State Feedback $H_{\infty}$ Control of Power Units Based on an Improved Particle Swarm Optimization

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A new state feedback $H_{\infty}$ control scheme is presented used in the boiler-turbine power units based on an improved particle swarm optimizing algorithm. Firstly, the nonlinear system is transformed into a linear time-varying system; then the $H_{\infty}$ control problem is transformed into the solution of a Riccati equation. The control effect of $H_{\infty}$ controller depends on the selection of matrix $P$, so an improved particle swarm optimizing (PSO) algorithm by introducing differential evolution algorithm is used to solve the Riccati equation. The main purpose is that mutation and crossover are introduced for a new population, and the population diversity is improved. It is beneficial to eliminate stagnation caused by premature convergence, and the algorithm convergence rate is improved. Finally, the real-time optimizing of the controller parameters is realized. Theoretical analysis and simulation results show that a state feedback $H_{\infty}$ controller can be obtained, which can ensure asymptotic stability of the system, and the double objectives of stabilizing system and suppressing the disturbance are reached. The system can work well over a larger range working point.

1. Introduction

Boiler and turbine are main equipment of coal-fired power generation units, so the coordination control plays an important role in improving the performance of the generation system. The nonlinear control method which has been used in the coordinated control system can solve the optimizing operation problem over large range working point effectively. It becomes the hot topic in recent years. Backstepping method [1, 2] has better rapidity and guarantees the stability of system. Adaptive decoupling control is used [3], and the good tracking and adaptive ability of system is shown. The terminal sliding mode controller can improve the precision of the system [4].

In this paper, firstly, the nonlinear coordinate system for boiler and steam turbine is transformed into a linear time-varying system with external disturbance. Then the $H_{\infty}$ control problem can be transformed into the solution of Riccati equation [5, 6]. Since the selection of the Riccati equation solution has a great influence on the control effect, the improved particle swarm optimizing (PSO) algorithm is applied to seek the solution of Riccati equation. Particle swarm optimizing (PSO) algorithm [7–9] can adaptively adjust parameters according to speed vector, which is different from other evolution algorithms. Each particle executes a kind of “consciousness” variation in the evolutionary process, namely, particles only toward some good directions flight according to the experience of the populations, so that the PSO algorithm has more opportunities to fly to more optimal solution area faster. The other advantage of PSO algorithm is ease of implementation, because there are very few parameters needed to be adjusted. PSO algorithm is able to expand the scope of the feasible solution and ensure the existence of the solution. In order to eliminate the stagnation and avoid premature convergence to stop, an improved particle swarm optimizing algorithm is adopted by introducing differential evolution algorithm [10–13], which guarantees the population converging to the global optimal solution continuously. Finally, the real-time optimizing of the controller parameters is realized. Theoretical analysis and simulation results show that the $H_{\infty}$ controller obtained can ensure asymptotic stability of system and the double objectives of stabilizing system and suppressing the disturbance are reached. The system can work well over a large range working point.
2. The Linearization of Boiler-Steam Turbine Model

The model of boiler-turbine is the foundation of coordinate control. The Astrom model [14] is a very typical three-order MIMO nonlinear coordination system model of machine furnace, which has been quoted widely. The model can be described as the following dynamic equations:

\[
\frac{dp}{dt} = -0.0018u_2 p^{9/8} + 0.9u_1 - 0.15u_3,
\]

\[
\frac{dP_0}{dt} = (0.073u_2 - 0.016) p^{9/8} - 0.1P_0,
\]

\[
\frac{d\rho_f}{dt} = \frac{(141u_3 - (1.1u_2 - 0.19) p)}{85},
\]

where \( p \) is the drum pressure, \( P_0 \) is the output power, \( \rho_f \) is the fluid density inside the system, \( u_1 \) is the opening degree of the fuel flow regulator, \( u_2 \) is the opening degree of the steam flow regulator, and \( u_3 \) is the opening degree of the water flow regulator.

Considering the actual situation of the regulator and dynamic process itself, the opening position of the regulator and its changing rate are limited [14].

The opening degrees of the three regulators should satisfy the conditions below:

\[
\frac{du_1}{dt} \leq 0.007/s, \quad 0 \leq u_1 \leq 1; \tag{2}
\]

\[-2/s \leq \frac{du_2}{dt} \leq 0.02/s, \quad 0 \leq u_2 \leq 1; \tag{3}
\]

\[
\frac{du_3}{dt} \leq 0.05/s, \quad 0 \leq u_3 \leq 1.
\]

The output of the system is the drum pressure \( p \) (kg/cm\(^2\)), the output power \( P_0 \) (MW), and the drum level \( X_w \) (m). The drum pressure and the output power are two state variables and the drum water level is given by the following equation:

\[
X_w = 0.05 \left( 0.13073\rho_f + 100\alpha_{cs} + \frac{q_e}{9} \right) - 67.975, \tag{4}
\]

where \( \alpha_{cs} \) is the coefficient of steam quality and \( q_e \) (kg/s) is the evaporation rate:

\[
\alpha_{cs} = \frac{(1 - 0.001538\rho_f)(0.8p - 25.6)}{\rho_f (1.0394 - 0.0012304p)},
\]

\[
q_e = (0.854u_2 - 0.147) p + 45.9u_1 - 2.514u_3 - 2.096.
\]

Set

\[
x = [x_1 \; x_2 \; x_3]^T = [p \; P_0 \; \rho_f]^T, \tag{5}
\]

\[
u = [u_1 \; u_2 \; u_3]^T,
\]

\[
y = [y_1 \; y_2 \; y_3]^T = [p \; P_0 \; X_w]^T.
\]

At the selected working point \((x^0, u^0)\), the nonlinear function is expanded into a Taylor series and the first-order derivative item holds; the linear time-varying model can be got:

\[
\ddot{x} = A\ddot{x} + Bu,
\]

\[
\ddot{y} = C\ddot{x} + Du,
\]

where \( x = x^0, \dot{y} = y^0, \ddot{u} = u^0, \) and the expression of \( A, B, C, \) and \( D \) could be obtained by partial derivative operation:

\[
A = \begin{bmatrix}
-0.002025u_2 x_1^{1/8} & 0 & 0 \\
9/8 (0.073u_2 - 0.016) x_1^{1/8} & -0.1 & 0 \\
-1.1u_2 - 0.19 & 0 & 0 \\
85 & \\
0 & 0.9 & -0.0018 x_1^{9/8} \\
0 & 0.073x_1^{9/8} & 0 \\
0 & -0.01294x_1 & 0 \\
0.25328 & 0.0047444x_1 & -0.013967
\end{bmatrix},
\]

\[
B = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0.25328 & 0.0047444x_1 & -0.013967
\end{bmatrix},
\]

\[
C = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
c_{31} & 0 & c_{33}
\end{bmatrix},
\]

\[
D = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0.25328 & 0.0047444x_1 & -0.013967
\end{bmatrix},
\]

where \( x = x^0, u = u^0, \) and

\[
c_{31} = 0.05 \begin{bmatrix}
80.002 (1 - 0.001538x_3) \\
80.002 (1 - 0.001538x_3) \\
0.854u_2 - 0.147
\end{bmatrix},
\]

\[
c_{33} = 0.05 \begin{bmatrix}
100 (0.8x_1 - 25.6) \\
0.854u_2 - 0.147
\end{bmatrix},
\]

The coefficient matrix of model above will change with the states of working points selected, so that the whole system is transformed into a linear system with time-varying parameters. The opening degrees of the three regulators are restrained by the following conditions:

\[
\frac{du_1}{dt} \leq 0.007/s, \quad -0.34 \leq u_1 \leq 0.66;
\]

\[-2/s \leq \frac{du_2}{dt} \leq 0.02/s, \quad -0.69 \leq u_2 \leq 0.31; \tag{6}
\]

\[
\frac{du_3}{dt} \leq 0.05/s, \quad -0.435 \leq u_3 \leq 0.565.
\]
3. State Feedback $H_\infty$ Control

The actual system with external disturbance is considered, so the state feedback $H_\infty$ control scheme is used to realize the stable control of machine furnace coordinate system. The linear time-varying model of (6) is expressed as follows:

$$\begin{align*}
\dot{x} &= A\bar{x} + B\bar{u} + B_1w, \\
y &= C\bar{x} + D\bar{u},
\end{align*}$$

where $w \in \mathbb{R}^{3 \times 1}$ is the external disturbance and $B_1 \in \mathbb{R}^{3 \times 3}$ is the corresponding coefficient matrix.

The $H_\infty$ control problem above is equivalent to design a feedback control law

$$\bar{u} = K\bar{x} + r$$

which can ensure the correspondingly closed-loop system stable $(A + BK)$ stable, and

$$\| T_{yw}(s) \|_{\infty} < \gamma,$$

where $T_{yw}(s)$ represents the closed-loop transfer function matrix form $w$ to $y$, $\gamma > 0$ is a given constant, $K \in \mathbb{R}^{3 \times 3}$ is the control matrix, and $r \in \mathbb{R}^{3 \times 1}$ is the reference input.

Assumption 1 ($(A, B)$ is internally stable). For the system (10) meeting Assumption 1, the necessary and sufficient conditions of the $H_\infty$ control question above are that for a constant $\epsilon > 0$ and a positively definite matrix $Q$ the Riccati equation

$$\begin{align*}
(A - BH_FD^T C)\Sigma &+ \Sigma (A - BH_FD^T C)^T + P \left( A - BH_FD^T C \right)^T - \frac{1}{\epsilon} \left( PB\Sigma B^T P - PBH_FB^T P + \gamma^2 PB_1B_1^T P \right) \\
&+ C^T \left( I - DH_FD^T \right) C + Q = 0
\end{align*}$$

has positive-definite symmetric solution $P > 0$. If the system (13) has solution, then closed-loop system is internally stable. The state feedback matrix of (12) is given by the following equation:

$$K = -\left( \frac{1}{2\epsilon} \Phi_F \Sigma \Phi_F + H_F \right) B^T P - H_F D^T C,$$

where $\Phi_F = \Sigma \left( \Sigma \Sigma^T \right)^{-1} \left( U^T U \right)^{-1} \left( \Sigma \Sigma^T \right)^{-1} \Sigma$, $U$ and $\Sigma$ are any matrix to satisfy under equations [5] $D = U \Sigma$, $U \in \mathbb{R}^{3 \times 1}$, $\Sigma \in \mathbb{R}^{3 \times 3}$, and rank $U = \text{rank} \Sigma = 1$, where matrix $\Phi_F \in \mathbb{R}^{3 \times 3}$ satisfied $\Phi_F \Phi_F^T = I$ and $\Phi_F \Sigma^T = 0$.

Combining (10) and (11), the standard state feedback $H_\infty$ control can be got:

$$\begin{align*}
\dot{x} &= G \begin{bmatrix} x \\ \bar{u} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} x \\ \bar{u} \end{bmatrix}, \\
\bar{u} &= K\bar{x} + r.
\end{align*}$$

Figure 1: The block diagram of the standard state feedback $H_\infty$ control.

The block diagram of the standard state feedback $H_\infty$ control is shown in Figure 1. The state feedback $H_\infty$ control problem above can be transformed into the solution of Riccati equation (13). The control effect of $H_\infty$ controller depends on the selection of matrix $P$; in this paper an improved particle swarm optimizing (PSO) algorithm is used for solving the Riccati equation. This scheme can expand the scope of the feasible solution and ensure the existence of the solution, and the real-time optimizing of the controller parameters is realized.

4. Particle Swarm Algorithm Is Used to Solve the Matrix Equation

In this paper, an improved particle swarm optimization algorithm is applied to seek the solution of Riccati equation, rewriting Riccati equation (13) into the following form:

$$\begin{align*}
(A - BH_FD^T C)^T P + P (A - BH_FD^T C) \\
&- \frac{1}{\epsilon} \left( PB \Sigma B^T P - PBH_FB^T P + \gamma^2 PB_1B_1^T P \right) \\
&+ C^T \left( I - DH_FD^T \right) C + Q = E.
\end{align*}$$

where $\gamma$ and $\epsilon$ are the given constants and matrixes $H_F$, $\Phi_F$, and $Q$ are the given coefficient matrix.

The fitness value of the objective function is

$$\min f(E) = \sum_{i=1}^{m} \sum_{j=1}^{n} |E_{ij}|,$$
The particle number of populations is \( s \); for the \( i \)th particle, \( X_i(t) = \{ x_{i1}(t), x_{i2}(t), \ldots, x_{is}(t) \} \) and \( V_i(t) = \{ v_{i1}(t), v_{i2}(t), \ldots, v_{is}(t) \} \) are the current position and velocity, respectively. The best position of the \( i \)th particle is recorded as \( Z_i(t) = \{ z_{i1}(t), z_{i2}(t), \ldots, z_{is}(t) \} \), which is also the position with the best fitness value of the \( i \)th particle recorded and called \( pbest \). For minimizing problems, the smaller the fitness value of objective function, the better the corresponding fitness value.

In this paper set the objective function of minimizing problem to be \( f(X_i) \); the best position of \( i \)th particle depends on the following scheme:

\[
Z_i(t + 1) = \begin{cases} 
Z_i(t), & f \left( X_i(t + 1) \right) \geq f \left( Z_i(t) \right) \\
X_i(t + 1), & f \left( X_i(t + 1) \right) < f \left( Z_i(t) \right) .
\end{cases}
\] (18)

Set the best position of all particles recorded as \( Z_g(t) \), which is called the global best position \( gbest \); then

\[
f \left( Z_g(t) \right) = \min \{ f \left( Z_0(t) \right), f \left( Z_1(t) \right), \ldots, f \left( Z_s(t) \right) \} .
\] (19)

The standard evolving equation of particle swarm algorithm can be described as follows:

\[
V_i(t + 1) = \xi V_i(t) + c_1 \text{rand}_1(t) \left[ Z_i(t) - X_i(t) \right] + c_2 \text{rand}_2(t) \left[ Z_g(t) - X_i(t) \right],
\] (20)

\[
X_i(t + 1) = X_i(t) + V_i(t + 1),
\] (21)

where \( c_1 \) and \( c_2 \) are acceleration constants and \( \text{rand}_1(t) \) and \( \text{rand}_2(t) \) are the random number within [0, 1]. \( \xi \) is the inertial weight which is similar to the temperature of simulated annealing, the bigger \( \xi \) has good global convergence ability, and the smaller \( \xi \) has strong local convergence ability. Therefore, with the increase of the number of iterations, the inertia weight should decrease constantly. Thereby, the particle swarm algorithm has strong global convergence ability in early and strong local convergence ability in later period. In this paper self-adjusting of the inertia weight \( \xi \) meets

\[
\xi = 0.9 - \frac{t}{\text{MaxNumber}} \times 0.5 .
\] (22)

MaxNumber is the biggest iterating number.

For (20), the right hand can be divided into three parts: the first part represents the original velocity, the second part is the impact of the best position of the \( i \)th particle (\( pbest \)) on the position of the current particle, and the third part is the impact of the global best position (\( gbest \)) on the position of the current particle, and the second part and the third part represent the modification of the original velocity.

Traditional PSO algorithm has a potential property which would cause the algorithm entrapping into a local optimum. In addition, the parameters of traditional PSO algorithm have stronger dependence. In order to eliminate the stagnation and avoid premature convergence, the differential evolution (DE) algorithm is introduced, and an improved PSO algorithm guarantees the populations converging to the global optimal solution continuously.

4.2. Differential Evolution Algorithm. DE algorithm has characteristics of remembering individual optimal solution and can share the information in populations [15–18]. DE algorithm initializes the population randomly in the feasible solution space of the problem, by means of mutation and crossover to produce a new population for the current population. Then, an one-to-one selection is made among the two populations consequently to generate a new final population.

In this paper a six-dimensional \( X_i(t) = \{ x_{i1}(t), x_{i2}(t), \ldots, x_{i6}(t) \} \) is used to denote a particle individual, so in each generation of the populations, the particle number is \( s \). Execute mutation operation to each target individual \( X_i(t) \) to obtain corresponding variation individual \( M_i(t + 1) = \{ m_{i1}(t + 1), m_{i2}(t + 1), \ldots, m_{is}(t + 1) \} \) according to the following equation:

\[
M_i(t + 1) = X_{r1}(t) + F \left( X_{r2}(t) - X_{r3}(t) \right),
\] (23)

where \( F \) is set in (0, 2], called mutation constant which controls the ratio of the father generation difference vector \( (X_{r2} - X_{r3}) \) to avoid search stagnation. \( r_1, r_2, \) and \( r_3 \) are randomly selected from the set \{1, 2, \ldots, s\}, which are different from each other and are different from index \( i \).

Crossover operation is executed to target individual and its variation individual for acquiring trial individual \( U_i(t + 1) = \{ u_{i1}(t + 1), u_{i2}(t + 1), \ldots, u_{is}(t + 1) \} \) according to the following equation:

\[
u_{ij}(t + 1) = \begin{cases} 
m_{ij}(t + 1), & \text{rand}(j) \leq C_R \quad \text{or} \quad j = \text{rand}(n(i)) \\
x_{ij}(t), & \text{otherwise},
\end{cases}
\] (24)

where \( \text{rand}(j) \in [0, 1] \) is uniform distributing random number and \( j \in [1, 2, \ldots, 6] \) is the \( j \)th position coordinate. \( \text{rand}(n(i)) \in [1, 2, \ldots, 6] \) is a variable index selected randomly, which ensures at least one variable quantity contributed by mutation ones. \( C_R \) is a crossover constant within [0, 1], which controls the dispersion of the population.

Compare the target individual \( X_i(t) \) with the trial individual \( U_i(t + 1) \) to decide whether the trial individual is a member of the next generation \( X_i(t + 1) \) according to the following equation:

\[
X_i(t + 1) = \begin{cases} 
U_i(t + 1), & f \left( U_i(t + 1) \right) \geq f \left( X_i(t) \right) \\
X_i(t), & \text{otherwise}.
\end{cases}
\] (25)
4.3. Improved Particle Swarm Optimizing Algorithm. The improved particle swarm optimizing algorithm by introducing differential evolution algorithm can eliminate stagnation and avoid premature convergence, and the procedure is as follows.

**Step 1.** Initialize a population which contains \( s \) particles randomly, and set mutation and crossover constants, and determine the maximum number of generations and the maximum flight velocity.

**Step 2.** The dimension of the particle is six which is judged by the matrix \( P \). For the \( i \)th particle, it has random position \( X_i(t) \) and flight speed \( V_i(t) \) in the 6-dimensional search space. \( V_i(t) = \text{Rand}(t) \ast V_{\text{max}} \), where \( \text{Rand}(t) \) is within \([-1, 1]\). Suppose the initial population is the position with the best fitness value \( p\text{best} \).

**Step 3.** Compute the fitness value \( f(X_i) \) of each particle \( X_i(t) \), and compare the fitness value of each current particle with its \( p\text{best} \). If the current fitness value is smaller than the \( p\text{best} \) fitness value, regard the current position as current \( p\text{best} \) position.

**Step 4.** Compare the fitness value of each current particle with the \( g\text{best} \). If the current fitness value is smaller than the \( g\text{best} \), regard current position as current \( g\text{best} \) position.

**Step 5.** The position and speed of each particle implement evolving according to (20), (21), and (22).

**Step 6.** Mutation and crossover operation are carried out for the position of particle according to (23) and (24), and then a new position is generated according to (25).

**Step 7.** If the maximum number of generations has not been reached, or a good enough fitness value has not been obtained, return to Step 3.

The flow chart of improved particle swarm optimizing algorithm is shown in Figure 2.
5. Simulation Research

Set \( x^0 = [108.00, 66.65, 428.0]^T \) and \( u^0 = [0.340, 0.690, 0.435]^T \) is the initial working point. \( c_1 = 1.25 \), \( c_2 = 1.5 \), \( s = 50 \), \( \gamma = 1 \), \( \varepsilon = 0.01 \), and MaxNumber = 200 are previously selected. In this paper

\[
Q = I,
\]

\[
\Phi_F = \begin{bmatrix}
0.44 & -0.242 & -0.86 \\
0.784 & -0.374 & 0.495
\end{bmatrix},
\]

\[
B_1 = \begin{bmatrix}
0.3 & 0 & 0 \\
0 & 0.2 & 0 \\
0 & 0 & 0
\end{bmatrix},
\]

\[
H_F = \begin{bmatrix}
0.6003 & 1.2165 & -0.0331 \\
1.2165 & 2.4569 & -0.067 \\
-0.0331 & -0.067 & 0.0018
\end{bmatrix}.
\]

The improved particle swarm algorithm is applied in (10) for simulation test. MATLAB software is adopted. \( w = [0.8 \sin 2t, 0.5 \cos t, \sin 2t]^T \) is external disturbance, and the reference input signal of system is taken as

\[
\begin{align*}
r_1 &= 108 + 10.8 \times 1 (t - 200) + 10.8 \times 1 (t - 1000), \\
r_2 &= 66.65 + 18.41 \times 1 (t - 600) + 20.74 \times 1 (t - 1400), \\
r_3 &= 0.
\end{align*}
\]

At \( t = 200 \) s, the drum pressure \( p \) is stable in 118.8 (kg/cm\(^2\)). At \( t = 600 \) s, the output power \( P_0 \) is stable in 85.06 (MW). At \( t = 1000 \) s, the drum pressure \( p \) is stable in 129.0 (kg/cm\(^2\)). At \( t = 1400 \) s, the output power \( P_0 \) is stable in 105.8 (MW).

The response of the system outputs with two particle swarm optimizing algorithms is shown in Figures 3–5. It can be seen from Figures 3–5 that the change of drum pressure at 200 s and 1000 s causes the tiny change of output power, but it soon becomes smooth. The change of output power at 200 s and 1000 s combining the change of drum pressure at 200 s and 1000 s causes the tiny change of drum water level, but it soon becomes smooth. The small peak in the changing states is caused by the change of the reference input at different times. It is to show the effect of the decoupling. At the same time there are three disturbances \( 0.8 \sin 2t, 0.5 \cos t, \) and \( \sin 2t \) added in this system. The simulation results are acceptable.

The simulation results show that the method can guarantee the closed-loop system asymptotical stability in a large range working point. The system has strong robustness for external disturbance.

6. Conclusions

In this paper, a new state feedback \( H_{\infty} \) control scheme based on an improved particle swarm optimizing algorithm is presented and applied to the boiler-turbine unit with external disturbance. The following results are got:

(1) In this paper, the machine furnace nonlinear coordinate system is translated into a linear time-varying system; then the \( H_{\infty} \) control problem is transformed into the solution of Riccati equation.

(2) With the improved particle swarm algorithm to solve the Riccati equation, the parameters of the controller real-time optimizing are achieved. The method can enhance the precision of optimizing and ensure the stability in large range working point.

(3) The control system has stronger robustness for external disturbance, and the double purposes of stabilizing system and suppressing the disturbance have been realized.
The output responses with nice steady state performance and decoupling effect are got.

**Conflict of Interests**

The authors declare that there is no conflict of interests regarding the publication of this paper.

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