Research Article

Parallel-Machine Scheduling with Time-Dependent and Machine Availability Constraints

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We consider the parallel-machine scheduling problem in which the machines have availability constraints and the processing time of each job is simple linear increasing function of its starting times. For the makespan minimization problem, which is NP-hard in the strong sense, we discuss the Longest Deteriorating Rate algorithm and List Scheduling algorithm; we also provide a lower bound of any optimal schedule. For the total completion time minimization problem, we analyze the strong NP-hardness, and we present a dynamic programming algorithm and a fully polynomial time approximation scheme for the two-machine problem. Furthermore, we extended the dynamic programming algorithm to the total weighted completion time minimization problem.

1. Introduction

Consider the following scheduling problem with deteriorating job and machine availability constraints. There are n independent deteriorating jobs $J = \{J_1, \dots, J_n\}$ to be processed on m identical parallel machines. The actual processing time of job J_j ($j = 1, \dots, n$) is $p_j = \alpha_j t$, where α_j (> 0) and t are the deteriorating rate and the starting time of job J_j , respectively. Each job J_j has a weight w_j . All jobs are released at time t_0 (> 0). The case with $t_0 = 0$ is not considered because the job would have its processing times equal to zero if $t_0 = 0$. We assume that the jobs are nonresumable in our problems. Machine M_i ($1 \leq i \leq m$) is not continuously available and unavailable during the period $I_i = [H_1^{(i)}, H_2^{(i)}]$. In addition, we assume that $t_0(1 + \alpha_{\max}) < H_1^i < H_2^i$ and $H_1^{(i)} < t_0 \prod_{j=1}^n (1 + \alpha_j)$, where $\alpha_{\max} = \max_{j=1, \dots, n} \{\alpha_j\}$. Otherwise, all the jobs can be finished before the nonavailable period and the problem becomes trivial. Without loss of generality, we assume that all parameters are integral unless stated otherwise.

Our objective is to minimize the makespan and the total (weighted) completion time. Following Gawiejnowicz [1], we denote our problems as $P_m|nr - a, p_j = \alpha_j t|C_{\max}$ and $P_m|nr - a, p_j = \alpha_j t|\sum(w_j)C_j$, where $nr - a$ means nonresumable.

The model described above falls into the categories of the scheduling with deteriorating job and the machine

scheduling with availability constraints. The scheduling with deteriorating job was first considered by J. N. D. Gupta and S. K. Gupta [2] and Browne and Yechiali [3]. Cheng et al. [4] gave a survey and the monograph by Gawiejnowicz [1] presented this scheduling from different perspectives and covers results and examples. Graves and Lee [5] pointed out that machine scheduling with an availability constraint is very important and is still relatively unexplored. They studied this problem whose maintenance needs to be performed within a fixed period. Lee [6] presented extensive study of single and parallel machine scheduling problems with an availability constraint, with respect to various performance measures and two cases are considered: resumable and nonresumable. A job is said to be resumable if it cannot be finished before the nonavailable interval of a machine and can continue after the machine is available again. On the other hand, a job is said to be nonresumable if it has to restart rather than continue. Ma et al. [7] gave a survey of this scheduling.

In this paper, we consider the deteriorating job scheduling with machine availability constraints on m identical parallel machines. The jobs are nonresumable and our objective is to minimize the makespan and the total (weighted) completion time.

Relevant Previous Work. Wu and Lee [8] initiated the deteriorating job scheduling with machine availability constraints;

they showed that minimizing the makespan of scheduling deteriorating jobs on a single machine with an availability constraint can be transformed into 0-1 integer programming. Ji et al. [9] gave some results for the linear deteriorating jobs with an availability constraint on a single machine. Gawiejnowicz and Kononov [10] considered the complexity and approximability of scheduling resumable proportionally deteriorating jobs. Fan et al. [11] considered the scheduling resumable deteriorating jobs on a single machine with nonavailability constraints. Li and Fan [12] addressed the nonresumable scheduling problem $1|nr-a, p_j = \alpha_j t| \sum w_j C_j$. The problems they considered are on the single machine. In this paper, we consider the parallel-machine scheduling problem with deteriorating jobs and machine availability constraints, and we show that the problems are strongly NP-hard and present some algorithms.

2. Minimizing the Makespan

In this section, we first show that problem $P_m|nr-a, p_j = \alpha_j t|C_{\max}$ is NP-hard in the strong sense. Ji and Cheng [13] showed that problem $P_m|p_j = \alpha_j t|C_{\max}$ is NP-hard in the strong sense when m is arbitrary. In their problem, all the machines are available all the time. Thus, our problem $P_m|nr-a, p_j = \alpha_j t|C_{\max}$ is NP-hard in the strong sense when m is arbitrary.

In the following, we discuss the Longest Deteriorating Rate (LDR for short) algorithm and List Scheduling (LS for short) algorithm and analyze a lower bound of any optimal schedule.

2.1. LDR and LS Algorithms

LS Algorithm. Given a sequence of jobs $J_1, \dots, J_n$, assign the jobs one by one according to the list. Each job is assigned to the machine where the job can be finished as early as possible.

LDR Algorithm. Sort the jobs in the nonincreasing order of their deteriorating rates, and then assign the jobs by LS algorithm.

$C_{\max}^{\text{LDR}}, C_{\max}^{\text{LS}}$, and $C_{\max}^*$ denote the makespan corresponding LDR, LS, and the optimal solution, respectively.

Theorem 1. $C_{\max}^{\text{LDR}}/C_{\max}^*$ can be arbitrarily large even for the two-machine problem $P_2|nr-a, p_j = \alpha_j t|C_{\max}$.

Proof. Consider a problem with the following instance: $t_0 = 1, \alpha_1 = \alpha_2 = 7, \alpha_3 = \alpha_4 = \alpha_5 = 3, H_1^{(1)} = H_1^{(2)} = 64, H_2^{(1)} = H_2^{(2)} = n$, and $m = 2$. In the optimal schedule with makespan $C_{\max}^* = 64$, jobs J_1 and J_2 are scheduled before $H_1^{(1)}$ on the first machine, and jobs J_3, J_4 , and J_5 are scheduled before $H_1^{(2)}$ on the second machine. However, $C_{\max}^{\text{LDR}} = 4n$. As a consequence, $C_{\max}^{\text{LDR}}/C_{\max}^* = 4n/64 \rightarrow \infty$ when $n \rightarrow \infty$. $\square$

Theorem 2. If $\min_{i=1, \dots, m} \{H_1^{(i)}\} \geq t_0 \prod_{j=1}^n (1 + \alpha_j)$, then $C_{\max}^{\text{LS}}/C_{\max}^* \leq (1 + \alpha_{\max})^{(m-1)/m}$ and $C_{\max}^{\text{LDR}}/C_{\max}^* \leq (1 +$

$\alpha_{\min})^{(m-1)/m}$, where $\alpha_{\max} = \max_{j=1, \dots, n} \{\alpha_j\}$ and $\alpha_{\min} = \min_{j=1, \dots, n} \{\alpha_j\}$.

The proof of this theorem is similar to the proofs of Theorems 1 and 3 in Liu et al. [14].

2.2. Lower Bound of Any Optimal Schedule. Without loss of generality, let $t_0 = 1, H_1 = \max_{i=1, \dots, m} \{H_1^{(i)}\}$, and $H_2 = \min_{i=1, \dots, m} \{H_2^{(i)}\}$.

Theorem 3.

$$C_{\max}^* \geq \frac{H_2 (1 + \alpha_{\min})^{n/m}}{H_1}. \quad (1)$$

Proof. In any optimal schedule, let J_{S_i} and $J_{S_i}^B$ denote the set of jobs scheduled on machine M_i and before H_1^i on M_i ($i = 1, \dots, m$), respectively. Then, the set of jobs scheduled after H_2^i is $J_{S_i}/J_{S_i}^B$, and $\prod_{J_j \in J_{S_i}^B} (1 + \alpha_j) \leq H_1^i$. We have the load of machine M_i denoted by L_i holding $L_i = H_2^i \prod_{J_j \in J_{S_i}/J_{S_i}^B} (1 + \alpha_j) \geq H_2^i \prod_{J_j \in J_{S_i}} (1 + \alpha_j)/H_1^i$ for $i = 1, \dots, m$. Note that $C_{\max}^* = \max_{i=1, \dots, m} \{L_i\}$. Thus, $C_{\max}^* \geq \sqrt[m]{\prod_{i=1}^m L_i} \geq \sqrt[m]{H_2^1 \cdots H_2^m \prod_{j=1}^n (1 + \alpha_j)/H_1^1 \cdots H_1^m} \geq H_2 (1 + \alpha_{\min})^{n/m}/H_1$. $\square$

3. Minimizing the Total Completion Time

In this section, we discuss the total completion time minimization problem.

Ji and Cheng [13] showed that problem $P_m|p_j = \alpha_j t| \sum C_j$ is NP-hard in the strong sense when m is arbitrary, which implies that our problem $P_m|nr-a, p_j = \alpha_j t| \sum C_j$ is NP-hard in the strong sense when m is arbitrary.

3.1. Dynamic Programming Algorithm. In this subsection, we present a dynamic programming algorithm for $P_2|nr-a, p_j = \alpha_j t| \sum C_j$ when machine M_2 is always available. For convenience, let $t_0 = 1$, and let the only nonavailable interval on machine M_1 be $[H_1^{(1)}, H_2^{(1)})$, $\alpha = \prod_{j=1}^n (1 + \alpha_j)$, and $\alpha^j = \prod_{i=1}^j (1 + \alpha_i)$.

Smallest Deteriorating Rate (SDR for Short). Sort the jobs in the nondecreasing order of their deteriorating rates such that $\alpha_1 \leq \alpha_2 \leq \dots \leq \alpha_n$.

Lemma 4. In any optimal solution to problem $P_2|nr-a, p_j = \alpha_j t| \sum C_j$, the jobs scheduled before the nonavailable interval are processed by the SDR order and so are the jobs scheduled after the nonavailable interval and on machine M_2 .

We can proof this lemma by the interchanging argument.

We assume that the jobs are reindexed in the SDR order. Let $f_j(v, u)$ denote the optimal value of the objective function

satisfying the following conditions:

- (i) The jobs in consideration are $J_1, \dots, J_j$.
- (ii) The total processing time of M_1 before $H_1^{(1)}$ is v .
- (iii) The total processing time of M_2 is u .

To get $f_j(v, u)$, we distinguish three cases as follows.

Case 1. Job J_j is scheduled before $H_1^{(1)}$.

In this case, u does not change. The starting time of J_j is $(v+1)/(1+\alpha_j)$, the total processing time of M_1 before $H_1^{(1)}$ is $(v+1)/(1+\alpha_j) - 1 = (v-\alpha_j)/(1+\alpha_j)$ before inserting job J_j , and the completion of J_j is $v+1$. Thus, $f_j(v, u) = f_{j-1}((v-\alpha_j)/(1+\alpha_j), u) + v+1$.

Case 2. Job J_j is scheduled on machine M_2 .

In this case, v does not change. The starting time of J_j is $(u+1)/(1+\alpha_j)$, the total processing time of M_2 is $(u+1)/(1+\alpha_j) - 1 = (u-\alpha_j)/(1+\alpha_j)$ before inserting job J_j , and the completion of J_j is $u+1$. Thus, $f_j(v, u) = f_{j-1}(v, (u-\alpha_j)/(1+\alpha_j)) + u+1$.

Case 3. Job J_j is scheduled after $H_2^{(1)}$.

In this case, both u and v do not change. The completion of J_j is $H_2^{(1)}\alpha^j/(v+1)(u+1)$ since $\alpha^j = \prod_{i=1}^j (1+\alpha_i) = (v+1)(u+1)$

1) $\prod_{J_i \in J_{H_2^{(1)}}^j} (1+\alpha_i)$, where $J_{H_2^{(1)}}^j$ denotes the set of jobs scheduled after $H_2^{(1)}$. Thus, $f_j(v, u) = f_{j-1}(v, u) + H_2^{(1)}\alpha^j/(v+1)(u+1)$.

Combining the above cases, we design a dynamic programming algorithm as follows.

Algorithm DPI.

Step 1. Reindex the jobs in nondecreasing order of their deteriorating rates such that $\alpha_1 \leq \alpha_2 \leq \dots \leq \alpha_n$.

Step 2 (Initialization).

$$f_1(v, u) = \begin{cases} v+1 & v = \alpha_1 \text{ and } u = 0, \\ u+1 & v = 0 \text{ and } u = \alpha_1, \\ H_2^{(1)}(1+\alpha_1) & v = 0 \text{ and } u = 0, \\ \infty & \text{otherwise.} \end{cases} \quad (2)$$

Step 3 (Iteration).

$f_j(v, u)$

$$= \min \begin{cases} \min \left\{ f_{j-1} \left(\frac{v-\alpha_j}{1+\alpha_j}, u \right) + v+1, f_{j-1} \left(v, \frac{u-\alpha_j}{1+\alpha_j} \right) + u+1, f_{j-1}(v, u) + \frac{H_2^{(1)}\alpha^j}{(v+1)(u+1)} \right\} & \text{if } \frac{v-\alpha_j}{1+\alpha_j}, \frac{u-\alpha_j}{1+\alpha_j}, \frac{H_2^{(1)}\alpha^j}{(v+1)(u+1)} \in \mathbb{Z}, \\ \min \left\{ f_{j-1} \left(\frac{v-\alpha_j}{1+\alpha_j}, u \right) + v+1, f_{j-1} \left(v, \frac{u-\alpha_j}{1+\alpha_j} \right) + u+1 \right\} & \text{if } \frac{v-\alpha_j}{1+\alpha_j}, \frac{u-\alpha_j}{1+\alpha_j} \in \mathbb{Z}, \frac{H_2^{(1)}\alpha^j}{(v+1)(u+1)} \notin \mathbb{Z}, \\ \min \left\{ f_{j-1} \left(\frac{v-\alpha_j}{1+\alpha_j}, u \right) + v+1, f_{j-1}(v, u) + \frac{H_2^{(1)}\alpha^j}{(v+1)(u+1)} \right\} & \text{if } \frac{v-\alpha_j}{1+\alpha_j}, \frac{H_2^{(1)}\alpha^j}{(v+1)(u+1)} \in \mathbb{Z}, \frac{u-\alpha_j}{1+\alpha_j} \notin \mathbb{Z}, \\ \min \left\{ f_{j-1} \left(v, \frac{u-\alpha_j}{1+\alpha_j} \right) + u+1, f_{j-1}(v, u) + \frac{H_2^{(1)}\alpha^j}{(v+1)(u+1)} \right\} & \text{if } \frac{u-\alpha_j}{1+\alpha_j}, \frac{H_2^{(1)}\alpha^j}{(v+1)(u+1)} \in \mathbb{Z}, \frac{v-\alpha_j}{1+\alpha_j} \notin \mathbb{Z}, \\ f_{j-1} \left(\frac{v-\alpha_j}{1+\alpha_j}, u \right) + v+1 & \text{if } \frac{v-\alpha_j}{1+\alpha_j} \in \mathbb{Z}, \frac{u-\alpha_j}{1+\alpha_j}, \frac{H_2^{(1)}\alpha^j}{(v+1)(u+1)} \notin \mathbb{Z}, \\ f_{j-1} \left(v, \frac{u-\alpha_j}{1+\alpha_j} \right) + u+1 & \text{if } \frac{u-\alpha_j}{1+\alpha_j} \in \mathbb{Z}, \frac{v-\alpha_j}{1+\alpha_j}, \frac{H_2^{(1)}\alpha^j}{(v+1)(u+1)} \notin \mathbb{Z}, \\ f_{j-1}(v, u) + \frac{H_2^{(1)}\alpha^j}{(v+1)(u+1)} & \text{if } \frac{H_2^{(1)}\alpha^j}{(v+1)(u+1)} \in \mathbb{Z}, \frac{v-\alpha_j}{1+\alpha_j}, \frac{u-\alpha_j}{1+\alpha_j} \notin \mathbb{Z}, \\ \infty & \text{otherwise.} \end{cases} \quad (3)$$

Step 4 (Solution). The optimal value is $\min\{f_n(v, u) \mid v = 0, 1, \dots, H_1^{(1)} - 1; u = 0, 1, \dots, \alpha - 1\}$.

Theorem 5. The problem $P_2|nr - a, p_j = \alpha_j t| \sum C_j$ is solvable in $O(n\alpha H_1^{(1)})$ time by Algorithm DPI.

Proof. The correctness of Algorithm DPI is guaranteed by the above discussion. Note that $v = 0, 1, \dots, H_1^{(1)} - 1$ and $u = 0, 1, \dots, \alpha - 1$. Thus, the recursive function has at most $O((\alpha - 1)(H_1^{(1)} - 1)) = O(\alpha H_1^{(1)})$ states. Each iteration takes $O(n)$ time to execute. Hence, the running time is $O(n\alpha H_1^{(1)})$. $\square$

3.2. A Fully Polynomial Time Approximation Scheme. In this subsection, we present a fully polynomial time approximation scheme for problem $P_2|nr - a, p_j = \alpha_j t| \sum C_j$ when machine M_2 is always available. Following Woeginger [15], we show that $P_2|nr - a, p_j = \alpha_j t| \sum C_j$ is DP-benevolent, which follows that there exists a fully polynomial time approximation scheme for our problem. For convenience, let $t_0 = 1$.

The fully polynomial time approximation scheme is based on Lemma 4 stated in Section 3.2. Thus, we first sort the jobs in the nondecreasing order of their deteriorating rates such that $\alpha_1 \leq \alpha_2 \leq \dots \leq \alpha_n$. The dynamic programming algorithm proposed in the following goes through n phases. In the k th ($k = 1, 2, \dots, n$) phase, we input the vector $X_k = [\alpha_k]$; meanwhile, a state set S_k is generated. Any state in S_k is a vector $[l_1, l_2, l_3, z]$ which encodes a partial schedule for the first k jobs $J_1, \dots, J_k$. The component l_1 represents the total processing time before $H_1^{(1)}$ on machine M_1 , l_2 represents the total processing time after $H_2^{(1)}$ on machine M_1 , l_3 represents the total processing time on machine M_2 , and the component z represents the objective value of the current schedule. The initial set S_0 contains the only state $[1, H_2^{(1)}, 1, 0]$. The state S_k is generated from the state S_{k-1} by three mappings F_1, F_2 , and F_3 which are defined as follows:

$$\begin{aligned} F_1(\alpha_k, l_1, l_2, l_3, z) &= [l_1(1 + \alpha_k), l_2, l_3, z + l_1(1 + \alpha_k)], \\ F_2(\alpha_k, l_1, l_2, l_3, z) &= [l_1, l_2(1 + \alpha_k), l_3, z + l_2(1 + \alpha_k)], \\ F_3(\alpha_k, l_1, l_2, l_3, z) &= [l_1, l_2, l_3(1 + \alpha_k), z + l_3(1 + \alpha_k)]. \end{aligned} \quad (4)$$

Intuitively, function F_1 puts job J_k at the end before $H_1^{(1)}$ on machine M_1 if it is possible for the given state; and function F_2 puts job J_k at the end after $H_2^{(1)}$ on machine M_1 and function F_3 puts job J_k at the end on machine M_2 . Finally, set $G(l_1, l_2, l_3, z) = z$.

Combining the above discussion, we design a dynamic programming algorithm as follows.

Algorithm DP2.

Initialize $s_0 := \{[1, H_2^{(1)}, 1, 0]\}$

For $k = 1$ to n do

Let $s_k := \emptyset$

For every $[l_1, l_2, l_3, z] \in s_{k-1}$ do

$[l_1, l_2(1 + \alpha_k), l_3, z + l_2(1 + \alpha_k)]$ assignments s_k
and

$[l_1, l_2, l_3(1 + \alpha_k), z + l_3(1 + \alpha_k)]$ assignments s_k

If $l_1(1 + \alpha_k) \leq H_1^{(1)}$

Then $[l_1(1 + \alpha_k), l_2, l_3, z + l_1(1 + \alpha_k)]$ assignments s_k

Endfor

Endfor

Output $\min\{G(l_1, l_2, l_3, z) : [l_1, l_2, l_3, z] \text{ assignments } s_n\}$.

Note that the number of states in the above dynamic programming is bounded by $H_2^{(1)} n \prod_{j=1}^n (1 + \alpha_j)$. There holds the following result.

Theorem 6. *There exists a fully polynomial time approximation scheme for problem $P_2|nr - a, p_j = \alpha_j t| \sum C_j$ when one machine is always available.*

Proof. The functions F_1, F_2 , and F_3 are vectors of polynomials with nonnegative coefficients and the polynomial functions in F_1, F_2 , and F_3 that yield the components are polynomials. Moreover, all polynomials linearly depend on l_1, l_2, l_3 , and z . The inequality inside operator “if” can be checked in polynomial time. The objective function G is a polynomial with nonnegative coefficients. Therefore, similar to the example in Section 5.3 of Woeginger [15], it is not hard to verify that the above dynamic programming satisfies the conditions of Lemma 6.1 and Theorem 2.5 from [15]. Thus, problem $P_2|nr - a, p_j = \alpha_j t| \sum C_j$ is DP-benevolent. As a result, there exists a fully polynomial time approximation scheme for problem $P_2|nr - a, p_j = \alpha_j t| \sum C_j$ when one machine is always available. $\square$

4. Minimizing the Total Weighted Completion Time

In this section, we extend the dynamic programming algorithm to the total weighted completion time minimization problem, that is, $P_2|nr - a, p_j = \alpha_j t| \sum w_j C_j$. We also assume that machine M_2 is always available.

We assume that the jobs are reindexed such that $w_1(1 + \alpha_1)/\alpha_1 \geq w_2(1 + \alpha_2)/\alpha_2 \geq \dots \geq w_n(1 + \alpha_n)/\alpha_n$. For convenience, denote this order by *Weight Deteriorating Rate* (WDR for short).

Lemma 7. *In any optimal solution to problem $P_2|nr - a, p_j = \alpha_j t| \sum w_j C_j$, the jobs scheduled before the nonavailable interval are processed by the WDR order and so are the jobs scheduled after the nonavailable interval and on machine M_2 .*

We can prove this lemma by the interchanging argument.

We assume that the jobs are reindexed in the WDR order. Similar to the context of Section 3.1, let $f_j(v, u)$ denote the optimal value of the objective function satisfying the following conditions:

- (i) The jobs in consideration are $J_1, \dots, J_j$.

- (ii) The total processing time of M_1 before $H_1^{(1)}$ is v .
- (iii) The total processing time of M_2 is u .

We design a dynamic programming algorithm as follows.

Algorithm DP3.

Step 1. Reindex the jobs such that $w_1(1 + \alpha_1)/\alpha_1 \geq w_2(1 + \alpha_2)/\alpha_2 \geq \dots \geq w_n(1 + \alpha_n)/\alpha_n$.

Step 2 (Initialization).

$$f_1(v, u) = \begin{cases} w_1(v + 1) & v = \alpha_1 \text{ and } u = 0, \\ w_1(u + 1) & v = 0 \text{ and } u = \alpha_1, \\ w_1 H_2^{(1)}(1 + \alpha_1) & v = 0 \text{ and } u = 0, \\ \infty & \text{otherwise.} \end{cases} \quad (5)$$

Step 3 (Iteration).

$f_j(v, u)$

$$= \min \left\{ \begin{array}{l} \min \left\{ f_{j-1} \left(\frac{v - \alpha_j}{1 + \alpha_j}, u \right) + w_j(v + 1), f_{j-1} \left(v, \frac{u - \alpha_j}{1 + \alpha_j} \right) + w_j(u + 1), f_{j-1}(v, u) + \frac{w_j H_2^{(1)} \alpha^j}{(v + 1)(u + 1)} \right\} \\ \min \left\{ f_{j-1} \left(\frac{v - \alpha_j}{1 + \alpha_j}, u \right) + w_j(v + 1), f_{j-1} \left(v, \frac{u - \alpha_j}{1 + \alpha_j} \right) + w_j(u + 1) \right\} \\ \min \left\{ f_{j-1} \left(\frac{v - \alpha_j}{1 + \alpha_j}, u \right) + w_j(v + 1), f_{j-1}(v, u) + \frac{w_j H_2^{(1)} \alpha^j}{(v + 1)(u + 1)} \right\} \\ \min \left\{ f_{j-1} \left(v, \frac{u - \alpha_j}{1 + \alpha_j} \right) + w_j(u + 1), f_{j-1}(v, u) + \frac{w_j H_2^{(1)} \alpha^j}{(v + 1)(u + 1)} \right\} \\ f_{j-1} \left(\frac{v - \alpha_j}{1 + \alpha_j}, u \right) + w_j(v + 1) \\ f_{j-1} \left(v, \frac{u - \alpha_j}{1 + \alpha_j} \right) + w_j(u + 1) \\ f_{j-1}(v, u) + \frac{w_j H_2^{(1)} \alpha^j}{(v + 1)(u + 1)} \\ \infty \end{array} \right. \quad \begin{array}{l} \text{if } \frac{v - \alpha_j}{1 + \alpha_j}, \frac{u - \alpha_j}{1 + \alpha_j}, \frac{H_2^{(1)} \alpha^j}{(v + 1)(u + 1)} \in Z, \\ \text{if } \frac{v - \alpha_j}{1 + \alpha_j}, \frac{u - \alpha_j}{1 + \alpha_j} \in Z, \frac{H_2^{(1)} \alpha^j}{(v + 1)(u + 1)} \notin Z, \\ \text{if } \frac{v - \alpha_j}{1 + \alpha_j}, \frac{H_2^{(1)} \alpha^j}{(v + 1)(u + 1)} \in Z, \frac{u - \alpha_j}{1 + \alpha_j} \notin Z, \\ \text{if } \frac{u - \alpha_j}{1 + \alpha_j}, \frac{H_2^{(1)} \alpha^j}{(v + 1)(u + 1)} \in Z, \frac{v - \alpha_j}{1 + \alpha_j} \notin Z, \\ \text{if } \frac{v - \alpha_j}{1 + \alpha_j} \in Z, \frac{u - \alpha_j}{1 + \alpha_j}, \frac{H_2^{(1)} \alpha^j}{(v + 1)(u + 1)} \notin Z, \\ \text{if } \frac{u - \alpha_j}{1 + \alpha_j} \in Z, \frac{v - \alpha_j}{1 + \alpha_j}, \frac{H_2^{(1)} \alpha^j}{(v + 1)(u + 1)} \notin Z, \\ \text{if } \frac{H_2^{(1)} \alpha^j}{(v + 1)(u + 1)} \in Z, \frac{v - \alpha_j}{1 + \alpha_j}, \frac{u - \alpha_j}{1 + \alpha_j} \notin Z, \\ \text{otherwise.} \end{array} \quad (6)$$

Step 4 (Solution). The optimal value is $\min\{f_n(v, u) \mid v = 0, 1, \dots, H_1^{(1)} - 1; u = 0, 1, \dots, \alpha - 1\}$.

Theorem 8. The problem $P_2|nr - a, p_j = \alpha_j t| \sum w_j C_j$ is solvable in $O(n\alpha H_1^{(1)})$ time by Algorithm DP3.

5. Conclusions

In this paper, we considered the parallel-machine scheduling with time-dependent and machine availability constraints. We showed that our two problems $P_m|nr - a, p_j = \alpha_j t|C_{\max}(\sum C_j)$ are NP-hard in the strong sense. We analyzed the LDR and LS algorithms and the lower bound of any optimal schedule and presented dynamic programming algorithm and fully polynomial time approximation scheme for the problem $P_2|nr - a, p_j = \alpha_j t| \sum w_j C_j$. Furthermore, we extended the dynamic programming algorithm to problem $P_2|nr - a, p_j = \alpha_j t| \sum w_j C_j$.

For future research, the other objectives are worth considering. The design of PTAS for our problems is another worthy topic.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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