Research Article

Multisoliton Solutions and Breathers for the Coupled Nonlinear Schrödinger Equations via the Hirota Method

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Under investigation in this paper are the coupled nonlinear Schrödinger (CNLS) equations with dissipation terms by the Hirota method, which are better than the formal Schrödinger equation in eliciting optical solitons. The bilinear form has been constructed, via which multisolitons and breathers are derived. In particular, the three-bright soliton solution and breathers are derived and simulated via some pictures. The propagation characters are analysed with the change of the parameters.

1. Introduction

Since predicted and demonstrated experimentally, the propagation of temporal soliton envelopes in nonlinear optical media has attracted a great deal of attention due to the applications in the optical fiber long-distance transmission systems [1–7]. In the picosecond regime, optical solitons are based on the balance between the group velocity dispersion and self-phase modulation [8]. The dynamics of such solitons are described by the nonlinear Schrödinger (NLS) equation [9, 10]

$$iu_t + \frac{1}{2}u_{xx} + |u|^2 u = 0, \quad (1)$$

where u denotes the slowly varying complex envelope of the wave and x, t depend on the longitudinal distance and retarded time, respectively. Taking account of third-order dispersion, self-steepening, and other nonlinear effects, by Darboux transformation [11] described the rogue waves and rational solutions of the Hirota equation in the following form:

$$iu_t + \alpha(2|u|^2 u + u_{xx}) + i\beta(u_{ttt} + 6|u|^2 u_t) = 0, \quad (2)$$

where $\alpha = 1/2$ and the two terms in (2) that enter with a real coefficient β are responsible for the third-order

dispersion and a time-delay correction to the cubic term. For the further study of the higher-order terms and nonlinear terms influence on soliton propagation mechanism, Guo have constructed the Lax Pair through Darboux transformation and derived the breathers and multisoliton solutions for the fourth-order generalized nonlinear Schrödinger equation as follows [12]:

$$iu_t + u_{xx} + 2|u|^2 u + \tau(u_{xxx} + 8|u|^2 u_{xx} + 2u^2 u_{xx}^* + 6u^* u_x^2 + 4|u_x|^2 u + 6|u|^4 u) = 0. \quad (3)$$

Even though (1), (2), and (3) may adequately describe the propagation in a single-mode waveguide, switching operations, and routing, other optical effects which involve soliton pulses require the interaction between two or more modes [13, 14]. For several past decades, soliton propagation in the coupled nonlinear Schrödinger (CNLS) equations with different effective terms has been investigated from numerical simulation through applying Painlevé analysis [14], constructing Lax pair [15, 16], and carrying on gauge transformation [17–19]. In particular, [20] reveals the AB- and Ma-breathers and localized solitons for the Hirota-Maxwell-Bloch system on constant backgrounds in erbium doped fibers in detail. Recently, many solutions including bright solitons, bounded solitons, and collisions between

two solitons have been derived via Darboux transformation [21, 22] and Bäcklund transformation [23, 24]. The bound states with periodic attraction and repulsion between two solitons have been derived by the Lax Pair in [25], which are contained in the coupled higher-order nonlinear Schrödinger equations with variable coefficients. Subsequently, multidimensional coupled nonlinear equations attract the attention of so many science researchers by a variety of methods [26–28]. References [29, 30], especially, have investigated the $(2 + 1)$ -dimensional coupled nonlinear Schrödinger equations via the Hirota method and analysed two-solitons and collisions between solitons in detail. Considering the above, we investigate the following CNLS equations as the form [6]

$$\begin{aligned} iu_x + c_1 u_{tt} + 2(\alpha |u|^2 + \beta |v|^2)u + (\Delta_1 + \Delta_2)u \\ + (K_1 + iK_2)v = 0, \\ iv_x + c_2 v_{tt} + 2(\beta |u|^2 + \gamma |v|^2)v + (\Delta_1 - \Delta_2)u \\ + (K_1 - iK_2)v = 0, \end{aligned} \quad (4)$$

where c_1 and c_2 are the group velocity coefficients, α , β , and γ are the parameters related to nonlinear effects, and Δ_j , K_j ($j = 1, 2$) are related to the linear effects. It has been verified that (4) is completely integrable for the case $c_1 = c_2$, $\alpha = \beta = \gamma$ in [6]. Thus, we consider the complete integrable CNLS equations as

$$\begin{aligned} iu_x + c_1 u_{tt} + 2\alpha(|u| + |v|)u + \Delta_+ u + K_+ v = 0, \\ iv_x + c_1 v_{tt} + 2\alpha(|u| + |v|)v + \Delta_- u + K_- v = 0, \end{aligned} \quad (5)$$

with $\Delta_{\pm} = \Delta_1 \pm \Delta_2$, $K_{\pm} = K_1 \pm iK_2$, where c_1 is the group velocity coefficient, α is the parameter related to nonlinear effects, and $\Delta_{\pm}$, $K_{\pm}$ are related to linear effects. Although the investigations into the CNLS equations have been conducted via the different methods including Darboux transform [17], the Hirota method [31, 32], extended Jacobi's elliptic function method [18], Painlevé's analysis [14], and Bäcklund transformation [33–36] and many soliton solutions have been derived including one-soliton solutions, there are still several points to continue to explore as follows:

- (1) Analysis of the CNLS equations and extraction of the bilinear system based on the Hirota method.
- (2) Seeking breather solutions and three- (two-) soliton solutions and describing the propagation characters through the numerical simulations.
- (3) Analysing the propagation characteristics of collision between multisoliton solutions and breather solutions with different values of parameters.

Thus, we will seek multisolitons, breathers, and collisions between several solitons, respectively, via the Hirota method introduced in [37] and describing the propagation characteristics with figures in the next section. In addition, the

conclusion will be given in Section 3, which illustrates the transmission characteristics of the exact solutions contained in the CNLS equations synthetically.

2. Bilinear Forms and Soliton Solutions

The Hirota method is a direct analytic tool to acquire the soliton solutions which has been applied to many nonlinear evolution equations (NLEEs) [37–39]. Once the bilinear form of the NLEE was given, multisoliton solutions can be derived through the truncated formal perturbation expansion at different levels [40]. In this section, we will apply the method to construct N -soliton solutions for (5).

Introducing the dependent variable transformation

$$\begin{aligned} u &= \frac{g}{f}, \\ v &= \frac{h}{f}, \end{aligned} \quad (6)$$

into (5), the bilinear form can be derived as follows:

$$\begin{aligned} (iD_x + c_1 D_t^2 + \Delta_+ - i\sqrt{K_+ \Delta_-})(g \cdot f) &= 0, \\ (iD_x + c_1 D_t^2 + K_- - i\sqrt{K_+ \Delta_-})(h \cdot f) &= 0, \\ D_t^2(f \cdot f) - 2\frac{\alpha}{c_1}(|g|^2 + |h|^2) &= 0, \end{aligned} \quad (7)$$

where g , h are all complex differential functions to be determined, f is a real one, and the bilinear operators D_x and D_t^2 are defined by

$$\begin{aligned} D_x^n D_t^m (f \cdot g) &= \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'} \right)^n \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'} \right)^m \\ &\times f(x, t) g(x', t') \Big|_{t'=t, x'=x}, \end{aligned} \quad (8)$$

where m and n are both positive integers.

Next, we will analyse (7) through expanding g , h , and f with the parameter ε , obtain the soliton solutions by symbolic computation, and analyse the propagation characters based on numerical simulation.

2.1. One-Soliton Solutions for (5). In this case, we truncate the perturbation expansion of g , h , and f in relation to an expansion parameter ε as follows:

$$\begin{aligned} g &= \varepsilon g_1, \\ h &= \varepsilon h_1, \\ f &= 1 + \varepsilon^2 f_1. \end{aligned} \quad (9)$$

Substituting (9) into (7), setting $\varepsilon = 1$, we solve the bilinear system recursively and obtain the analytical one-soliton solution as

$$u = \frac{e^{t\omega_1 + x(\sqrt{\Delta_- K_+} + i(\Delta_+ + c_1 \omega_1^2))}}{1 + (X/c_1) \left(e^{t(\omega_3 + \bar{\omega}_3) + x\sqrt{\Delta_- K_+} + x(\sqrt{\Delta_- K_+} + i(\Delta_+ - \bar{\Delta}_+ + c_1(\omega_1^2 - \bar{\omega}_1^2)))} / (\omega_3 + \bar{\omega}_3)^2 \right)},$$

$$v = \frac{-i\sqrt{\Delta_- K_+} e^{t\omega_1 + x(\sqrt{\Delta_- K_+} + i(\Delta_+ + c_1 \omega_1^2))}}{1 + (X/c_1) \left(e^{t(\omega_3 + \bar{\omega}_3) + x\sqrt{\Delta_- K_+} + x(\sqrt{\Delta_- K_+} + i(\Delta_+ - \bar{\Delta}_+ + c_1(\omega_1^2 - \bar{\omega}_1^2)))} / (\omega_3 + \bar{\omega}_3)^2 \right)},$$
(10)

with

$$\begin{aligned} \theta_1 &= k_1 x + \omega_1 t, \\ \theta_3 &= k_3 x + \omega_3 t, \\ X &= \alpha \left(1 + \sqrt{\frac{\Delta_-}{K_+}} \sqrt{\frac{\bar{\Delta}_-}{\bar{K}_+}} \right), \\ K_- &= \Delta_+, \\ k_3 &= \sqrt{\Delta_- K_+} + \sqrt{\bar{\Delta}_- \bar{K}_+} \\ &\quad + i(\Delta_+ - \bar{\Delta}_+ + c_1(\omega_1^2 - \bar{\omega}_1^2)), \\ k_1 &= \sqrt{\Delta_- K_+} + i(\Delta_+ + c_1 \omega_1^2), \\ g_1 &= \exp(\theta_1), \\ h_1 &= -i\sqrt{\frac{\Delta_-}{K_+}} \exp(\theta_1), \\ f_1 &= \delta \exp(\theta_3 + \bar{\theta}_3), \\ \delta &= \frac{X}{c_1(\omega_3 + \bar{\omega}_3)^2}, \end{aligned} \quad (11)$$

where c_1 , δ , and $\Delta_{\pm}$ are real constants and $K_{\pm}$ are both complex ones.

By choosing different parameters, soliton solutions can be derived. Figure 1(a) displays the propagation characteristics of the bright soliton solution. It can be seen in Figure 1(b) that presents boundedness in amplitude and periodicity in the process of propagation, which is similar to the one in Figure 1(c) with the modification of the parameter ω_3 .

2.2. Two-Soliton Solutions for (5). Similar to the procedure in Section 2.1, we truncate the perturbation expansion of g , h , and f with respect to the parameter ε as

$$\begin{aligned} g &= \varepsilon g_1 + \varepsilon^3 g_3, \\ h &= \varepsilon h_1 + \varepsilon^3 h_3, \\ f &= 1 + \varepsilon^2 f_2 + \varepsilon^4 f_4. \end{aligned} \quad (12)$$

Substituting (12) into (7), solving the bilinear system recursively, and setting $\varepsilon = 1$, $\varphi_1 = 0$, and $\varphi_2 = 0$, the analytical two-soliton solutions can be derived as follows:

u

$$\begin{aligned} &= \frac{e^{p_1} + e^{p_2} + (\alpha\Lambda/c_1) \left(e^{p_1+r_2+\bar{r}_1} / (\omega_1 + \bar{\omega}_1)^2 (\omega_2 + \bar{\omega}_1)^2 + e^{p_1+r_2+\bar{r}_2} / (\omega_2 + \bar{\omega}_2)^2 (\omega_1 + \bar{\omega}_2)^2 \right)}{1 + (\alpha\Lambda/c_1) \left(e^{r_1+\bar{r}_1} / (\omega_1 + \bar{\omega}_1)^2 + e^{r_2+\bar{r}_1} / (\omega_2 + \bar{\omega}_1)^2 + e^{r_1+\bar{r}_2} / (\omega_1 + \bar{\omega}_2)^2 + e^{r_2+\bar{r}_2} / (\omega_2 + \bar{\omega}_2)^2 \right) + (\alpha^2\Gamma/\Phi) e^{r_1+\bar{r}_1+r_2+\bar{r}_2}}, \\ v &= \frac{i\sqrt{\Delta_- K_+} \left(e^{p_1} + e^{p_2} + (\alpha\Lambda/c_1) \left(e^{p_1+r_2+\bar{r}_1} / (\omega_1 + \bar{\omega}_1)^2 (\omega_2 + \bar{\omega}_1)^2 + e^{p_1+r_2+\bar{r}_2} / (\omega_2 + \bar{\omega}_2)^2 (\omega_1 + \bar{\omega}_2)^2 \right) \right)}{1 + (\alpha\Lambda/c_1) \left(e^{r_1+\bar{r}_1} / (\omega_1 + \bar{\omega}_1)^2 + e^{r_2+\bar{r}_1} / (\omega_2 + \bar{\omega}_1)^2 + e^{r_1+\bar{r}_2} / (\omega_1 + \bar{\omega}_2)^2 + e^{r_2+\bar{r}_2} / (\omega_2 + \bar{\omega}_2)^2 \right) + (\alpha^2\Gamma/\Phi) e^{r_1+\bar{r}_1+r_2+\bar{r}_2}}, \end{aligned} \quad (13)$$

with

$$\begin{aligned} \theta_1 &= k_1 x + \omega_1 t + \varphi_1, \\ \theta_2 &= k_2 x + \omega_2 t + \varphi_2, \\ f_4 &= \sigma_5 \exp(\theta_1 + \bar{\theta}_1 + \theta_2 + \bar{\theta}_2), \\ f_2 &= \sigma_1 \exp(\theta_1 + \bar{\theta}_1) + \sigma_2 \exp(\theta_2 + \bar{\theta}_2) + \sigma_3 \\ &\quad \cdot \exp(\theta_1 + \bar{\theta}_2) + \sigma_4 \exp(\theta_2 + \bar{\theta}_1), \end{aligned}$$

$$g_1 = \exp(\theta_1) + \exp(\theta_2),$$

$$g_3 = \eta_1 \exp(\theta_1 + \bar{\theta}_1 + \theta_2) + \eta_2 \exp(\theta_2 + \bar{\theta}_2 + \theta_1),$$

$$k_1 = \sqrt{\Delta_- K_+} + i(\Delta_+ + c_1 \omega_1^2),$$

$$k_2 = \sqrt{\Delta_- K_+} + i(\Delta_+ + c_1 \omega_2^2),$$

$$\bar{\Delta}_+ = \Delta_+ - i \left(\sqrt{\Delta_- K_+} + \sqrt{\bar{\Delta}_- \bar{K}_+} \right),$$

$$\begin{aligned}
\Lambda &= \left(1 + \sqrt{\frac{\Delta_-}{K_+}} \sqrt{\frac{\bar{\Delta}_-}{\bar{K}_+}} \right) (\omega_1 - \omega_2)^2, \\
p_1 &= t\omega_1 + x \left(\sqrt{\Delta_- K_+} + i(\Delta_+ + c_1 \omega_1^2) \right), \\
p_2 &= t\omega_2 + x \left(\sqrt{\Delta_- K_+} + i(\Delta_+ + c_1 \omega_2^2) \right), \\
r_1 &= t\omega_1 + ix c_1 \omega_1^2, \\
r_2 &= t\omega_2 + ix c_1 \omega_2^2, \\
\bar{r}_1 &= t\bar{\omega}_1 - ix c_1 \bar{\omega}_1^2, \\
\bar{r}_2 &= t\bar{\omega}_2 - ix c_1 \bar{\omega}_2^2, \\
K_- &= \Delta_+, \\
\Gamma &= \left(\Delta_- \bar{\Delta}_- + K_+ \bar{K}_+ \left(\frac{2\Lambda}{(\omega_1 - \omega_2)^2} - 1 \right) \right) (\omega_1 - \omega_2)^2 \\
&\quad \cdot (\bar{\omega}_1 - \bar{\omega}_2)^2, \\
\Phi &= c_1^2 K_+ \bar{K}_+ (\omega_1 + \bar{\omega}_1)^2 (\omega_2 + \bar{\omega}_1)^2 (\omega_2 + \bar{\omega}_2)^2 \\
&\quad \cdot (\omega_1 + \bar{\omega}_2)^2,
\end{aligned} \tag{14}$$

where c_1 , $K_{\pm}$, and $\Delta_{\pm}$ are the same as those in (10); other parameters can be seen in Appendix A. Obviously, Figure 2(a) describes the propagation characteristics of the two-soliton solution. Changing the parameters α , K_+ , and ω_2 , Figure 2(b) can be described certainly, which reveals the propagation process with the higher impact velocity than that in Figure 2(a) due to decreasing of the values of α , K_+ , and ω_2 ; and setting the values of ω_1 , ω_2 , and ω_3 to real constants, soliton solutions are derived in Figures 2(c) and 2(d), which describe the bright soliton and the breather propagate in parallel. The relationship in the space coordinate between the bright soliton and the breather also varies as changing the value of ω_2 .

2.3. Three-Soliton Solutions for (5). By treating the truncated perturbation expansions of g , h , and f as

$$\begin{aligned}
g &= \varepsilon g_1 + \varepsilon^3 g_3 + \varepsilon^5 g_5, \\
h &= \varepsilon h_1 + \varepsilon^3 h_3 + \varepsilon^5 h_5, \\
f &= 1 + \varepsilon^2 f_2 + \varepsilon^4 f_4 + \varepsilon^6 f_6,
\end{aligned} \tag{15}$$

we give the three-soliton solutions for (5) as follows:

$$\begin{aligned}
u &= \frac{g_1 + g_3 + g_5}{1 + f_2 + f_4 + f_6}, \\
v &= \frac{h_1 + h_3 + h_5}{1 + f_2 + f_4 + f_6},
\end{aligned} \tag{16}$$

with

$$\begin{aligned}
\theta_1 &= k_1 x + \omega_1 t + \varphi_1, \\
\theta_2 &= k_2 x + \omega_2 t + \varphi_2, \\
\theta_3 &= k_3 x + \omega_3 t + \varphi_3, \\
g_1 &= \exp(\theta_1) + \exp(\theta_2) + \exp(\theta_3), \\
f_6 &= \Omega \exp(\theta_1 + \theta_2 + \theta_3 + \bar{\theta}_1 + \bar{\theta}_2 + \bar{\theta}_3), \\
g_3 &= \eta_1 \exp(\theta_1 + \bar{\theta}_1 + \theta_2) + \eta_2 \exp(\theta_1 + \bar{\theta}_2 + \theta_2) \\
&\quad + \eta_3 \exp(\theta_1 + \bar{\theta}_3 + \theta_2) \\
&\quad + \eta_4 \exp(\theta_1 + \bar{\theta}_1 + \theta_3) \\
&\quad + \eta_5 \exp(\theta_1 + \bar{\theta}_2 + \theta_3) \\
&\quad + \eta_6 \exp(\theta_3 + \bar{\theta}_1 + \theta_2) \\
&\quad + \eta_7 \exp(\theta_3 + \bar{\theta}_2 + \theta_2) \\
&\quad + \eta_8 \exp(\theta_1 + \bar{\theta}_3 + \theta_3) \\
&\quad + \eta_9 \exp(\theta_3 + \bar{\theta}_3 + \theta_2), \\
g_5 &= \chi_1 \exp(\theta_1 + \theta_2 + \theta_3 + \bar{\theta}_1 + \bar{\theta}_2) \\
&\quad + \chi_2 \exp(\theta_1 + \theta_2 + \theta_3 + \bar{\theta}_1 + \bar{\theta}_3) \\
&\quad + \chi_3 \exp(\theta_1 + \theta_2 + \theta_3 + \bar{\theta}_3 + \bar{\theta}_2), \\
f_2 &= \sigma_1 \exp(\theta_1 + \bar{\theta}_1) + \sigma_2 \exp(\theta_2 + \bar{\theta}_2) \\
&\quad + \sigma_3 \exp(\theta_1 + \bar{\theta}_2) + \sigma_4 \exp(\theta_2 + \bar{\theta}_1) \\
&\quad + \sigma_5 \exp(\theta_3 + \bar{\theta}_3) + \sigma_6 \exp(\theta_3 + \bar{\theta}_1) \\
&\quad + \sigma_7 \exp(\theta_3 + \bar{\theta}_2) + \sigma_8 \exp(\theta_2 + \bar{\theta}_3) \\
&\quad + \sigma_9 \exp(\theta_1 + \bar{\theta}_3), \\
f_4 &= \rho_1 \exp(\theta_1 + \theta_2 + \bar{\theta}_2 + \bar{\theta}_1) \\
&\quad + \rho_2 \exp(\theta_1 + \theta_3 + \bar{\theta}_3 + \bar{\theta}_1) \\
&\quad + \rho_3 \exp(\theta_3 + \theta_2 + \bar{\theta}_2 + \bar{\theta}_3) \\
&\quad + \rho_4 \exp(\theta_1 + \theta_2 + \bar{\theta}_3 + \bar{\theta}_1) \\
&\quad + \rho_5 \exp(\theta_1 + \theta_2 + \bar{\theta}_2 + \bar{\theta}_3) \\
&\quad + \rho_6 \exp(\theta_1 + \theta_3 + \bar{\theta}_2 + \bar{\theta}_1) \\
&\quad + \rho_7 \exp(\theta_1 + \theta_3 + \bar{\theta}_2 + \bar{\theta}_3) \\
&\quad + \rho_8 \exp(\theta_3 + \theta_2 + \bar{\theta}_2 + \bar{\theta}_1) \\
&\quad + \rho_9 \exp(\theta_3 + \theta_2 + \bar{\theta}_3 + \bar{\theta}_1),
\end{aligned} \tag{17}$$

FIGURE 1: Evolution of the solutions of (10). (a) Parameters are $c_1 = 1$, $\alpha = 1$, $K_+ = 1 - i$, $\Delta_- = -1$, $\Delta_+ = 1$, $\omega_1 = 2$, and $\omega_3 = 3$. (b) With the same parameters as those in (a) except for $c_1 = -1$, $K_+ = -1$, $\Delta_- = 1.5$, and $\Delta_+ = 2$. (c) With the same parameters as those in (b) except for $\omega_3 = 1$.

where φ_1 , φ_2 , and φ_3 are the same as those in (13); other parameters can be seen in Appendix B. According to (16), with the parameters ω_1 , ω_2 , and ω_3 being complex numbers, we can obtain a precise three-bright-soliton in Figure 3(a); with the parameters ω_1 , ω_2 , and ω_3 being real ones, we can derive a precise three-breather in Figure 3(b). Decreasing the value of K_+ , α , and ω_3 while keeping other parameters as those in Figure 3(b), we obtain the colliding bright soliton with breather in Figure 3(c). Keeping the parameters the same as those in Figure 3(a), except for $\omega_1 = 2$, Figure 4(a) is derived. In contrast to the case in Figure 3(a), if setting $\omega_2 = -2$, we can obtain the opposite propagation process on x coordinate in Figure 4(b).

3. Conclusions

In this paper, our main focus is on (5) with the linear effects. Through the constructed Hirota system, we obtain

one-soliton solutions in Figure 1, two-soliton solutions in Figure 2, and three-soliton solutions in Figures 3 and 4 via symbolic computation, which describe the propagation characters of solitons with different values of parameters. Figure 1 describes one bright soliton in (a) and the bound soliton in (b) and (c) as the parameter ω_3 changes. Obviously, it shows in Figure 2 that two-bright-soliton turns to the parallel propagation between a bright soliton and a breather with change of the values of Δ_+ , ω_1 , and ω_3 . Choosing suitable values of ω_1 and ω_3 in Figure 2(a), the three-bright-soliton is derived in Figure 3(a). In particular, we seek out the three-breathers in Figure 3(b) with the real constants of ω_1 , ω_2 , and ω_3 . However, when one of ω_1 , ω_2 , and ω_3 is a complex one, we can receive the propagation process of the evolution between the bright soliton and the breather in Figure 4. Using all the above, via setting the key parameters to different values, we obtain optical multisoliton solutions with distinctive propagation characteristics.

FIGURE 2: Evolution of the solutions of (13). (a) Parameters are $c_1 = -1$, $\alpha = 1$, $K_+ = 1$, $\Delta_- = -2$, $\Delta_+ = 2$, $\omega_1 = 1 - i$, and $\omega_3 = -2 + i$. (b) Parameters are $c_1 = -1$, $\alpha = -1$, $K_+ = -3$, $\Delta_- = 2$, $\Delta_+ = 3$, $\omega_1 = 1$, and $\omega_3 = 2$. (c) With the same parameters as those in (b) except for $\Delta_+ = -3$, $\omega_1 = -1$, and $\omega_3 = -2$. (d) With the same parameters as those in (b) except for $\Delta_+ = -1$, $\omega_1 = 1 - i$, and $\omega_3 = -1$.

FIGURE 3: Evolution of the solutions of (16). (a) Parameters are $c_1 = -1$, $\alpha = 1$, $K_+ = 1$, $\Delta_- = -2$, $\Delta_+ = 2$, $\omega_1 = 2 - i$, $\omega_2 = -2 + i$, and $\omega_3 = -3$. (b) With the same parameters as those in (a) except for $\omega_1 = 1$, $\omega_2 = 2$, and $\omega_3 = 3$. (c) With the same parameters as those in (b) except for $\alpha = -1$, $K_+ = -3$, $\Delta_- = 2$, and $\omega_3 = 3 + i$.

Appendix

A. The Parameters of the Two-Soliton Solutions for (5)

Corresponding parameters in solution equations (13) are expressed as

$$\Gamma = (\Delta_- \bar{\Delta}_- + K_+ \bar{K}_+ (2\Lambda - 1)),$$

$$\Lambda = \left(1 + \sqrt{\frac{\Delta_-}{K_+}} \sqrt{\frac{\bar{\Delta}_-}{\bar{K}_+}} \right),$$

$$\sigma_1 = \frac{\alpha \Lambda}{c_1 (\omega_1 + \bar{\omega}_1)^2},$$

$$\sigma_2 = \frac{\alpha \Lambda}{c_1 (\omega_2 + \bar{\omega}_2)^2},$$

$$\sigma_3 = \frac{\alpha \Lambda}{c_1 (\omega_1 + \bar{\omega}_2)^2},$$

$$\sigma_4 = \frac{\alpha \Lambda}{c_1 (\omega_2 + \bar{\omega}_1)^2},$$

$$\eta_1 = \frac{\alpha \Lambda (\omega_1 - \omega_2)^2}{c_1 (\omega_1 + \bar{\omega}_1)^2 (\bar{\omega}_1 + \omega_2)^2},$$

$$\begin{aligned}
\sigma_5 &= \frac{\alpha^2 \Gamma (\omega_1 - \omega_2)^2 (\bar{\omega}_1 - \bar{\omega}_2)^2}{K_+ \bar{K}_+ c_1^2 (\omega_2 + \bar{\omega}_2)^2 (\omega_1 + \bar{\omega}_1)^2 (\omega_1 + \bar{\omega}_2)^2 (\omega_2 + \bar{\omega}_1)^2}, \\
\eta_2 &= \frac{\alpha \Lambda (\omega_1 - \omega_2)^2}{c_1 (\omega_1 + \bar{\omega}_2)^2 (\bar{\omega}_2 + \omega_2)^2},
\end{aligned} \tag{A.1}$$

where c_1 and α are real constants.

B. The Parameters of the Three-Soliton Solutions for (5)

With the same value of Γ and Λ as those in Appendix A, corresponding parameters in solution equations (16) are expressed as

$$\begin{aligned}
\sigma_1 &= \frac{\alpha \Lambda}{c_1 (\omega_1 + \bar{\omega}_1)^2}, \\
\sigma_2 &= \frac{\alpha \Lambda}{c_1 (\omega_2 + \bar{\omega}_2)^2}, \\
\sigma_3 &= \frac{\alpha \Lambda}{c_1 (\omega_1 + \bar{\omega}_2)^2}, \\
\sigma_4 &= \frac{\alpha \Lambda}{c_1 (\omega_2 + \bar{\omega}_1)^2}, \\
\sigma_5 &= \frac{\alpha \Lambda}{c_1 (\omega_3 + \bar{\omega}_3)^2}, \\
\sigma_6 &= \frac{\alpha \Lambda}{c_1 (\omega_3 + \bar{\omega}_1)^2}, \\
\sigma_7 &= \frac{\alpha \Lambda}{c_1 (\omega_3 + \bar{\omega}_2)^2}, \\
\sigma_8 &= \frac{\alpha \Lambda}{c_1 (\omega_2 + \bar{\omega}_3)^2}, \\
\sigma_9 &= \frac{\alpha \Lambda}{c_1 (\omega_1 + \bar{\omega}_3)^2}, \\
\chi_1 &= \frac{\alpha^2 \Gamma (\omega_1 - \omega_2)^2 (\omega_1 - \omega_3)^2 (\omega_2 - \omega_3)^2 (\bar{\omega}_1 - \bar{\omega}_2)^2}{c_1^2 K_+ \bar{K}_+ (\omega_1 + \bar{\omega}_1)^2 (\omega_2 + \bar{\omega}_1)^2 (\omega_3 + \bar{\omega}_1)^2 (\omega_1 + \bar{\omega}_2)^2 (\omega_2 + \bar{\omega}_2)^2 (\omega_3 + \bar{\omega}_2)^2}, \\
\chi_2 &= \frac{\alpha^2 \Gamma (\omega_1 - \omega_2)^2 (\omega_1 - \omega_3)^2 (\omega_2 - \omega_3)^2 (\bar{\omega}_1 - \bar{\omega}_3)^2}{c_1^2 K_+ \bar{K}_+ (\omega_1 + \bar{\omega}_1)^2 (\omega_2 + \bar{\omega}_1)^2 (\omega_3 + \bar{\omega}_1)^2 (\omega_1 + \bar{\omega}_3)^2 (\omega_2 + \bar{\omega}_3)^2 (\omega_3 + \bar{\omega}_3)^2}, \\
\chi_3 &= \frac{\alpha^2 \Gamma (\omega_1 - \omega_2)^2 (\omega_1 - \omega_3)^2 (\omega_2 - \omega_3)^2 (\bar{\omega}_2 - \bar{\omega}_3)^2}{c_1^2 K_+ \bar{K}_+ (\omega_2 + \bar{\omega}_2)^2 (\omega_1 + \bar{\omega}_2)^2 (\omega_3 + \bar{\omega}_2)^2 (\omega_1 + \bar{\omega}_3)^2 (\omega_2 + \bar{\omega}_3)^2 (\omega_3 + \bar{\omega}_3)^2}, \\
\rho_1 &= \frac{\alpha^2 \Gamma (\omega_1 - \omega_2)^2 (\bar{\omega}_1 - \bar{\omega}_2)^2}{c_1^2 K_+ \bar{K}_+ (\omega_1 + \bar{\omega}_1)^2 (\omega_2 + \bar{\omega}_1)^2 (\omega_1 + \bar{\omega}_2)^2 (\bar{\omega}_2 + \omega_2)^2}, \\
\eta_1 &= \frac{\alpha \Lambda (\omega_1 - \omega_2)^2}{c_1 (\omega_1 + \bar{\omega}_1)^2 (\bar{\omega}_1 + \omega_2)^2}, \\
\rho_2 &= \frac{\alpha^2 \Gamma (\omega_1 - \omega_3)^2 (\bar{\omega}_1 - \bar{\omega}_3)^2}{c_1^2 K_+ \bar{K}_+ (\omega_1 + \bar{\omega}_1)^2 (\omega_3 + \bar{\omega}_1)^2 (\omega_1 + \bar{\omega}_3)^2 (\bar{\omega}_3 + \omega_3)^2}, \\
\eta_2 &= \frac{\alpha \Lambda (\omega_1 - \omega_2)^2}{c_1 (\omega_1 + \bar{\omega}_2)^2 (\bar{\omega}_2 + \omega_2)^2},
\end{aligned}$$

FIGURE 4: Evolution of the solutions of (13). (a) With the same parameters as those in Figure 3(a) except for $\omega_1 = 2$. (b) With the same parameters as those in (a) except for $\omega_2 = -2$.

$$\rho_3 = \frac{\alpha^2 \Gamma (\omega_2 - \omega_3)^2 (\bar{\omega}_2 - \bar{\omega}_3)^2}{c_1^2 K_+ \bar{K}_+ (\omega_2 + \bar{\omega}_2)^2 (\omega_3 + \bar{\omega}_2)^2 (\omega_2 + \bar{\omega}_3)^2 (\bar{\omega}_3 + \omega_3)^2},$$

$$\eta_3 = \frac{\alpha \Lambda (\omega_1 - \omega_2)^2}{c_1 (\omega_1 + \bar{\omega}_3)^2 (\bar{\omega}_3 + \omega_2)^2},$$

$$\rho_4 = \frac{\alpha^2 \Gamma (\omega_1 - \omega_2)^2 (\bar{\omega}_1 - \bar{\omega}_3)^2}{c_1^2 K_+ \bar{K}_+ (\omega_1 + \bar{\omega}_1)^2 (\omega_2 + \bar{\omega}_1)^2 (\omega_1 + \bar{\omega}_3)^2 (\bar{\omega}_2 + \omega_3)^2},$$

$$\eta_4 = \frac{\alpha \Lambda (\omega_1 - \omega_3)^2}{c_1 (\omega_1 + \bar{\omega}_1)^2 (\bar{\omega}_1 + \omega_3)^2},$$

$$\rho_5 = \frac{\alpha^2 \Gamma (\omega_1 - \omega_2)^2 (\bar{\omega}_2 - \bar{\omega}_3)^2}{c_1^2 K_+ \bar{K}_+ (\omega_1 + \bar{\omega}_2)^2 (\omega_2 + \bar{\omega}_2)^2 (\omega_1 + \bar{\omega}_3)^2 (\bar{\omega}_3 + \omega_2)^2},$$

$$\eta_5 = \frac{\alpha \Lambda (\omega_1 - \omega_3)^2}{c_1 (\omega_1 + \bar{\omega}_2)^2 (\bar{\omega}_2 + \omega_3)^2},$$

$$\rho_6 = \frac{\alpha^2 \Gamma (\omega_1 - \omega_3)^2 (\bar{\omega}_2 - \bar{\omega}_1)^2}{c_1^2 K_+ \bar{K}_+ (\omega_1 + \bar{\omega}_1)^2 (\omega_3 + \bar{\omega}_2)^2 (\omega_1 + \bar{\omega}_2)^2 (\omega_3 + \bar{\omega}_2)^2},$$

$$\begin{aligned}
\eta_6 &= \frac{\alpha \Lambda (\omega_3 - \omega_2)^2}{c_1 (\omega_3 + \bar{\omega}_1)^2 (\bar{\omega}_1 + \omega_2)^2}, \\
\rho_7 &= \frac{\alpha^2 \Gamma (\omega_1 - \omega_3)^2 (\bar{\omega}_2 - \bar{\omega}_3)^2}{c_1^2 K_+ \bar{K}_+ (\omega_1 + \bar{\omega}_2)^2 (\omega_3 + \bar{\omega}_2)^2 (\omega_1 + \bar{\omega}_3)^2 (\omega_3 + \bar{\omega}_3)^2}, \\
\eta_7 &= \frac{\alpha \Lambda (\omega_2 - \omega_3)^2}{c_1 (\omega_2 + \bar{\omega}_2)^2 (\bar{\omega}_2 + \omega_3)^2}, \\
\rho_8 &= \frac{\alpha^2 \Gamma (\omega_2 - \omega_3)^2 (\bar{\omega}_1 - \bar{\omega}_2)^2}{c_1^2 K_+ \bar{K}_+ (\omega_2 + \bar{\omega}_1)^2 (\omega_3 + \bar{\omega}_1)^2 (\omega_2 + \bar{\omega}_2)^2 (\omega_3 + \bar{\omega}_2)^2}, \\
\eta_8 &= \frac{\alpha \Lambda (\omega_1 - \omega_3)^2}{c_1 (\omega_1 + \bar{\omega}_3)^2 (\bar{\omega}_3 + \omega_3)^2}, \\
\rho_9 &= \frac{\alpha^2 \Gamma (\omega_2 - \omega_3)^2 (\bar{\omega}_1 - \bar{\omega}_3)^2}{c_1^2 K_+ \bar{K}_+ (\omega_2 + \bar{\omega}_1)^2 (\omega_3 + \bar{\omega}_1)^2 (\omega_2 + \bar{\omega}_3)^2 (\omega_3 + \bar{\omega}_3)^2}, \\
\eta_9 &= \frac{\alpha \Lambda (\omega_3 - \omega_2)^2}{c_1 (\omega_3 + \bar{\omega}_2)^2 (\bar{\omega}_3 + \omega_3)^2},
\end{aligned} \tag{B.1}$$

where c_1 and α are real constants.

Competing Interests

The authors declare that they have no competing interests.

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