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Research Article

Robust Finite-Time H_∞ Control for Linear Time-Varying Descriptor Systems with Jumps

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The finite-time H_∞ control problem is addressed for uncertain time-varying descriptor system with finite jumps and time-varying norm-bounded disturbance. Firstly, a sufficient condition of finite-time boundedness for the abovementioned class of system is obtained. Then the result is extended to finite-time H_∞ for the system. Based on the condition, state feedback controller is designed such that the closed-loop system is finite-time boundedness and satisfies L_2 gain. The conditions are given in terms of differential linear matrix inequalities (DLMIs) and linear matrix inequalities (LMIs), and such conditions require the solution of a feasibility problem involving DLMIs and LMIs, which can be solved by using existing linear algorithms. Finally, a numerical example is given to illustrate the effectiveness of the method.

1. Introduction

In practice a system could be stable but completely useless because it possesses undesirable transient performances. Thus it is useful to consider the stability of such systems only over a finite time. The concept of finite-time stability (FTS) was first introduced in the Russian literature [1–3]. Later during the 1960s this concept appeared in the literature [4, 5]. Until now, there are many valuable results for this type of stability. In [6] the FTS problem for continuous-time linear time-varying system with finite jumps is dealt with and the finite-time analysis and designed problems for continuous-time time-varying linear system are dealt with in [7]. Sufficient conditions for FTS and finite-time stabilization have been provided in the control literature; see [8–10]. The problem of FTS of linear system via impulsive control at fixed times and variable times was considered in [11]. FTS in the presence of exogenous inputs leads to the concept of finite-time boundedness (FTB). In other words a system is said to be FTB if, given a bound on the initial condition and a characterization of the set of admissible inputs, the state variables remain below the prescribed limit for all inputs in the set. Necessary and sufficient conditions for FTS and FTB are presented, and both the state feedback and the output feedback problems are considered in [12].

On the other hand, in the past three decades, descriptor system theory has been well studied since it often better describe physical systems than regular ones. In time-varying cases, many problems based on descriptor system have been extensively studied and many interesting results have been extended. The controllability, observability, impulsive controllability, and impulsive observability problem of time-varying singular system has been discussed in [13, 14]. The finite-time stability (FTS) problems of time-varying linear singular system have been studied [15–17]. The strict LMI solving problem for descriptor system has been discussed in [18–20].

The system we consider in this paper is a class of uncertain linear time-varying descriptor system with finite state jumps and time-varying norm-bounded disturbance. This class of system exists in the real world which displays a certain kind of dynamics with impulse effect at time instant, that is, the state jumps, which cannot be described by pure continuous or pure discrete models. Recently, some results on time-varying descriptor system with jumps have been reported. Stability, robust stabilization, and H_∞ control of singular impulsive system were studied in [21]. The problem of stability and stabilization of singular Markovian jump system with external discontinuities and saturating inputs were addressed in [22]. Paper [23] formulated and studied a model for singular

impulsive delayed systems with uncertainty from nonlinear perturbations. In [24, 25], sufficient conditions for FTS of linear time-varying singular system with impulses at fixed times were given in terms of matrix inequalities.

We tackle the problem of the FTB, finite-time H_∞ control, and the controller design. Our results are different from those previous results. In Section 2, the problem we deal with is precisely stated and some preliminary definitions and notations are provided. In Section 3, first, the result of FTB and finite-time H_∞ control problem analysis is given, and then based on the result, the controller design problem is considered. In this section, we also discuss the numerical algorithms to solve the D/DLMIs. In Section 4, simple examples are presented to illustrate the applicability of our results. Finally, some concluding remarks are provided in Section 5.

2. Problem Statement

We consider a time-varying descriptor system with jumps at fixed time described by

$$\begin{aligned} E\dot{x}(t) &= A(t)x(t) + B(t)u(t) + G(t)\omega(t) \quad (t \neq \tau_k), \\ \Delta x(\tau_k) &= A_k x(\tau_k^-) \quad (t = \tau_k), \\ y(t) &= C(t)x(t) + D(t)u(t) + F(t)\omega(t), \end{aligned} \tag{1}$$

where $x(t) \in R^n$ is the state vector; $u(t) \in R^l$ is the input; $\omega(t) \in R^m$ is the disturbance input; $y(t) \in R^q$ is the output. $Ex(t_0) = x_0$ is the initial value of the system state. $A(t) \in R^{n\times n}$, $B(t) \in R^{n\times l}$, $G(t) \in R^{n\times m}$, $C(t) \in R^{q\times n}$, $D(t) \in R^{q\times l}$, $F(t) \in R^{q\times m}$, and $E \in R^{n\times n}$ are continuous matrix functions while E is singular:

$$\begin{aligned} A(t) &= \overline{A}(t) + \Delta A(t) \\ B(t) &= \overline{B}(t) + \Delta B(t), \end{aligned} \tag{2}$$

where $\Delta A(t)$, $\Delta B(t)$ are unknown matrices representing time-varying parameter uncertainties that can be described as

$$\left[\Delta A(t) \quad \Delta B(t)\right] = M\Delta(t)\left[N_a \quad N_b\right], \tag{3}$$

where M, N_a, N_b are known real constant matrix and $F(t) \in R^{q\times s}$ is unknown matrix function with Lebesgue-measurable elements and satisfies $\Delta^T(t)\Delta(t) \leq I$; when $\Delta A(t)$ and $\Delta B(t)$ are zero matrix, system (1) is called norminal system.

$A_k \in R^{n\times n}, k = 1, 2, \ldots, N$ is time-invariant matrix which reflects the discontinuity of the state trajectory of (1), $-1 < \lambda_{\max} A_k < 0$. System (1) exhibits a finite jump from $x(\tau_k)$ to $x(\tau_k^+)$ at fixed time sequence $\{\tau_k\}$ $(t_0 < \tau_1 < \cdots < \tau_m \leq T < \tau_{m+1} < \cdots)$ and $\Delta x(\tau_k) = x(\tau_k^+) - x(\tau_k^-)$; we assume that the evolution of the state vector $x(t)$ is left continuous at each τ_k, namely,

$$\begin{aligned} x(\tau_k) &= x(\tau_k^-) = \lim_{h\to 0^+} x(\tau_k - h), \\ x(\tau_k^+) &= \lim_{h\to 0^+} x(\tau_k + h). \end{aligned} \tag{4}$$

Moreover, $\omega(t) \in L_2[0, T]$ is the disturbance input and satisfies

$$\int_0^T \omega^T(t)\omega(t)\,dt \leq d, \quad d \geq 0. \tag{5}$$

In this paper we investigate the behavior of system (1) within a finite time interval $[t_0, T]$. Firstly, we propose the following definitions and lemmas.

Definition 1 (regular and impulse-free). (1) The time-varying descriptor system $E\dot{x}(t) = A(t)x(t) + G(t)\omega(t)$ is said to be uniformly regular in time interval $[t_0, T]$, if for any $t \in [t_0, T]$ there exists a scalar s, such that $\det(sE - A(t)) \neq 0$.

(2) The time-varying descriptor system $E\dot{x}(t) = A(t)x(t) + G(t)\omega(t)$ is said to be impulse-free in time interval $[t_0, T]$, if for any $t \in [t_0, T]$ there exists a scalar s, such that $\deg(\det(sE - A(t))) = \operatorname{rank}(E(t))$.

From [26], we know that the continuity of matrix functions $A(t)$, $B(t)$, and $G(t)$ and the uniform regularity of system (1) assure the existence and uniqueness of the solution to $(t_0, T]$. So in this paper we suppose (1) is uniformly regular.

Definition 2 ([8] (finite-time boundedness (FTB) for time–varying descriptor system with jumps)). Given a positive scalar T and a positive definite matrix-valued function $R(\cdot)$, system (1) is said to be FTB with respect to $(T, R(\cdot), d)$ if

$$x_0^T R(t_0) x_0 \leq 1 \Longrightarrow x^T(t) R(t) x(t) \leq 1, \quad t \in [t_0, t_0 + T]. \tag{6}$$

Lemma 3. *Given matrices Ω, Γ, and Ξ with appropriate dimensions and with Ω symmetrical, then*

$$\Omega + \Gamma\Delta(t)\Xi + \Xi^T\Delta^T(t)\Gamma^T < 0 \tag{7}$$

for any $\Delta(t)$ satisfying $\Delta^T(t)\Delta(t) \leq I$ if and only if there exists a scalar $\varepsilon > 0$ such that

$$\Omega + \varepsilon\Gamma\Gamma^T + \varepsilon^{-1}\Xi^T\Xi < 0. \tag{8}$$

The problem of finite-time H_∞ to be addressed in the paper can be formulated as finding a state feedback controller, $u = K(t)x$, such that the following conclusions are held for the close-loop system below:

$$\begin{aligned} E\dot{x}(t) &= A_c(t)x(t) + G(t)\omega(t) \quad (t \neq \tau_k) \\ \Delta x(\tau_k) &= A_k x(\tau_k^-) \quad (t = \tau_k) \\ y(t) &= C_c(t)x(t) + F(t)\omega(t). \end{aligned} \tag{9}$$

(1) The closed-loop system (9) is FTB with respect to $(T, R(\cdot), d)$.

(2) The controlled output $y(t)$ satisfies $\int_{t_0}^T (y^T(t)y(t) - \gamma^2\omega^T(t)\omega(t))dt < M(x_0, t_0)$, for any nonzero $\omega(t)$, where $\gamma > 0$ is a prescribed scalar.

A system is called finite-time H_∞ if the two conditions above are satisfied.

3. Main Result

The system can be decomposed (1) into two subsystems as follows:

$$\begin{aligned}\dot{x}_1(t) &= A_{11}(t)x_1(t) + A_{22}(t)x_2(t) + B_1(t)u(t) \\ &\quad + G_1(t)\omega(t), \\ 0 &= A_{21}(t)x_1(t) + A_{22}(t)x_2(t) + B_2(t)u(t) + G_2(t)\omega(t), \\ y(t) &= C_1(t)x_1(t) + C_2(t)x_2(t) + D(t)u(t) + F(t)\omega(t),\end{aligned} \tag{10}$$

where

$$\begin{gathered} PEQ = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \qquad PA(t)Q = \begin{bmatrix} A_{11}(t) & A_{12}(t) \\ A_{21}(t) & A_{22}(t) \end{bmatrix}, \\ PG(t) = \begin{bmatrix} G_1(t) \\ G_2(t) \end{bmatrix} \\ Q^{-1}x(t) = \begin{bmatrix} x_1(t) & x_2(t) \end{bmatrix}, \qquad C(t)Q = \begin{bmatrix} C_1(t) & C_2(t) \end{bmatrix}. \end{gathered} \tag{11}$$

P, Q are nonsingular matrix, and it is easy to find that system (1) is impulse-free in time interval $[t_0, T]$, for any inital value x_0, if matrix function $A_{22}(t)$ is invertible.

The following theorem gives a sufficient condition for FTB of system (1)

Theorem 4. *Uncertain time-varying descriptor system with jumps (1) is said to be FTB with respect to $(T, R(\cdot), d)$, if there exists a nonsingular and piecewise continuously differential matrix-valued function $\Gamma(\cdot)$ such that*

$$E^T\Gamma(t) = \Gamma^T(t)E \geq R(t) \geq 0 \tag{12a}$$

$$\begin{bmatrix} \Omega(t) & \Gamma^T(t)G(t) & \Gamma^T(t)M \\ G^T(t)\Gamma(t) & -\dfrac{1}{1+d}I & 0 \\ M^T\Gamma(t) & 0 & \varepsilon I \end{bmatrix} < 0, \tag{12b}$$

$$E^T\Gamma(t_0) = \Gamma^T(t_0)E \leq \frac{1}{1+d}R(t_0) \tag{12c}$$

are fulfilled over $[t_0, T]$, where

$$\Omega(t) = \overline{A}^T(t)\Gamma(t) + \Gamma^T(t)\overline{A}(t) + E^T\dot{\Gamma}(t) + \varepsilon N_a^T N_a. \tag{13}$$

Proof. By Schur complement (12b) is equivalent to

$$\begin{bmatrix} \Omega_0(t) & \Gamma^T(t)G(t) \\ G^T(t)\Gamma(t) & -\dfrac{1}{1+d}I \end{bmatrix} < 0, \tag{14}$$

where $\Omega_0(t) = \overline{A}^T(t)\Gamma(t) + \Gamma^T(t)\overline{A}(t) + E^T\dot{\Gamma}(t) + \varepsilon N_a^T N_a + (1/\varepsilon)\Gamma^T(t)MM^T\Gamma(t)$.

Decompose (14) as

$$\begin{aligned}&\begin{bmatrix} \overline{A}^T(t)\Gamma(t) + \Gamma^T(t)\overline{A}(t) + E^T\dot{\Gamma}(t) & \Gamma^T(t)G(t) \\ G^T(t)\Gamma(t) & -\dfrac{1}{1+d}I \end{bmatrix} \\ &\quad + \varepsilon\begin{bmatrix} N_a^T N_a & 0 \\ 0 & 0 \end{bmatrix} + \frac{1}{\varepsilon}\begin{bmatrix} \Gamma^T(t)MM^T\Gamma(t) & 0 \\ 0 & 0 \end{bmatrix}.\end{aligned} \tag{15}$$

From Lemma 3 it is easy to derive

$$\begin{aligned}&\begin{bmatrix} \overline{A}^T(t)\Gamma(t) + \Gamma^T(t)\overline{A}(t) + E^T\dot{\Gamma}(t) & \Gamma^T(t)G(t) \\ G^T(t)\Gamma(t) & -\dfrac{1}{1+d}I \end{bmatrix} \\ &\quad + \begin{bmatrix} \Gamma^T(t)M & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \Delta(t) & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} N_a & 0 \\ 0 & 0 \end{bmatrix} \\ &\quad + \left(\begin{bmatrix} \Gamma^T(t)M & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \Delta(t) & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} N_a & 0 \\ 0 & 0 \end{bmatrix}\right)^T,\end{aligned} \tag{16}$$

so (14) is equivalent to the following LMI:

$$\begin{bmatrix} \Omega_1(t) & \Gamma^T(t)G(t) \\ G^T(t)\Gamma(t) & -\dfrac{1}{1+d}I \end{bmatrix} < 0, \tag{17}$$

where $\Omega_1(t) = A^T(t)\Gamma(t) + \Gamma^T(t)A(t) + E^T\dot{\Gamma}(t)$. From (12a), (12b), (12c), and

$$Q^{-T}\Gamma(t)P = \begin{bmatrix} \Gamma_1(t) & \Gamma_2(t) \\ \Gamma_3(t) & \Gamma_4(t) \end{bmatrix} \tag{18}$$

we have $\Gamma_2(t) = 0$ and $\Gamma_1(t)$ is symmetric; from (17) we have $\Omega_1(t) < 0$,

$$\begin{aligned}\Omega_1(t) &= Q^T A^T(t)P^T P^{-T}\Gamma(t)Q \\ &\quad + Q^T\Gamma^T(t)P^{-1}PA(t)Q + Q^T E^T P^T P^{-T}\dot{\Gamma}(t)Q \\ &= \begin{bmatrix} A_{11}^T(t) & A_{21}^T(t) \\ A_{12}^T(t) & A_{22}^T(t) \end{bmatrix}\begin{bmatrix} \Gamma_1(t) & \Gamma_2(t) \\ \Gamma_3(t) & \Gamma_4(t) \end{bmatrix} \\ &\quad + \begin{bmatrix} \Gamma_1^T(t) & \Gamma_3^T(t) \\ \Gamma_2^T(t) & \Gamma_4^T(t) \end{bmatrix}\begin{bmatrix} A_{11}(t) & A_{12}(t) \\ A_{21}(t) & A_{22}(t) \end{bmatrix} \\ &\quad + \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{\Gamma}_1(t) & \dot{\Gamma}_2(t) \\ \dot{\Gamma}_3(t) & \dot{\Gamma}_4(t) \end{bmatrix} \\ &= \begin{bmatrix} \star & \star \\ \star & A_{22}^T(t)\Gamma_4(t) + \Gamma_4^T(t)A_{22}(t) \end{bmatrix} < 0.\end{aligned} \tag{19}$$

Obviously $A_{22}^T(t)\Gamma_4(t) + \Gamma_4^T(t)A_{22}(t) < 0$. So $A_{22}(t)$ is invertible, and system is impulse-free in time interval $[t_0, T]$ for any initial value.

Consider the Lyapunov function

$$V(t, x(t)) = x^T(t) E^T \Gamma(t) x(t). \tag{20}$$

Then, differentiating $V(t, x(t))$ with respect to time t on the time interval $t \in (t_0, \tau_1)$, we obtain

$$\begin{aligned}\dot{V}(t, x(t)) &= x^T(t)\left(A^T(t)\Gamma(t) + \Gamma^T(t)A(t) + E^T\dot{\Gamma}(t)\right)x(t) \\ &\quad + \omega^T(t)G^T(t)\Gamma(t)x(t) \\ &\quad + x^T(t)\Gamma^T(t)G(t)\omega(t) \\ &= x^T(t)\Omega_1(t)x(t) + \omega^T(t)G^T(t)\Gamma(t)x(t) \\ &\quad + x^T(t)\Gamma^T(t)G(t)\omega(t).\end{aligned} \tag{21}$$

Construct a new vector as follows:

$$z(t) = \begin{bmatrix} x(t) \\ \omega(t) \end{bmatrix}. \tag{22}$$

By (17), it is easy to see that

$$\begin{aligned} & z^T(t)\begin{bmatrix} \Omega_1(t) & \Gamma^T(t)G(t) \\ G^T(t)\Gamma(t) & -\dfrac{1}{1+d}I \end{bmatrix} z(t) \\ &= x^T(t)\Omega_1(t)x(t) + \omega^T(t)G^T(t)\Gamma(t)x(t) \\ &\quad + x^T(t)\Gamma^T(t)G(t)\omega(t) - \frac{1}{1+d}\omega^T\omega(t) \\ &= \dot{V}(t, x(t)) - \frac{1}{1+d}\omega^T(t)\omega(t) < 0.\end{aligned} \tag{23}$$

It follows that

$$\dot{V}(t, x(t)) < \frac{1}{1+d}\omega^T(t)\omega(t). \tag{24}$$

Integrating both sides of (24) from t_0 to t in which $t \in (t_0, \tau_1)$ and noting that $x^T(t_0)R(t_0)x(t_0) < 1$, we obtain

$$\begin{aligned} V(t, x(t)) &< V(t_0, x(t_0)) + \frac{1}{1+d}\int_{t_0}^{t}\omega^T(\tau)\omega(\tau)\,d\tau \\ &< \frac{1}{1+d}x_0^T R(t_0)x_0 + \frac{1}{1+d}\int_{t_0}^{t}\omega^T(\tau)\omega(\tau)\,d\tau \\ &< \frac{1}{1+d} + \frac{1}{1+d}\int_{t_0}^{t}\omega^T(\tau)\omega(\tau)\,d\tau \qquad t \in (t_0, \tau_1).\end{aligned} \tag{25}$$

When $t = \tau_1$, since $x(\tau_1^-) = x(\tau_1)$ and $\tau_1^- \in (t_0, \tau_1)$, by (25) we also have

$$V(\tau_1, x(\tau_1)) < \frac{1}{1+d} + \frac{1}{1+d}\int_{t_0}^{\tau_1}\omega^T(\tau)\omega(\tau)\,d\tau. \tag{26}$$

Now we consider the situation $t \in (\tau_1, \tau_2)$. Integrating both sides of (24) from τ_1^+ to t it is obvious to have

$$V(t, x(t)) < \frac{1}{1+d}\int_{\tau_1}^{t}\omega^T(\tau)\omega(\tau)\,d\tau + V(\tau_1^+, x(\tau_1^+)) \qquad t \in (\tau_1, \tau_2) \tag{27}$$

and according to $-1 < \|A_k\|_2 < 0$

$$\begin{aligned} V(\tau_1^+, x(\tau_1^+)) &= x^T(\tau_1^+)E^T\Gamma(\tau_1^+)x(\tau_1^+) \\ &= [(I + A_1)x(\tau_1)]^T E^T\Gamma(\tau_1^+) \\ &\quad \cdot [(I + A_1)x(\tau_1)] \\ &< x^T(\tau_1)E^T\Gamma(\tau_1)x(\tau_1) \\ &\le \frac{1}{1+d} + \frac{1}{1+d}\int_{t_0}^{\tau_1}\omega^T(\tau)\omega(\tau)\,d\tau\end{aligned} \tag{28}$$

so when $t \in (\tau_1, \tau_2]$ we have

$$V(t, x(t)) < \frac{1}{1+d} + \frac{1}{1+d}\int_{t_0}^{t}\omega^T(\tau)\omega(\tau)\,d\tau. \tag{29}$$

By the same progress, it is easy to have

$$x^T(t)R(t)x(t) \le V(t, x(t)) \le 1. \tag{30}$$

□

For norminal system it is easy to obtain the following result.

Corollary 5. *Norminal time-varying descriptor system with jumps is said to be FTB with respect to $(T, R(\cdot), d)$, if there exists a nonsingular and piecewise continuously differential matrix-valued function $\Gamma(\cdot)$ such that (12a), (12b), (12c), (17), and (24) are fulfilled over $[t_0, T]$.*

Theorem 6. *Given a scalar $\gamma > 0$, uncertain time-varying descriptor system with jumps (1) is said to be FTB with respect to $(T, R(\cdot), d)$ and satisfies $\int_{t_0}^{T}(y^T(t)y(t) - \gamma^2\omega^T(t)\omega(t))dt < 0$. There exists a nonsingular and piecewise continuously differential matrix-valued function $\Gamma(\cdot)$ such that*

$$E^T\Gamma(t) = \Gamma^T(t)E \ge R(t) \ge 0 \tag{31a}$$

$$\begin{bmatrix} \Xi(t) & \Gamma^T(t)G(t) & C^T(t)F(t) & \Gamma^T(t)M \\ G^T(t)\Gamma(t) & -\dfrac{1}{1+d}I & 0 & 0 \\ F^T(t)C(t) & 0 & F^T(t)F(t) - \gamma^2 I & 0 \\ M^T\Gamma(t) & 0 & 0 & \varepsilon I \end{bmatrix} < 0, \tag{31b}$$

$$E^T\Gamma(t_0) = \Gamma^T(t_0)E \le \frac{1}{1+d}R(t_0) \tag{31c}$$

are fulfilled over $[t_0, T]$, *where*

$$\Xi(t) = E^T\dot{\Gamma}(t) + \overline{A}^T(t)\Gamma(t) + \Gamma^T(t)\overline{A}(t) + C^T(t)C(t) + \varepsilon N_a^T N_a. \tag{32}$$

Proof. By the proof of Theorem 4, (31b) is equivalent to the following LMI:

$$\begin{bmatrix} \Xi_0(t) & \Gamma^T(t)G(t) & C^T(t)F(t) \\ G^T(t)\Gamma(t) & -\dfrac{1}{1+d}I & 0 \\ F^T(t)C(t) & 0 & F^T(t)F(t) - \gamma^2 I \end{bmatrix} < 0, \tag{33}$$

where $\Xi_0(t) = \overline{A}^T(t)\Gamma(t) + \Gamma^T(t)\overline{A}(t) + E^T\dot{\Gamma}(t) + \varepsilon N_a^T N_a + (1/\varepsilon)\Gamma^T(t)MM^T\Gamma(t) + C^T(t)C(t)$.

By Lemma 3, it is easy to have

$$\Theta(t) = \begin{bmatrix} \Xi_1(t) + C^T(t)C(t) & \Gamma^T(t)G(t) & C^T(t)F(t) \\ G^T(t)\Gamma(t) & -\dfrac{1}{1+d}I & 0 \\ F^T(t)C(t) & 0 & F^T(t)F(t) - \gamma^2 I \end{bmatrix} < 0, \tag{34}$$

where

$$\Xi_1(t) = E^T\dot{\Gamma}(t) + A^T(t)\Gamma(t) + \Gamma^T(t)A(t) < 0. \tag{35}$$

Since $C^T(t)C(t) > 0$, we have

$$\begin{bmatrix} \Xi_1(t) & \Gamma^T(t)G(t) \\ G^T(t)\Gamma(t) & -\dfrac{1}{1+d}I \end{bmatrix} < 0. \tag{36}$$

By (31a), (31b), and (36) and the proof of Theorem 4, system (1) is FTB.

Now we will show that the following performance function is bounded:

$$J = \int_{t_0}^{T} \left(y^T(t)y(t) - \gamma^2\omega^T(t)\omega(t)\right) < V(x_0, t_0). \tag{37}$$

Construct a new vector as follows:

$$z(t) = \begin{bmatrix} x(t) \\ \omega(t) \\ \omega(t) \end{bmatrix}. \tag{38}$$

By (34), we have

$$\begin{aligned} z^T(t)\Theta(t)z(t) &= x^T(t)\Xi_1(t)x(t) + \omega^T(t)G^T(t)\Gamma(t)x(t) \\ &\quad + x^T(t)\Gamma^T(t)G(t)\omega(t) + x^T(t)C^T(t)C(t)x(t) \\ &\quad + \omega^T(t)F^T(t)C(t)x(t) + x^T(t)C^T(t)F(t)\omega(t) \\ &\quad + \omega^T(t)F^T(t)F(t)\omega(t) - \left(\frac{1}{1+d} + \gamma^2\right)\omega^T(t)\omega(t) \\ &= x^T(t)\Xi_1(t)x(t) + \omega^T(t)G^T(t)\Gamma(t)x(t) \\ &\quad + x^T(t)\Gamma^T(t)G(t)\omega(t) - \frac{1}{1+d}\omega^T(t)\omega(t) \\ &\quad + y^T(t)y(t) - \gamma^2\omega^T(t)\omega(t) \\ &= \dot{V}(t) - \frac{1}{1+d}\omega^T(t)\omega(t) + y^T(t)y(t) \\ &\quad - \gamma^2\omega^T(t)\omega(t) < 0. \end{aligned} \tag{39}$$

Therefore, for $t \in (\tau_k, \tau_{k+1})$, $k = 0, 1, 2, \ldots, m$, we have

$$\begin{aligned} J_{\tau_{k+1}} &= \int_{\tau_k}^{\tau_{k+1}} z^T(t)\Theta(t)z(t)\,dt \\ &\quad - \left(\int_{\tau_k}^{\tau_{k+1}} \dot{V}(t) - \frac{1}{1+d}\omega^T(t)\omega(t)\right)dt \end{aligned} \tag{40}$$

with $-V(T) - \int_{t_0}^{T}(1/(1+d))\omega^T(t)\omega(t) < 0$ and for $\Theta(t) < 0$ we have

$$\begin{aligned} J &= \sum_{i=0}^{m+1} J_{\tau_{i+1}} \\ &= \int_{t_0}^{T} z^T(t)\Theta(t)z(t)\,dt + V(t_0) - V(T) \\ &\quad - \int_{t_0}^{T} \frac{1}{1+d}\omega^T(t)\omega(t)\,dt < V(t_0) \\ &\qquad (\tau_0 = t_0,\ \tau_{m+1} = T). \end{aligned} \tag{41}$$

This completes the proof of the theorem. □

For norminal system it is easy to obtain the following result.

Corollary 7. *Given a scalar* $\gamma > 0$, *norminal time-varying descriptor system with jumps (1) is said to be FTB with respect to* $(T, R(\cdot), d)$ *and satisfies* $\int_{t_0}^{T}(y^T(t)y(t) - \gamma^2\omega^T(t)\omega(t))dt < 0$. *There exists a nonsingular and piecewise continuously differential matrix-valued function* $\Gamma(\cdot)$ *such that (34), (42a), (42b), (42c), and (45) are fulfilled over* $[t_0, T]$.

Theorem 8. *Given a scalar $\gamma > 0$ and a state feedback controller $u = K(t)x$, the close-loop system (9) is said to be FTB with respect to $(T, R(\cdot), d)$ and satisfies $J = \int_{t_0}^{T}(y^T(t)y(t) - \gamma^2\omega^T(t)\omega(t))dt < V(x_0, t_0)$. If there exists a nonsingular and piecewise continuously differential matrix-valued function $\overline{\Gamma}(\cdot)$ and a continuously differential matrix-valued function $L(\cdot)$ such that*

$$\overline{\Gamma}^T(t)E^T = E\overline{\Gamma}(t) \geq R(t) \geq 0 \tag{42a}$$

$$\begin{bmatrix} \Pi(t) & G(t) & 0 & \overline{\Gamma}^T(t)C^T(t) + L^T(t)D^T(t) & \overline{\Gamma}^T(t)N_a^T + L^T(t)N_b^T \\ G^T(t) & -\frac{1}{1+d}I & 0 & 0 & 0 \\ 0 & 0 & -\gamma^2 I & F^T(t) & 0 \\ C(t)\overline{\Gamma}(t) + D(t)L(t) & 0 & F(t) & -I & 0 \\ N_a\overline{\Gamma}(t) + N_bL(t) & 0 & 0 & 0 & -\varepsilon I \end{bmatrix} < 0, \tag{42b}$$

$$\overline{\Gamma}^T(t_0)E^T = E\overline{\Gamma}(t_0) \leq \frac{1}{1+d}R(t_0) \tag{42c}$$

are fulfilled over $[t_0, T]$, the state feedback gain is obtained with $K(t) = L(t)\overline{\Gamma}^{-1}(t)$, where

$$\Pi(t) = -E\dot{\overline{\Gamma}}(t) + \overline{\Gamma}^T(t)\overline{A}^T(t) + \overline{A}(t)\overline{\Gamma}(t) + L^T(t)\overline{B}^T(t) + \overline{B}(t)L(t) + \varepsilon MM^T. \tag{43}$$

Proof. By Schur complement and (42b), we have

$$\begin{bmatrix} \Pi(t) & G(t) & 0 & \overline{\Gamma}^T(t)C^T(t) + L^T(t)D^T(t) \\ G^T(t) & -\frac{1}{1+d}I & 0 & 0 \\ 0 & 0 & -\gamma^2 I & F^T(t) \\ C(t)\overline{\Gamma}(t) + D(t)L(t) & 0 & F(t) & -I \end{bmatrix} + \frac{1}{\varepsilon}\begin{bmatrix} \Gamma^T(t)N_a^T + L^T(t)N_b^T \\ 0 \\ 0 \\ 0 \end{bmatrix}\begin{bmatrix} N_a\overline{\Gamma}(t) + N_bL(t) & 0 & 0 & 0 \end{bmatrix} < 0. \tag{44}$$

Continue using Schur complement, and then (44) is equivalent to

$$\begin{bmatrix} \Pi_0(t) & \star & \star \\ \star & \star & \star \\ \star & \star & \star \end{bmatrix} + \begin{bmatrix} \overline{\Gamma}^T(t)C^T(t) + L^T(t)D^T(t) \\ 0 \\ F^T(t) \end{bmatrix}\begin{bmatrix} C(t)\overline{\Gamma}(t) + D(t)L(t) & 0 & F(t) \end{bmatrix} = \begin{bmatrix} \Pi_0(t) & \star & \left(\overline{\Gamma}^T(t)C^T(t) + L^T(t)D^T(t)\right)F(t) \\ \star & \star & \star \\ F^T(t)\left(C(t)\overline{\Gamma}(t) + D(t)L(t)\right) & \star & F^T(t)F(t) - \gamma^2 I \end{bmatrix} < 0, \tag{45}$$

where $\Pi_0(t) = \Pi(t) + (1/\varepsilon)(\Gamma^T(t)N_a^T + L^T(t)N_b^T)(N_a\overline{\Gamma}(t) + N_bL(t))$.

We rewrite (45) as

$$\begin{bmatrix} \Pi_1(t) & G(t) & \left(\overline{\Gamma}^T(t)C^T(t) + L^T(t)D^T(t)\right)F(t) \\ G^T(t) & -\dfrac{1}{1+d}I & 0 \\ F^T(t)\left(C(t)\overline{\Gamma}(t) + D(t)L(t)\right) & 0 & F^T(t)F(t) - \gamma^2 I \end{bmatrix} < 0 \tag{46}$$

since $K(t) = L(t)\overline{\Gamma}^{-1}(t)$ and

$$\begin{aligned} \Pi_1(t) = {} & -E\dot{\overline{\Gamma}}(t) + \overline{\Gamma}^T(t)\left(\overline{A}^T(t) + K^T(t)\overline{B}^T(t)\right) \\ & + \left(\overline{A}(t) + \overline{B}(t)K(t)\right)\overline{\Gamma}(t) + \varepsilon MM^T \\ & + \frac{1}{\varepsilon}\overline{\Gamma}^T(t)\left(N_a + N_bK(t)\right)^T\left(N_a + N_bK(t)\right)\overline{\Gamma}(t) \\ & + \overline{\Gamma}^T(t)\left(C^T(t) + K^T(t)D^T(t)\right) \\ & \cdot (C(t) + D(t)K(t))\overline{\Gamma}(t) \end{aligned} \tag{47}$$

By Lemma 3 we have

$$\begin{aligned} \Pi_1(t) = {} & -E\dot{\overline{\Gamma}}(t) + \overline{\Gamma}^T(t)C_c^T(t)C_c(t)\overline{\Gamma}(t) \\ & + \overline{\Gamma}^T(t)\left(\overline{A}^T(t) + K^T(t)\overline{B}^T(t)\right) \\ & + \left(\overline{A}(t) + \overline{B}(t)K(t)\right)\overline{\Gamma}(t) \\ & + \overline{\Gamma}^T(t)\left(M\Delta(t)N_a + M\Delta(t)N_bK(t)\right)^T \\ & + \left(M\Delta(t)N_a + M\Delta(t)N_bK(t)\right)\overline{\Gamma} \\ = {} & -E\dot{\overline{\Gamma}}(t) + \overline{\Gamma}^T(t)C_c^T(t)C_c(t)\overline{\Gamma}(t) + \overline{\Gamma}^T(t)A_c^T(t) \\ & + A_c(t)\overline{\Gamma}(t). \end{aligned} \tag{48}$$

Taking $\Gamma(t) = \overline{\Gamma}^{-1}(t)$, since $\overline{\Gamma}^T(t)E = E^T\overline{\Gamma}(t)$ and $I = \overline{\Gamma}(t)\Gamma(t)$, $0 = \dot{\overline{\Gamma}}(t)\Gamma(t) + \overline{\Gamma}(t)\dot{\Gamma}(t)$, it is easy to have

$$-\overline{\Gamma}^{-T}(t)E\dot{\overline{\Gamma}}(t)\overline{\Gamma}^{-1}(t) = -E^T\overline{\Gamma}(t)\dot{\overline{\Gamma}}(t)\overline{\Gamma}(t) = E^T\dot{\Gamma}(t). \tag{49}$$

Pre- and postmultiplying (46) by $\mathrm{diag}(\overline{\Gamma}^{-T}(t), I, I)$ and $\mathrm{diag}(\overline{\Gamma}^{-1}(t), I, I)$, we have

$$\begin{bmatrix} \Pi_2(t) & \Gamma^T(t)G(t) & C_c^T(t)F(t) \\ G^T(t)\Gamma(t) & -\dfrac{1}{1+d}I & 0 \\ F^T(t)C_c(t) & 0 & F^T(t)F(t) - \gamma^2 I \end{bmatrix} < 0, \tag{50}$$

where $\Pi_2(t) = E\dot{\Gamma}(t) + C_c^T(t)C_c(t) + \Gamma^T(t)A_c(t) + A_c^T(t)\Gamma(t)$.

By Theorem 6, close-loop system is FTB and satisfies $J < V(x_0, t_0)$. □

For norminal system it is easy to obtain the following result.

Corollary 9. *Given a scalar $\gamma > 0$ and a state feedback controller $u = K(t)x$, the norminal close-loop system is said to be FTB with respect to $(T, R(\cdot), d)$ and satisfies $J = \int_{t_0}^{T}(y^T(t)y(t) - \gamma^2\omega^T(t)\omega(t))dt < V(x_0, t_0)$. If there exists a nonsingular and piecewise continuously differential matrix-valued function $\overline{\Gamma}(\cdot)$ and a continuously differential matrix-valued function $L(\cdot)$ such that (50), (42a), and (42c) are fulfilled over $[t_0, T]$, the state feedback gain is obtained with $K(t) = L(t)\overline{\Gamma}^{-1}(t)$.*

All the conditions in this paper are expressed in terms of time-varying D/DLMIs. However, with an appropriate choice of the structure of the unknown matrix function $P(t)$, it can be turned into a "standard" LMI problem. The unknown matrix function $P(t)$ has been assumed to be piecewise affine; that is,

$$\begin{gathered} P(0)\left(\text{or } \left(\overline{P}(t)\right)\right) = \Pi_1^0 \\ P(t)\left(\text{or } \left(\overline{P}(t)\right)\right) = \Pi_k^0 + \Pi_k^s\left(t - (k-1)T_s\right) \\ \dot{P}(t)\left(\text{or } \left(\overline{P}(t)\right)\right) = \Pi_k^s, \\ k \in N : k < \overline{k}, \quad t \in \left[(k-1)T_s, kT_s\right] \\ P(t)\left(\text{or } \left(\overline{P}(t)\right)\right) = \Pi_{\overline{k}+1}^0 + \Pi_{\overline{k}+1}^0\left(t - \overline{k}T_s\right), \\ \dot{P}(t)\left(\text{or } \left(\overline{P}(t)\right)\right) = \Pi_{\overline{k}+1}^0, \\ t \in \left[\overline{k}T_s, T\right], \end{gathered} \tag{51}$$

where $\overline{k} = \max k \in N^+ : k < T/T_s$.

Hence, the conditions are reduced to a set of LMIs. Note that the conditions are not strict LMI conditions due to $E^T\Gamma(t) = \Gamma^T(t)E \geq 0$, and this may cause a big trouble in checking the conditions numerically. In order to translate the nonstrict LMI into strict LMI, we can use the following lemma.

Lemma 10 (see [27, 28]). *Let $X \in R^{n\times n}$ be symmetric such that $E_L^T X E_L > 0$ and let $T \in R^{(n-r)\times(n-r)}$ be nonsingular. Then, $XE + H^T T S^T$ is nonsingular and its inverse is expressed as*

$$\left(XE + H^T T S^T\right)^{-1} = \mathbb{X}E^T + S\mathbb{T}H, \tag{52}$$

where $\mathbb{X}$ is symmetric and $\mathbb{T}$ is a nonsingular matrix with

$$E_R^T \mathbb{X} E_R = \left(E_L^T X E_L\right)^{-1}, \qquad \mathbb{T} = \left(S^T S\right)^{-1} T^{-1} \left(HH^T\right)^{-1}, \tag{53}$$

where H and S are any matrix with full row rank and satisfy $ME = 0$ and $ES = 0$, respectively; E is decomposed as $E = E_L E_R^T$ with $E_L \in R^{n\times r}$ and $E_R \in R^{n\times r}$ are of full column rank.

Let $\Pi_k^0 = X_k^0 E + H^T T_k^0 S^T$ and $\Pi_k^s = X_k^s E + H^T T_k^s S^T$. Using Lemma 10, we can get $(X_k^0 E + H^T T_k^0 S^T)^{-1} = \mathbb{X}_k^0 E^T + S\mathbb{T}_k^0 H$ and $(X_k^s E + H^T T_k^s S^T)^{-1} = \mathbb{X}_k^s E^T + S\mathbb{T}_k^0 H$. In this way the condition $E^T\Gamma(t) = \Gamma^T(t)E \geq 0$ is satisfied, so the nonstrict LMIs have been translated into strict LMIs. Exploiting the MATLAB LMI toolbox, it is possible to find matrices $X_k^s, T_k^0, T_k^s, X_k^0$ (or $\mathbb{X}_k^0, \mathbb{X}_k^s, \mathbb{T}_k^0, \mathbb{T}_k^s$), such that $\Gamma(t)$ (or $\overline{\Gamma}(t)$) and $L(t)$ verify the conditions.

4. Numerical Example

Consider the linear time-varying descriptor system with jumps defined by

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \qquad A = \begin{bmatrix} 0 & 1 \\ 2(t+1) & 3(t+1)^2 \end{bmatrix},$$
$$B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \qquad F = \begin{bmatrix} 0.5 & 1 \end{bmatrix}, \qquad A_k = \begin{bmatrix} 0.01 & 0 \\ 0 & 0 \end{bmatrix},$$
$$G = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad D = 1, \qquad C = \begin{bmatrix} 1 & 1 \end{bmatrix},$$
$$M = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, \qquad N_a = \begin{bmatrix} 0.2 & 0.1 \end{bmatrix}, \qquad N_b = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}. \tag{54}$$

Given $\omega(t) = \left[\sin(2\pi t + 1) \quad \cos(2\pi t - 2)\right]^T$, $\Omega = [1.5, 2.5]$, $R = \mathrm{diag}(1.2, 1)$, $\gamma = 1.5$, and $\varepsilon = 1.654$. It is easy to see

$$E_R = E_L = \begin{bmatrix} 1 \\ 0 \end{bmatrix}. \tag{55}$$

We choose $H^T = S = [0 \quad 1]^T$. Solving the LMIs by the numerical algorithm in the previous section using MATLAB LMI toolbox, it is possible to find two piecewise affine matrix functions $\overline{\Gamma}(t)$ and $L(t)$ which verify the conditions of Theorem 8. Therefore, the following state feedback control law can be obtained (see Figure 1).

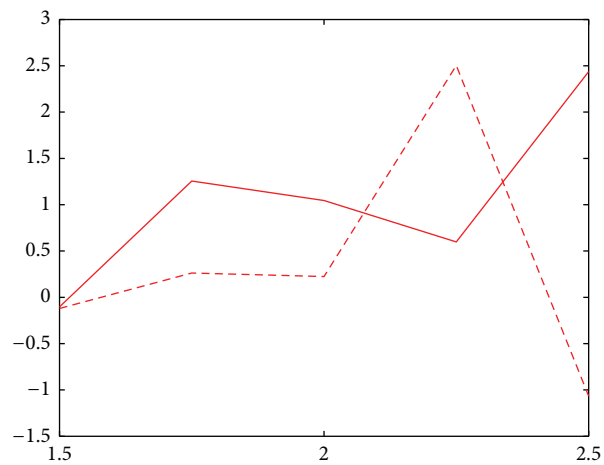

Figure 1: $K_1(t)$ (solid line) and $K_2(t)$ (dashed line).

5. Conclusions

In this paper, we have formulated and studied the finite-time H_∞ control problem of uncertain time-varying descriptor system with jumps at fixed times. A sufficient condition for FTB and a sufficient condition for finite-time H_∞ of the system have been presented in terms of DLMIs and LMIs. Then, the state feedback controller has been designed to guarantee the closed-loop system's FTB and satisfy the L_2 gain. At last, two numerical examples have been used to illustrate the main result.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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