Research Article

Discrete-Time Nonlinear Control of VSC-HVDC System

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Because VSC-HVDC is a kind of strong nonlinear, coupling, and multi-input multioutput (MIMO) system, its control problem is always attracting much attention from scholars. And a lot of papers have done research on its control strategy in the continuous-time domain. But the control system is implemented through the computer discrete sampling in practical engineering. It is necessary to study the mathematical model and control algorithm in the discrete-time domain. The discrete mathematical model based on output feedback linearization and discrete sliding mode control algorithm is proposed in this paper. And to ensure the effectiveness of the control system in the quasi sliding mode state, the fast output sampling method is used in the output feedback. The results from simulation experiment in MATLAB/SIMULINK prove that the proposed discrete control algorithm can make the VSC-HVDC system have good static, dynamic, and robust characteristics in discrete-time domain.

1. Introduction

Voltage source converter based high voltage direct current (VSC-HVDC) technology which uses advanced (insulated gate bipolar transistor) IGBT device and pulse width modulation (PWM) method overcomes the disadvantage of traditional HVDC. It can not only adjust the active and reactive power independently, but also supply power to passive network system. Therefore the VSC-HVDC technology has a broad application prospect in the fields of distributed generation, asynchronous AC network interconnection, remote-area power supply, and so forth [1–5].

Because VSC-HVDC is a multiple input multiple output, strong coupling nonlinear system, its control problem is always attracting much attention from the scholars. In chronological order, its research process can be classified into the following two phases.

- (1) The steady state mathematical model and basic control strategy of VSC-HVDC system: [6] developed the steady state model and proposed control strategy which combined an inverse model controller with a PI controller. Reference [7] presented an equivalent continuous-time state space model of VSC-HVDC in the synchronous d - q reference frame and

proposed a decoupled PI control strategy using the feed-forward compensation method. Reference [8] presented the elements of VSC-HVDC and proposed a feed-forward decoupled current control strategy. In this phase, the study is mainly based on the traditional PI controllers under the linear decoupling control strategy. The design method of this kind of control strategy is easy. And it shows good adjustment ability. But the parameters of PI controllers are fixed, and their adjustment ability is limited. When system suffers from large disturbance, they show weak robustness and dynamic characteristics. Some severe cases can lead to sustained oscillation of the system. Therefore some scholars started the second phase study.

- (2) Optimizing and nonlinear control strategy of the VSC-HVDC system: [9] proposed an adaptive control design to improve dynamic performances of VSC-HVDC systems. The adaptive controllers designed for nonlinear characteristics of VSC-HVDC systems, which were based on back stepping method, considered parameters uncertainties. Reference [10] presented a robust nonlinear controller for VSC-HVDC transmission link using input-output linearization

and sliding mode control strategy. The feedback linearization was used to cancel nonlinearity and the sliding mode control offered invariant stability to modeling uncertainties. Reference [11] proposed the controller design based on H_∞ control methodologies to deal with the nonlinearities introduced by requirements to power flow and line voltage. Reference [12] addressed two different robust nonlinear control methodologies based on sliding mode control for the VSC-HVDC transmission system's performance enhancement and stability improvement. In this phase, the studies are based on the nonlinear control, robust control, and optimizing control of the VSC-HVDC system. The experimental results from research papers show these control strategies can improve static, dynamic, and robust performance of VSC-HVDC system.

However, the referred literatures are based on the continuous-time state. In practice, nowadays most controllers are implemented in discrete time. It is known that the realization of a controller using digital elements and complex programmable logic devices can achieve maximum reproducibility at minimum cost. So it is necessary to do research on the discrete model and control strategy. In [13], the discrete PI controllers of VSC-HVDC system were established. Reference [14] studied the discrete PI control algorithm of VSC-HVDC system which supplied power to the passive network.

But so far, the discrete-time mathematic model and control strategy based on nonlinear control methods are seldom studied by scholars. Through proper feedback linearization control strategy, complex nonlinear system synthesis problems can be transformed to linear system synthesis problems. As a kind of robust control method, sliding mode control is versatile to linear and nonlinear system. It is easy to be designed and carried out. Because of its complete robustness to the parameters change and external disturbances which meet the match condition, it gets extensive attention from scholars and engineers [15].

Therefore, the discrete mathematical model of the VSC-HVDC system based on nonlinear feedback linearization and discrete sliding mode control algorithm are proposed in this paper. And the fast output sampling (FOS) technique is adopted in the output feedback, which ensures the stability of the closed loop system in the condition of quasi sliding mode control. The simulation results performed in MATLAB/SIMULINK show that the proposed discrete control strategy can make the VSC-HVDC system have good operation performance.

2. The Mathematical Model of VSC-HVDC

2.1. Continuous-Time State Space Model. The structure diagram of VSC-HVDC is shown in Figure 1. The more widely used continuous-time mathematical models based on the synchronous reference coordinates are employed in this paper, which are shown as (1) and (7).

2.1.1. Rectifier Side. Choose state variables $\mathbf{x}_{1,2} = (i_{rd} \ i_{rq})^T$, controlled variables $\mathbf{u}_{1,2} = (U_{crd} \ U_{crq})^T$, and output variables $\mathbf{y}_{1,2} = (i_{rd} \ i_{rq})^T$. The mathematical model of the rectifier side is shown as

$$\begin{aligned}\dot{x}_1 &= \frac{-R_r}{L_r}x_1 + \omega_r x_2 + \frac{U_{srd}}{L_r} - \frac{1}{L_r}u_1, \\ \dot{x}_2 &= \frac{-R_r}{L_r}x_2 - \omega_r x_1 + \frac{U_{srq}}{L_r} - \frac{1}{L_r}u_2, \\ y_1 &= x_1, \\ y_2 &= x_2,\end{aligned}\quad (1)$$

where i_{rd} , i_{rq} and U_{srd} , U_{srq} are the d - q axis currents and voltages on the rectifier side, respectively. U_{crd} and U_{crq} are, respectively, the d - q axis control inputs on the rectifier side. R_r and L_r are the corresponding equivalent resistance and the inductance on the rectifier side. ω_r is the AC system frequency on the rectifier side.

Here, the output variables on the rectifier side should be $(P_r \ Q_r)^T$. P_r and Q_r are, respectively, the output values of active and reactive power on the rectifier side. For convenience, define U_{sr} as the line voltage effective value of the rectifier power supply. Define the d axis in the synchronization reference frame coincidence with a axis in the three-phase reference coordinate, so $U_{srd} = U_{sr}$, $U_{srq} = 0$, $\begin{bmatrix} P_r \\ Q_r \end{bmatrix} = \begin{bmatrix} U_{srd} & U_{srq} \\ -U_{srq} & U_{srd} \end{bmatrix} \begin{bmatrix} i_{rd} \\ i_{rq} \end{bmatrix} = \begin{bmatrix} U_{sr}i_{rd} \\ U_{sr}i_{rq} \end{bmatrix}$. Therefore the output variables can be chosen as $\mathbf{y}_{1,2} = (i_{rd} \ i_{rq})^T$.

Define

$$\begin{aligned}f_1 &= \frac{-R_r}{L_r}x_1 + \omega_r x_2 + \frac{U_{srd}}{L_r}, \\ f_2 &= \frac{-R_r}{L_r}x_2 - \omega_r x_1 + \frac{U_{srq}}{L_r}, \\ \mathbf{f}_r &= (f_1, f_2)^T, \\ \mathbf{g}_1 &= \left(-\frac{1}{L_r}, 0\right)^T, \\ \mathbf{g}_2 &= \left(0, -\frac{1}{L_r}\right)^T.\end{aligned}\quad (2)$$

Therefore (1) can be reorganized by

$$\begin{aligned}\dot{\mathbf{x}}_{1,2} &= \mathbf{f}_r + \mathbf{g}_1 u_1 + \mathbf{g}_2 u_2, \\ \mathbf{y}_{1,2} &= \begin{pmatrix} h_1(x) \\ h_2(x) \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}.\end{aligned}\quad (3)$$

Then, do exact feedback linearization on system equation (3). By calculation, the relative degree of y_1 is denoted by $\gamma_1 = 1$. The relative degree of y_2 is denoted by $\gamma_2 = 1$. Based on the feedback linearization theory [16], the exact feedback

FIGURE 1: Structure diagram of VSC-HVDC system.

linearization state equations of the rectifier side are obtained as

$$\begin{aligned} \begin{pmatrix} \dot{y}_1 \\ \dot{y}_2 \end{pmatrix} &= \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \\ &= \begin{pmatrix} L_{f_r}^{\gamma_1} h_1 \\ L_{f_r}^{\gamma_2} h_2 \end{pmatrix} \\ &\quad + \begin{pmatrix} L_{g_1} L_{f_r}^{\gamma_1-1} h_1 & L_{g_2} L_{f_r}^{\gamma_1-1} h_1 \\ L_{g_1} L_{f_r}^{\gamma_2-1} h_2 & L_{g_2} L_{f_r}^{\gamma_2-1} h_2 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}, \quad (4) \\ \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} &= \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} f_1 \\ f_2 \end{pmatrix} + \begin{pmatrix} -\frac{1}{L_r} & 0 \\ 0 & -\frac{1}{L_r} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}. \end{aligned}$$

Because $\begin{vmatrix} -1/L_r & 0 \\ 0 & -1/L_r \end{vmatrix} = 1/L_r^2 \neq 0$, $\begin{pmatrix} -1/L_r & 0 \\ 0 & -1/L_r \end{pmatrix}$ is nonsingular.

The controlled variables u_1 and u_2 can be denoted by virtual control variables v_1 and v_2 :

$$\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} L_r (f_1 - v_1) \\ L_i (f_2 - v_2) \end{pmatrix}. \quad (5)$$

To make it convenient for using FOS (fast output sampling) technique in Section 3.1 [17], substitute (5) into (3); then the system equation of rectifier side can be denoted by

$$\begin{aligned} \dot{x}_1 &= v_1, \\ \dot{x}_2 &= v_2, \\ y_1 &= x_1, \\ y_2 &= x_2. \end{aligned} \quad (6)$$

2.1.2. Inverter Side. Choose state variables $\mathbf{x}_{3,4,5} = (i_{id} \ i_{iq} \ U_{dc2})^T$, controlled variables $\mathbf{u}_{3,4} = (U_{cid} \ U_{ciq})^T$, and output variables $\mathbf{y}_{3,4} = (U_{dc2} \ i_{iq})^T$. The mathematical model of inverter side is shown as

$$\begin{aligned} \dot{x}_3 &= \frac{-R_i}{L_i} x_3 + \omega_i x_4 + \frac{U_{sid}}{L_i} - \frac{1}{L_i} u_3, \\ \dot{x}_4 &= \frac{-R_i}{L_i} x_4 - \omega_i x_3 + \frac{U_{siq}}{L_i} - \frac{1}{L_i} u_4, \end{aligned}$$

$$\begin{aligned} \dot{x}_5 &= \frac{U_{sid} x_3 + U_{siq} x_4 - R_i (x_3^2 + x_4^2)}{C x_5} + \frac{U_{dc1} - x_5}{C R_{dc}}, \\ y_3 &= x_5, \\ y_4 &= x_4, \end{aligned} \quad (7)$$

where i_{id} , i_{iq} and U_{sid} , U_{siq} are the d - q axis currents and voltages on the inverter side, respectively. U_{cid} and U_{ciq} are, respectively, the control inputs on the inverter side. R_i and L_i are the corresponding equivalent resistance and the inductance on the inverter side. U_{dc1} and U_{dc2} are, respectively, the DC voltages on the rectifier side and inverter side. C and R_{dc} are, respectively, the converter station capacitance and DC line resistance. ω_i is the AC system frequency on the inverter side.

Here, the output variables on the inverter side should be $(U_{dc2} \ Q_i)$. Q_i is the output value of reactive power on the inverter side. For convenience, define U_{si} as the line voltage effective value of the inverter side power source. Define the d axis in the synchronization reference frame that coincides with a axis in the three-phase reference coordinate, so $U_{sid} = U_{si}$, $U_{siq} = 0$, and $Q_i = -U_{siq} i_{id} + U_{sid} i_{iq} = U_{si} i_{iq}$. Therefore the output variables can be chosen as $\mathbf{y}_{3,4} = (U_{dc2} \ i_{iq})^T$.

Define

$$\begin{aligned} f_3 &= \frac{-R_i}{L_i} x_3 + \omega_i x_4 + \frac{U_{sid}}{L_i}, \\ f_4 &= \frac{-R_i}{L_i} x_4 - \omega_i x_3 + \frac{U_{siq}}{L_i}, \\ f_5 &= \frac{U_{sid} x_3 + U_{siq} x_4 - R_i (x_3^2 + x_4^2)}{C x_5} + \frac{U_{dc1} - x_5}{C R_{dc}}, \\ \mathbf{f}_i &= (f_3, f_4, f_5)^T, \\ \mathbf{g}_3 &= \left(-\frac{1}{L_i}, 0, 0 \right)^T, \\ \mathbf{g}_4 &= \left(0, -\frac{1}{L_i}, 0 \right)^T. \end{aligned} \quad (8)$$

Therefore (7) can be reorganized by

$$\begin{aligned}\dot{\mathbf{x}}_{3,4,5} &= \mathbf{f}_i + \mathbf{g}_3 u_3 + \mathbf{g}_4 u_4, \\ \mathbf{y}_{3,4} &= \begin{pmatrix} h_3(x) \\ h_4(x) \end{pmatrix} = \begin{pmatrix} x_5 \\ x_4 \end{pmatrix}.\end{aligned}\quad (9)$$

Then, do exact feedback linearization on system (9). By calculation, the relative degree of y_3 is denoted by $\gamma_3 = 2$. The relative degree of y_4 is denoted by $\gamma_4 = 1$. Based upon the feedback linearization theory [16], the exact feedback linearization state equations of inverter side are obtained as

$$\begin{aligned}\begin{pmatrix} \ddot{y}_3 \\ \dot{y}_4 \end{pmatrix} &= \begin{pmatrix} \ddot{x}_5 \\ \dot{x}_4 \end{pmatrix} = \begin{pmatrix} v_3 \\ v_4 \end{pmatrix} \\ &= \begin{pmatrix} L_{f_i}^{\gamma_3} h_3 \\ L_{f_i}^{\gamma_4} h_4 \end{pmatrix} \\ &\quad + \begin{pmatrix} L_{g_3} L_{f_i}^{\gamma_3-1} h_3 & L_{g_4} L_{f_i}^{\gamma_3-1} h_3 \\ L_{g_3} L_{f_i}^{\gamma_4-1} h_4 & L_{g_4} L_{f_i}^{\gamma_4-1} h_4 \end{pmatrix} \begin{pmatrix} u_3 \\ u_4 \end{pmatrix}, \\ \begin{pmatrix} \ddot{x}_5 \\ \dot{x}_4 \end{pmatrix} &= \begin{pmatrix} v_3 \\ v_4 \end{pmatrix} \\ &= \begin{pmatrix} a_1 f_3 + a_2 f_4 + a_3 f_5 \\ f_4 \end{pmatrix} \\ &\quad + \begin{pmatrix} \frac{-a_1}{L_i} & \frac{-a_2}{L_i} \\ 0 & \frac{-1}{L_i} \end{pmatrix} \begin{pmatrix} u_3 \\ u_4 \end{pmatrix},\end{aligned}\quad (10)$$

where

$$\begin{aligned}a_1 &= \frac{\partial f_5}{\partial x_3} = \frac{U_{sid} - 2R_i x_3}{Cx_5}, \\ a_2 &= \frac{\partial f_5}{\partial x_4} = \frac{U_{sid} - 2R_i x_4}{Cx_5}, \\ a_3 &= \frac{\partial f_5}{\partial x_5} = -\frac{U_{sid} x_3 + U_{sid} x_4 - R_i (x_3^2 + x_4^2)}{Cx_5^2} - \frac{1}{CR_{dc}}.\end{aligned}\quad (11)$$

Because $\begin{vmatrix} -a_1/L_i & -a_2/L_i \\ 0 & -1/L_i \end{vmatrix} = a_1/L_i^2 \neq 0$, $\begin{pmatrix} -a_1/L_i & -a_2/L_i \\ 0 & -1/L_i \end{pmatrix}$ is nonsingular.

The controlled variables u_3 and u_4 can be denoted by virtual control variables v_3 and v_4 :

$$\begin{pmatrix} u_3 \\ u_4 \end{pmatrix} = \begin{pmatrix} (a_1 f_1 + a_3 f_3 - v_3 + a_2 v_4) \cdot \frac{L_i}{a_1} \\ L_i (f_4 - v_4) \end{pmatrix}.\quad (12)$$

To make it convenient for using FOS (fast output sampling) technology in Section 3.1 [17], substitute a_1 , a_2 , a_3 , and (12)

into (7); then the system equation of inverter side can be denoted by

$$\begin{aligned}\dot{x}_3 &= \left(\frac{a_3^2}{a_1} + \frac{a_3}{a_1 CR_{dc}} \right) x_5 + \frac{1}{a_1} v_3 - \frac{a_2}{a_1} v_4, \\ \dot{x}_4 &= v_4, \\ \dot{x}_5 &= \left(-a_3 - \frac{1}{CR_{dc}} \right) x_5, \\ y_3 &= x_5, \\ y_4 &= x_4.\end{aligned}\quad (13)$$

Here, $\dot{x}_5$ should be $\dot{x}_5 = (-a_3 - 1/CR_{dc})x_5 + (U_{dc1} - x_5)/CR_{dc}$. Since, during normal operation, U_{dc1} is approximately equal to x_5 , for convenient calculation, $\dot{x}_5$ is simplified into $\dot{x}_5 = (-a_3 - 1/CR_{dc})x_5$.

2.2. Discrete-Time State Space Model. Discretize the virtual control variables v_1 , v_2 , v_3 , and v_4 with sampling time τ :

$$\begin{aligned}\begin{pmatrix} v_1(k) \\ v_2(k) \end{pmatrix} &= \begin{pmatrix} \frac{y_1(k+1) - y_1(k)}{\tau} \\ \frac{y_2(k+1) - y_2(k)}{\tau} \end{pmatrix} \\ &= \begin{pmatrix} \frac{x_1(k+1) - x_1(k)}{\tau} \\ \frac{x_2(k+1) - x_2(k)}{\tau} \end{pmatrix} \\ &= \begin{pmatrix} f_1(k) \\ f_2(k) \end{pmatrix} + \begin{pmatrix} \frac{-1}{L_r} & 0 \\ 0 & \frac{-1}{L_r} \end{pmatrix} \begin{pmatrix} u_1(k) \\ u_2(k) \end{pmatrix},\end{aligned}\quad (14)$$

$$\begin{aligned}\begin{pmatrix} v_3(k) \\ v_4(k) \end{pmatrix} &= \begin{pmatrix} \frac{y_3(k+2) - 2y_3(k+1) + y_3(k)}{\tau^2} \\ \frac{y_4(k+1) - y_4(k)}{\tau} \end{pmatrix} \\ &= \begin{pmatrix} \frac{x_5(k+2) - 2x_5(k+1) + x_5(k)}{\tau^2} \\ \frac{x_4(k+1) - x_4(k)}{\tau} \end{pmatrix} \\ &= \begin{pmatrix} a_1 f_3(k) + a_2 f_4(k) + a_3 f_5(k) \\ f_4(k) \end{pmatrix} \\ &\quad + \begin{pmatrix} \frac{-a_1}{L_i} & \frac{-a_2}{L_i} \\ 0 & \frac{-1}{L_i} \end{pmatrix} \begin{pmatrix} u_3(k) \\ u_4(k) \end{pmatrix}.\end{aligned}\quad (15)$$

Discretize system equations (6), (13) with sampling time τ , shown as

$$\begin{aligned}
 x_1(k+1) &= x_1(k) + \tau v_1(k), \\
 x_2(k+1) &= x_2(k) + \tau v_2(k), \\
 y_1(k) &= x_1(k), \\
 y_2(k) &= x_2(k), \\
 x_3(k+1) &= x_3(k) + \left(\frac{\tau a_3^2}{a_1} + \frac{\tau a_3}{a_1 C R_{dc}} \right) x_5(k) \\
 &\quad + \frac{\tau}{a_1} v_3(k) - \frac{\tau a_2}{a_1} v_4(k), \\
 x_4(k+1) &= x_4(k) + \tau v_4(k), \\
 x_5(k+1) &= \left(1 - \tau a_3 - \frac{\tau}{C R_{dc}} \right) x_5(k), \\
 y_3(k) &= x_5(k), \\
 y_4(k) &= x_4(k).
 \end{aligned} \tag{16}$$

Rearrange (16), and then get

$$\begin{aligned}
 \mathbf{x}_{1,2}(\mathbf{k}+1) &= \mathbf{A}_{rr} \mathbf{x}_{1,2}(\mathbf{k}) + \mathbf{B}_{rr} \mathbf{v}_{1,2}(\mathbf{k}), \\
 \mathbf{y}_{1,2}(\mathbf{k}) &= \mathbf{C}_r \mathbf{x}_{1,2}(\mathbf{k}), \\
 \mathbf{x}_{3,4,5}(\mathbf{k}+1) &= \mathbf{A}_{ir} \mathbf{x}_{3,4,5}(\mathbf{k}) + \mathbf{B}_{ir} \mathbf{v}_{3,4}(\mathbf{k}), \\
 \mathbf{y}_{3,4}(\mathbf{k}) &= \mathbf{C}_i \mathbf{x}_{3,4,5}(\mathbf{k}),
 \end{aligned} \tag{17}$$

where

$$\begin{aligned}
 \mathbf{A}_{rr} &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \\
 \mathbf{B}_{rr} &= \begin{pmatrix} \tau & 0 \\ 0 & \tau \end{pmatrix}, \\
 \mathbf{C}_r &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \\
 \mathbf{A}_{ir} &= \begin{pmatrix} 1 & 0 & \frac{\tau a_3^2}{a_1} + \frac{\tau a_3}{a_1 C R_{dc}} \\ 0 & 1 & 0 \\ 0 & 0 & 1 - \tau a_3 - \frac{\tau}{C R_{dc}} \end{pmatrix}, \\
 \mathbf{B}_{ir} &= \begin{pmatrix} \frac{\tau}{a_1} & -\frac{\tau a_2}{a_1} \\ 0 & \tau \\ 0 & 0 \end{pmatrix}, \\
 \mathbf{C}_i &= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}.
 \end{aligned} \tag{18}$$

Assume that the pairs $(\mathbf{A}_{rr}, \mathbf{B}_{rr})$, $(\mathbf{A}_{ir}, \mathbf{B}_{ir})$ are controllable and the pairs $(\mathbf{A}_{rr}, \mathbf{C}_r)$, $(\mathbf{A}_{ir}, \mathbf{C}_i)$ are observable through properly sampling output variables.

3. Discrete Sliding Mode Control of VSC-HVDC System

In ideal continuous-time case, the SMC (sliding mode control) switches at infinite frequency and forces the states to slide on the so-called switching hyperplane. In practical applications, direct implementation of continuous-time SMC schemes using digital elements, which are considered as the device for imperfect switching, will inevitably induce chattering phenomenon and deteriorate performance or even induce instability. Chattering will cause serious harmonics which is undesirable in VSC-HVDC systems. Hence, the controller design using the discrete-time SMC (DSMC) algorithm is desirable for a successful implementation of the VSC-HVDC control systems. And due to the finite sampling frequency, the controller inputs are calculated once per sampling period and held constant during that interval. Under such a circumstance, the trajectories of the system states of interest are unable to precisely move along the sliding surface, which will lead to a quasi sliding mode motion only [18]. Therefore, only using static output feedback technology has not effectively ensured the control effect of discrete sliding mode control. The fast output sampling technology should be used [17, 19].

3.1. Fast Output Sampling (FOS) Technology. Compared with the static output feedback technology, FOS not only keeps its advantage, but also can randomly configure the system poles and always make the closed loop system stable. And then FOS can ensure the effectiveness of the discrete sliding mode control. In the FOS, every sampling period τ is divided into N subintervals. Here $\Delta = \tau/N$ and N is equal to or greater than the observable index of system $(\mathbf{A}, \mathbf{B})$. The output variables are measured at time instants $t = l\Delta$, $l = 0, 1, \dots, N-1$. Consider the discrete-time system having be at time $t = k\tau$; the fast output samples are obtained as [17, 19]

$$\mathbf{y}_k = \begin{bmatrix} y((k-1)\tau) \\ y((k-1)\tau + \Delta) \\ \vdots \\ y((k-1)\tau + (N-1)\Delta) \end{bmatrix}. \tag{19}$$

Then the rectifier side can be expressed as

$$\begin{aligned}
 \mathbf{x}_{1,2}(\mathbf{k}) &= \mathbf{A}_{rr} \mathbf{x}_{1,2}(\mathbf{k}-1) + \mathbf{B}_{rr} \mathbf{v}_{1,2}(\mathbf{k}-1), \\
 \mathbf{y}_{1,2k} &= \mathbf{C}_{r0} \mathbf{x}_{1,2}(\mathbf{k}-1) + \mathbf{D}_{r0} \mathbf{v}_{1,2}(\mathbf{k}-1),
 \end{aligned} \tag{20}$$

where

$$\begin{aligned} \mathbf{C}_{r0} &= \begin{bmatrix} \mathbf{C}_r \\ \mathbf{C}_r \mathbf{A}_r \\ \vdots \\ \mathbf{C}_r \mathbf{A}_r^{N_1-1} \end{bmatrix}, \\ \mathbf{D}_{r0} &= \begin{bmatrix} \mathbf{0} \\ \mathbf{C}_r \mathbf{B}_r \\ \vdots \\ \mathbf{C}_r \sum_{j=0}^{N_1-2} \mathbf{A}_r^j \mathbf{B}_r \end{bmatrix}. \end{aligned} \quad (21)$$

And assume that $\mathbf{C}_{r0}$ and $\mathbf{D}_{r0}$ are invertible through appropriate choice. $(\mathbf{A}_r \ \mathbf{B}_r \ \mathbf{C}_r)$ is the system parameter matrix with sampling rate $1/\Delta_1$, $\Delta_1 = \tau/N_1$.

Define $\mathbf{y}_{1,2k} = \begin{pmatrix} y_1((k-1)\tau) \\ y_2((k-1)\tau) \end{pmatrix}$. This means y_1 is sampled once and y_2 is sampled once in each sampling period τ .

The inverter side can be expressed as

$$\begin{aligned} \mathbf{x}_{3,4,5}(k) &= \mathbf{A}_{ir} \mathbf{x}_{3,4,5}(k-1) + \mathbf{B}_{ir} \mathbf{v}_{3,4}(k-1), \\ \mathbf{y}_{3,4k} &= \mathbf{C}_{i0} \mathbf{x}_{3,4,5}(k-1) + \mathbf{D}_{i0} \mathbf{v}_{3,4}(k-1), \end{aligned} \quad (22)$$

where

$$\begin{aligned} \mathbf{C}_{i0} &= \begin{bmatrix} \mathbf{C}_i \\ \mathbf{C}_i \mathbf{A}_i \\ \vdots \\ \mathbf{C}_i \mathbf{A}_i^{N_2-1} \end{bmatrix}, \\ \mathbf{D}_{i0} &= \begin{bmatrix} \mathbf{0} \\ \mathbf{C}_i \mathbf{B}_i \\ \vdots \\ \mathbf{C}_i \sum_{j=0}^{N_2-2} \mathbf{A}_i^j \mathbf{B}_i \end{bmatrix}. \end{aligned} \quad (23)$$

And assume that $\mathbf{C}_{i0}$ and $\mathbf{D}_{i0}$ are invertible through appropriate choice. $(\mathbf{A}_i \ \mathbf{B}_i \ \mathbf{C}_i)$ is the system parameter matrix with sampling rate $1/\Delta_2$, $\Delta_2 = \tau/N_2$.

Define $\mathbf{y}_{3,4k} = \begin{pmatrix} y_3((k-1)\tau) \\ y_4((k-1)\tau + \Delta_2) \end{pmatrix}$. This means y_3 is sampled twice and y_4 is sampled once in each sampling period τ .

(a) The reference value of d axis current

(b) The reference value of q axis current

FIGURE 2: Control block diagrams of output references value at the rectifier station.

According to (20) and (22), state vectors $\mathbf{x}_{1,2}(k)$ and $\mathbf{x}_{3,4,5}(k)$ can be deduced as

$$\begin{aligned} \mathbf{x}_{1,2}(k) &= \mathbf{A}_{rr} \mathbf{C}_{r0}^{-1} \mathbf{y}_{1,2k} \\ &\quad + (\mathbf{B}_{rr} - \mathbf{A}_{rr} \mathbf{C}_{r0}^{-1} \mathbf{D}_{r0}) \mathbf{v}_{1,2}(k-1), \end{aligned} \quad (24)$$

$$\begin{aligned} \mathbf{x}_{3,4,5}(k) &= \mathbf{A}_{ir} \mathbf{C}_{i0}^{-1} \mathbf{y}_{3,4k} \\ &\quad + (\mathbf{B}_{ir} - \mathbf{A}_{ir} \mathbf{C}_{i0}^{-1} \mathbf{D}_{i0}) \mathbf{v}_{3,4}(k-1). \end{aligned} \quad (25)$$

3.2. The Rectifier Side Control. The differences between output and reference variables are denoted by (26). Because the relative degrees of y_1 and y_2 are, respectively, equal to 1, 1, the sliding mode surfaces are defined as (27). Consider

$$\mathbf{e}_{1,2} = \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \begin{pmatrix} y_1(k) - i_{rdref}(k) \\ y_2(k) - i_{rqref}(k) \end{pmatrix}, \quad (26)$$

where $i_{rdref}(k)$ and $i_{rqref}(k)$ are the reference values. Their calculation processes are shown in Figure 2. P_r , P_{rref} , Q_r , and Q_{rref} are, respectively, the output value and reference value of active and reactive power. Consider

$$\mathbf{s}_{1,2} = \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} = \begin{pmatrix} e_1(k) \\ e_2(k) \end{pmatrix}. \quad (27)$$

FIGURE 3: Control block diagram of output reference value at the inverter station.

Desired state trajectories of the discrete variable structure system shown as in (28) can be obtained by control law based on reaching law method:

$$\begin{aligned} s_1(k+1) - s_1(k) &= -\rho_1 \tau s_1(k) - \varepsilon_1 \tau \operatorname{sgn}(s_1(k)), \\ s_2(k+1) - s_2(k) &= -\rho_2 \tau s_2(k) - \varepsilon_2 \tau \operatorname{sgn}(s_2(k)), \end{aligned} \quad (28)$$

where $\rho_1, \rho_2, \varepsilon_1$, and ε_2 are all greater than zero. And $1 - \rho_1 \tau > 0$, $1 - \rho_2 \tau > 0$.

Now replacing (26) with (27) and then with (28), the virtual control variables can be denoted by

$$\begin{aligned} v_1(k) &= -\rho_1 s_1(k) - \varepsilon_1 \operatorname{sgn}(s_1(k)), \\ v_2(k) &= -\rho_2 s_2(k) - \varepsilon_2 \operatorname{sgn}(s_2(k)). \end{aligned} \quad (29)$$

The control variables shown as in (30) can be deduced by (14), (24), and (29):

$$\begin{aligned} u_1(k) &= -R_r Q_{r1} + \omega L_r Q_{r2} - L_r v_1 + U_{srd}, \\ u_2(k) &= -\omega L_r Q_{r1} - R_r Q_{r2} - L_r v_2 + U_{srq}, \end{aligned} \quad (30)$$

where Q_{r1} and Q_{r2} are the first and second row of $\mathbf{x}_{1,2}(\mathbf{k})$ in Section 3.1. $u_1(k)$, $u_2(k)$ can be expressed by $\mathbf{y}_{1,2}(\mathbf{k})$ through FOS.

3.3. The Inverter Side Control. The differences between output and reference variables are denoted by (31). Because the relative degrees of y_3 and y_4 are, respectively, equal to 2, 1, the sliding mode surfaces are defined as (32). Consider

$$\mathbf{e}_{3,4} = \begin{pmatrix} e_3 \\ e_4 \end{pmatrix} = \begin{pmatrix} y_3(k) - U_{dc\text{ref}}(k) \\ y_4(k) - i_{iq\text{ref}}(k) \end{pmatrix}, \quad (31)$$

where $U_{dc\text{ref}}(k)$ and $i_{iq\text{ref}}(k)$ are the reference values. Usually $U_{dc\text{ref}}(k)$ is known. The calculation process of $i_{iq\text{ref}}(k)$ is shown in Figure 3. Q_i and $Q_{i\text{ref}}$ are, respectively, the output and reference value of reactive power:

$$\mathbf{s}_{3,4} = \begin{pmatrix} s_3 \\ s_4 \end{pmatrix} = \begin{pmatrix} c_3 e_3(k) + \frac{e_3(k+1) - e_3(k)}{\tau} \\ e_4(k) \end{pmatrix}. \quad (32)$$

Similar to the rectifier side, desired state trajectories of the discrete variable structure system shown as in (33) can be obtained by control law based on reaching law method:

$$\begin{aligned} s_3(k+1) - s_3(k) &= -\rho_3 \tau s_3(k) - \varepsilon_3 \tau \operatorname{sgn}(s_3(k)), \\ s_4(k+1) - s_4(k) &= -\rho_4 \tau s_4(k) - \varepsilon_4 \tau \operatorname{sgn}(s_4(k)), \end{aligned} \quad (33)$$

where $\rho_3, \rho_4, \varepsilon_3$, and ε_4 are all greater than zero. And $1 - \rho_3 \tau > 0$, $1 - \rho_4 \tau > 0$.

Now replacing (31) with (32) and then with (33), the virtual control variables can be denoted by

$$\begin{aligned} v_3(k) &= -\rho_3 s_3(k) - \varepsilon_3 \operatorname{sgn}(s_3(k)) \\ &\quad - \frac{c_3 [y_3(k+1) - y_3(k)]}{\tau}, \\ v_4(k) &= -\rho_4 s_4(k) - \varepsilon_4 \operatorname{sgn}(s_4(k)). \end{aligned} \quad (34)$$

The control variables shown as in (35) can be deduced by (15), (25), and (34):

$$\begin{aligned} u_3(k) &= -R_i Q_{i3} + \omega_i L_i Q_{i4} - \frac{L_i a_3}{a_1} \left(a_3 + \frac{2}{CR_{dc}} \right) Q_{i5} \\ &\quad - \frac{L_i}{a_1} v_3 + \frac{L_i a_2}{a_1} v_4 + \frac{L_i a_3 U_{dc1}}{a_1 CR_{dc}} + U_{sid}, \\ u_4(k) &= -\omega_i L_i Q_{i3} - R_i Q_{i4} - L_i v_4 + U_{siq}, \end{aligned} \quad (35)$$

where Q_{i3} , Q_{i4} , and Q_{i5} are the first, second, and third row of $\mathbf{x}_{3,4,5}(\mathbf{k})$ in Section 3.1. $u_3(k)$ and $u_4(k)$ can be expressed by $\mathbf{y}_{3,4}(\mathbf{k})$ through FOS.

4. Simulation Results

The typical VSC-HVDC system composed of two converter stations is taken as example and the detailed parameters are shown in Table 1. The simulation experiment is performed in MATLAB/SIMULINK. In the per-unit value system, the based power is 200 MW, the based voltage at AC side is 81.65 KV, and the based voltage at DC side is 100 KV. The sampling period $\tau = 74 \mu\text{s}$. Also $N_1 = 1$ and $N_2 = 2$. The discrete SMC controllers parameters are $\rho_1 = \rho_2 = 1100$, $\varepsilon_1 = \varepsilon_2 = 3$, $\rho_3 = \rho_4 = 1000$, $\varepsilon_3 = \varepsilon_4 = 3$, and $c_3 = 1200$.

4.1. The Steady State Operation. In steady state operation, $P_{r\text{ref}}$, $U_{dc\text{ref}}$, $Q_{i\text{ref}}$, and $Q_{i\text{ref}}$ are, respectively, 1 pu, 0 pu, 1 pu, and 0 pu. As shown in Figure 4, the active, reactive power and DC voltage can track their reference values effectively. The proposed mathematical model and control strategy can be proved to make the system operate well in steady state condition.

FIGURE 4: Responses of the rectifier and inverter in steady state operation.

TABLE 1: The system parameters of three-level VSC-HVDC.

Parameters	Value
AC voltages U_{sr}, U_{si}	230 kv
ω_r, ω_i	$2 * \pi * 50$ rad/s
Transformer ratio T_1, T_2	230 kv/100 kv
R_r, R_i	0.325 Ω
L_r, L_i	0.072 H
Rated voltage in DC side	± 100 kv
C	70 μ F
Unit resistance of DC line	$1.39e - 002$ Ω /km
DC line length	75 * 2 km
Switching frequency	1350 HZ

4.2. *Reversion and Step Changes.* As shown in Figure 5, the change process of P_{rref} is as follows: the P_{rref} keeps 1 pu from

0 s to 1.5 s. At 1.5 s, it steps to -1 pu, and then keeps this value until 3.0 s. At last, it steps to 1 pu at 3.0 s. From the experimental results, the referred changes least affect the reactive power on the rectifier side and reactive power on the inverter side. As shown in Figure 6, the change process of Q_{rref} is as follows: the Q_{rref} keeps 0 pu from 0 s to 1.5 s. And then steps to 0.1 pu at 1.5 s. The change process of Q_{iref} is as follows: the Q_{iref} keeps 0 pu from 0 s to 2.5 s. And then steps to -0.1 pu at 2.5 s. From the experimental results, the referred changes least affect the active power on the rectifier side and the active power on the inverter side. These can prove that the proposed discrete SMC strategy can make the active and reactive power decoupled and independent.

4.3. *Robustness Test.* The equivalent resistance and inductance at the converter stations both reduce 20%. The experimental results are shown in Figure 7. The reference values

(a) The active and reactive power at rectifier side

(b) The active and reactive power at inverter side

FIGURE 5: Responses of the rectifier and inverter when active power reverses.

(a) The active and reactive power at rectifier side

(b) The active and reactive power at inverter side

FIGURE 6: Responses of the rectifier and inverter when the reactive power step changes.

are the same as those in Section 4.1. The active, reactive power and DC voltage can track their reference values smoothly and quickly. This can prove that the proposed control strategy has good robustness.

5. Conclusions

This paper deduces the discrete mathematical model of VSC-HVDC system by nonlinear input-output feedback linearization method. Based on this, the discrete sliding mode robust controllers are designed. And to ensure the effectiveness of the controllers in quasi sliding mode condition, the FOS

technology is used in output feedback. The results from simulation experiment in MATLAB/SIMULINK prove the effectiveness of proposed discrete mathematical model and control strategy. Since the actual computer control system is discrete sampling and SMC has a promising prospect, the proposed discrete mathematical model and control strategy have certain practical application prospect.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

FIGURE 7: Responses of the rectifier and inverter when the inner parameters change at both converter stations.

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