1. Introduction

Broadband noise reduction has been investigated over decades. Many researchers have demonstrated that the addition of foams or fiber batting can effectively attenuate sound at certain frequencies [1, 2]. Impedance mismatch of gap layer and the use of microperforated panels/mass inclusions also have been feasible tools for enhancing sound insulation [3–6]. Recently, negative dynamic mass density structures become popular candidates for attenuating elastic waves [7–12] or acoustic waves [13–22] over a certain band of frequencies. For the attenuation of elastic waves, the first proposed promising candidate was a one-dimensional mass-in-mass system usually comprised of a host mass connected to an auxiliary mass by massless and elastic springs. This class of metamaterials is able to create a resonant-type bandgap where waves cannot propagate freely. To acquire multiple bandgaps at desired frequency range, two-dimensional designs of metamaterials, such as metacomposites [11] and multiscale mass-in-mass systems [12], were subsequently introduced. For sound isolation, the basic unit of the pioneering composite is usually comprised of an elastic membrane and a centrally loaded mass. The desired frequency band where sound waves cannot propagate can be achieved by tailoring the mass weight. To eliminate sounds with multiple frequencies, multicelled structures were also proposed [16–18]. Different magnitudes of mass attached to each of the cells in the array can create a multipeak transmission loss profile. In addition to using four-celled array, geometric variation of inclusions is an alternative solution to effectively increase frequency bandwidth and produce multiple transmission loss peaks [19]. Their findings show that the characteristics of transmission loss profile can be manipulated by the number of rings and the distribution of mass. Phononic crystals constituted by periodical repetition of inclusions in a background material also can create frequency gaps which are known as Bragg bandgaps and usually occurred at high frequencies [23, 24]. Because Bragg-type bandgaps are not practice for filtering low-frequency waves, a new design of phononic crystals with locally resonant structures, which can generate flexible resonant-type bandgaps, was developed [25–28]. By altering the properties of the resonant units, the opening and closing
of the low-frequency gaps and the gap frequency ranges can be controlled or shifted. Other types of acoustic metamaterials, such as an array of side holes on a tube or an array of thin elastic membranes, have demonstrated their potential capability of filtering low-frequency sound [29, 30]. Those structures exhibit either a negative effective modulus or a negative effective density in a certain frequency range where acoustic waves produced by the excitation are completely blocked.

Sandwich panels containing lightweight honeycomb core are widely used in aircraft and satellite launch vehicles [6, 31]. Although sandwich panels have good benefit of static strength, they are relatively weak in sound insulation. To improve their performance in acoustic absorption, the use of the aforementioned efficient tools becomes necessary. Since the honeycomb core usually has a shape of square form or hexagon form, membrane-frame acoustic metamaterials might be more suitable selections. In this study, the acoustic response of square membranes with multiple frame masses is presented. The governing equation of the proposed vibroacoustic problem is obtained by using the energy method. Parameter studies including frame mass per unit area of the membrane, \( \rho_{\text{membrane}} \) denotes the density per unit area of the ith frame mass, \( \rho_{\text{mass}} \) denotes the density per unit area of the ith frame mass, \( F \) is the intensity of tension, \( P^- \) and \( P^+ \) are acoustic pressure field in the negative and positive z direction, \( \omega = \omega(x, y, t) \) is the transverse displacement in the z direction of the membrane-frame structure, and \( h_i - h_i' \) represents the area of the ith frame mass as shown as follows:

\[
h_i - h_i' = \left[ H (x - x_{0i}) - H (x - x_{0i} - c_i) \right] \cdot \left[ H (y - y_{0i}) - H (y - y_{0i} - d_i) \right] - \left[ H (x - x'_{0i}) - H (x - x'_{0i} - c_i') \right] \cdot \left[ H (y - y'_{0i}) - H (y - y'_{0i} - d_i') \right],
\]

where \( H \) is Heaviside function. Substituting (2) into (1) gives the equation of motion; namely,

\[
\rho_{\text{membrane}} \frac{\partial^2 \omega}{\partial t^2} + \sum_{i=1}^{Q} \rho_{\text{mass}, i} \left( h_i - h_i' \right) \frac{\partial^2 \omega}{\partial t^2} - F \nabla^2 \omega = P^- - P^+,
\]

in which \( a, b, \rho_{\text{membrane}} \) denote the length, width, and density per unit area of the membrane, \( \rho_{\text{mass}} \) denotes the density per unit area of the ith frame mass, \( F \) is the intensity of tension, \( P^- \) and \( P^+ \) are acoustic pressure field in the negative and positive z direction, \( \omega = \omega(x, y, t) \) is the transverse displacement in the z direction of the membrane-frame structure, and \( h_i - h_i' \) represents the area of the ith frame mass as shown as follows:

\[
h_i - h_i' = \left[ H (x - x_{0i}) - H (x - x_{0i} - c_i) \right] \cdot \left[ H (y - y_{0i}) - H (y - y_{0i} - d_i) \right] - \left[ H (x - x'_{0i}) - H (x - x'_{0i} - c_i') \right] \cdot \left[ H (y - y'_{0i}) - H (y - y'_{0i} - d_i') \right],
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\[
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\]

Figure 1 shows a novel design of a basic unit that can be used to construct a membrane-type locally resonant acoustic metamaterial. This structure unit consists of a thin tensioned membrane with one/multiple frame masses (mass \( m \)) embedded in the membrane. Consider a plane sound wave \( P^- = P^+ \) produced by the excitation are completely blocked.

Figure 1: A sketch of membrane-frame structure.
in which \( A \) is plane sound wave pressure amplitude, \( \omega \) is the wave frequency, \( \rho_{\text{air}} \) is the density of the air, and \( c_{\text{air}} \) is the speed of sound. To apply Galerkin discretization procedure [34] to (4), the transverse displacement function \( w \) is expressed as

\[
w(x, y, t) = \sum_{n=1}^{N} W_n(x, y) u_n(t),
\]

where \( W_n(x, y) \) is the mode function and \( u_n(t) \) is the harmonic time function excited by the acoustic load; that is,

\[
u_n(t) = \tilde{u}_n e^{i \omega t}.
\]

The mode shape of the membrane simply supported on all four sides can be expressed as [35]

\[
W_n(x, y) = \sin \frac{k_n x}{a} \sin \frac{l_n y}{b}, \quad k, l = 1, 2, \ldots
\]

Substituting (6) and (7) into (4), multiplying each term by \( W_m(x, y) \), and integrating all terms in the equation over the domain \((0 \leq x \leq a, 0 \leq y \leq b)\) give

\[
-\omega^2 \left( \rho_{\text{membrane}} \int_0^a \int_0^b W_n \sum_{n=1}^{N} W_n^2 dx \, dy \right) \tilde{u}_m \\
-\omega^2 \sum_{i=1}^{Q} (\rho_{\text{mass}}) \int_0^a \int_0^b \left( \int_{y_0}^{y_0 + d} \int_{x_0}^{x_0 + r_c} W_m W_n dx \, dy \right) \tilde{u}_n \\
-\int_{y_0}^{y_0 + d} \int_{x_0}^{x_0 + r_c} W_m W_n \left( \int_{x_0}^{x_0 + r_c} \int_{y_0}^{y_0 + d} W_n \tilde{u}_n dx \, dy \right) \tilde{u}_n \\
+ \frac{j \omega}{2} \rho_{\text{air}} c_{\text{air}} \int_0^a \int_0^b W_m \sum_{n=1}^{N} W_n^2 dx \, dy \tilde{u}_m \\
+ \left( -F \int_0^a \int_0^b W_m V^2 \sum_{n=1}^{N} W_n^2 dx \, dy \right) \tilde{u}_m \\
= 2A \left( \int_0^a \int_0^b W_m^2 dx \, dy \right).
\]

Let

\[
C_m = 2\rho_{\text{air}} c_{\text{air}} \int_0^a \int_0^b W_m \sum_{n=1}^{N} W_n^2 dx \, dy,
\]

\[
K_m = -F \int_0^a \int_0^b W_m V^2 \sum_{n=1}^{N} W_n^2 dx \, dy,
\]

\[
D_m = \int_0^a W_m^2 dx \, dy.
\]

Equation (9) can be simplified as

\[
-\omega^2 \left\{ M_m \tilde{u}_m + \sum_{i=1}^{Q} \left( \rho_{\text{mass}} \sum_{n=1}^{N} (S_{mn}) \tilde{u}_n \right) \right\} \\
+ j \omega C_m \tilde{u}_m + K_m \tilde{u}_m = 2AD_m
\]

\((m = 1, 2, \ldots, N)\).

Rewriting (11) in the matrix form gives

\[
\left\{ -\omega^2 \left[ [M] + [R] \right] + j \omega \left[ [C] + [K] \right] \right\} \tilde{\mathbf{u}} = 2A \{ \mathbf{d} \},
\]

where \([M], [K], \) and \([C]\) are \(N \times N\) diagonal matrices, \([D]\), \([W]\), and \([\tilde{\mathbf{u}}]\) are \(N \times 1\) matrices, and \([R]\) is also \(N \times N\) matrix and has a form as follows:

\[
[R] = \sum_{i=1}^{Q} (\rho_{\text{mass}}) \left[ \begin{array}{c} (S_{11})_i \\ (S_{12})_i \\ \vdots \\
(S_{N1})_i \\ (S_{N2})_i \\ \vdots \\
(S_{NN})_i \end{array} \right].
\]

The solution of displacement amplitude \(\tilde{w}(x, y)\) also can be written in the matrix form as

\[
\tilde{w}(x, y) = \sum_{n=1}^{N} W_n(x, y) \tilde{u}_n = [W]^T \{ \tilde{\mathbf{u}} \},
\]

where \(\{ \tilde{\mathbf{u}} \}\) is acquired from (12). Hence we can get the sound power transmission coefficient \(\tau\) defined as the ratio of transmitted to incident intensities [33]:

\[
\tau = \frac{\rho_{\text{air}} c_{\text{air}} \langle \tilde{v} \rangle^2}{A},
\]

where \(\langle \tilde{v} \rangle\) is the average velocity amplitude of the entire membrane. Then the sound transmission loss (TL) can be obtained by

\[
\text{TL} = 10 \log_{10} \left( \frac{1}{\tau} \right).
\]

To treat the whole structure as a homogeneous membrane, the dynamic effective mass density is introduced and acquired by

\[
\rho_{\text{eff}} = \frac{\langle \tilde{p} - \langle \tilde{p} \rangle \rangle}{\langle \tilde{a} \rangle},
\]

where \(\langle \tilde{a} \rangle\) is the averaged out-of-plane acceleration of the structure, which is normal to the membrane surface at rest; \(\langle \tilde{p} \rangle - \langle \tilde{p} \rangle\) is the averaged pressure difference between the transmitted wave and the incident wave.
Table 1: Physical properties of membrane material [16].

<table>
<thead>
<tr>
<th>Property</th>
<th>Polyetherimide (PEI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass density (kg/m³)</td>
<td>1200</td>
</tr>
<tr>
<td>Young’s modulus (Pa)</td>
<td>3.6 × 10⁶</td>
</tr>
<tr>
<td>Poisson ratio</td>
<td>0.36</td>
</tr>
<tr>
<td>Thickness (mm)</td>
<td>0.076</td>
</tr>
<tr>
<td>Length (mm)</td>
<td>27.4</td>
</tr>
<tr>
<td>Tension (Pa)</td>
<td>6.4 × 10⁶</td>
</tr>
</tbody>
</table>

3. Numerical Results

Consider a single-celled structure comprised of a square membrane with an embedded frame mass as shown in Figure 1. The characteristics of sound transmission of the present structure were evaluated using the analytical approach and finite element method. In the finite element analysis, the commercial code COMSOL Multiphysics is used to capture the dynamic behavior of the membrane-frame structure. All the physical properties of the membrane material are displayed in Table 1. Table 2 lists the properties (mass, width, and location) of frame-shaped inclusions embedded in the membrane.

Figure 2 shows the transmission loss of four illustrative examples. Analytical results are obtained by using (16); FE results are calculated from the ratio of the incident power to the transmitted power. It is found that the ratio between analytical and finite element methods is quite good. In Figures 2(a) and 2(b), only one TL peak and two TL dips occur while in Figures 2(c) and 2(d), two TL peaks and three TL dips are found. It is evident that the addition of frame masses can produce more TL peaks. The TL dips correspond to resonance behavior where sound transmission reaches a maximum, while the TL peaks correspond to antiresonance where sound transmission is minimized.

Figures 3(a)–3(d) exhibit the dynamic effective mass density of four membrane structures (Model I, Model II, Model III, and Model IV), respectively, as a function of the frequency. Analytical results are acquired by using (17); FE results are obtained by dividing the average pressure on the membrane surface by the average membrane acceleration. It is seen that near the TL peak frequencies, negative effective mass occurs. Observation of the results shown in Figures 3(c) and 3(d) presents that the addition of a frame mass leads to an additional jump where the effective mass turns from positive to negative.

Figure 4 illustrates the effect of the material and geometrical properties of the embedded frame on the TL plot of the membrane-frame structure. The results of Figure 4(a) indicate that the first TL peak and valley have a strong dependency on the magnitude of the frame mass. With an increase of the frame mass, the first peak and valley move toward low-frequency region. As the frame width increases, the first TL peak and valley slightly shift to higher frequencies (Figure 4(b)). The first TL valley is from the frame mass and membrane vibrating coherently while the second TL valley is from the membrane vibration with the motionless frame mass. The first TL peak is viewed as the result of superposition of two resonant eigenmodes and has a strong dependency on the frame mass. Hence, as shown in Figure 4(a), the change in frame mass magnitude greatly influences the first TL peak and valley frequencies but has no effect on the second TL valley. On the contrary, with the same mass magnitude and frame location, the change in frame width significantly influences the second TL valley but has a relatively small impact on the first TL peak and valley (as shown in Figure 4(b)). Figure 4(c) exhibits the transmission loss varying with the locations of the frame masses. The frame widths for three cases remain the same. The distances from the center for Frame I, Frame II, and Frame III are 2.14 mm, 3.85 mm, and 5.56 mm, respectively. As the frame moves away from the center, a notable increase in the first TL peak and valley frequencies is observed. The reason is possibly that different frame mass locations result in different eigenmodes of the membrane-frame structure. The characteristic frequencies of the first resonance and TL peak varying with the location of the frame mass are presented in Figure 5. It seems that the first TL peak and resonance frequencies are almost proportional to the mass location which is measured from the membrane center. Also, it is seen that the former rising rate is higher than the latter.

The schematics of the stacked membrane-central-mass structure and stacked membrane-frame structure are displayed in Figures 6(a) and 6(b), respectively. Four different mass magnitudes are considered: 0.08 g, 0.16 g, 0.24 g, and 0.32 g. Figure 7 shows the comparison of sound transmission of the stacked structure, obtained by FEA analysis, with that of four single-celled structures, obtained by analytical models. It is seen that the TL peaks and valley frequencies of the stacked structure coincide with the ones of single-celled structures. In other words, the characteristic frequencies of the series-type structure can be precisely predicted by single-layer structures. It is also seen that the width of frequency band in which sound wave cannot propagate is effectively enlarged. Compared with the results shown in Figures 7(a) and 7(b), it seems that the acoustic performances of two stacked structures present a similar behavior. Moreover, more frequency bands due to the periodicity of the structure are created in both cases. Figure 8 exhibits the effects of air spacing on sound transmission of two structures. It is evident that air spacing does affect the frequency band caused by the periodic assembly but has no effect on the TL peak and valley frequencies, caused by resonance and antiresonance of the local resonator. With larger air spacing, the band width becomes wider and the beginning frequency of the band moves toward lower frequency regime. Figure 9 depicts two sketches of the array-type structures. Two unit cells are considered and shown in Figures 9(a) and 9(b), respectively, a membrane with a central mass and the one with a frame mass. The results show that a multiphase TL profile can be created by using different masses at adjacent cells. Sound transmission of the array-type structure and four single-celled structures are compared and displayed in Figure 10. From the results of Figures 10(a) and 10(b), it is seen that the TL valleys for four-celled array correspondingly match the ones for single-celled structures. In other words, the resonances of the local
Table 2: Properties of frame-shaped mass inclusions.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Central mass (g)</th>
<th>Frame I (g)</th>
<th>Frame II (g)</th>
<th>Distance from the center: Frame I (mm)</th>
<th>Distance from the center: Frame II (mm)</th>
<th>Frames I and II width (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model I</td>
<td>0.32</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Model II</td>
<td>—</td>
<td>0.32</td>
<td>—</td>
<td>2.14</td>
<td>—</td>
<td>0.856</td>
</tr>
<tr>
<td>Model III</td>
<td>0.32</td>
<td>0.32</td>
<td>—</td>
<td>2.14</td>
<td>—</td>
<td>0.856</td>
</tr>
<tr>
<td>Model IV</td>
<td>—</td>
<td>0.32</td>
<td>0.32</td>
<td>2.14</td>
<td>3.85</td>
<td>0.856</td>
</tr>
</tbody>
</table>

Figure 2: Transmission loss of (a) Model I, (b) Model II, (c) Model III, and (d) Model IV.
Figure 3: Dynamic effective mass of (a) Model I, (b) Model II, (c) Model III, and (d) Model IV.
resonators do not depend on the cell arrangement whereas the antiresonance frequencies are affected. As expected, series-type structures can produce stronger attenuation in sound wave than array-type structures and single-celled structures.

4. Conclusions

This study addresses the acoustic behavior of thin membranes with square frame-shaped masses. Analytical results are consistent with FE results in transmission loss and effective mass.
Figure 5: The dependence of the first TL peak and TL valley upon the mass location.

Figure 6: Schematics of the structures stacked in series with (a) membrane-central-mass units and (b) membrane-frame units.

Figure 7: (a) Comparison of membrane-central-mass structures stacked in series with four single-celled structures and (b) comparison of membrane-frame structures stacked in series with four single-celled structures.
The results indicate that a multi peak TL profile can be created by adding more frame masses in the membrane. Frequencies where negative effective mass occurs coincide with the TL peak frequencies. The TL peak and valley frequencies can be tailored by altering the geometrical and material properties of the frame. Two types of structure arrangements, a series-type structure and an array-type structure, are also evaluated and compared with single-celled structures. It is found that structures stacked in series can produce a frequency band with wide bandwidth and enhance attenuation in sound wave.

**Competing Interests**

The authors declare that they have no competing interests.

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