Research Article

Damage Evaluation of Critical Components of Tilted Support Spring Nonlinear System under a Rectangular Pulse

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Dimensionless nonlinear dynamical equations of a tilted support spring nonlinear packaging system with critical components were obtained under a rectangular pulse. To evaluate the damage characteristics of shocks to packaged products with critical components, a concept of the damage boundary surface was presented and applied to a tilted support spring system, with the dimensionless critical acceleration of the system, the dimensionless critical velocity, and the frequency parameter ratio of the system taken as the three basic parameters. Based on the numerical results, the effects of the frequency parameter ratio, the mass ratio, the dimensionless peak pulse acceleration, the angle of the system, and the damping ratio on the damage boundary surface of critical components were discussed. It was demonstrated that with the increase of the frequency parameter ratio, the decrease of the angle, and/or the increase of the mass ratio, the safety zone of critical components can be broadened, and increasing the dimensionless peak pulse acceleration or the damping ratio may lead to a decrease of the damage zone for critical components. The results may lead to a thorough understanding of the design principles for the tilted support spring nonlinear system.

1. Introduction

A tilted support spring nonlinear system usually shows excellent energy absorption performance and was used to protect precise instruments since 1960s. In the transport process, a product is damaged mainly because of shock and vibration. So, to evaluate the damage potential of shocks to the system, the concept of the damage boundary is proposed and modified by many specialists. Newton and Burgess [1–3] presented the damage boundary theory for guiding the design of a cushion packaging. Based on Newton's theory of an acceleration damage boundary (ADB), a displacement damage boundary was introduced and combined with that of ADB to determine an actual damage boundary by Wang et al. [4]. Then, the concept was extended to evaluate the damage potential of shocks and drops to the nonlinear packaging system [5–8]. Assuming the damage of products occurs firstly at the so-called critical components, the damage boundary surface concept was proposed and applied to typical nonlinear systems by Wang [9, 10]. This damage boundary surface concept leads

to a noticeable reduction of disparity between the theoretical findings and the real-test results. Wang [11, 12] established the model of the nonlinear packaging system and evaluated the damage boundary of critical components. Wang [13, 14] gained the model of a suspension packaging system under a rectangular pulse and further discussed the effects of the angle and the damping ratio on the shock response spectra, the acceleration response, and the damage boundary surface. The shock dynamical model of the tilted support spring system with critical components under a rectangular pulse was established by Duan et al. [15] and Chen [16] to evaluate the dynamical characteristics of the system. Wu et al. [17, 18] analyzed the vibration of the tilted support spring system in order to discuss the decreasing shock characteristics of the system.

In this paper, the dimensionless nonlinear dynamical equations for the tilted support spring nonlinear system with critical components are obtained under the action of a rectangular pulse. According to the evaluation methodology [14, 19, 20], the damage boundary surface is gained and the effects of the dimensionless peak pulse acceleration, the angle,

the mass ratio, the frequency parameter ratio, and the damping ratio on the damage boundary of critical components are discussed. The results provide theoretical foundations for the design of the tilted support spring nonlinear system.

2. Modeling and Equations

The model of the tilted support spring nonlinear system with critical components is shown in Figure 1, where m_1 and m_2 denote the mass of critical components and the main body; k_1 and c_1 are the linear elastic coefficient and the damping coefficient between critical components and the main body; k_2 and c_2 are the linear elastic coefficient and the damping coefficient of the tilted support spring nonlinear system; l_0 and φ_0 are the length and the support angle of the four tilted support springs before they are compressed, respectively.

To evaluate the damage characteristics of the system under a rectangular pulse, the equations of the pulse is expressed as

$$\ddot{u}_0 = \begin{cases} \ddot{u}_{0m} & 0 \leq t \leq t_0 \\ 0 & t > t_0, \end{cases} \quad (1)$$

where t_0 is the pulse duration. To facilitate the numerical analysis, the coordinate system is established, the static equilibrium position is treated as the original points, and the downward direction is regarded as the positive direction. Then, the shock dynamical equations of the system with damping under a rectangular pulse can be obtained as

$$\begin{aligned} m_1 \frac{d^2 x_1}{dt^2} - k_1 (x_2 - x_1) + c_1 \left(\frac{dx_1}{dt} - \frac{dx_2}{dt} \right) &= 0, \\ m_2 \frac{d^2 x_2}{dt^2} + 4k_2 \left[a_0 (x_2 + u_0) + \frac{b_0}{l_0} (x_2 + u_0)^2 \right. \\ &\quad \left. + \frac{c_0}{l_0^2} (x_2 + u_0)^3 \right] \\ &\quad + k_1 (x_2 - x_1) + c_1 \left(\frac{dx_2}{dt} - \frac{dx_1}{dt} \right) \\ &\quad + c_2 \left(\frac{dx_2}{dt} + \frac{du_0}{dt} \right) = 0, \end{aligned} \quad (2)$$

where $a_0 = \sin^2 \varphi_0$, $b_0 = -3/2 \sin \varphi_0 \cos^2 \varphi_0$, and $c_0 = 1/2(1 - 6\sin^2 \varphi_0 + 5\sin^4 \varphi_0)$. The initial conditions of the displacement and the velocity are $x_{1,2}(0) = 0$ and $dx_{1,2}(0)/dt = 0$, respectively.

Letting $y_1 = (x_2 - x_1)/l_0$, $y_2 = (x_2 + u_0)/l_0$, $\omega_1 = \sqrt{k_1/m_1}$, and $\omega_2 = 2\sqrt{k_2/m_2}$ and combining with (2), the dimensionless

FIGURE 1: The model of the tilted support spring nonlinear system with critical components.

nonlinear shock dynamical equations of the system under a rectangular pulse can be shown as

$$\begin{aligned} \frac{d^2 y_1}{d\tau^2} + (a_0 y_2 + b_0 y_2^2 + c_0 y_2^3) \\ + 2c_2 \frac{dy_2}{d\tau} + \lambda_1^2 (1 + \lambda_2) y_1 \\ + 2c_1 \lambda_1 (1 + \lambda_2) \frac{dy_1}{d\tau} &= 0, \\ \frac{d^2 y_2}{d\tau^2} + (a_0 y_2 + b_0 y_2^2 + c_0 y_2^3) \\ + \lambda_1^2 \lambda_2 y_1 + 2c_2 \frac{dy_2}{d\tau} \\ + 2c_1 \lambda_1 \lambda_2 \frac{dy_1}{d\tau} &= \beta \ddot{u}_0, \end{aligned} \quad (3)$$

where $T = 1/\omega_2$ is taken as the period parameter; $\tau = t/T$ is the dimensionless time parameter; $\tau_0 = t_0/T$ is the dimensionless pulse duration parameter; $\lambda_1 = \omega_1/\omega_2$ is the frequency parameter ratio of the system; $\lambda_2 = m_1/m_2$ is the mass ratio of the system; $\beta = T^2/l_0$ is defined as a characteristic parameter of the system; $c_1 = c_1/2\sqrt{k_1 m_1}$ is the damping ratio between critical components and the main body; $c_2 = c_2/2\sqrt{2k_2 m_2}$ is the damping ratio of the tilted support spring nonlinear system; $\beta \ddot{u}_{0m}$ is the dimensionless peak pulse acceleration, respectively.

According to (3), the initial conditions of the dimensionless displacement and velocity can be transformed as $y_{1,2}(0) = 0$ and $dy_{1,2}(0)/d\tau = 0$, respectively.

The expression of the dimensionless rectangular pulse is then in (3):

$$\beta \ddot{u}_0 = \begin{cases} \beta \ddot{u}_{0m} & 0 \leq \tau \leq \tau_0 \\ 0 & \tau > \tau_0. \end{cases} \quad (4)$$

FIGURE 2: The damage boundary curve of critical components under the action of a rectangular pulse when (a) $\varphi_0 = 70^\circ, 75^\circ, 80^\circ, 85^\circ, 90^\circ$, (b) $\lambda_2 = 0.01, 0.05, 0.1, 0.2, 0.3$, (c) $\lambda_1 = 1, 2, 3, 5, 10$, (d) $\varsigma_2 = 0.01, 0.05, 0.1, 0.15, 0.2$, and (e) $\beta \ddot{u}_{0m} = 0.01, 0.05, 0.1, 0.15, 0.2$.

FIGURE 3: The effect of the dimensionless peak pulse acceleration on the damage boundary surface of critical components when (a) $\beta\ddot{u}_{0m} = 0.01$, (b) $\beta\ddot{u}_{0m} = 0.05$, (c) $\beta\ddot{u}_{0m} = 0.1$, and (d) $\beta\ddot{u}_{0m} = 0.2$.

3. Damage Evaluation

3.1. Damage Boundary Curve. According to (3), it demonstrates that the damage characteristic of a product and its critical components is relevant to the angle of the system, the dimensionless peak pulse acceleration, the damping ratio, the mass ratio, and the frequency parameter ratio of the system. This section will chiefly concentrate on the damage boundary of critical components, which can clearly reflect the damage characteristics of the tilted support spring nonlinear system caused by the geometry nonlinearity of the structure.

Integrating the dimensionless displacement parameters of the system $y_1 = (x_2 - x_1)/l_0$ and $y_2 = (x_2 + u_0)/l_0$ with (3), the dimensionless critical acceleration of the system, where the ratio of the peak pulse acceleration to the maximum shock response acceleration of critical components $\Gamma = \ddot{u}_{0m}/(\ddot{x}_1)_m$ is taken as, can be written as

$$\Gamma = \frac{\ddot{u}_{0m}}{(\ddot{x}_1)_m} = \frac{\beta\ddot{u}_{0m}}{(\lambda_1^2 y_1 + 2\zeta_1 \lambda_1 (dy_1/d\tau))_m}. \quad (5)$$

The product of the dimensionless critical acceleration and the dimensionless pulse duration $V = \Gamma\tau_0$ is taken as the dimensionless critical velocity of the system.

To evaluate the damage boundary of critical components, the dimensionless critical acceleration of the system Γ and the dimensionless critical velocity of the system V are regarded as two basic parameters of the damage boundary curve of critical components. The dimensionless shock dynamic equations (3) are solved using fourth order Runge-Kutta method. Based on the numerical results, the influence of the angle, the frequency parameter ratio, the mass ratio of the system, the dimensionless peak pulse acceleration, and the damping ratio on the damage boundary curve of critical components are analyzed.

Just like Figure 2(a), the effect of the angle on the damage boundary curve of critical components is revealed, based on the frequency parameter ratio $\lambda_1 = 10$, the mass ratio $\lambda_2 = 0.02$, the dimensionless peak pulse acceleration $\beta\ddot{u}_{0m} = 0.2$, the damping ratio between critical components and the main body $\zeta_1 = 0.05$, and the damping ratio of the tilted support spring system $\zeta_2 = 0.1$.

FIGURE 4: The effect of the angle of the tilted support spring system on the damage boundary surface of critical components when (a) $\varphi_0 = 60^\circ$, (b) $\varphi_0 = 70^\circ$, (c) $\varphi_0 = 80^\circ$, and (d) $\varphi_0 = 90^\circ$.

The effect of the mass ratio on the damage boundary curve of critical components is presented in Figure 2(b), setting the angle $\varphi_0 = 70^\circ$, the frequency parameter ratio $\lambda_1 = 10$, the dimensionless peak pulse acceleration $\beta\ddot{u}_{0m} = 0.1$, the damping ratio between critical components and the main body $\zeta_1 = 0.05$, and the damping ratio of the tilted support spring system $\zeta_2 = 0.1$.

Figure 2(c) shows the influence of the frequency parameter ratio on the damage boundary curve of critical components, when we set the angle $\varphi_0 = 70^\circ$, the mass ratio $\lambda_2 = 0.02$, the dimensionless peak pulse acceleration $\beta\ddot{u}_{0m} = 0.1$, the damping ratio between critical components and the main body $\zeta_1 = 0.05$, and the damping ratio of the tilted support spring system $\zeta_2 = 0.1$.

Under conditions of the angle $\varphi_0 = 70^\circ$, the frequency ratio $\lambda_1 = 10$, the dimensionless peak pulse acceleration $\beta\ddot{u}_{0m} = 0.2$, the mass ratio $\lambda_2 = 0.01$, and the damping ratio between critical components and the main body $\zeta_1 = 0$, the influence of the damping ratio of the tilted support spring system on the damage boundary curve of critical components is presented in Figure 2(d).

Figure 2(e) shows the effect of the dimensionless peak pulse acceleration on the damage boundary curve of critical components, when we choose the mass ratio $\lambda_2 = 0.01$, the frequency ratio $\lambda_1 = 10$, the angle $\varphi_0 = 70^\circ$, the damping ratio between critical components and the main body $\zeta_1 = 0.05$, and the damping ratio of the tilted support spring system $\zeta_2 = 0.1$.

3.2. Damage Boundary Surface. To further evaluate the damage characteristics of the system under the action of a rectangular pulse, the damage boundary surface is proposed, where the ratio of the peak pulse acceleration to the maximum shock response acceleration of critical components $\Gamma = \ddot{u}_{0m}/(\ddot{x}_1)_m$ is regarded as the dimensionless critical acceleration of the system. The dimensionless critical acceleration of the system Γ , the dimensionless critical velocity of the system V , and the frequency parameter ratio λ_1 are regarded as three basic parameters of the damage boundary surface of critical components.

When we choose the mass ratio $\lambda_2 = 0.01$, the angle of the system $\varphi_0 = 70^\circ$, the damping ratio between critical

FIGURE 5: The effect of the mass ratio on the damage boundary surface of critical components when (a) $\lambda_2 = 0.01$, (b) $\lambda_2 = 0.05$, (c) $\lambda_2 = 0.1$, and (d) $\lambda_2 = 0.2$.

components and the main body $\zeta_1 = 0.05$, and the damping ratio of the tilted support spring system $\zeta_2 = 0.1$, the effect of the dimensionless peak pulse acceleration on the damage boundary surface of critical components is given in Figure 3.

Figure 4 reveals the effect of the angle on the damage boundary surface of critical components, under conditions of the mass ratio $\lambda_2 = 0.02$, the dimensionless peak pulse acceleration $\beta\ddot{u}_{0m} = 0.1$, the damping ratio between critical components and the main body $\zeta_1 = 0.05$, and the damping ratio of the tilted support spring system $\zeta_2 = 0.1$.

Setting the angle of the system $\varphi_0 = 70^\circ$, the dimensionless peak pulse acceleration $\beta\ddot{u}_{0m} = 0.1$, the damping ratio between critical components and the main body $\zeta_1 = 0.05$, and the damping ratio of the tilted support spring system $\zeta_2 = 0.1$, the effect of the mass ratio on the damage boundary surface of critical components is presented in Figure 5.

Figure 6 shows the influence of the damping ratio between critical components and the main body on the damage boundary surface of critical components, under conditions of the mass ratio of the system $\lambda_2 = 0.1$, the dimensionless peak pulse acceleration $\beta\ddot{u}_{0m} = 0.1$, the angle of the system $\varphi_0 = 70^\circ$, and the damping ratio of the tilted support spring system $\zeta_2 = 0$.

Under conditions of the angle $\varphi_0 = 70^\circ$, the dimensionless peak pulse acceleration $\beta\ddot{u}_{0m} = 0.1$, the mass ratio of the system $\lambda_2 = 0.1$, and the damping ratio between critical components and the main body $\zeta_1 = 0$, the influence of the damping ratio of the tilted support spring nonlinear system on the damage boundary surface of critical components is shown in Figure 7.

4. Discussion

4.1. The Effects of Angle of Tilted Support Spring System. According to the evaluation results of the damage boundary surface of the tilted support spring nonlinear system with critical components, Figures 2(a) and 4 demonstrate that under the action of the fixed mass ratio, the peak pulse acceleration and the damping ratio, comparing with the linear system ($\varphi_0 = 90^\circ$), decreasing the angle can enlarge the safety zone of critical components, especially at a high frequency parameter ratio.

4.2. The Effects of Mass Ratio. It shows in Figures 2(b) and 5 that increasing the mass ratio of the system can obviously

FIGURE 6: The effect of the damping ratio between critical components and the main body on the damage boundary surface of critical components when (a) $\zeta_1 = 0.01$, (b) $\zeta_1 = 0.05$, (c) $\zeta_1 = 0.1$, and (d) $\zeta_1 = 0.2$.

move the critical velocity boundary to the right and increase the safety zone of critical components.

4.3. The Effects of Frequency Parameter Ratio. Figures 2(c) and 6 can clearly show that the damage boundary is sensitive to the frequency parameter ratio and increasing the frequency parameter ratio can decrease the damage zone of critical components. On the basis of numerical results, the frequency parameter ratio of the system is an important parameter. It is particularly significant for advancing shock resistance characteristics of critical components to control the frequency parameter ratio suitably and increase the frequency parameter ratio of the system as possible in the permissive condition (it is suggested that frequency ratio is $\lambda_1 > 5$).

4.4. The Effects of the Damping Ratio. By analyzing Figures 2(d) and 7, it presents that increasing the damping ratio of the tilted support spring system can increase the safety zone of critical components. Figure 6 demonstrates that at a low frequency parameter ratio, increasing the damping ratio

between critical components and the main body can bring about the increase of the critical velocity boundary value and the safety zone of critical components, while the critical acceleration boundary of the system tends to be stable at a high frequency ratio.

4.5. The Effects of the Peak Pulse Acceleration. Based on the numerical results of Figures 2(e) and 3, it shows that under conditions of the given mass ratio, the angle, and the damping, with the dimensionless peak pulse acceleration $\beta \ddot{u}_{0m}$ increasing, the dimensionless critical acceleration moves up, resulting in the corresponding increase of the safety zone of critical components. Furthermore, when the maximum pulse acceleration $\ddot{u}_{0m}$ is fixed, increasing the characteristic parameter of the system β can improve the shock resistance characteristics of critical components.

5. Conclusions

The damage potential of shocks to packaged product for the titled support spring nonlinear system was studied and

FIGURE 7: The effect of the damping ratio of the tilted support spring system on the damage boundary surface of critical components when (a) $\zeta_2 = 0.01$, (b) $\zeta_2 = 0.05$, (c) $\zeta_2 = 0.1$, and (d) $\zeta_2 = 0.2$.

the damage boundary surface for the system was obtained. The results show that with the increase of the frequency parameter ratio, the decrease of the angle, and/or the increase of the mass ratio, the safety zone of critical components can be broadened, and increasing the dimensionless peak pulse acceleration or the damping ratio may lead to a decrease of the damage zone for critical components. The results may lead to a thorough understanding of the design principles for the tilted support spring nonlinear system.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

References

- [1] R. E. Newton, *Fragility Assessment Theory and Practice*, Monterey Research Laboratory, Monterey, Calif, USA, 1968.
- [2] G. J. Burgess, "Product fragility and damage boundary theory," *Packaging Technology and Science*, vol. 1, no. 1, pp. 5–10, 1988.
- [3] G. J. Burgess, "Effects of fatigue on fragility testing and the damage boundary curve," *Journal of Testing and Evaluation*, vol. 24, no. 6, pp. 419–426, 1996.
- [4] Z. L. Wang, C. F. Wu, and D. C. Xi, "Damage boundary of a packaging system under rectangular pulse excitation," *Packaging Technology and Science*, vol. 11, no. 4, pp. 189–202, 1998.
- [5] Z.-W. Wang and C.-Y. Hu, "Shock spectra and damage boundary curves for non-linear package cushioning systems," *Packaging Technology and Science*, vol. 12, no. 5, pp. 207–217, 1999.
- [6] Z. W. Wang, "Shock spectra and damage boundary curves for hyperbolic tangent cushioning system and their important features," *Packaging Technology and Science*, vol. 14, no. 4, pp. 149–157, 2001.
- [7] Z.-W. Wang, "On evaluation of product dropping damage," *Packaging Technology and Science*, vol. 15, no. 3, pp. 115–120, 2002.
- [8] Z.-W. Wang, "Dropping damage boundary curves for cubic and tangent package cushioning systems," *Packaging Technology and Science*, vol. 15, no. 5, pp. 263–266, 2002.
- [9] J. Wang, F. Duan, J.-H. Jiang, L.-X. Lu, and Z.-W. Wang, "Dropping damage evaluation for a hyperbolic tangent cushioning

- system with a critical component,” *Journal of Vibration and Control*, vol. 18, no. 10, pp. 1417–1421, 2012.
- [10] J. Wang, J. H. Jiang, L. X. Lu, and Z. W. Wang, “Dropping damage evaluation for a tangent nonlinear system with a critical component,” *Computers and Mathematics with Applications*, vol. 61, no. 8, pp. 1979–1982, 2011.
- [11] J.-H. Jiang and Z.-W. Wang, “Dropping damage boundary curves for cubic and hyperbolic tangent packaging systems based on key component,” *Packaging Technology and Science*, vol. 25, no. 7, pp. 397–411, 2012.
- [12] J. Wang, Z.-W. Wang, L.-X. Lu, Y. Zhu, and Y.-G. Wang, “Three-dimensional shock spectrum of critical component for nonlinear packaging system,” *Shock and Vibration*, vol. 18, no. 3, pp. 437–445, 2011.
- [13] L. Wang and A. J. Chen, “The shock response spectra of the suspension packaging system under rectangular pulse,” *Journal of Applied Packaging Research*, vol. 5, no. 4, pp. 237–246, 2012.
- [14] L. Wang and A.-J. Chen, “The damage boundary curve of the suspension packaging system under rectangular pulse,” *Applied Mechanics and Materials*, vol. 105–107, pp. 70–73, 2011.
- [15] N. Duan, S. Song, and A. Chen, “The dynamic evaluation of tilted support spring nonlinear system with critical components under the action of a rectangular pulse,” *Mathematical Problems in Engineering*, vol. 2014, Article ID 404163, 6 pages, 2014.
- [16] A.-J. Chen, “The shock characteristics of tilted support spring packaging system with critical components,” *Shock and Vibration*, vol. 2014, Article ID 496035, 8 pages, 2014.
- [17] X. Wu, Y. X. Luo, and Y. Wu, “Study on vertical nonlinear natural vibration of shock absorber system with tilted support spring,” *Journal of Vibration and Shock*, vol. 27, no. 8, pp. 85–87, 2008 (Chinese).
- [18] X. Wu, Y. X. Luo, and L. J. Yang, “Vibration of shock absorber system with tilted support spring under displacement excitation of foundation,” *Journal of Vibration and Shock*, vol. 28, no. 11, pp. 115–117, 2009 (Chinese).
- [19] Z.-W. Wang and J.-H. Jiang, “Evaluation of product dropping damage based on key component,” *Packaging Technology and Science*, vol. 23, no. 4, pp. 227–238, 2010.
- [20] S. Goyal, J. M. Papadopoulos, and P. A. Sullivan, “Shock protection of portable electronic products: shock response spectrum, damage boundary approach, and beyond,” *Shock and Vibration*, vol. 4, no. 3, pp. 169–191, 1997.