Research Article

$H_\infty$ Excitation Control Design for Stochastic Power Systems with Input Delay Based on Nonlinear Hamiltonian System Theory

Weiwei Sun, 1,2 Lianghong Peng, 3 Ying Zhang, 4 and Huaidan Jia 1

1 Institute of Automation, Qufu Normal University, Qufu 273165, China
2 School of Engineering, Qufu Normal University, Rizhao 276826, China
3 School of Automation, Southeast University, Nanjing 210096, China
4 Basic Teaching Department, Shandong Water Polytechnic, Rizhao 276826, China

Correspondence should be addressed to Weiwei Sun; wwsun@hotmail.com

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This paper presents $H_\infty$ excitation control design problem for power systems with input time delay and disturbances by using nonlinear Hamiltonian system theory. The impact of time delays introduced by remote signal transmission and processing in wide-area measurement system (WAMS) is well considered. Meanwhile, the systems under investigation are disturbed by random fluctuation. First, under prefeedback technique, the power systems are described as a nonlinear Hamiltonian system. Then the $H_\infty$ excitation controller of generators connected to distant power systems with time delay and stochasticity is designed. Based on Lyapunov functional method, some sufficient conditions are proposed to guarantee the rationality and validity of the proposed control law. The closed-loop systems under the control law are asymptotically stable in mean square independent of the time delay. And we through a simulation of a two-machine power system prove the effectiveness of the results proposed in this paper.

1. Introduction

Time delay always exists in power systems control area. It is often ignored when controller is mainly applied in local systems where the communication time delay is very small compared to the system time constants (see, e.g., [1, 2] and the references therein). Due to the further study of phase measurement unit (PMU) and WAMS, coordinated stability control has got a lot of attention. It uses remote measuring information given by WAMS/PMU. Unlike the small delay in local control, the time delay in wide-area power systems can vary from tens to several hundred milliseconds or more. Since that the large time delay will go against the stability of the system and reduce the performance of the system, so it is very necessary to consider the influence of it on the power system stability analysis and controller design. Besides, the generators are interfered with speed regulation, fluctuation of load, mechanical torsional vibration, the changes of damping coefficients, and so on in the transient process. These random fluctuations can be regarded as a kind of random process [3]. However, the application of the Itô differential formula will lead to the appearance of gravitation and the Hessian term. What is more, the stochastic disturbance (Wiener process) will cause no definition of the system states’ derivative [4]. Therefore, stochastic and delay factors increase the difficulties of the analysis and synthesis [5]. Some results, which took signal transmission time delays or stochasticity in power systems into account, have been obtained. Reference [6] presented a free-weighting matrix method based on linear control design approach for the wide-area robust damping controller associated with flexible alternating current transmission system device to improve the dynamical performance of the large-scale power systems. Reference [7] proposed a delay-independent decentralized coordinated robust approach to design excitation controller in terms of $H_\infty$ optimization method incorporating linear matrix inequality (LMI) technique. Considering the nonlinear effects of randomized torsional oscillation on the excitation regulation dynamic process of a generator rotor and exploiting Monte-Carlo principle and numerical methods,
the algorithms and workflow of the proposed excitation control system's transient stability analysis approach were presented in [3]. Reference [8] presented a stochastic cost model and a solution technique for optimal scheduling of the generators in a wind integrated power system considering the demand and wind generation uncertainties.

Based on the linearization at steady state operating point, lots of the techniques are by far achieved and applied to controller design in power systems. These techniques have some disadvantages, such as ignoring some nonlinearities of the system and just expressing the partial structures of the system. What is more, the designed controllers are generally relatively complicated and not very easy to realize online operation. Therefore, some nonlinear methods should be worked out to achieve good control performance for the power systems in consideration of time-delay, stochastic, and disturbances. In recent years, energy-based Lyapunov function method has obtained numerous attention, and remarkable achievements have been reached with this method in the analysis and synthesis of nonlinear systems, as well as in the power systems (see, e.g., [9–13] and the references therein). The method can thoroughly take advantage of the internal structural properties of the systems and make the control design relatively simple. An important step in using energy-based control strategy is to transform the system into a dissipative Hamiltonian system formulation. This kind of system, proposed by [14], has great benefits for that its Hamilton function can be used as the sum of its Hamilton function can be used as the sum of semidefinite (resp., positive definite), \( \text{tr}[X] \) denotes the trace for square matrix \( X \). \( \lambda_{\max}(P) \) (resp., \( \lambda_{\min}(P) \)) denotes the maximum (minimum) of eigenvalue of a real symmetric matrix \( P \).

2. Problem Formulation and Nonlinear Hamilton Realization

Consider the following \( n \)-machine power systems, each generator of which is described by a third-order dynamic model (see [1, 20]):

\[
\dot{\delta}_i = \omega_i - \omega_0, \\
\dot{\omega}_i = \frac{\omega_0}{M_i} P_{mi} - \frac{D_i}{M_i} (\omega_i - \omega_0) - \frac{\omega_0}{M_i} P_{ei} + e_{i1},
\]

\[
E_{qi}' = -\frac{1}{T d_{ti}} E_{qi} + \frac{1}{T d_{ti}} u_{fi}(t) + e_{i2},
\]

where

\[
E_{qi} = E_{qi}' + I_{di}(x_{di} - x_{di}'),
\]

\[
I_{di} = B_{ij} E_{qi}' - \sum_{j=k,j\neq i}^n B_{ij} E_{qj}' \cos(\delta_i - \delta_j),
\]

\[
P_{ei} = G_{ei} E_{qi}^2 + E_{qi}' \sum_{j=1, j\neq i}^n B_{ij} E_{qj}' \sin(\delta_i - \delta_j),
\]

\( i = 1, 2, \ldots, n, \)

\( \delta_i \) is the power angle of the \( i \)-th generator (radians), \( \omega_i \) is the rotor speed of the \( i \)-th generator (rad/s), \( \omega_0 = 2\pi f_0 \). \( E_{qi}' \) is the \( q \)-axis internal transient voltage of the \( i \)-th generator (per unit), \( x_{di} \) is the \( d \)-axis transient reactance (per unit), \( x_{di}' \) is the \( d \)-axis transient reactance of the \( i \)-th generator (per unit),
$u_{fi}$ is the voltage of the field circuit of the $i$th generator, the control input (per unit), $M_i$ is the inertia coefficient of the $i$th generator $(s)$, $D_i$ is the damping constant (per unit), $T_{d0i}$ is the field circuit time constant (per unit), $P_{mi}$ is the mechanical power, assumed to be constant (per unit), $P_{oi}$ is the active electrical power (per unit), $e_1$ and $e_2$ are bounded disturbances, $I_{gi}$ is the $d$-axis current (per unit), $E_{qi}$ is the internal voltage (per unit), $V_{gi} = G_{gi} + jB_{gi}$ is the admittance of line $i$-$j$ (per unit), and $Y_{ii} = G_{ii} + jB_{ii}$ is the self-admittance of bus $i$ (per unit).

There are signal transmission delays and random process in the modern power systems. The delays in the measuring data exist in such a case that the exciter inputs are taken from remote buses. And assume that all the feedback wide-area signals have the time delay $\tau$. Meanwhile, the generator torque can be regarded as a kind of random process because of random fluctuation in transient process, such as speed regulation, fluctuation of load, mechanical torsional vibration, and the changes of damping coefficients. Moreover, considering the imaginary control input is $u_{fi}$ which feeds back both the local measurement information and the wide-area measurement signals, so the power system (1) should be modeled into differential-algebraic equations with time delay and stochasticity as follows:

$$
\begin{align*}
    d\delta_i &= (\omega_i - \omega_0)\,dt, \\
    d\omega_i &= \left[ \frac{\omega_i}{M_i} P_{mi} - \frac{D_i}{M_i} (\omega_i - \omega_0) - \frac{\omega_i}{M_i} P_{oi} + e_1 \right] dt \\
    &\quad+ \frac{\xi}{M_i} (\omega_i - \omega_0)\,dw(t), \\
    dE_{qij}' &= \left[ -\frac{1}{T_{d0i}} E_{qij}' + \frac{1}{T_{d0i}} u_{fi}(t - \tau) + e_2 \right] dt,
\end{align*}
$$

where $\xi$ is random disturbance intensity and $w(t)$ is a zero-mean Wiener process on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ relative to an increasing family $(\mathcal{F}_t)_{t \geq 0}$ of $\sigma$ algebras $(\mathcal{F}_t)_{t \geq 0} \subset \mathcal{F}$; here $\Omega$ is the samples space, $\mathcal{F}$ is a $\sigma$ algebra of subsets of the sample space, and $P$ is the probability measure on $\mathcal{F}$. Moreover, we assume $E[dw(t)] = 0$, $E[|dw(t)|^2] = dt$, where $E$ is the expectation operator.

Assume that $(\delta_{i0}, \omega_0, E_{qij}'(0), i = 1, 2, \ldots, n)$ are the preassigned operating points of system (3).

Setting $x_{i1} = \delta_i$, $x_{i2} = \omega_i - \omega_0$, $x_{i3} = E_{qij}'$, $a_i = (\omega_0/M_i) T_{d0i} = e_i$, $b_i = (D_i/M_i) G_{gi}$, $c_i = e_i$, $w_0 = 1/T_{d0i} = e_i$, $(x_{i1} - x_{i1}'(0))/T_{d0i} = h_i$, and $(1/T_{d0i}) u_{fi}(t - \tau) = v_i(t - \tau)$, then system (3) can be rewritten as follows:

$$
\begin{align*}
    dx_{i1} &= x_{i2} dt, \\
    dx_{i2} &= \left[ a_i + b_i x_{i2} - c_i x_{i3}^2 + e_1 \right] dt \\
    &\quad- d_i x_{i3} \sum_{j=1, j \neq i}^n B_{ij} x_{j3} \sin (x_{i1} - x_{j1}) dt + \frac{\xi}{M_i} x_{i3} dw(t), \\
    dx_{i3} &= \left[ -(e_i + h_i B_{ij}) x_{i3} + v_i(t - \tau) + e_2 \right] dt + h_i \sum_{j=1, j \neq i}^n B_{ij} x_{j3} \cos (x_{i1} - x_{j1}) dt, \\
    i &= 1, 2, \ldots, n.
\end{align*}
$$

(4)

Inspired by [11], we introduce a prefeedback control law:

$$
\begin{align*}
    v_i(t - \tau) &= -\frac{2c_i h_i}{d_i} x_{i3}(t - \tau) x_{i3}(t - \tau) - k_i x_{i3}(t - \tau) \\
    &\quad+ \bar{u}_i + u_i(t - \tau), \quad i = 1, 2, \ldots, n,
\end{align*}
$$

where the first term is to make system (4) have a Hamilton structure, the second and third terms are to guarantee the operating point of the system unchanged, $u_i(t - \tau)$ is the new reference input, and $\bar{u}_i$ and $k_i$ are undetermined constants. To make the operating point of the system invariant, $\bar{u}_i$ and $k_i$ have to satisfy

$$
\begin{align*}
    - (e_i + h_i B_{ij}) E_{qij}'(0) &= -\frac{2c_i h_i}{d_i} \delta_{ij}'(0) E_{qij}'(0) - k_i E_{qij}'(0) + \bar{u}_i, \\
    i &= 1, 2, \ldots, n,
\end{align*}
$$

and $k_i = k_{i0}$ which is spelled out in [11]; what is more, this reference provides a kind of choice of $\bar{u}_i$ and $k_i$.

Furthermore, (5) can be rewritten as

$$
\begin{align*}
    v_i(t - \tau) &= -\frac{2c_i h_i}{d_i} x_{i1}(t) x_{i3}(t) - k_i x_{i3}(t - \tau) + \bar{u}_i \\
    &\quad+ \frac{2c_i h_i}{d_i} \left[ x_{i3}(t - \tau) x_{i3}(t - \tau) - x_{i1}(t) x_{i3}(t) \right] \\
    &\quad- k_i \left[ x_{i3}(t - \tau) - x_{i3}(t) \right] + u_i(t - \tau), \\
    i &= 1, 2, \ldots, n.
\end{align*}
$$

Let $x_i = [x_{i1}, x_{i2}, x_{i3}]^T$, $e_i = [e_{i1}, e_{i2}]^T$, then system (4) can be expressed as a dissipative Hamiltonian system as follows:

$$
\begin{align*}
    dx_i &= \left( (I - R_i) VH_i(x_i) + g_i u_i(t - \tau) + \frac{2c_i h_i}{d_i} g_i^T x_{i3}(t) g_i x_{i3}(t) \right) dt \\
    &\quad+ \frac{1}{h_i} \left[ g_i^T x_{i3}(t) g_i^T x_{i3}(t) \right] dt \\
    &\quad- g_i^T x_{i3}(t - \tau) g_i x_{i3}(t - \tau) + k_i g_i^T x_{i3}(t - \tau) + g_i^T x_{i3}(t) \right] dt + \frac{1}{h_i} g_i h_i^T (x_i) dw(t), \\
    i &= 1, 2, \ldots, n.
\end{align*}
$$

(8)
where

\[
H_i(x_i) = -\frac{a_i}{d_i}x_{i1} + \frac{c_i}{d_i}x_{i1}x_{i2} + \frac{e_i + h_iB_i}{2h_i}x_{i2}^2 \\
+ \frac{1}{2d_i}x_{i2}^2 - \frac{\pi_i}{h_i}x_{i3} \\
- x_{i3} \sum_{j=1, j\neq i}^n B_{ji}x_{j3} \cos (x_{i1} - x_{j1}),
\]

which imply that \(H(x)\) is the common Hamilton function for the \(n\) generators. Furthermore, \(H(x) \in C^2\) holds obviously.

Setting

\[
\begin{align*}
\bar{u} &= [u_1, \ldots, u_n]^T, \\
\bar{e} &= [e_1^T, \ldots, e_n^T]^T, \\
\bar{y} &= [y_1^T, \ldots, y_n^T]^T,
\end{align*}
\]

then system (8) can be rewritten as follows:

\[
\begin{align*}
\dot{x}(t) &= \left( (J - R) \nabla H(x) + G_1\bar{u}(t - \tau) \\
&+ 2G_1C \left[ G_1^T x(t) G_1 x(t) - G_1^T x(t - \tau) G_1 x(t - \tau) \right] \\
&+ G_1KG_1^T [x(t) - x(t - \tau)] + G_2\bar{e} \right) dt + G_4(x)dw(t),
\end{align*}
\]

where \(J = \text{Diag}(J_1, \ldots, J_n), R = \text{Diag}(R_1, \ldots, R_n), C = \text{Diag}(c_1h_1/d_1, \ldots, c_nh_n/d_n), K = \text{Diag}(k_1, \ldots, k_n),\)

\[
\begin{align*}
G_1 &= \begin{pmatrix} g_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & g_n \end{pmatrix}_{3n \times n}, \\
G_2 &= \begin{pmatrix} g_2 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & g_n \end{pmatrix}_{3n \times 2n}, \\
G_3 &= \begin{pmatrix} g_3 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & g_n \end{pmatrix}_{3n \times n}, \\
G_4(x) &= \begin{pmatrix} g_4^{(1)}(x_1) & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & g_4^{(n)}(x_n) \end{pmatrix}_{3n \times n}.
\end{align*}
\]

Obviously, \(J\) is a skew-symmetric matrix, and \(R\) is a positive semidefinite matrix. In addition, we can choose \(y = G_2^T \nabla H(x)\) and \(z = PG_1^{T} \nabla H(x)\) as the output and the penalty signal, respectively, where \(P\) is a full column rank weighting matrix.

**Definition 1.** The stochastic time delay Hamiltonian system (13) is said to be robustly asymptotically stable in mean square, if there exists a controller \(u(t - \tau)\) such that

\[
\lim_{t \to \infty} \mathbb{E} \left[ \left\| x(t) - x_0 \right\|^2 \right] = 0,
\]

where \(x_0\) is the preassigned equilibrium and \(x(t)\) is the solution of system (13) at time \(t\) under initial condition.
Consider the following cost function:
\[
C(T_0) = E \left\{ \int_0^{T_0} z^T(t) z(t) dt \right\} - \gamma^2 E \left\{ \int_0^{T_0} e^T(t) e(t) dt \right\}, \quad \forall T_0 > 0.
\]
(16)

Then $H_\infty$ control objective of system (13) is to find a feedback controller:
\[
u(t - \tau) = \alpha (t - \tau)
\]
(17)
such that
\[
C(\infty) < 0 \quad (T_0 \rightarrow \infty),
\]
(18)
for given $\gamma > 0$ and at the same time the closed-loop system is asymptotically stable when $\epsilon = 0$.

We conclude this section by recalling some auxiliary results to be used in this paper.

Lemma 2 (see [21]). For system
\[
dx(t) = f(x(t), x(t - \tau)) dt + g(x(t), x(t - \tau)) dw(t), \quad \forall t \geq 0,
\]
(19)
assume that $f(x, y)$ and $g(x, y)$ are locally Lipschitz in $(x, y)$. If there exists a function $V(x, t) \in C^2(\mathbb{R}^n \times [-\tau, \infty); \mathbb{R}_+)$ such that for some constant $K > 0$ and any $t \geq 0$,
\[
\mathcal{L} V \leq K (1 + V(x(t), t) + V(x(\tau), \tau - \tau)),
\]
(20)
where the differential operator $\mathcal{L}$ is defined as
\[
\mathcal{L} = \frac{\partial V}{\partial t} + \nabla V(x(t), x(t - \tau)) g(x(t), x(t - \tau)) + \frac{1}{2} \text{tr} \left\{ g^T(x(t), x(t - \tau)) \frac{\partial^2 V}{\partial x^2} g(x(t), x(t - \tau)) \right\},
\]
(21)
then there exists a unique solution on $[-\tau, \infty)$ for any initial data $x(t) = \phi(t) : t \in [-\tau, 0] \in C^2_{\infty}([-\tau, 0]; \mathbb{R}^n)$.

Lemma 3. For any given matrices $A \in \mathbb{R}^{n \times r}$ and $B \in \mathbb{R}^{n \times s}$, there holds
\[
\text{tr} \left( A^T B \right) \leq \frac{1}{2} \left[ \text{tr} \left( A^T A \right) + \text{tr} \left( B^T B \right) \right].
\]
(22)

Proof. This proof can be achieved by using the properties of matrix's trace.

\[\square\]

3. Main Results

3.1. Hamiltonian System. The $H_\infty$ controller is given below for the stochastic Hamiltonian system (13) with input delay.

Theorem 4. Consider system (13) and the following assumptions are satisfied:

(A1) $V H(x_0) = 0$;

(A2) $\text{Hess}(H(x_0)) > 0$;

(A3) $H(x) - H(x_0) \geq (\alpha_1/2) \|x - x_0\|^2$;

(A4) $\nabla^T H(x) \cdot \nabla H(x) \geq \beta_1 \|x - x_0\|^2$.

If
\[
\begin{align*}
2R + \frac{1}{\gamma^2} G_1 G_1^T &= \frac{1}{\gamma^2} G_2 G_2^T \geq 0
\end{align*}
\]
(23)
holds, then the $H_\infty$ control problem of system (13) can be solved by the feedback control law:
\[
u(t - \tau) = -\frac{1}{2} \left( \frac{1}{\gamma^2} G_1^T + P^T P G_1^T \right) \nabla H(x(t - \tau))
\]
\[
- \frac{1}{4} (G_1^T G_1)^{-1} G_1^T G_5 \nabla H(x(t - \tau))
\]
\[
+ 2C X(t - \tau) + K G_1^T x(t - \tau) - M - 2N
\]
\[
- \frac{1}{2} \tau \lambda_1 \lambda_2 T - \frac{1}{4} \tau \lambda_1 \lambda_2 (G_1^T G_1)^{-1} G_1^T G_5,
\]
(24)
where $x_0$ is the preassigned equilibrium of system (13), $G_5 = \text{Diag}(g_3^{(1)}, \ldots, g_3^{(n)})$,
\[
g_5 = \begin{pmatrix}
0 & 0 & 0 \\
(\sigma_1^2 + 1) \epsilon^2 & M_1^2 & 0 \\
0 & 0 & 0
\end{pmatrix}_{3 \times 3}
\]
(25)
\[
X(t) = \begin{pmatrix}
x_{11}(t) x_{13}(t) \\
x_{21}(t) x_{23}(t) \\
\vdots \\
x_{n1}(t) x_{n3}(t)
\end{pmatrix}_{n \times 1}
\]
\[
M, N, T \text{ are all positive constant matrices which satisfy}
\]
\[
\|M\| \geq \|KG_1^T x(t)\|, \|N\| \geq \|CX(t)\|, \|T\| \geq \|(1/\gamma^2)G_1^T + P^T P G_1^T\|,
\]
\[
\text{and } \lambda_1 \text{ and } \lambda_2 \text{ are constants which satisfy } \lambda_1 \geq \sup_{t \geq -\tau} \|\text{Hess}(H(x(t)))\|, \lambda_2 \geq \sup_{t \geq -\tau} \|\dot{x}(t)\|.
\]

Proof. Take a Lyapunov candidate function as follows:
\[
V(x) = 2H(x) - 2H(x_0).
\]
(26)

According to Itô differential formula, it follows that
\[
dV(x) = \mathcal{L} V(x) dt + \nabla V(x) G_4(x) dw(t).
\]
(27)
According to (21) in Lemma 2, one has

$$\mathcal{L} V(x) = \frac{1}{2} \text{tr} \left\{ g^T (x(t), x(t - \tau)) \frac{\partial^2 V}{\partial x^2} g(x(t), x(t - \tau)) \right\} + \frac{\partial V}{\partial t} f(x(t), x(t - \tau))$$

$$= \text{tr} \left[ G_4^T (x) \text{Hess} (H (x)) G_4 (x) \right] + 2 \nabla^T H (x) (J - R) \nabla H (x)$$

$$+ 2 \nabla^T H (x) G_1 KG_1^T [x(t) - x(t - \tau)] + 4 \nabla^T H (x) G_1 C [X(t) - X(t - \tau)]$$

$$+ 2 \nabla^T H (x) G_2 e$$

$$- \nabla^T H (x) G_1 \left( \frac{1}{y^2} G_1^T + P^T P G_1^T \right) \nabla H (t - \tau)$$

$$- \frac{1}{2} \nabla^T H (x) G_1 (G_1^T G_4)^{-1} G_1^T G_5 \nabla H (t - \tau) + 4 \nabla^T H (x) G_1 C X (t - \tau)$$

$$+ 2 \nabla^T H (x) G_1 C X (t - \tau)$$

$$- 2 \nabla^T H (x) G_1 (M + 2N) - \tau \lambda_1 \lambda_2 \nabla^T H (x) G_1 T$$

$$- \frac{1}{2} \tau \lambda_1 \lambda_2 \nabla^T H (x) G_1 (G_1^T G_4)^{-1} G_1^T G_5.$$

Based on the facts of Lemma 3 and Condition (22), we can achieve

$$\text{tr} \left[ G_4^T (x) \text{Hess} (H (x)) G_4 (x) \right]$$

$$\leq \frac{1}{2} \text{tr} \left[ G_4^T (x) \text{Hess} (H (x)) \text{Hess}^T (H (x)) G_4 (x) \right] + \frac{1}{2} \text{tr} \left[ G_4^T (x) G_4 (x) \right] = \nabla^T H (x) G_2 \nabla H (x).$$

According to Newton-Leibniz formula, it follows that

$$\nabla H (x, t) = \nabla H (x) - \int_{t-\tau}^{t} \text{Hess} (H (x(s))) \dot{x} (s) \, ds.$$

Therefore, the following equalities hold:

$$- \nabla^T H (x) G_1 \left( \frac{1}{y^2} G_1^T + P^T P G_1^T \right) \nabla H (x) (t - \tau)$$

$$= - \nabla^T H (x) G_1 \left( \frac{1}{y^2} G_1^T + P^T P G_1^T \right)$$

$$\cdot \left[ \nabla H (x) - \int_{t-\tau}^{t} \text{Hess} (H (x(s))) \dot{x} (s) \, ds \right].$$

According to the Mean Value Theorem of Integrals, there exists $\theta \in [t - \tau, t]$ that satisfies

$$\int_{t-\tau}^{t} \text{Hess} (H (x(s))) \dot{x} (s) \, ds = \tau \nabla^T H (x) \dot{x} (\theta).$$

Consequently, we have

$$\nabla^T H (x) G_1 \left( \frac{1}{y^2} G_1^T + P^T P G_1^T \right) \cdot \int_{t-\tau}^{t} \text{Hess} (H (x(s))) \dot{x} (s) \, ds = \frac{\tau}{2} \nabla^T H (x) \dot{x} (\theta).$$

Similarly, we further obtain

$$\frac{1}{2} \nabla^T H (x) G_1 (G_1^T G_4)^{-1} G_1^T G_5 \nabla H (x)$$

$$\cdot \tau \lambda_1 \lambda_2 \nabla^T H (x) G_1 (G_1^T G_4)^{-1} G_1^T G_5,$$

Combining the above inequalities, we can conclude that

$$\mathcal{L} V(x) \leq -2 \nabla^T H (x) R \nabla H (x) + 2 \nabla^T H (x) G_2 e$$

$$- \nabla^T H (x) G_1 \left( \frac{1}{y^2} G_1^T + P^T P G_1^T \right) \nabla H (x)$$

$$= -2 \nabla^T H (x) R \nabla H (x) - \left[ ye - \frac{1}{y^2} G_4^T \nabla H (x) \right]$$

$$- \frac{1}{y^2} \nabla^T H (x) G_1 G_1^T \nabla H (x) + \left( y^2 e^T e - z^T z \right)$$

$$+ \frac{1}{y^2} \nabla^T H (x) G_2 G_2^T \nabla H (x)$$

$$\leq -2 \nabla^T H (x) \left( 2R + \frac{1}{y^2} G_1^T G_1^T - \frac{1}{y^2} G_2 G_2^T \right) \nabla H (x)$$

$$+ \left( y^2 e^T e - z^T z \right).$$
Taking (23) into account, it yields
\[
\mathcal{L} V(x) \leq \gamma^2 e^r - z^T z.
\] (36)
Integrating (36) from 0 to \( T \) leads to (18) which holds as \( T_0 \to \infty \).
Next step we prove the closed-loop system where system (13) under the control law (24) is asymptotically stable in mean square when \( \varepsilon = 0 \).
When \( \varepsilon = 0 \), from (35), we can easily get that
\[
\mathcal{L} V(x) \leq -\gamma^2 e^r + \frac{1}{\varepsilon^2} - z^T z.
\]
Integrating this inequality from 0 to \( T \), we have
\[
\mathcal{L} V(x) \leq -\gamma^2 e^r + \frac{1}{\varepsilon^2} - z^T z.
\]
Set \( \varepsilon = 0 \), then we have
\[
\mathcal{L} V(x) \leq -\gamma^2 e^r + \frac{1}{\varepsilon^2} - z^T z.
\]
Furthermore, owing to (A4) holding, there is
\[
\mathcal{L} V(x) \leq -\gamma^2 e^r + \frac{1}{\varepsilon^2} - z^T z.
\]
which implies
\[
E \{ \mathcal{L} V(x) \} \leq -\gamma^2 e^r + \frac{1}{\varepsilon^2} - z^T z.
\]
From condition (A3), one has
\[
\alpha_1 \| x - x_0 \|^2 \leq V(x) = 2(H(X) - H(x_0)),
\]
\[
E \{ \alpha_1 \| x(T) - x_0 \|^2 \} \leq E \{ V(T) \},
\] (45)
\[
\frac{d}{dt} E \{ \alpha_1 \| x(T) - x_0 \|^2 \} \leq \frac{d}{dt} E \{ V(T) \}
\]
\[
\leq c_0 \beta_1 E \{ \| x(T) - x_0 \|^2 \}.
\]
Set \( c_1 = -c_0 \beta_1 / \alpha_1 \); it follows that
\[
\frac{d}{dt} E \{ \| x(T) - x_0 \|^2 \} \leq c_1 E \{ \| x(T) - x_0 \|^2 \}.
\] (46)
Multiplying \( e^{-\gamma T} \) to the two sides of inequality (44) yields
\[
e^{-\gamma T} \frac{d}{dt} E \{ \| x(T) - x_0 \|^2 \} - e^{-\gamma T} c_1 E \{ \| x(T) - x_0 \|^2 \}
\] \leq 0
which implies that
\[
\frac{d}{dt} \left( e^{-\gamma T} E \{ \| x(T) - x_0 \|^2 \} \right) \leq 0.
\] (48)
Integrating inequality (48) from \( t_0 \) to \( T \), we have
\[
e^{-\gamma T} E \{ \| x(T) - x_0 \|^2 \} - e^{-\gamma t_0} c_1 E \{ \| x(t_0) - x_0 \|^2 \} \leq 0;
\] (49)
that is,
\[
E \{ \| x(T) - x_0 \|^2 \} \leq e^{\gamma (T-t_0)} E \{ \| x(t_0) - x_0 \|^2 \},
\] \quad \forall T > t_0. (50)
Due to \( c_1 < 0 \), there is
\[
\lim_{T \to \infty} E \{ \| x(T) - x_0 \|^2 \} = 0.
\] (51)
According to Definition 1, we can conclude that system (13) under the control law (24) is robustly asymptotically stable in mean square with respect to \( x_0 \). This completes the proof.

\textbf{Remark 5.} \( \nabla H(x_0) = 0 \) and \( \text{Hess}(H(x_0)) > 0 \) guarantee that the equilibrium \( x_0 \) is the minimal point of \( H(x) \). Moreover, in view of conditions (A1)–(A4), there hold \( \nabla V(x_0) = 0 \) and \( \text{Hess}(V(x_0)) > 0 \), which together with \( V(x_0) = 0 \) lead to the fact that \( V(x) \) is a positive definite function in some neighborhood of equilibrium \( x_0 \).

\textbf{Remark 6.} Owing to the fact of \( H(x) \in C^2 \), the solution of the closed-loop system (13) under the control law (24) is existent and unique on \([-\tau, \infty)\) for any initial data \( x(t) = \phi(t) : t \in [-\tau, 0] \in C_0^b([-\tau, 0]; \mathbb{R}^n) \) in some neighborhood of equilibrium \( x_0 \).
3.2. $N$-Machine Power System. In this subsection, we consider the $n$-machine power system (3).

First, we can verify that

$$H(x) = \sum_{i=1}^{n} \left[ -P_{mi} \delta_i + G_{ii} \delta_i E_{qi}^2 + \frac{M_i}{2 \omega_i} (\omega_i - \omega_0)^2 
+ \frac{1}{2} \left( \frac{\mu_i T_{d0i}}{x_{di} - x_{di}'} \right) E_{qi}^2 - \frac{\mu_i T_{d0i}}{x_{di} - x_{di}'} E_{qi}' \right] (52)$$

$$- \frac{1}{2} E_{qi}' \sum_{j=1,j\neq i}^{n} B_{ij} E_{qi}^0 \cos (\delta_i - \delta_j) \in C^2.$$ 

Choose the preassigned equilibrium

$$x_0 = (\delta_i^{(0)}, \omega_0, E_{qi}^{(0)}), \quad i = 1, 2, \ldots, n$$

satisfying

$$\text{Hess} \left( H(x_0) \right) = \text{Hess} \left\{ \sum_{i=1}^{n} \left[ -P_{mi} \delta_i^{(0)} + G_{ii} \delta_i^{(0)} E_{qi}^{(0)^2} + \frac{1}{2} \left( \frac{\mu_i T_{d0i}}{x_{di} - x_{di}'} \right) E_{qi}^{(0)^2} \right. 
- \frac{1}{2} E_{qi}^{(0)^2} \sum_{j=1,j\neq i}^{n} B_{ij} E_{qi}^{(0)} \cos \left( \delta_i^{(0)} - \delta_j^{(0)} \right) \left. \right] \right\} > 0$$

and $\nabla^T H(x) = 0$; that is

$$P_{mi} + G_{ii} E_{qi}^{(0)} + E_{qi}^{(0)} \sum_{j=1,j\neq i}^{n} B_{ij} E_{qi}^{(0)} \sin \left( \delta_i^{(0)} - \delta_j^{(0)} \right) = 0,$$

$$1 + \left( \frac{\mu_i T_{d0i}}{x_{di} - x_{di}'} \right) E_{qi}^{(0)} - \frac{\mu_i T_{d0i}}{x_{di} - x_{di}'} E_{qi}'$$

$$- \sum_{j=1,j\neq i}^{n} B_{ij} E_{qi}^{(0)} \cos \left( \delta_i^{(0)} - \delta_j^{(0)} \right)$$

$$+ 2 G_{ii} \delta_i^{(0)} E_{qi}'(0) = 0.$$ 

Meanwhile, we assume that there exist positive constants $\alpha_1, \beta_1$ such that $H(x) - H(x_0) \geq (\alpha_1/2)\|x - x_0\|^2$ and $\nabla^T H(x) \cdot \nabla H(x) \geq \beta_1 \|x - x_0\|^3$ hold.

An $H_{\infty}$ controller for system (3) is given in the following theorem.

**Theorem 7.** Consider power system (3). If

$$u_{fi}(t-\tau) = -2 G_{ii} \left( x_{di} - x_{di}' \right) \delta_i (t-\tau) E_{qi}'(t-\tau)$$

$$- k_i T_{d0i} E_{qi}' (t-\tau) + T_{d0i} \pi_i - \frac{1}{2} T_{d0i} \left[ \frac{1}{y^2} + \beta_i^2 \right]$$

$$+ \left( \omega_0^2 + M_i^2 \right) \epsilon^2$$

$$+ \sum_{j=1,j\neq i}^{n} B_{ij} E_{qi}^0 \cos \left( \delta_i - \delta_j \right)$$

$$+ \left( k_i E_{qi}' (t-\tau) - \pi_i \right) T_{d0i}$$

$$- \tau E_{qi}'(t-\tau) + T_{d0i} k_i E_{qi}' (t-\tau) - (m_i + 2 n_i)$$

$$- \frac{1}{2} \tau \lambda_1 \lambda_2 t_i - \frac{1}{4} \tau \lambda_1 \lambda_2 \frac{\epsilon^2}{M_i^4},$$

$$i = 1, 2, \ldots, n,$$

where $\lambda_1, \lambda_2, m_i, n_i, \text{ and } t_i, \text{ are constants, which satisfy}$

$$\lambda_1 \geq \sup_{t_2 - \tau} \left\| \text{Hess} \left[ \sum_{i=1}^{n} \left( \frac{\mu_i T_{d0i}}{x_{di} - x_{di}'} \right) E_{qi}' - \frac{\mu_i T_{d0i}}{x_{di} - x_{di}'} E_{qi}' \right] \right\|$$

$$+ G_{ii} \delta_i E_{qi}' - \frac{\mu_i T_{d0i}}{x_{di} - x_{di}'} E_{qi}' - P_{mi} \delta_i$$

$$- \sum_{j=1,j\neq i}^{n} B_{ij} E_{qi}^0 \cos \left( \delta_i - \delta_j \right) \right\|,$$

$$\lambda_2 \geq \sup_{t_2 - \tau} \left\| \left( \delta_i (t) \omega_i (t) E_{qi}'(t) \right) \right\|,$$

$$m_i \geq |k_i E_{qi}'|,$$
\[ n_i \geq \left| \frac{2G_{ii} (x_{di} - x_{di}') \delta_i (t - \tau) E_{qi}' (t - \tau)}{T_{dii}} \right|, \]
\[ t_i \geq \left( \frac{1}{T_i^2} + p_i^2 \right), \quad i = 1, 2, \ldots, n. \]  

(58)

\((\delta_i(t), \omega_i(t), E_{qi}'(t)), i = 1, 2, \ldots, n, \) is the solution of the closed-loop system at time \( t \) under initial condition.

**Proof.** Taking
\[
y_i = \left( \begin{array}{cc}
\frac{M_i}{\omega_0} (\omega_i (t) - \omega_0) \\
2G_i \delta_i (t) E_{qi}' (t) + \frac{1 + (x_{di} - x_{di}') B_{ii} E_{qi}' (t)}{x_{di} - x_{di}'},
\end{array} \right)
\]
\[
- \left( \begin{array}{c}
0 \\
\sum_{j=1, j\neq i}^n B_{ij} E_{qj}' (t) \cos (\delta_i (t) - \delta_j (t))
\end{array} \right) + \left( \begin{array}{c}
0 \\
(k_i E_{qi}' (t) - \pi_i) T_{dii},
\end{array} \right),
\]
\[
z_i = p_i \left[ 2G_i \delta_i (t) E_{qi}' (t) - \sum_{j=1, j\neq i}^n B_{ij} E_{qj}' (t) \cos (\delta_i (t) - \delta_j (t)) + \frac{1 + (x_{di} - x_{di}') B_{ii} E_{qi}' (t)}{x_{di} - x_{di}'} \delta_i (t) \right]
\]
\[
+ \left( \begin{array}{c}
0 \\
\sum_{j=1, j\neq i}^n B_{ij} E_{qj}' (t) \cos (\delta_i (t) - \delta_j (t)) \\
+ \frac{1 + (x_{di} - x_{di}') B_{ii} E_{qi}' (t)}{x_{di} - x_{di}'} \delta_i (t) \right)
\]
\[
+ \frac{1 + (x_{di} - x_{di}') B_{ii} E_{qi}' (t)}{x_{di} - x_{di}'} \delta_i (t) 
\]
into consideration, then we can prove the result using the similar method in the proof of Theorem 4, where \( p_i \geq 0, i = 1, 2, \ldots, n \) are the weighting constants.

### 4. Illustrative Examples

To show the effectiveness of the proposed control strategy, we give a two-machine power system as shown in Figure 1. The generators \( G_1, G_2 \) are assumed to be connected to distant power systems and disturbed by random fluctuation. In simulating, a temporary short-circuit fault occurs at point \( K \) (see Figure 1) during the time 0.5 sec–1 sec. The system parameters used in this simulation are given in Table 1. Choose \( \omega_0 = 1, \xi = 1. \)

Choosing the following preassigned operating point
\[
\left[ \delta_i^{(0)}, \omega_i^{(0)}, E_{qi}^{(0)} \right], \quad i = 1, 2, \ldots, n
\]

then \( \overline{\omega}_1 = 0.5, \overline{\omega}_2 = 16/15, \) and \( k_1 = k_2 = -0.1. \)

It is easy to verify that system (60) with the above values satisfies conditions (A1)–(A4) of Theorem 4.

**Table 1:** Generators’ data (all per unit except \( M_i, T_{dii}, i = 1, 2, \ldots, n \) in seconds).

<table>
<thead>
<tr>
<th>( M_i )</th>
<th>( P_{mi} )</th>
<th>( D_1 )</th>
<th>( x_{d1} )</th>
<th>( x_{d1}' )</th>
<th>( T_{dii} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>1</td>
<td>5</td>
<td>1</td>
<td>0.5</td>
<td>5</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>6</td>
<td>1</td>
<td>0.4</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
<td>10</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

**Figure 1:** Two-machine power system.

\[
dE_{q1}' = \left[ \frac{-1}{5} E_{q1} + \frac{1}{5} u_{f1} (t - \tau) + \epsilon_{12} \right] dt,
\]
\[
dE_{q2}' = \left[ \frac{-1}{6} E_{q2} + \frac{1}{6} u_{f2} (t - \tau) + \epsilon_{22} \right] dt,
\]
\[
dE_{q1}' = E_{q1}' + 0.51d_1,
\]
\[
P_{e1}' = E_{q1} + 2 \sum_{j=1, j\neq i}^n E_{qj}' \sin (\delta_i - \delta_j),
\]
\[
I_{d1} = 4E_{q1}' - \sum_{j=1, j\neq i}^n E_{qj}' \cos (\delta_i - \delta_j),
\]
\[
E_{q2}' = E_{q2}' + 0.61d_2,
\]
\[
P_{e2}' = E_{q2} + 2 \sum_{j=1, j\neq i}^n E_{qj}' \sin (\delta_i - \delta_j),
\]
\[
I_{d2} = 10E_{q2}' - \sum_{j=1, j\neq i}^n E_{qj}' \cos (\delta_i - \delta_j).
\]

(60)
This paper studied the $H_\infty$ excitation controller design problem of a class of stochastic power systems with time-delay and stochastic disturbances.

The fault indicates a unit step function; that is, $\epsilon_{t1} = \epsilon_{t2} = \epsilon_{t1} = \epsilon_{t2} = -1(t - 0.5) + 1(t - 1)$. For given $\gamma' = 4$, we can find $p_1 = p_2 = 1$ such that inequality (56) is satisfied.

We will test the effectiveness of the proposed control configuration at two different time delays $\tau = 0.5$ s and $\tau = 0.05$ s. The initial condition is $(\delta_1(0), \omega_1(0), E'_{q1}(0), \delta_2(0), \omega_2(0), E'_{q2}(0)) = [1.2 \ 1 \ 2 \ 1 \ 2 \ 1 \ 2 \ 1 \ 2 \ 1 \ 2]$. Take $\mu = 1/8, \lambda^+ = 40, m'_{11} = m'_{12} = 100, m'_{21} = m'_{22} = 100, \text{ and } m_1 = 40$. According to Theorem 7 proposed in this paper, system (60) is asymptotically stable in mean square for all $\tau \geq 0$ and $\epsilon = 0$ under the feedback control law

$$u_{j1}(t - \tau) = (4096)^{-1} \left[ -29858.5\delta_1(t - \tau) E'_{q1}(t - \tau) ight. $$

$$- 62358.25E'_{q1}(t - \tau) + 12881.25E'_{q2}(t - \tau) $$

$$- \cos(\delta_1(t - \tau) - \delta_2(t - \tau)) - 320200\tau $$

$$- \text{sgn}(5\omega_1 - 5) + 74646.25 $$

$$- 4480\gamma \text{sgn}[5E'_{q1}(t) + 2\delta_1(t - \tau) E'_{q1}(t - \tau) - \delta_2(t - \tau))] - 5], $$

$$u_{j2}(t - \tau) = (4096)^{-1} \left[ -35011\delta_2(t - \tau) E'_{q2}(t - \tau) ight. $$

$$- 162422.4E'_{q2}(t - \tau) + 17915.1E'_{q1}(t - \tau) $$

$$- \cos(\delta_1(t - \tau) - \delta_2(t - \tau)) - 384240\tau $$

$$- \text{sgn}(6\omega_2 - 6) + 191094.4 $$

$$- 5376\tau $$

$$\cdot \text{sgn}\left[\left(\frac{2}{3} + 10\right)E'_{q2}(t) + 2\delta_2(t - \tau) E'_{q2}(t) ight. $$

$$- E'_{q1}(t) \cos(\delta_1(t - \tau) - \delta_2(t - \tau)) - \frac{64}{6} \right].$$

Simulations with the above initial condition and the delay $\tau = 0.5$ s and $\tau = 0.05$ s are given in Figures 2-4 and Figures 5-7, separately. Through Figures 2-7, we can see that the states of the system converge to the equilibrium $(\delta_1(0), \omega_1(0), E'_{q1}(0), \delta_2(0), \omega_2(0), E'_{q2}(0)) = [1.2 \ 1 \ 2 \ 1 \ 2 \ 1 \ 2 \ 1 \ 2 \ 1 \ 2]$. Eventually, obviously, under the delayed feedback controller by using the proposed method, the robustness of the closed-loop system is guaranteed. It is also seen that the controller possesses insensitivity in regard to the types of time delay and stochastic disturbances.

5. Conclusion

This paper studied the $H_\infty$ excitation controller design problem of a class of stochastic power systems with time-delay and...
disturbances. In the design process, we used the prefedback technique, Newton-Leibniz formula, and a few properties of norm. Besides, we obtain these results by nonlinear Hamilton function approach due to the special structural properties of the Hamiltonian systems. We also give a two-machine power system simulation and it shows that the results achieved in this paper are practicable in analyzing the $H_{\infty}$ excitation control problem of stochastic power system in consideration of time-delay and disturbances.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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