pi_{p2}} Y_{p1}^T & \sqrt{\pi_{p2}} Y_{p1}^T & \sqrt{\pi_{p2}} Y_{p1}^T & \sqrt{\pi_{p2}} Y_{p1}^T \\
\end{bmatrix}$, $T_p = \text{diag}(Q_1, Q_{p-1}, Q_p, ..., Q_N)$, $I_p = \text{diag}(\Phi_1, ..., \Phi_{p-1}, \Phi_p, ..., \Phi_N)$, $\Phi_i = \begin{bmatrix} I_r & 0 \end{bmatrix}$ GEY_q G^T \begin{bmatrix} I_r \end{bmatrix}$, $\overline{R} \in \mathbb{R}^{r \times (n-r)}$ is any matrix with full column rank and satisfies $\overline{E} \overline{R} = 0$, and $G, H$ are nonsingular matrices that make $GEH = \begin{bmatrix} 0 \vert \overline{0} \end{bmatrix}$.

Proof. From ??, it can be concluded that $Y_p$ and $Y_{p3}$ are nonsingular matrices. Because $Y_p = (\overline{E} Y_p + \overline{S}_p \overline{R}^T)^T$,
\[
Y_p^T \gamma_{p2} = E Y_p = \overline{E} Y_p \gamma_{p2} \geq 0.
\]

Denote $H^{-1} Y_p G^T = \begin{bmatrix} Y_{p11} & Y_{p12} \\
Y_{p21} & Y_{p22} \end{bmatrix}$ from (17), it is easy to obtain that $Y_{p22} = 0$ and $Y_{p11}$ is symmetric; then $H^{-1} Y_p G^T = \begin{bmatrix} Y_{p11} & 0 \\
Y_{p21} & Y_{p22} \end{bmatrix}$. So it can be concluded that $Y_{p11}$ and $Y_{p22}$ are nonsingular; furthermore, $G^{-T} Y_p H = \begin{bmatrix} Y_{p11} & 0 \\
-Y_{p21} & Y_{p22} \end{bmatrix}$. Let
\[
\tilde{Y}_p = \begin{bmatrix} Y_p & 0 \end{bmatrix}. \text{So } [I_r, 0] \text{ diag}(G, I_m) \tilde{Y}_q \text{ diag}(G^T, I_m) \begin{bmatrix} 0 \end{bmatrix} = Y_{q11} \text{ is nonsingular. By Lemma 2, } X_p = \begin{bmatrix} Y_p & 0 \end{bmatrix} = (E^T Y_p + \overline{S}_p \overline{R}^T)^T, \text{ where } P_p > 0, S_p \in \mathbb{R}^{n \times (n-r)}, \text{ and } R \in \mathbb{R}^{r \times (n-r)} \text{ is a matrix with full column rank and satisfies } E^T R = 0. \text{ Denote } X_{p2} \triangleq Y_{p1} - \overline{Y}_p, X_{p2} \triangleq \overline{Y}_p = Y_{p1}^T Y_p^{-1} p, \text{ and } \overline{X}_p \triangleq \overline{Y}_p^{-1} = \begin{bmatrix} X_{p2} & 0 \end{bmatrix}. \text{ So }
\]
\[
\begin{bmatrix}
H^{-T} \begin{bmatrix} I_r \\
0 \end{bmatrix} \\
0_{msr} \end{bmatrix} \begin{bmatrix} 0 \end{bmatrix} \text{ diag}(H^{-T}, I_m) = \text{ diag}(H^{-T}, I_m)
\]
Denote $Q_p \triangleq Q^{-1}$, $Q \triangleq Q^{-1}$, and $Z \triangleq Z^{-1}$. By Lemma 3, it can be obtained that
\[
\begin{bmatrix}
-E^TZE & E^TZE \\
E^TZE & -E^TZE
\end{bmatrix}
= \begin{bmatrix}
I_n & -I_n \\
-I_n & (E^TZE) [I_n - I_n]
\end{bmatrix},
\]
(19)

Now pre- and postmultiplying (18) by $diag(\bar{X}_p^T, X_p^T, Z, I_n, I_n, I_n, \ldots, I_n)$ and its transpose, by Schur complement lemma, and (18)-(19), it is easy to see that
\[
\begin{bmatrix}
\Gamma_{1p1} & * & * \\
E^T[Z\ 0] \bar{E} + [A_{d,pi}^T \ 0] \bar{X}_p - E^TZE - Q_p & * \\
\bar{d}Z \ A_{pi} & \bar{d}ZA_{d,pi} & -Z
\end{bmatrix}
< 0,
\]
where $\Gamma_{1p1} = N_p \left( \sum_{i=1}^{N} \pi_{pi} E^T \bar{X}_p + \bar{X}^T_p A_{pi} + \bar{A}^T_p \bar{X}_p + \text{diag}(Q_p, 0) + \bar{d} \text{diag}(Q, 0) - E^T \text{diag}(Z, 0) \bar{E} \right)$. Pre- and postmultiplying (18) by $diag(\bar{X}_p^T, l_1, \ldots, l_1)$ and its transposition by Schur complement lemma, it can be seen that
\[
\sum_{q=1}^{N} \pi_{pq} Q_q < Q.
\]
(21)

From (20), it can be concluded that
\[
\sum_{i=1}^{k} \pi_{pp} \left( E^T \bar{X}_p + \bar{X}^T_p A_{pi} + \bar{A}^T_p \bar{X}_p \right) - E^T \text{diag}(Z, 0) \bar{E} < 0.
\]
(22)

On the other hand, $\text{diag}(G, I_m) \bar{E} \text{diag}(H, I_m) = \begin{bmatrix}
[I_n \ 0] & 0 \\
0 & 0
\end{bmatrix}_{m \times m}$. Then
\[
E^T \bar{X}_p = \bar{X}^T_p \bar{E} = \begin{bmatrix}
E^T & 0 \\
0 & X_p \end{bmatrix} = \begin{bmatrix}
E^T X_p & 0 \\
0 & 0
\end{bmatrix}
= \begin{bmatrix}
E^T P_p E & 0 \\
0 & 0
\end{bmatrix} \geq 0.
\]
(23)

Denote $\bar{A}_p(t) \triangleq \left( \sum_{i=1}^{k} \lambda_i \bar{A}_{pi} \right) = \left[ \bar{A}_{pi}(t), \bar{A}_{pi}(t) \right]$; from (22), it can be obtained that
\[
X_p^T \bar{A}_p(t) + \bar{A}_p^T(t) X_p < 0,
\]
(24)
for every $p \in \delta$, which implies that $\bar{A}_p(t)$ is nonsingular. Thus, the pair $(\bar{E}, \sum_{i=1}^{k} \bar{A}_{pi})$ is regular and impulse-free for every $p \in \delta$. By (20), it is easy to see that
\[
\begin{bmatrix}
\bar{X}_p & \bar{X}^T_p \bar{A}_{pi} + \bar{A}_{d,pi} \end{bmatrix}
< 0.
\]
(25)

Pre- and postmultiplying (25) by $\begin{bmatrix}
I_m \ [\bar{g} \ 0] \\
0 \ \bar{I}_n
\end{bmatrix}$ and its transpose, it can be obtained that
\[
\sum_{q=1}^{N} \pi_{pq} E^T \bar{X}_p + \bar{X}^T_p \left( \bar{A}_{pi} + \bar{A}_{d,pi} \right) + \left( \bar{A}_{pi} + \bar{A}_{d,pi} \right)^T \bar{X}_p < 0.
\]
(26)

Hence,
\[
\sum_{i=1}^{k} \pi_{pi} \left( E^T \bar{X}_p + \bar{X}^T_p \left( \bar{A}_{pi} + \bar{A}_{d,pi} \right) + \left( \bar{A}_{pi} + \bar{A}_{d,pi} \right)^T \bar{X}_p < 0. \right.
\]
(27)

Equation (27) implies that the pair $(\bar{E}, \sum_{i=1}^{k} \lambda_i (\bar{A}_{pi} + \bar{A}_{d,pi}))$ is regular and impulse-free for every $p \in \delta$. Thus, by Definition 1, system (12) is regular and impulse-free. By Remark 7, this implies that system (11) is regular and impulse-free.

Now, it will be shown that system (11) is stochastically stable. Define a new process $\{X(t), r(t), t \geq 0\}$ by $X_t = x(t + \theta), -2d \leq \theta \leq 0$; then $\{X(t), r(t), t \geq 0\}$ is a Markovian process with the initial state $(\phi(t), r_0)$. Now, for $t \geq d$, choose the following stochastic Lyapunov-Krasovskii candidate for this system:
\[
V(x(t), p(t)) = \sum_{m=1}^{N} V_m(x(t), p(t)),
\]
(28)

where
\[
\begin{align*}
V_1(x(t), p(t)) &= x^T(t) E^T P_t E x(t) = x^T(t) E^T X_p x(t) \\
&= x^T(t) E^T \bar{X}_p x(t), \\
V_2(x(t), p(t)) &= \int_{t-d}^{t} x^T(s) Q_P x(s) ds, \\
V_3(x(t), p(t)) &= d \int_{t-d}^{t} \bar{x}^T(s) E^T Z E x(s) ds d\theta, \\
V_4(x(t), p(t)) &= \int_{t-d}^{t} \int_{t+\theta}^{t} \bar{x}^T(s) Q x(s) ds d\theta,
\end{align*}
\]
(29)
Let $\mathcal{L}$ be the weak infinitesimal generator of the random process $\{(x_t, p), t \geq 0\}$. Then, for each $p \in \mathcal{S}$,

$$\mathcal{L}V(x_t, p, t) \leq 2\bar{Z}^T(t) \bar{X}_p \bar{E}(t)$$

$$+ \bar{x}^T(t) \left( \sum_{q=1}^{N} \pi_{pq} E^T \bar{X}_q \right) \bar{x}(t)$$

$$+ x^T(t) Q_p x(t)$$

$$- x^T(t - d) Q_p x(t - d)$$

$$+ \int_{t-d}^{t} x^T(s) \sum_{q=1}^{N} \pi_{pq} Q_q x(s) ds$$

$$+ d\bar{x}^T(t) Q x(t)$$

$$- \int_{t-d}^{t} x^T(s) Q x(s) ds$$

$$+ d \int_{t-d}^{t} x^T(s) E^T Z \bar{E} x(s) ds.$$ 

From (21), it is clear that

$$\int_{t-d}^{t} x^T(s) \left( \sum_{q=1}^{N} \pi_{pq} Q_q \right) x(s) ds$$

$$< \int_{t-d}^{t} x^T(s) Q x(s) ds.$$ 

From Lemma 4, it follows that

$$- d \int_{t-d}^{t} x^T(s) E^T Z \bar{E} x(s) ds$$

$$\leq \begin{bmatrix} x(t) & t \end{bmatrix}^T \begin{bmatrix} -E^T Z & E^T Z \\ E^T Z & -E^T Z \end{bmatrix} \begin{bmatrix} x(t) \\ x(t - d) \end{bmatrix}.$$ 

So it can be concluded that

$$\mathcal{L}V(x_t, p, t) \leq \sum_{i=1}^{k} \eta_i^T(t) \Phi_{pi} \eta(t),$$ 

where

$$\eta^T(t) = \begin{bmatrix} \bar{x}^T(t) & x^T(t - d) \end{bmatrix},$$

$$\Phi_{pi} = \begin{bmatrix} Y_{1pi} & * \\ Y_{2pi} & Y_{3pi} \end{bmatrix},$$

$$Y_{1pi} = \tilde{Y}_{1pi} + \begin{bmatrix} A_{1pi} \\ B_{1pi} \end{bmatrix} \tilde{d} \begin{bmatrix} A_{pi} \\ B_{pi} \end{bmatrix}.$$ 

Using (20), it is easy to see that there exists a scalar $\kappa > 0$ such that, for every $p \in \mathcal{S}$, $\mathcal{L}V(x_t, p, t) \leq -\kappa ||x(t)||^2$, where $\kappa = \min_{p \in \mathcal{S}} \min_{p \in \mathcal{E}} (\lambda_{\min}(-\Phi_{pi})).$

So, for $t \geq d$, by Dynkin’s formula, it can be obtained that

$$\mathcal{E} \{ V(x_t, p, t) \} - \mathcal{E} \{ V(x_{t-d}, p, d) \}$$

$$\leq -\kappa \mathcal{E} \left\{ \int_{d}^{t} ||x(s)||^2 ds \right\},$$ 

which yields

$$\mathcal{E} \left\{ \int_{d}^{t} ||x(s)||^2 ds \right\} \leq \kappa^{-1} \mathcal{E} \{ V(x_{t-d}, p, d) \}. $$

Because $GEH = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, denote

$$A_p(t) = \sum_{i=1}^{k} \sum_{j=1}^{k} \lambda_{ij} \begin{bmatrix} A_{pi} + B_{pi} K_{pj} \end{bmatrix}.$$

$$= \begin{bmatrix} A_{p1}(t) & A_{p2}(t) \\ A_{p3}(t) & A_{p4}(t) \end{bmatrix}$$

$$= \begin{bmatrix} \sum_{i=1}^{k} \sum_{j=1}^{k} \lambda_{ij} A_{pi1} \sum_{i=1}^{k} \sum_{j=1}^{k} \lambda_{ij} \tilde{A}_{pij2} \\ \sum_{i=1}^{k} \sum_{j=1}^{k} \lambda_{ij} \tilde{A}_{pij3} & \sum_{i=1}^{k} \sum_{j=1}^{k} \lambda_{ij} \tilde{A}_{pij4} \end{bmatrix},$$

$$A_{d,p}(t) = \sum_{i=1}^{k} \lambda_{i} A_{dp} = \begin{bmatrix} A_{dp1}(t) & A_{dp2}(t) \\ A_{dp3}(t) & A_{dp4}(t) \end{bmatrix}$$

$$= \begin{bmatrix} \sum_{i=1}^{k} \lambda_{i} A_{dp1} \sum_{i=1}^{k} \lambda_{i} A_{dp2} \\ \sum_{i=1}^{k} \lambda_{i} A_{dp3} & \sum_{i=1}^{k} \lambda_{i} A_{dp4} \end{bmatrix}. $$

By the regularity and nonimpulse of system (II), $A_{dp}(t)$ is nonsingular; for each $p \in \mathcal{S}$, set $\overline{G}_p = \begin{bmatrix} 1 & -A_{dp}(t) A_{d,p}^{-1}(t) \end{bmatrix} G.$

It is easy to obtain

$$\overline{G}_p E H = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},$$

$$\overline{G}_p A_p(t) H = \begin{bmatrix} \tilde{A}_{p1}(t) & 0 \\ \tilde{A}_{p3}(t) & I_{n-r} \end{bmatrix},$$

$$\overline{G}_p A_p(t) H = \begin{bmatrix} \tilde{A}_{p1}(t) & 0 \\ \tilde{A}_{p3}(t) & I_{n-r} \end{bmatrix}.$$ 


where
\[
\begin{align*}
A_{p_1}(t) &= A_{p_1}(t) - A_{p_2}(t) A_{p_4}^{-1}(t) A_{p_3}(t), \\
A_{p_3}(t) &= A_{p_4}^{-1}(t) A_{p_3}(t), \\
A_{d,p_1}(t) &= A_{d,p_1}(t) - A_{p_2}(t) A_{p_4}^{-1}(t) A_{d,p_3}(t), \\
A_{d,p_2}(t) &= A_{d,p_2}(t) - A_{p_2}(t) A_{p_4}^{-1}(t) A_{d,p_4}(t), \\
A_{d,p_3}(t) &= A_{p_4}^{-1}(t) A_{d,p_3}(t), \\
A_{d,p_4}(t) &= A_{p_4}^{-1}(t) A_{d,p_4}(t).
\end{align*}
\]

Then, for each \( p \in \mathcal{P} \), system (11) is equal to
\[
\begin{align*}
\psi_1(t) &= \bar{A}_{p_1}(t) \psi_1(t) + \bar{A}_{d,p_1}(t) \psi_1(t-d) \nonumber \\
&+ \bar{A}_{d,p_2}(t) \psi_2(t-d), \\
-\psi_2(t) &= \bar{A}_{p_3}(t) \psi_1(t) + \bar{A}_{d,p_3}(t) \psi_1(t-d) \\
&+ \bar{A}_{d,p_4}(t) \psi_2(t-d), \\
\psi(t) &= \varphi(t) = H^{-1} x(t), \\
&\quad t \in [-d, 0],
\end{align*}
\]

where \( \psi(t) = \left[ \begin{array}{c} \psi_1(t) \\ \psi_2(t) \end{array} \right] = H^{-1} x(t) \).

For any \( t \geq 0 \), using Lemma 5, there exists a scalar \( \rho > 0 \) such that \( \| A_{p_4}(t) \| < \delta_p \), and \( A_i(\xi(t)) \geq 0 \), and \( \sum_{i=1}^k \lambda_i(\xi(t)) = 1 \); it follows from (40) that
\[
\begin{align*}
\sup_{0 \leq s \leq d} \| \psi_1(s) \| &\leq \| \psi_1(0) \| \\
&+ k_1 \int_0^t \left[ \| \psi_1(s) \| + \| \psi_1(s-d) \| + \| \psi_2(s-d) \| \right] ds,
\end{align*}
\]

where
\[
\begin{align*}
k_1 &= \max_{p \in \mathcal{P}} \left\{ \max_{i,j \in \mathcal{E}} \| \bar{A}_{pij} \| \\
&+ \delta \max_{i,j \in \mathcal{E}} \| \bar{A}_{pij} \| \max_{i,j \in \mathcal{E}} \| A_{d,pi} \| \\
&+ \delta \max_{i,j \in \mathcal{E}} \| \bar{A}_{pij} \| \max_{i,j \in \mathcal{E}} \| A_{d,pj} \| \\
&+ \delta \max_{i,j \in \mathcal{E}} \| \bar{A}_{pij} \| \max_{i,j \in \mathcal{E}} \| A_{d,pj} \| \right\}.
\end{align*}
\]

Then, for any \( 0 \leq t \leq d \),
\[
\| \psi_1(t) \| \leq (2k_1 d + 1) \| \psi_1(0) \| + k_1 \int_0^t \| \psi_1(s) \| ds.
\]

 Applying the Gronwall–Bellman lemma, it can be obtained, for any \( 0 \leq t \leq d \), that
\[
\| \psi_1(t) \| \leq (2k_1 d + 1) \| \psi_1(0) \| e^{k_1 d}.
\]
Theorem 10. For a prescribed scalar $d > 0$, there exists a state feedback controller (10) with $w(t) = \sum_{i=1}^{k} \lambda_i L_i Y_{p,i}^T x(t)$ such that system (II) is stochastically admissible with an $H_\infty$ performance $\gamma$ for any constant time delay $d$ satisfying $d \in [0, \bar{d}]$, if there exist matrices $\bar{P}_p > 0$, $\bar{Q}_p > 0$, $\bar{Q} > 0$, $\bar{Z} > 0$, $L_{pi}$, $S_p$, $Y_{p,2}$, and $Y_{p,3}$, $i \in \mathcal{I}$, $p \in \mathcal{S}$, such that
\[
\begin{bmatrix}
\Xi_{p,1} & * & * & * \\
L_p + Y_p^T B_{p,1}^T + Y_{p,2} - Y_{p,3}^T - Y_{p,3} & * & * & * \\
(Y_p^T A_{d,p,i}^T + EY_p)^T + Y_p^T E^T - \bar{Z} & 0 & ( - Y_p^T Y_p + \bar{Q}_p) & * & * & * \\
\bar{d} A_{p,i} Y_p - \bar{d} B_{p,i} Y_{p,2} & \bar{d} B_{p,i} Y_{p,3} & - \bar{Z} & * & * & * \\
C_{p,i} Y_p - D_{p,i} Y_{p,2} & D_{p,i} Y_{p,3} & 0 & 0 & 0 & - \gamma^2 I \\
[I_r \ 0] H^{-1} M_p^T & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix} < 0,
\]
where
\[
\Xi_{p,2} = \begin{bmatrix} Y_p & 0 & 0 & 0 & 0 \\
\bar{d} Y_p & 0 & 0 & 0 & 0 \\
[I_r \ 0] H^{-1} M_p^T & 0 & 0 & 0 & 0 \\
\end{bmatrix},
\]
\[
\Xi_{p,3} = \text{diag}\{-\bar{Q}_p, -\bar{d} Q, -J_p\},
\]
and the other notations are the same as in Theorem 8.

Proof. From Theorem 8 when $w(t) = 0$ system (II) is stochastically admissible. Let
\[
I_{zw}(t) = \mathbb{E}\left\{\int_{0}^{t}\left[ z^T(s) z(s) - \gamma^2 w^T(s) w(s)\right] ds\right\}. \tag{55}
\]
Under zero initial condition, it is easy to see that
\[
I_{zw}(t) \leq \mathbb{E}\left\{\int_{0}^{t}\left[ z^T(s) z(s) - \gamma^2 w^T(s) w(s)\right] ds\right\} + \mathcal{D}(x_s, p_s) \right\} ds\right\} \leq \mathbb{E}\left\{\int_{0}^{t}\sum_{i=1}^{k} \lambda_i \left[ \zeta^T(s) \left( \Omega_p + \Theta_p \Xi_p \right) \zeta(s) \right] ds\right\}, \tag{56}
\]
where
\[
\zeta^T(t) = \left[ x^T(t) \ x^T(t - d) \ w^T(t) \right],
\]
\[
\Omega_p = \begin{bmatrix} Y_{1,p} & * & * \\
Y_{2,p} & Y_{3,p} & * \\
[B_{w,p,1}^T \ 0] & X_p & 0 & - \gamma^2 I \\
\end{bmatrix}, \tag{57}
\]
\[
\Theta_p = \begin{bmatrix} C_{p,1} & C_{d,p} & C_{w,p} \end{bmatrix},
\]
and notations of $Y_{1,p}$, $Y_{2,p}$, and $Y_{3,p}$ are the same as in Theorem 8. Hence, by Schur complement lemma and using the similar method in the proof of Theorem 8, from (55) and (53), it can be obtained that $I_{zw}(t) < 0$ for all $t > 0$. Therefore, for any nonzero $w(t) \in L^2_{\infty}(0, \infty)$, (52) holds. Hence, according to Definition 9, the system is stochastically admissible with an $H_\infty$ performance $\gamma$. This completes the proof. \hfill $\square$

Remark 11. Compared with methods in [21, 22], because of the method of the augmented matrix adopted in Theorems 8 and 10, the number of LMIs needed to solve is relatively small in this paper. When the value of $k$ is relatively large, the quality of the computation is greatly reduced. Some new relaxation matrices added will reduce the conservatism of control conditions compared with previous literatures, which can be seen from Example 2.

4. The Design of the Static Output Feedback Controller

When $r_i = p \in \mathcal{S}$, consider the overall SMJFS as follows:
\[
E \dot{x}(t) = \sum_{i=1}^{k} \lambda_i \left( A_{p,i} x(t) + A_{d,p,i} x(t - d) + B_{p,i} u(t) \right) + B_{w,p,i} w(t),
\]
\[
y(t) = \sum_{i=1}^{k} \lambda_i C_{y,p,i} x(t),
\]
\[ z(t) = \sum_{i=1}^{k} \lambda_i \left( C_{pi} x(t) + C_{d,pi} x(t-d) + D_{pi} u(t) + C_{w,pi} w(t) \right), \]
\[ x(t) = \phi(t), \quad \forall t \in [-d,0], \quad i \in \mathcal{T} \triangleq \{1, 2, \ldots, k\}, \]

where \( y(t) \in \mathbb{R}^{p_1} \) is the system output, \( C_{d,pi} (i \in \mathcal{S}) \) are known constant matrices with appropriate dimensions, and the other notations are the same as in (3).

The following static output feedback controller will be considered here:
\[ u_p(t) = \sum_{i=1}^{k} \lambda_i K_{pi} y(t), \quad (59) \]

where \( K_{pi} (p \in \mathcal{S}, i \in \mathcal{T}) \) are local controller gains, such that the closed-loop system is

\[ E \dot{x}(t) = \sum_{i=1}^{k} \sum_{j=1}^{k} \sum_{s=1}^{k} \lambda_i \lambda_j \left( A_{pi} + B_{pi} K_{pj} C_{y,ps} \right) x(t) + A_{d,pi} x(t-d) + B_{w,pi} w(t), \]
\[ z(t) = \sum_{i=1}^{k} \sum_{j=1}^{k} \sum_{s=1}^{k} \lambda_i \lambda_j \left( C_{pi} + D_{pi} K_{pj} C_{y,ps} \right) x(t) + C_{d,pi} x(t-d) + C_{w,pi} w(t). \]

(60)

It is difficult to drive LMI-based conditions of the stochastic stabilization by employing the static output feedback control approach due to the appearance of crossing terms between system matrices and control gains. And system (60) can be rewritten in the following form:

\[ \dot{\bar{x}}(t) = \sum_{i=1}^{k} \sum_{j=1}^{k} \lambda_i \lambda_j \left( \bar{A}_{p,ji} \bar{x}(t) + \bar{B}_{w,pi} w(t) \right), \]
\[ z(t) = \sum_{i=1}^{k} \sum_{j=1}^{k} \sum_{s=1}^{k} \lambda_i \lambda_j \left( \bar{C}_{p,ji} \bar{x}(t) + C_{w,pi} w(t) \right), \]

where

\[ \bar{E} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \in \mathbb{R}^{(n+p_i) \times (n+p_i)}, \]
\[ \bar{C}_{p,ji} = \begin{bmatrix} C_{pi} & D_{pi} K_{pj} \end{bmatrix}, \]
\[ \bar{A}_{d,pi} = \begin{bmatrix} A_{d,pi} & 0 \\ 0 & 0 \end{bmatrix} \in \mathbb{R}^{(n+p_i) \times (n+p_i)}, \]
\[ \bar{x}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}, \]
\[ \bar{B}_{w,pi} = \begin{bmatrix} B_{w,pi} \\ 0 \end{bmatrix} \in \mathbb{R}^{(m+p_i) \times v}, \]
\[ \Lambda_{p,ji} = \begin{bmatrix} A_{pi} & B_{pi} K_{pj} \\ C_{y,pi} & -I \end{bmatrix}, \]

\[ \det \left( sE - \sum_{i=1}^{k} \sum_{j=1}^{k} \sum_{s=1}^{k} \lambda_i \lambda_j \left( A_{pi} + B_{pi} K_{pj} C_{y,ps} \right) \right) = \det \left( sE - \sum_{i=1}^{k} \sum_{j=1}^{k} \sum_{s=1}^{k} \lambda_i \lambda_j A_{p,ji} \right), \]
\[ \det \left( sE - \sum_{i=1}^{k} \sum_{j=1}^{k} \sum_{s=1}^{k} \lambda_i \lambda_j \left( A_{pi} + B_{pi} K_{pj} C_{y,ps} + A_{d,pi} \right) \right) = \det \left( sE - \sum_{i=1}^{k} \sum_{j=1}^{k} \sum_{s=1}^{k} \lambda_i \lambda_j \left( A_{p,ji} + \bar{A}_{d,pi} \right) \right). \]

As the discussion in Remark 7, the stochastic admissibility of system (60) can be studied by means of system (61).

**Remark 12.** For systems (60) and (61), it can be seen that
\[ \det \left( sE - \sum_{i=1}^{k} \sum_{j=1}^{k} \sum_{s=1}^{k} \lambda_i \lambda_j \left( A_{pi} + B_{pi} K_{pj} C_{y,ps} \right) \right) = \det \left( sE - \sum_{i=1}^{k} \sum_{j=1}^{k} \sum_{s=1}^{k} \lambda_i \lambda_j A_{p,ji} \right), \]
\[ \det \left( sE - \sum_{i=1}^{k} \sum_{j=1}^{k} \sum_{s=1}^{k} \lambda_i \lambda_j \left( A_{pi} + B_{pi} K_{pj} C_{y,ps} + A_{d,pi} \right) \right) = \det \left( sE - \sum_{i=1}^{k} \sum_{j=1}^{k} \sum_{s=1}^{k} \lambda_i \lambda_j \left( A_{p,ji} + \bar{A}_{d,pi} \right) \right). \]

**Theorem 13.** There exists an output feedback controller (59) with controller gains \( K_{pi} = L_{pi} \gamma_{pi}^{-1} \) \((p \in \mathcal{S}, i \in \mathcal{T})\) such that system (60) with \( u(t) = 0 \) is stochastically admissible, if there exist matrices \( \bar{P}_p > 0, \bar{Q}_p > 0, \bar{Z} > 0, \bar{Z} > 0, L_{pi}, \bar{S}_p, \) and \( Y_{p2}, \) \( p \in \mathcal{S}, 1 \leq i \neq j \leq k, \) such that
\[ \Theta_{p,ji} < 0, \]
\[ \frac{1}{k-1} \Theta_{p,ji} + \frac{1}{2} \left( \Theta_{p,ij} + \Theta_{p,ji} \right) < 0, \]

where...
\[
\Theta_{p,ij} = \begin{bmatrix}
\hat{\Gamma}_{p,ij} & * & * & * & * & * \\
L_p^TB_p^T + C_{y,p}Y_p & -Y_{p2} & Y_{p1} & * & * & * \\
Y_p^T A_{d,p} + EY_p & 0 & \Sigma_p & * & * & * \\
\hat{d}A_{p}Y_p & \hat{d}B_{p}L_{pj} & \hat{d}A_{d,p}Y_p & -Z & * & * \\
\hat{d}Y_p & 0 & 0 & 0 & 0 & -\hat{d}Q \\
[I_r, 0]H^{-1}M_p^T & 0 & 0 & 0 & 0 & -J_p
\end{bmatrix},
\]

5. Numerical Examples
Two examples will be given to illustrate the validity of developed methods.
Example 1. To illustrate the $H_{\infty}$ controller synthesis, the following nonlinear time delay system is considered:

\[
(1 + a \cos \theta(t)) \ddot{\theta}(t) = -b \dot{\theta}(t) + c \theta(t) + c_d (t - d) + \delta(r_t) e u(t) + f w(t).
\]  

(68)

The range of $\dot{\theta}(t)$ is assumed to satisfy $| \dot{\theta}(t) | < \psi$, $\psi = 2$, $a = b = e = f = 1$, $c = -1$, $c_d = 0.8$, $d \in [0, d]$, $d = 0.3$, and $u(t)$ is the control input. $w(t) = \cos(0.5t)e^{-0.01t}$ is the disturbance input. $r_t$ is a Markovian process taking values in a finite set $\{1, 2, 3\}$, $\delta(1) = 1$, $\delta(2) = 0.8$, $\delta(3) = 0.5$, and the output vector $z(t) = \theta(t)$.

Choose the vector $x(t) = [x_1(t) \ x_2(t) \ x_3(t)]^T$ with $x_1(t) = \theta(t)$, $x_2(t) = \dot{\theta}(t)$, and $x_3(t) = \ddot{\theta}(t)$. Then, the system is described by

\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0
\end{bmatrix}
\dot{x}(t)
= \begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
c & -b \psi^2 - 2 & -1 - a \cos x_1(t)
\end{bmatrix}
x(t)
+ \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
c_d & 0 & 0
\end{bmatrix}
x(t - d)
+ \begin{bmatrix}
0 & 0 & 0 \\
\delta(r_t) e & 0 & u(t)
\end{bmatrix}
+ \begin{bmatrix}
0 \\
0 \\
f
\end{bmatrix}
w(t).
\]  

(69)

It can be expressed exactly by the following fuzzy singular Markovian jump form:

\[
E \dot{x}(t) = \sum_{i=1}^{3} \lambda_i \left( A_{p_i} x(t) + A_{d,p_i} x(t - d) + B_{p_i} u(t) \right),
\]

\[
z(t) = \sum_{i=1}^{3} \lambda_i C_{p_i} x(t),
\]

\[
x(t) = \phi(t),
\]

\[t \in [-d, 0], \ p \in \{1, 2, 3\},\]  

where

\[
E = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0
\end{bmatrix},
\]

\[
A_{p_1} = \begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
c & -b (\psi^2 + 2) & a - 1
\end{bmatrix},
\]

\[
A_{p_2} = \begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
c & 0 & -a - 1 - a \psi^2
\end{bmatrix},
\]

\[
A_{p_3} = \begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
c & 0 & a - 1
\end{bmatrix},
\]

\[
A_{d,p_1} = A_{d,p_2} = A_{d,p_3} = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
c_d & 0 & 0
\end{bmatrix},
\]

\[
B_{11} = B_{12} = B_{13} = \begin{bmatrix}
0 \\
0 \\
e
\end{bmatrix},
\]

\[
B_{21} = B_{22} = B_{23} = \begin{bmatrix}
0 \\
0 \\
0.8e
\end{bmatrix},
\]

\[
B_{31} = B_{32} = B_{33} = \begin{bmatrix}
0 \\
0 \\
0.5e
\end{bmatrix},
\]

\[
B_{w,p_1} = B_{w,p_2} = B_{w,p_3} = \begin{bmatrix}
0 \\
0 \\
f
\end{bmatrix},
\]

\[
C_{p_1} = C_{p_2} = C_{p_3} = \begin{bmatrix}
1 & 0 & 0
\end{bmatrix},
\]

\[
\lambda_1 = \frac{x_1^2(t)}{\psi^2 + 2},
\]

\[
\lambda_2 = \frac{1 + \cos x_1(t)}{\psi^2 + 2},
\]

\[
\lambda_3 = \frac{\psi^2 - x_1^2(t) + 1 - \cos x_1(t)}{\psi^2 + 2}.
\]

It is seen that $0 \leq \lambda_i \leq 1$, $\sum_{i=1}^{3} \lambda_i = 1$. Let $\Pi = \begin{bmatrix}
-0.2 & 0.2 & 0.2 \\
0.2 & -0.3 & 0.2 \\
0.3 & -0.5 & 0.1
\end{bmatrix}$, $\nu = 1$; by solving $\Pi$ and (53) in Theorem 10, controller gains are given by

\[
K_{11} = [-14.2939 -14.0620 -3.6426],
\]

\[
K_{12} = [-14.2911 -14.4082 -3.2860],
\]

\[
K_{13} = [-14.2911 -14.4082 -3.2860].
\]
To demonstrate the effectiveness, assuming the initial condition \( \phi(t) = [-1.2 \ 0.8 \ -0.5]^T \), Figures 1 and 2 show state responses of the open-loop system and the closed-loop system controlled by (10), respectively. From Figure 1, it can be seen that the open-loop system is not stochastically admissible, and Figure 2 shows that when the controller obtained by Theorem 10 is exerted to this system it is stochastically admissible.

Example 2. Consider the example without uncertainties in [6].

Mode 1: \( A_1 = \begin{bmatrix} 1.5 & 1.4 \\ -3.5 & -4.5 \end{bmatrix}, A_{d_1} = \begin{bmatrix} 0.2 & 1.4 \\ -0.3 & 0.4 \end{bmatrix}, B_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, B_{w_1} = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}, C_1 = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}, C_{d_1} = \begin{bmatrix} -0.2 \\ 0.2 \end{bmatrix}, D_1 = 0.2, \) and \( C_{w_1} = 0.2. \)

Mode 2: \( A_2 = \begin{bmatrix} 1.7 & 1.6 \\ -1.3 & -2.5 \end{bmatrix}, A_{d_2} = \begin{bmatrix} 0.2 & 1.1 \\ -0.21 & -0.3 \end{bmatrix}, B_2 = \begin{bmatrix} 0.9 \\ 0.9 \end{bmatrix}, B_{w_2} = \begin{bmatrix} 1.5 \\ 1.3 \end{bmatrix}, C_2 = \begin{bmatrix} 0.4 \\ 0.3 \end{bmatrix}, C_{d_2} = \begin{bmatrix} -0.1 \\ 0.2 \end{bmatrix}, D_2 = 0.3, \) and \( C_{w_2} = 0.3. \)

\[ \Pi = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, E = \begin{bmatrix} 0 \ 0 \end{bmatrix}, d = 0.3, \gamma = 2.6, \text{ and in [6]} \ a = -0.5, b = 2.1, \text{ but in this paper} -2.4 \leq a \leq 2, -2 \leq b \leq 4.8 \text{ are taken.} \]

In Figure 3, “o” represents the range of the feasible solutions using Theorem 10 in this paper, and “*” represents the range of the feasible solutions using Theorem 3 in [6].

This illustrates that the method obtained in this paper has less conservatism.

6. Conclusions

In this paper, the problem of mode-dependent \( H_\infty \) control for singular Markovian jump fuzzy systems with time delay is considered. This class of systems under consideration is described by T-S fuzzy models. The main contribution of this paper is to design state feedback controllers and static output feedback controllers which can guarantee that resulting closed-loop systems are stochastically admissible with an \( H_\infty \) performance \( \gamma \) by the method of the augmented matrix. Finally, two examples are given to demonstrate the effectiveness of main results obtained here.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.
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