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*Research Article*

# Durability Evolution of RC Bridge under Coupling Action of Chloride Corrosion and Carbonization Based on DLA Model

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Chloride attack and carbonization are the main factors which affect the durability of concrete structures, and the respective theoretical models are systematically established. However, the quantitative analysis and models about the coupling effect of chloride attack and carbonization are less, so the precision and level of durability analysis of reinforced concrete are restricted. Diffusion-limited aggregation (DLA) model can finely simulate the process of gas diffusion and condensation with randomness and fractal characteristics, which is suitable for revealing the durability evolution process of the chloride attack, carbonization, and the coupling action in concrete. Based on the principle of DLA, considering the factors such as diffusion depth, concrete properties, and exposure conditions which influence the characteristics of chloride diffusion and carbonization, as well as the coupling effect, an integrated DLA model is established. The concentration of carbon dioxide and chloride at any time and any location can be obtained and dynamically displayed based on the DLA model. The performance predict method for concrete and steel bars considering fatigue effect is presented based on DLA, according to the demand for bridge durability analysis. Numerical examples show that the method can dynamically and intensively simulate the durability evolution process of reinforced concrete bridge.

## 1. Introduction

Reinforced concrete is an excellent construction material: highly durable, economical, under proper conditions, and semipermanent. However, unsuitable use of reinforced concrete in design or construction is likely to result in low performance and deterioration of the concrete long before the end of its intended service life [1]. Early deterioration of structures increases their maintenance costs and requires them to be rehabilitated or dismantled, thus producing building waste and greatly impacting local and global environments.

In recent years, various reinforced concrete structures worldwide have suffered rapid deterioration. Therefore, the durability of concrete structures, especially those exposed to aggressive environment, is of great concern [2]. Many deterioration causes and factors have been investigated. There are many factors that cause deterioration of reinforced concrete structures: neutralization, chloride damage, freezing and thawing, chemical corrosion, and alkali-aggregate reactions are often indicated as causes for the deterioration of reinforced concrete structures and each involves the corrosion and expansion of a reinforcing bar, which cracks the concrete and deteriorates the capacity and performance of the structure. The durability of reinforced concrete structure is one of the issues of common concern to civil engineering.

At present, the research on the durability of concrete includes three levels, which are microscopic level, mesoscopic level, and macroscopic level. The focus of microlevel research is the deterioration mechanics and damage rules of hardened cement paste and rebar. The scope of mesoscopic level research refers to the influence of material phase composition and structure on the durability. The scope of macroscopic level research involves the integrated performance and the structural durability evaluation and life prediction, and so forth. According to the achievements from above levels, the theoretical methods on the concrete structure durability are mainly divided into three types. The first one is mathematical model method, which calculates the evolution of the material parameters directly using the chloride ion diffusion equation and carbonation equation [3, 4]. The second method is numerical analysis, which derives the deterioration process of materials in time-space dimension by specified algorithms. For example, cellular automata algorithm has been proposed according to the given local rules [5]. The third method is the finite element method; the 3D model of reinforced concrete structures are usually created by the multiphysics coupling finite element software such as ANSYS and COMSOL, and the materials or the structures are simulated precisely [6]. Though extensive research is conducted on the durability of concrete structures, there are also many respective limitations in each type of method at present.

As a key type of infrastructure in civil engineering, bridge is an important part of the national transportation system, and the reinforced concrete bridge is the most common form in highway bridges. Due to the rigorous environment and the insufficient measurement of construction and maintenance worldwide, the durability of a great many reinforced concrete bridges reduced rapidly, the normal operation and safety are affected [7]. Therefore, the analysis and evaluation of the durability of the reinforced concrete bridge has become an important problem in engineering research.

The durability of the reinforced concrete bridge mainly involves the chloride ion erosion, the carbonation of concrete, and the reinforcement corrosion caused by the above factors and these factors are mainly affected by the diffusion depth, the characteristics of concrete, and the exposure conditions. Until now, the study on the durability degradation due to the single environmental factor such as carbonation and chloride ion penetration of reinforced concrete structure has been thoroughly developed [8]. However, it is worth noting that the actual durability of the concrete structure degradation in practical conditions is the long-term and coupling action of many factors in load, environment, and climate. The combined results are different with the law from a single factor, but the relevant research results and analysis methods, especially those that can be applied in the actual model or equations, are still very limited [9, 10].

In view of the above deficiency in the corresponding research limitations, a numerical analysis model based on diffusion-limited aggregation (DLA) is proposed, in order to reveal the coupling effect about the chloride ion erosion and concrete carbonization on the macroscopic capacity deterioration of the reinforced concrete structures. The change of the concrete and steel rebar in the microlevel, the evolution of the chemical composition and the interaction between the transmission performances in the mesoscopic level, and the durability in the macrolevel are studied. Finally, the chloride ion concentration and the carbonation degree on any location and at any time can be predicted; furthermore, the structural durability can also be calculated and the optimization strategy for maintenance can be provided.

## 2. Diffusion-Limited Agglomeration

Stochastic models of fractal growth have inspired a number of studies and applications in applied sciences, and the best known model is the diffusion-limited aggregation (DLA). DLA has been extensively employed since its proposition in 1981 by Witten and Sander to model cluster growth controlled by the random process of diffusion [11].

The basic model operates according to the following basic rule. The initial starting point for the growing cluster is fixed, so that the primary cluster consists of a single particle. At every step another particle is attached to the cluster according to some rule so that the cluster remains connected with respect to some neighboring relationship. This is replicated on every new step until the cluster reaches the predetermined size.

The mathematical description of the DLA model is as follows: on the system point group, fixed in a certain location layout particles as the initial state. From that position, different particles are produced once certain threshold value is surpassed, and the particles will move randomly until they are adsorbed by other fixed particles. The adsorption principle is as follows: in four nearest neighbors (up, down, left, and right lattice) that are fixed in particles, the movement of the particles will be adsorbed. And then a particle is produced, and a calculation is done according to the above rules. So it will be a coherent set design. The neighbors in DLA model are not equally likely to be attached to the cluster. Instead, the next particle is chosen among all neighbors with a distribution proportional to the equilibrium electrostatic potential on the boundary of the existing cluster, which is the solution of $\Delta u = 0$ where $\Delta = \nabla^2$ is the Laplace operator. Therefore, DLA captures the essential features of a typical dynamic growth process that is related to the Laplace equation. If the particles on the random walk are beyond the scope of limit state matrix, then the corresponding boundary condition treatment scheme should be considered as (1) the periodic boundary conditions, the particle will not escape, and it will produce a new particle after the old particle is adsorbed; (2) the absorption boundary condition, the particle will be given up when it is beyond the border anda new particle generates.

The advanced features of the DLA model are as follows: (1) with a very simple algorithm, the DLA embody the primary components in a wide range of natural phenomena but the physical mechanism does not need clear forms; (2) the self-similar fractal structure with invariance scale can be generated through a simple kinematics and dynamics process, and the mechanism of fractal growth in practical system is revealed in certain extent; (3) the interface has complex shape and instability, and the growth process is a process far from equilibrium dynamics, but cluster structure is stable and has determined fractal dimension. For more than 20 years, researchers with great interest in the mechanism of DLA model carried out extensive study, and rich theoretical research results are obtained, and the achievement is applied in the fields including fractal physics, environmental science, materials science, and urban planning [12].

The applications of diffusion-limited agglomeration theory in civil engineering are very few; in this paper, the particles are used to simulate the chemical material in microlevel, and the arrangement is controlled by the algorithm of DLA. The concentration of chlorine ion and carbon dioxin in the corresponding depth can be calculated according to the theoretical formula; thus a determined number of particles which randomly move in the horizontal direction are placed in the concrete. Hence, the randomness in the location and the precision in the calculation are simultaneously embodied through DLA model. In addition, the tiny cracks will occur when the reinforced concrete beam is subjected to the bending moment, and the chloride ion erosion and the carbonization will aggravate near the cracks, and the coupling phenomenon can be simulated in detail by the mutual adsorption display function in the DLA model. The durability analysis based on the diffusion-limited aggregation has important research significance and broad application prospects.

## 3. Chloride Ion Erosion Based on the DLA

If the pore distribution in concrete is assumed to be uniform, the chloride ions do not react with concrete, and the relative concrete exposed surface is a semi-infinite medium body; thus the chlorine ion in the intrusion of saturated concrete meet the second Fick diffusion law [2]. The one-dimensional diffusion equation is as follows:

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial x^2}, \tag{1}$$

where $t$ is the time of chloride ion erosion, $x$ is the distance from the concrete surface to the specified location, $c$ is the chloride ion concentration on the specified location, and $D$ is the chloride ion diffusion coefficient. The initial condition is $C(x,0) = 0$, and the boundary condition is $C(0,t) = C_s$ and $C(\infty,t) = C_0$. The solution of (1) is

$$C = C_0 + \left(C_s - C_0\right)\left[1 - \operatorname{erf}\frac{x}{2\sqrt{Dt}}\right], \tag{2}$$

where $C_0$ is the initial concentration of chloride ions in concrete, and $C_s$ is the chloride ion concentration on the exposed surface, and $\operatorname{erf}(z)$ is the error function written as follows:

$$\operatorname{erf}(z) = \frac{2}{\sqrt{\pi}}\int_0^z \exp\left(-u^2\right)\mathrm{d}u. \tag{3}$$

For the chloride ion diffusion process in complex condition, especially for the chloride ion diffusion based on the principle of diffusion-limited aggregation [13], the diffusion model considering multiple factors is advised to be written as follows:

$$\begin{aligned} C_f &= C_0 + \left(C_s - C_0\right) \\ &\quad \cdot \left[1 - \operatorname{erf}\right. \\ &\quad \cdot \left((x)\right. \\ &\quad \left.\left.\cdot \left(2\sqrt{\frac{KD_0Tt_0{}^m}{(1+R)(1-m)T_0}e^{q[1/T_0-1/T]}t^{1-m}}\right)^{-1}\right)\right], \end{aligned} \tag{4}$$

where $c_t$ is the total chlorine ion concentration at $t$ moment in the location which has the distance $x$ from the surface, $c_b$ is the combination of chlorine ion concentration, and $c_z$ is the free chlorine ion concentration. $R$ is the chloride ion combining ability, and $R = c_b/c_z = (c_t - c_z)/c_z$. $m$ is the experimental constant, $K$ is the degradation effect coefficient for the performance of concrete chloride ion diffusion, $T$ is temperature, $D_0$ is the chloride ion diffusion coefficient on temperature $T_0$, and $t_0$ is the concrete hydration age for $q$ is the activated constant, which associated with water cement ratio [14]; when $m_w/m_c = 0.4$, $q$ is 6000 K, when $m_w/m_c = 0.5$, $q$ is 5450 K, and when $m_w/m_c = 0.6$, $q$ is 3850 K.

The chlorine ion concentration in the location which has the distance $x$ from the surface can be calculated according to (4); thus, the corresponding number of chloride ions which moves at random in the horizontal direction can be arranged in the location of $x$, by using the theory of diffusion-limited aggregation. When the chloride ions approach others, adsorption will occur and the combination of the chloride ion diffusion equations and the DLA algorithm is realized.

According to the above basic idea, the flow chart of DLA model considering the chloride ion erosion calculation is proposed, as shown in Figure 1.

Assuming the concrete surface chloride ion concentration is known as 2.0 Kg/m$^3$, the original concentration of chloride ion in concrete was 0.01 Kg/m$^3$ and the concrete hydration age is 28 days. The protective layer thickness is 35 mm, the diameter of the steel bar is 25 mm, the water cement ratio is 0.4, and the temperature is 20 degrees Celsius; the diffusion coefficient can be obtained through the experiment, and $2.5 \times 10^{-11}$ m$^2$/s is determined in this study. When the chloride ion erosion time is 10 years, the chloride ion erosion model based on DLA simulation results is shown in Figure 2. It is obvious that the DLA model can intensively reveal the randomness and uncertainty in the process of chloride ion erosion in concrete, and it can simultaneously reflect the concentration variation and performance evolution in the process of chloride ion propagation accurately.

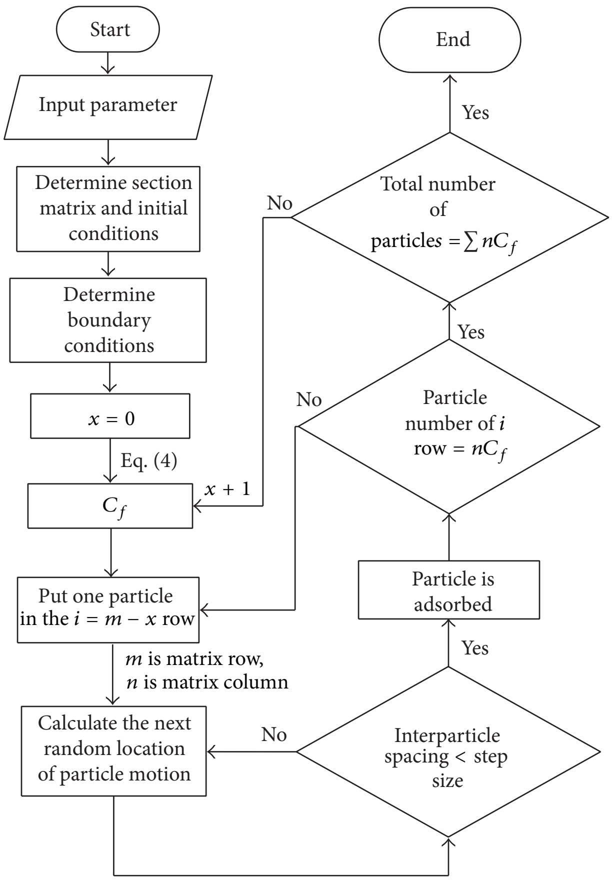



Figure 1: Flow chart of chloride attack in the DLA model.

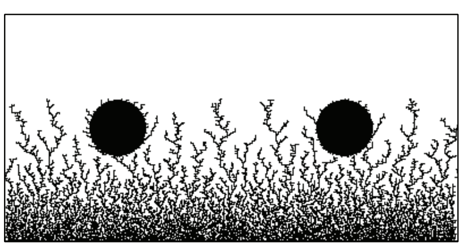


Figure 2: Chloride attack in 10 years based on DLA.

## 4. Reinforcement Corrosion under Chloride Ion Erosion

The chloride ion erosion accumulates in the concrete under severe conditions, and the corrosion will occur in the reinforcement bar when the chloride ion concentration reaches a critical value. There are two criteria to predicate the critical concentration, one is the chloride ion concentration, and the other is the ratio of chloride ion to hydroxide ion, that is, $[Cl^-/OH^-]$. A new criteria was proposed to apply in engineering structures; for a new structure, $0.6\,Kg/m^3$ is defined, and $0.6\,Kg/m^3$–$0.9\,Kg/m^3$ can be chosen for important existing structures, and $1.0\,Kg/m^3$ is for normal structures [15]. In this paper, $0.6\,Kg/m^3$ is defined as the critical concentration. The exposure condition of the case is in offshore atmospheric environment which is about 0.1 km far from the sea, and the average chloride concentration on concrete surface is $3.0\,Kg/m^3$ based on empirical data. Assuming that the chloride ion concentration on the concrete surface is $3.0\,Kg/m^3$, other parameters are the same as the previous section. The initial corrosion time is 9 years according to (4), and the destruction process of concrete structure is shown in Figure 3 as the DLA algorithm is used.

The quantity formula of the reinforcement corrosion in the corrosion time $t$ is as follows:

$$\Delta g = \frac{NS\int i_{\mathrm{corr}}\,\mathrm{d}t}{F}, \tag{5}$$

where $N$ is the molar mass of the reinforcement, and $N = g/n$; $F$ is Faraday constants, and $1F = 96500\,\mathrm{C/mol} = 26.8\,\mathrm{A{\cdot}h/mol}$. $S$ is the surface area of the reinforcement in the concrete and the unit is $cm^2$; $i_{\mathrm{corr}}$ is the corrosion current density and the unit is $A/cm^2$.

After considering the comprehensive function of factors such as chloride ion concentration, layer resistance, and temperature, the corrosion current density model is obtained by the regression analysis of the test results as follows:

$$\begin{aligned}\ln 1.08i &= 8.37 + 0.618\ln 169\mathrm{Cl}\\ &\quad - \frac{3034}{T} - 0.000105R_c + 2.32t^{-0.215},\end{aligned} \tag{6}$$

where $i$ is the corrosion current density and the unit is $\mu A/cm^2$. Cl is the chlorine ion around the reinforcement and the unit is $Kg/m^3$; $T$ is degree Fahrenheit; $R_c$ is the resistance of concrete cover and the unit is $\Omega$; $t$ is the time after rust begin and the unit is year.

The average corrosion rate based on the corrosion amount of the reinforcement is given by

$$\eta = \frac{\Delta g}{g_0} = \frac{NS\int i_{\mathrm{corr}}\,\mathrm{d}t}{Fg_0}, \tag{7}$$

where $g_0$ is the original quality of the rebar.

After studying the bending test of the reinforced concrete members under the coastal environment, the following model of steel corrosion current density is presented as [16]

$$i_{\mathrm{corr}}(t) = 0.3683\ln(t) + 1.1305. \tag{8}$$

Substituting (6) into (7) gives

$$\eta = \frac{\Delta g}{g_0} = \frac{NS\int(0.3683\ln(t) + 1.1305)\,\mathrm{d}t}{Fg_0}. \tag{9}$$

## 5. Corrosive Cracks Analysis under Chloride Ion Erosion

The reinforcement corrosion can cause the rust expansion and the cracking and spalling in the concrete cover, furthermore, inducing the deterioration of the structural bearing capacity and durability. The study on the cover-cracking

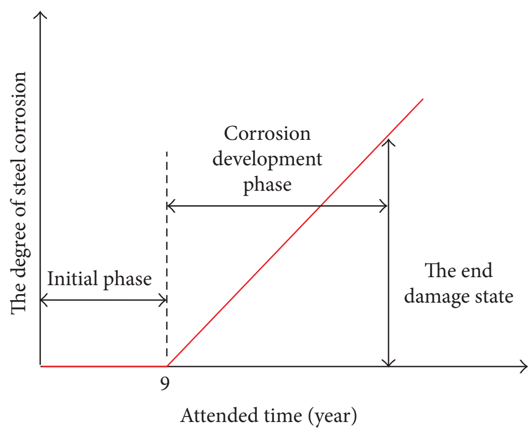



Figure 3: Corrosion process under chloride attack.

induced by steel corrosion mainly focuses on the early stage, and less research is carried out for the damage mechanism after the corrosion cracking. Because the corrosion state should be determined in the case of nondestructive operation in the actual engineering, the relationship between the corrosion rate and the corrosive crack width is necessary to the durability evaluation. At present, the corresponding function can be obtained by the methods, including normal test, elastic mechanics method, and finite element method (FEM), and mostly through the experienced formula of the accelerated corrosion experiments.

François and Arliguie [17] and Vidal et al. [18] focused on the effects on cracks to the reinforcement corrosion in reinforced concrete structure and established the life prediction models based on crack width and steel cross-sectional loss. Arya and Wood [19] studied the effect of crack spacing and different forms of cracks to the corrosion, and the cracks are divided into the lateral cracks, vertical cracks, live cracks, and dormant cracks; furthermore, the impact of different types of steel corrosion cracks was studied.

Assuming the reinforced depth rusty and crack width have a linearly proportional relationship and they are independent of the bar diameter, the cover thickness, the concrete strength, and other parameters; the equation on the steel corrosion rate and the corrosive crack width of concrete can be expressed in the form based on the impressed current accelerated corrosion of corrosive test according to [20] as

$$\eta_s = 4\left[\frac{1}{d}(0.222w - 0.011) - \frac{1}{d^2}(0.222w - 0.011)^2\right], \quad (10)$$

where $\eta_s$ is the rebar corrosion rate, $d$ is the rebar diameter, and $w$ is the corrosive crack width of concrete.

Assuming the material parameters are as stated above, the concrete chloride ion erosion model considering the corrosive cracks is established based on DLA algorithm and the formula above, as shown in Figure 4. It is obvious that the chloride ion erosion model based on DLA can dynamically reveal the diffusion state of chloride ions at different time points and the shape of the apparent derivative corrosion crack. Compared with the existing simulation methods, the DLA model has the advantages in the fineness, the accuracy, the computational efficiency, and the dynamic display.

## 6. Carbonation Analysis Based on DLA

As same as the chloride ion erosion, the concrete carbonation is also an important factor for the durability deterioration of the material and reinforced structure. Carbonation is the result of the carbon dioxide diffuse from the environment of the concrete inside, usually changing the chemical composition and organization of concrete structure and the mechanical properties of concrete vary significantly.

Peter et al. [21] theoretically analyzed the carbonation reaction and provide a general theory for carbonation assessment. Papadakis et al. [22] deduced the classic experiment carbonation coefficients according to the content of minerals and cement hydration products. Baroghel-Bouny [23] established a model based on high concentrations of carbonation of concrete porosity and water saturation rate, thus providing a theoretical basis for on-site testing for carbonation. Saetta and Vitaliani [24] and Isgor and Razaqpur [25] established the uncracked concrete carbonation established theoretical model according to the mathematical theory and finite element method. In addition, Song et al. [26], Alahmad et al. [27], and Niu [28] systematically analyzed the effect of the cracks on the carbon dioxide transport and studied the cracking concrete carbonation process and carbonation models.

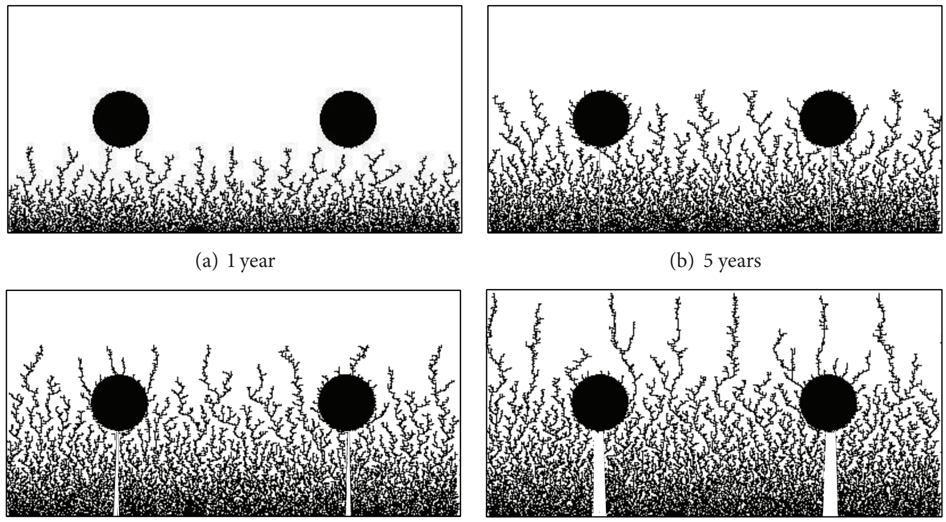
The theory on the concrete carbonation can be studied based on diffusion theory, and the assumptions are as follows: (1) the carbon dioxide concentration in the concrete is linear distribution; (2) the carbon dioxide concentration on the concrete surface is equal to the concentration of the external environmental condition and the concentration of noncarbonation area is 0; (3) the quantity of unit volume to absorb carbon dioxide is a constant value.

Under these assumptions, the concrete carbonation process follows Fick's first law of diffusion, and the formula for calculating the concrete carbonation depth can be derived as follows:

$$X = \sqrt{\frac{2D_{CO_2}C_{CO_2}}{M_{CO_2}}}\sqrt{t}, \quad (11)$$

where $X$ is the carbonation depth, $D_{CO_2}$ is the effective diffusion coefficient of the carbon dioxide in concrete, and $C_{CO_2}$ is the concrete surface carbon dioxide concentrations. $M_{CO_2}$ is the absorption amount of the carbon dioxide in unit volume concrete and $t$ is the carbonization time.

On the grounds of the theoretical model of concrete carbonation, a modified carbonation depth model is presented, and two major acting factors including the environmental conditions and the concrete quality are considered in the model. Besides, other factors such as the carbonation location, the casting surface, and the working stress are also



(a) 1 year

(b) 5 years

(c) 25 years

(d) 50 years

Figure 4: DLA evolution model for chloride attack.

involved [26–28]. The stochastic forecasting model is given by

$$X_c = K_{\mathrm{mc}} \cdot k_j \cdot k_{\mathrm{co}_2} \cdot k_p \cdot k_s \cdot K_e \cdot K_f \sqrt{t}, \tag{12}$$

where $K_{\mathrm{mc}}$ is the random variable which represents the difference between the carbonation model calculation and the actual test; at the same time, some other minor random factors are also included. $k_j$ is the correction coefficient on concrete corner, taken as 1.4 for corner and 1.0 for normal location. $k_{\mathrm{co}_2}$ is the efficient for carbon dioxide concentration; $k_p$ is the correction coefficient for casting surface, taken as 1.2 for the casting surface and 1.0 for normal surface. $k_s$ is the influence coefficient for working stress, taken as 1.1 for the concrete in tension, and 1.0 for the compressed concrete. $K_e$ is the environment influence coefficient and $K_f$ is the influence coefficient for concrete quality.

For the carbonation model using diffusion-limited aggregation algorithm, the modified carbonation depth can be calculated by (12), and the carbon dioxide concentration in the location which is $x$ far from the surface can be deduced, supposing the carbon dioxide concentration in concrete is linear distribution. In the DLA model, the corresponding amount of carbon dioxide is arranged to be in random motion and these carbon dioxide particles will adsorb together once their mutual distance is less than the given value.

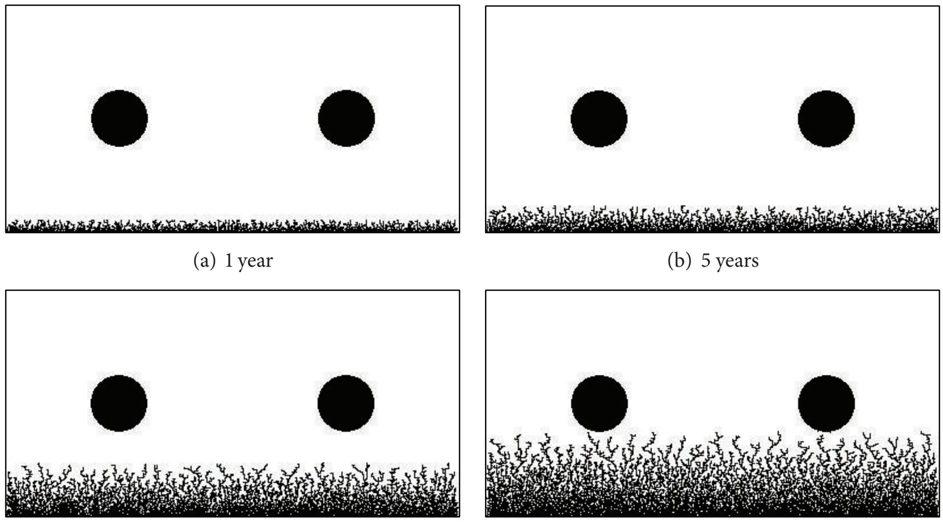
Assuming the carbonization time is 1 year, 10 years, 25 years, and 50 years, respectively, the concrete strength is 20 Mpa, the temperature is 21 degrees Celsius, and the relative humidity is 0.8. The efficient for carbon dioxide concentration is 1.5, the protective layer thickness is 35 mm, and the diameter of rebar is 25 mm. The carbonation model based on DLA is shown in Figure 5. It is apparent that the concrete carbonation based on DLA model can simulate the all evolutionary process and display the carbonizing state at different time points, perfectly having both accuracy and randomness.

## 7. DLA Model for Coupling Action of Chloride Ion Erosion and Carbonization

In actual environment, the decrease of structural performance due to the deterioration of the durability is usually the integrated consequence of multiple factors such as variable loads, climate, and service conditions. The deterioration process is not the simple superposition of each single factor but the interaction and accumulation of all the factors; this phenomenon causes the concrete deterioration which is more complicated and the related conclusions and experience formula based on the single factor effect have certain limitations. Thus, study on the durability under multiple factors is more intricate.

The study on the carbonization reaction can destruct the original filtering mechanism in the basal body of concrete [29] and promote the Friedel salt decomposition, and the chloride ion content increases. The carbonization can slightly reduce the chloride ion diffusion coefficient, and the chloride ion concentration has a maximum in the carbide cutting edge [30]. The experiments on the coupling effect of carbonation and chloride ion erosion are generally carried out considering limited factors and the specific conditions. The common qualitative conclusion is that the chloride ion concentration has less effect on the depth of carbonation but the carbonization can evoke the decline of the alkalinity in the pore solution, and the corrosion of the rebar placed on the carbonated concrete under the erosion of chloride ion will be more serious. Hence, the effect of carbonization on the chloride ion erosion is more obvious, which has more related factors.

The theoretical model and achievement of the coupling effect of carbonation and chloride ion erosion are little. Beaudoin [31] proposed the modified coefficient of the chloride ion diffusion under carbonation; the total chlorine ion content will increase under the carbonation; that is, the constrained chloride ion content on the surface is proportional



(a) 1 year (b) 5 years (c) 25 years (d) 50 years

Figure 5: DLA evolution model for carbonation.

to the relative increased free chloride ion content, and the modified carbonation coefficient is given by

$$\alpha = \frac{\mathrm{Cl}^-_{\mathrm{lib}}}{\mathrm{Cl}^-_{\mathrm{lib}} + \mathrm{Cl}^-_{\mathrm{free}}} = \frac{C_s' - C_s}{C_s'}, \tag{13}$$

where $\alpha$ is the carbonation modified coefficient, $\mathrm{Cl}^-_{\mathrm{lib}}$ is the free chlorine ion content under the action of carbonation, $\mathrm{Cl}^-_{\mathrm{free}}$ is the original free chlorine ion content, $C_s$ is the original chloride ion concentration on the concrete surface, and $C_s'$ is the chloride ion concentration on the concrete surface under carbonation.

The concentration of the internal free chloride ion is assumed to be higher than the concentration of the outside free chloride ion under carbonation action, and the internal free chloride ions will spread to the external environment; thus a modified factor $\beta$ is introduced as follows:

$$C_s' = \frac{\beta C_s}{1-\alpha}. \tag{14}$$

In this assumption, the chloride ion erosion depth is constant due to the fact that the carbonation depth is irrelevant to the carbon concentration on the concrete surface. However, the chloride ion erosion depth can actually increase and the initial corrosion time will be reduced; thus, the formula on the modified erosion depth is adopted as follows:

$$I_{\mathrm{cl;ca}} = \frac{\left(1 - \sqrt{c_{\mathrm{crit}}/c_s'}\right)^2}{\left(1 - \sqrt{c_{\mathrm{crit}}/c_s}\right)^2}, \tag{15}$$

where $c_{\mathrm{crit}}$ is the chloride ion concentration of the location whose distance from the surface is $x$.

According to the above discussion, the DLA model on the chloride ion erosion and the DLA model on the concrete carbonation can be combined and calculate the comprehensive damage and deterioration. The flow chart of DLA considering coupling action is shown in Figure 6. By determining the modified coefficient of carbonation, the surface chloride ion concentration under the carbonation action and chloride ion erosion depth can be calculated simultaneously. Thus, the coupling DLA model on both chloride ion erosion and carbonation can be established.

Assuming the example parameters are still as stated earlier, the influence coefficient of the carbon dioxide concentration is 1.5, and the evolution results obtained from the coupling model are shown in Figure 7. Compared with the results in Figure 4, it can be seen that the severity of the carbonation and the chloride ion erosion aggravates after considering the coupling effect, and the corrosive crack width which is induced by the chloride ion erosion in the concrete will obviously enlarge.

These results indicate that the durability degradation phenomenon can be precisely simulated considering the coupling action of both the chloride ion erosion and the carbonization, and DLA method has better ability than traditional methods. In addition, the chloride ion concentration increases with the chloride ion erosion depth and the chloride ion erosion is the primary factor and the influence of the carbonation is less, especially for the environment where the chlorine ion concentration is higher, such as the coastal region. Hence, only the effect of the carbonization on the chloride ion erosion is usually discussed and studied.

## 8. Durability Analysis for Bridge under Coupling Effect and Fatigue Condition

The reinforced bridges in service are subjected to coupling effect of the chloride ion erosion and the carbonization, so it is suitable to carry out the durability analysis by using the

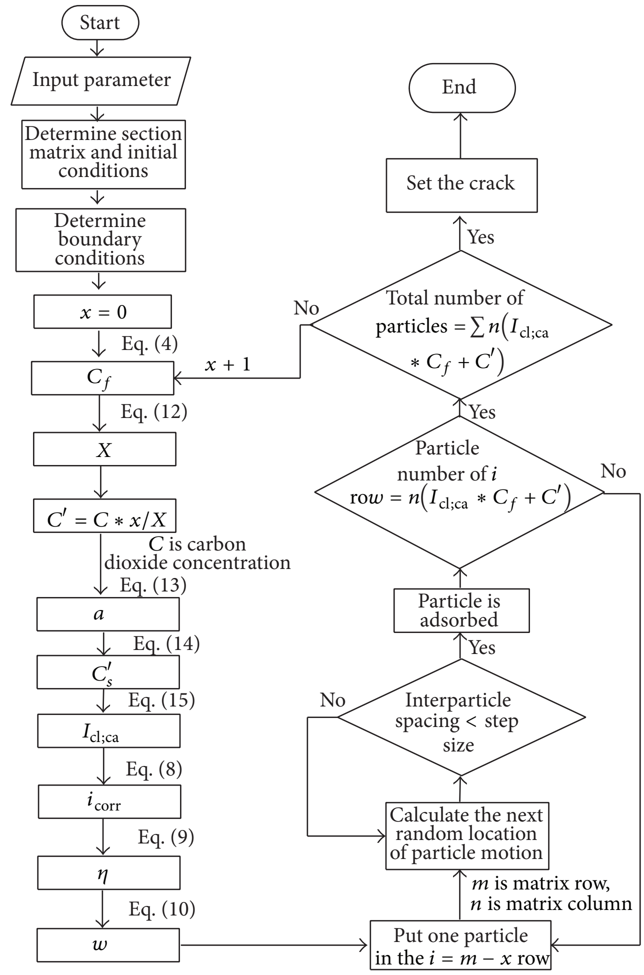



Figure 6: DLA flow chart of coupling action.

coupling DLA model proposed in this study. In addition, due to the reinforced concrete bridges bear also bear the effect of traffic load, the fatigue failure will occur under the action of cyclic loading although the actual stress is less than the yield strength. The rebar corrosion induced by the coupling effect of the chloride ion erosion and carbonization can not only reduces the effective cross section but also increases the fatigue stress amplitude; at the same time the steel fatigue strength decreases; thus, the service life of the structure will seriously shorten.

Therefore, for the reinforced concrete bridges under the condition of traffic load and complicated environment, the study on the time-varying fatigue durability considering the coupled action of chloride ion erosion and carbonization is significant.

In the effect of the chloride ion erosion, the corrosion rate will increase and the stress concentration will arise from the partial dents on the rebar. Under fatigue load, the chloride ion diffusion rate increases, the corrosion aggravates, and the fatigue resistance degrades. According to the data from the reinforcement degradation test with different corrosion states, the time-varying model for the corrosion depth and fatigue strength [32] is concluded as

$$\varphi(t) = \begin{cases} 1.066 & \Delta\delta < 1.23\,\text{mm} \\ 1.066 - 0.002e^{2.859\Delta\delta(t)} & \Delta\delta \geqslant 1.23\,\text{mm}, \end{cases} \tag{16}$$

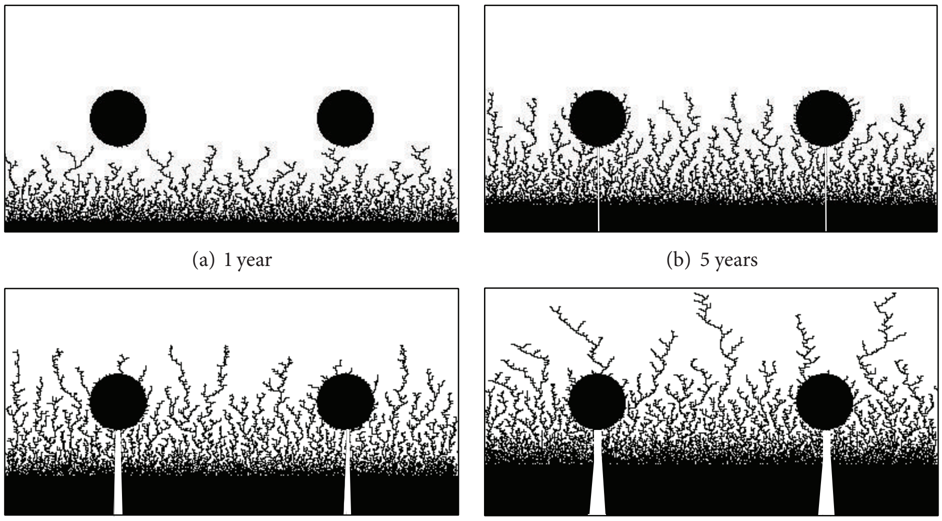
where $\varphi(t)$ is the steel fatigue strength coefficient; $\Delta\delta$ is the reinforcement corrosion depth, which can be obtained from the rebar corrosion rate. According to (10)–(16) and simulated results from the coupling DLA model, the reinforced time-varying fatigue strength and the integral durability index can be calculated.

In order to verify the accuracy of the method in this paper, a simply supported reinforced concrete highway bridge with T section is chosen as an example, and the cross section diagram is shown in Figure 8. The span is 16 m, the axial compression strength of the concrete is 20.7 N/mm$^2$, and protective layer thickness is 30 mm. The yield strength of the rebar is 235 N/mm$^2$, 12 rebars are set in the beam bottom, the diameter of the six main rebar is 32 mm, and the diameter of other rebar is 16 mm. The surface chloride ion concentration of the concrete is 3.0 Kg/m$^3$, and the original chloride ions concentration in the concrete is 0.01 Kg/m$^3$, the concrete hydration age is 28 days, the water cement ratio is 0.4, the environmental temperature is 20 degrees Celsius, and the humidity is 0.8. The diffusion coefficient can be obtained through the experiment, and $2.5 \times 10^{-11}$ m$^2$/s is assumed in this study. The carbon dioxide concentration influence coefficient is 1.5.

The action time for the environmental factors and fatigue load is from 1 year to 50 years, and the evolutionary process of the material performance of the main sections is calculated based on the coupling DLA model, considering the effect of both the chloride ion erosion and carbonization. The concrete carbonation, chloride ion erosion, and the corrosive crack propagation state of the midspan cross section after 25 years are shown in Figure 9. It can be seen that the cracks in the midspan cross section grown rapidly after the bridge served 25 years and the maximum width has been larger than the limit value 0.3 mm; thus major repair should adopt for the upper structure. Furthermore, it is verified that the DLA model can integrate the theoretical equations and simulation algorithm, and the coupling evolution process of the chloride ion erosion and carbonization can be dynamically and continuously displayed. Hence, the DLA model is appropriate for the engineering application.

The evolutionary process of the chloride ion concentration on the surface of the rebar located at the bottom of the cross section is shown in Figure 10. The evolution process about the concrete carbonation depth distribution is shown in Figure 11. The development trend about the rebar corrosion rate and the fatigue strength reduction coefficient of the rebar located at the bottom of the cross section under different service time is shown in Figures 12 and 13, respectively.

According to the results and comprehensive analysis, it can be seen that the increment speed of both the chloride ion concentration on the surface of the rebar and the concrete carbonation depth will diminish with the service time, and the fatigue strength of the rebar will reduce in the corrosion



(a) 1 year

(b) 5 years

(c) 25 years

(d) 50 years

Figure 7: DLA evolution model for coupling action.

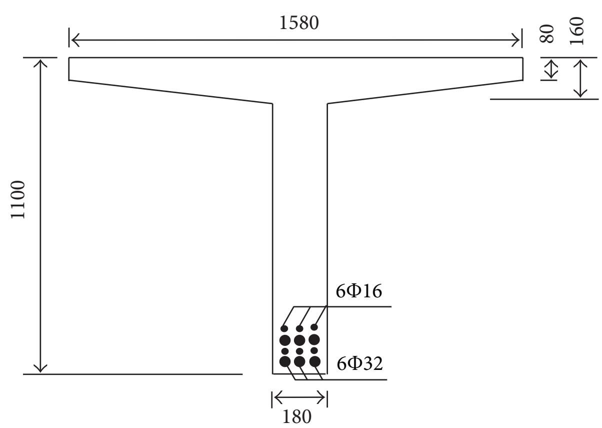



Figure 8: Section of T type RC beam.

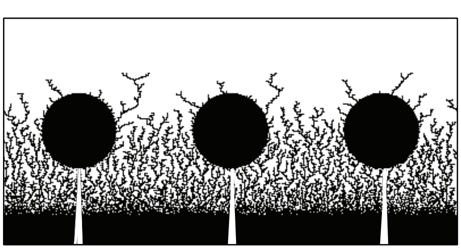


Figure 9: Evolutionary results for coupling action in 25 years.

environment induced by the chloride ion erosion. The rebar in the T type beam begun to rust in the 9th service year, and the corrosive cracks gradually expand under fatigue loads and environmental factors. The general durability of the bridge significantly decline until the 20th service year and necessary maintenance and repair strategy should be adopted.

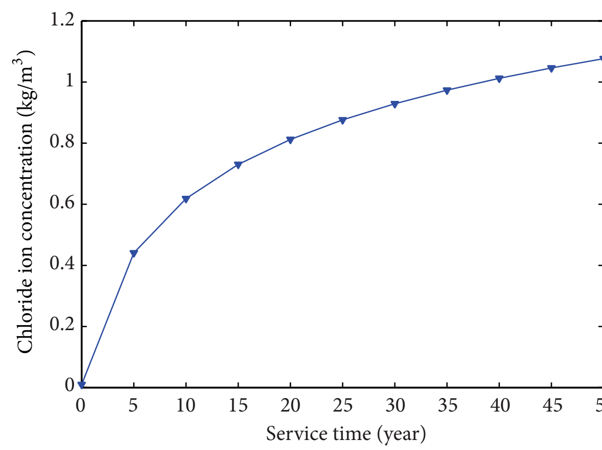



Figure 10: Evolutionary process of chloride concentration.

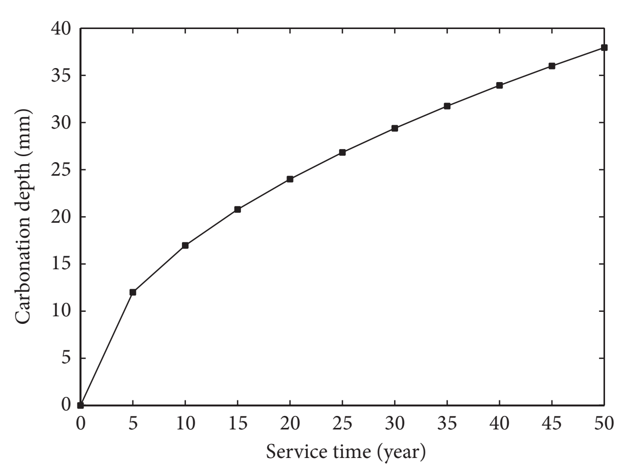



Figure 11: Evolutionary process of carbonation depth.

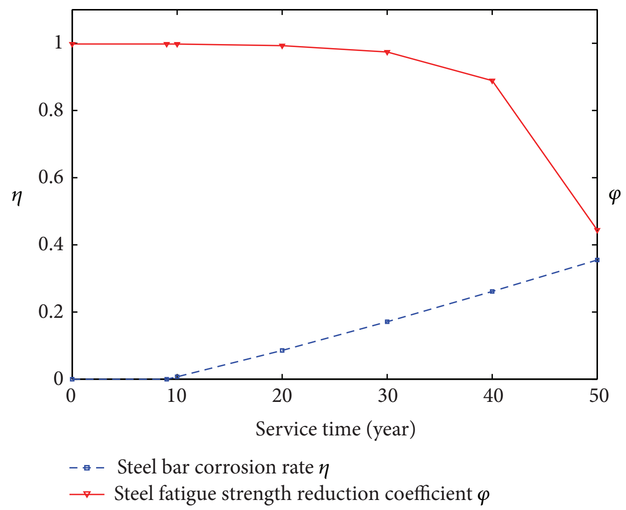



Figure 12: Rebar corrosion rate and fatigue strength reduction coefficient evolutionary process.

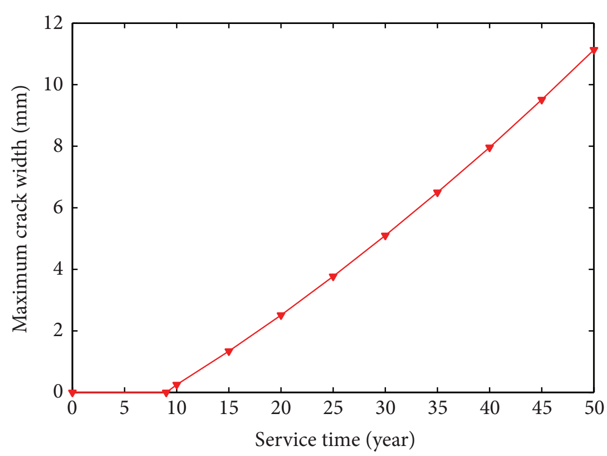



Figure 13: Evolutionary process of maximum width of cracks.

## 9. Conclusions

In this paper, an evolution model based on diffusion-limited aggregation is presented for the durability analysis of the reinforced concrete bridge, and the coupling action of the chloride ion erosion and the carbonization is considered. The evolutionary process of the chloride ion erosion, the carbonization, and the coupling action is simulated based on the chloride ion diffusion equation and carbonation depth equation.

The DLA model can meet the actual conditions of the diffusion of the chloride ion and carbon. Meanwhile, it also embodies the randomness by using the diffusion-limited aggregation principle. In addition, the DLA model can simulate the evolutionary process of microcracks, especially for the reinforced concrete structure subjected to bending moment, chloride ion erosion, and carbonation.

The DLA model can be used to simulate the concrete cracks of steel bar under the corrosive action according to the formula about the rebar corrosion rate and corrosive action, and the results are detailed and precise.

Through the relationship between the corrosion depth and the fatigue strength reduction, the fatigue strength under the coupling of the chloride ion erosion and the carbonization can be calculated, and the phenomena are also revealed by the DLA model. In general, the DLA model can be utilized to analyze the durability of the reinforced concrete bridge in all the service period.

It should be pointed out that there are many aspects need to be further studied in performance simulation by the DLA model, such as the application of the multidimensional diffusion equations, and the differences in different cross sections and lateral sections, and the effect on bond-slip effect between rebar and concrete in the final service time.

## Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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