Research Article

Compressive Sensing of Multichannel EEG Signals via $l_q$ Norm and Schatten-$p$ Norm Regularization

Jun Zhu,¹ Changwei Chen,² Shoubao Su,³ and Zinan Chang³

¹School of Computer Engineering, Jinling Institute of Technology, Nanjing 211169, China
²College of Computer and Information Engineering, Nanjing Xiaozhuang University, Nanjing 210017, China
³Jinling Institute of Technology, Nanjing, China

Correspondence should be addressed to Jun Zhu; xiaobaabcabc@126.com

Received 11 June 2016; Revised 25 September 2016; Accepted 19 October 2016

Academic Editor: Raffaele Solimene

Copyright © 2016 Jun Zhu et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

In Wireless Body Area Networks (WBAN) the energy consumption is dominated by sensing and communication. Recently, a simultaneous cosparsity and low-rank (SCLR) optimization model has shown the state-of-the-art performance in compressive sensing (CS) recovery of multichannel EEG signals. How to solve the resulting regularization problem, involving $l_0$ norm and rank function which is known as an NP-hard problem, is critical to the recovery results. SCLR takes use of $l_1$ norm and nuclear norm as a convex surrogate function for $l_0$ norm and rank function. However, $l_1$ norm and nuclear norm cannot well approximate the $l_0$ norm and rank because there exist irreparable gaps between them. In this paper, an optimization model with $l_q$ norm and schatten-$p$ norm is proposed to enforce cosparsity and low-rank property in the reconstructed multichannel EEG signals. An efficient iterative scheme is used to solve the resulting nonconvex optimization problem. Experimental results have demonstrated that the proposed algorithm can significantly outperform existing state-of-the-art CS methods for compressive sensing of multichannel EEG channels.

1. Introduction

The electroencephalogram (EEG) signal is one of the most frequently used biomedical signals [1, 2]. It is known that EEG signals are important health indicators for stroke and trauma; recent studies also indicate that EEG signals can be used for studying dementia and Alzheimer disease. Therefore the monitoring of these signals is of utmost importance. However, continuous EEG monitoring usually records a large number of data which is too large to be sampled and transmitted in many applications [3–5]. To overcome this issue, prior studies have proposed compressive sensing (CS) [6, 7].

From fewer measurements than suggested by the Nyquist theory, compressive sensing (CS) proves that a signal can be recovered when it is sparse in a transform domain. The sampling model is formulated as follows:

\[
y = \Phi x,
\]

where $y$ is the random measurement and $\Phi \in \mathbb{R}^{M \times N}$ ($M < N$) is the sampling matrix. CS assumes that the signal $x \in \mathbb{R}^N$ can be represented as $x = \Psi s$, where $\Psi$ is the transform domain and $s$ only contains a small number of nonzero elements. Then the synthesis based $l_0$-minimization model is formulated as

\[
\min_s \|s\|_0 \quad \text{s.t.} \quad y = \Phi \Psi s,
\]

where $\|s\|_0$ counts the number of nonzero elements in $s$. Many methods are proposed to solve problem (2), such as BP [6], OMP [8], and IHT [9].

Different from the traditional sparse or block sparse signal model, the cosparse signal model [10, 11] assumes that a signal multiplied by an analysis operator results in a sparse vector. The analysis based $l_0$-minimization problem can be formulated:

\[
\min_x \|Ox\|_0 \quad \text{s.t.} \quad y = \Phi x,
\]
where $O \in \mathbb{R}^{K \times N}$ ($K \geq N$) is the analysis operator and $a = Ox$ is the cosparse vector. The above minimization problem can be efficiently solved by many methods, including GAP [10], ABS [12], AIHT [13], and ACoSaMP [14].

The cosparse analysis method has a number of advantages for multichannel EEG signals, which has been demonstrated in [15]. First, compared with the sparse synthesis model which limits the incoherence of the sampling matrix, the cosparse analysis model allows the columns of the analysis operator to be coherent, which can obtain better recovery results. Second, the sparse synthesis model firstly estimates the sparse vector $s$ and then estimates the signal, but the cosparse analysis model directly estimates the EEG signal. In a word, the cosparse analysis method is more suitable than sparse synthesis approach for CS recovery of multichannel EEG signals.

Since the EEG signals from multiple channels are correlated with each other, they motivate us to recover multichannel EEG signals via low-rank regularization [16–18]. Recently, a simultaneous cosparsity and low-rank (SCLR) optimization model [15] has shown the state-of-the-art performance in CS recovery of multichannel EEG signals. SCLR chooses the second-order difference matrix as the analysis operator to enforce the approximate piecewise linear structure, and it takes use of $l_1$ norm and nuclear norm as a convex surrogate function for $l_0$ norm and rank function. However, SCLR approach may obtain suboptimal results in real application since the $l_1$ norm and nuclear norm may not be good surrogate functions for $l_0$ norm and rank. There exist irreparable gaps between $l_0$ norm, the real rank and $l_1$ norm, and nuclear norm, respectively. The optimization results based on convex surrogate functions essentially deviate from the real solution of original minimization problem.

Motivated by the fact that $l_q$ norm can obtain a more accurate result in sparse synthesis model [19, 20], schatten-$p$ norm can efficiently recover low-rank matrix in image denoising [21, 22]. They have been proved rigorously in theory that $l_q$ norm and schatten-$p$ norm are equivalent to $l_0$ norm and rank function, respectively, when $p$ and $q$ are tend to be 0. So it is desirable to take them together to better exploit cosparsity and low-rank property of multichannel EEG signals.

In this paper, a novel CS model based on $l_q$ norm and schatten-$p$ norm (LQSP) is proposed for the compressive sensing recovery of multichannel EEG signals reconstruction. We take use of $l_q$ norm for the $l_0$ norm to enforce cosparsity prior and employ schatten-$p$ norm for the matrix rank to enforce low-rank property prior. In addition, the alternating direction method of multipliers (ADMM) is used to efficiently solve the resulting nonconvex optimization problem.

The rest of the paper is organized as follows. In Section 2, we present our proposed LQSP in detail to exploit the cosparsity and low-rank property. In Section 3, we show that the optimization problems can be solved efficiently by the alternating direction multiplier method. Then we present the numerical experiments in Section 4. Section 5 provides some concluding remarks.

### 2. $l_q$ Norm and Schatten-$p$ Norm for CS Recovery of Multichannel EEG Signals

The cosparse recovery model for multichannel EEG signals can be represented as [10]

$$\min_x \|r(OX)\|_0 \quad \text{s.t.} \quad Y = \Phi X,$$

where $X \in \mathbb{R}^{N \times C}$ and $C$ is the number of the channels. $r(X)$ puts all the columns of $X$ into the column vector sequentially.

In Figure 1, we select chb01_31.edf which is used in our experiments as the test data and take the second-order difference matrix as the cosparse operator. From Figure 1, we can see that most entries of the cosparse vector are nearly zero and many singular values are close to 0, which have shown that our test data naturally have both cosparsity and low-rank property. So we simultaneously exploit these two useful priors in multichannel EEG signal recovery form the compressed measurement. Then the optimization model can be reformulated as [15]

$$\min_x \|r(OX)\|_0 + \lambda \text{rank}(X)$$

s.t. $Y = \Phi X.$

We cannot directly solve the above optimization problem that contained the $l_q$ norm and matrix rank function, which is known as an NP-hard problem. To obtain an approximated solution, SCLR employs $l_1$ norm and nuclear norm as surrogate functions for $l_0$ norm and matrix rank, where the $l_1$ norm sums all the absolute values of the entries and the nuclear norm sums all the singular values of the matrix. However, SCLR may obtain suboptimal results by using convex surrogate functions.

Motivated by the fact that $l_q$ norm can obtain a more accurate result in sparse synthesis model [19, 20] and schatten-$p$ norm can efficiently recover low-rank matrix in image denoising [21, 22], we propose to take use of $l_q$ norm and schatten-$p$ norm as nonconvex surrogate functions for $l_0$ norm and rank function. Then the problem can be reformulated as follows:

$$\min_x \|r(OX)\|_q^q + \lambda \|X\|_p^p$$

s.t. $Y = \Phi X,$

where $l_q$ norm sums all the absolute values of the entries to the power of $q$ and $\|X\|_p^p$ sums all the singular values of $X$ to the power of $p$.

### 3. Optimization Algorithm

It is very difficult to solve the above constrained optimization problem, so we employ ADMM, which has been widely used in compressive sensing [23–25], to divide this complicated problem into simpler subproblems and address them iteratively. Figure 2 gives the flow chart of the proposed approach.
By adding a set of auxiliary variables \( \{A, B\} \), the recovery problem can be reformulated as

\[
\begin{align*}
\min_{X} & \quad \|r(A)\|_q^p + \lambda \|B\|_p^p \\
\text{s.t.} & \quad Y = \Phi X, \\
& \quad A = OX, \\
& \quad X = B.
\end{align*}
\]

\( (7) \)

The corresponding augmented Lagrangian term is

\[
\begin{align*}
\min_{X,A,B} & \quad \|r(A)\|_q^p + \lambda \|B\|_p^p + \frac{\beta_1}{2} \|\Phi X - Y + \frac{f_1}{\beta_1}\|_F^2 \\
& \quad + \frac{\beta_2}{2} \|OX - A + \frac{f_2}{\beta_2}\|_F^2 + \frac{\beta_3}{2} \|X - B + \frac{f_3}{\beta_3}\|_F^2,
\end{align*}
\]

\( (8) \)
\( f_1 = \text{zeros}(M,C), f_2 = \text{zeros}(K,C), f_3 = \text{zeros}(N,C); \)

**Input:** \( \Phi, O \in \mathbb{R}^{K \times N} \) \((K = 2 \times N), \beta_1 = \beta_2 = \beta_3 = 1, \lambda = 1, \tau = 0.05, f_1, f_2, f_3 \) is a set of Lagrangian multipliers. Problem (8) consists of the following three subproblems:

\[
\begin{align*}
X^{k+1} &= \min_{X} \frac{\beta_1}{2} \left\| \Phi X - Y + \frac{f_1}{\beta_1} \right\|_F^2 \\
&\quad + \frac{\beta_2}{2} \left\| OX - A^k + \frac{f_2}{\beta_2} \right\|_F^2 \\
&\quad + \frac{\beta_3}{2} \left\| X - B^k + \frac{f_3}{\beta_3} \right\|_F^2, \\
A^{k+1} &= \min_{A} \left\| r(A) \right\|_F^2 + \frac{\beta_2}{2} \left\| OX^{k+1} - A + \frac{f_2}{\beta_2} \right\|_F^2, \\
B^{k+1} &= \min_{B} \lambda \left\| B_\delta \right\|_F^2 + \frac{\beta_3}{2} \left\| X^{k+1} - B + \frac{f_3}{\beta_3} \right\|_F^2.
\end{align*}
\]

Next, we present the details for solving each subproblem.

### 3.1. X Subproblem

X subproblem is a quadratic optimization problem admitting a closed-form solution

\[
X^{k+1} = \left( \beta_1 \Phi^T \Phi + \beta_2 O^T O + \beta_3 I \right)^{-1} \left( \beta_1 \Phi^T Y + \beta_2 O^T A^k + \beta_3 B^k - \Phi^T f_1 - O^T f_2 - f_3 \right),
\]

where \( I \in \mathbb{R}^{N \times N} \) is an identity matrix.

### 3.2. A Subproblem

A subproblem is a nonconvex problem; we cannot obtain the global minimizer. But we can solve it by using an iteratively reweighted approach. Assume

\[
A^{k+1,j+1} = \min_{A} \sum_{j} \alpha_2 \left\| r \left( OX - A + \frac{f_2}{\beta_2} \right) \right\|_2^2,
\]

where \( \alpha_2 \) is the weight of \( [\alpha_2]^{g+1} \) that is computed from the previous iterative \( A^{k+1,j} \) and \( \alpha_2 \) is the \( j \)th value of \( \alpha \). Then the problem admits a closed-form solution [26].

\[
a^{k+1,j+1} = \max \left( r \left( OX^{k+1} + \frac{f_2}{\beta_2} \right) - \frac{1}{\beta_2} u_2, 0 \right);
\]

when \( a^{k+1,j+1} \) satisfies the convergence condition, we set \( a^{k+1} = a^{k+1,j+1} \).

### 3.3. B Subproblem

Unfortunately, the B subproblem is also a nonconvex problem. The algorithm to solve this was derived in [23] which is called weighted singular value shrinkage.

\[
B^{k+1,c+1} = U \max \left\{ \Delta - \frac{\lambda}{\beta_3} \text{diag}(\delta_{j}), 0 \right\} V^T,
\]

where \( U \Delta V^T \) is the SVD of \( (X^{k+1} + f_3/\beta_3) \) and \( \delta_{j} = \beta_2 \left( \delta_{j}^k \right)^{-1} (X^{k+1} + f_3/\beta_3). \) \( \delta_{j}^k \) is the \( j \)th singular value of \( B^{k+1,c}. \) When \( B^{k+1,c+1} \) satisfies the convergence condition, we set \( B^{k+1} = B^{k+1,c+1} \) (see Algorithm 1).

### 4. Numerical Experimental Results

In this section, extensive experiments are conducted to verify the performance of the proposed LQSP approach. We compare our method with SCLR based on interior point method (SCLR-I) [15], ADMM method based SCLR (SCLR-A) [15], simultaneous orthogonal matching pursuit (SOMP) [27], BSBL [5], and simultaneous greedy analysis pursuit (SGAP) [28]. The experiments are carried out on the CHB-MIT scalp EEG database which is online available in the PhysioBank database: http://www.physionet.org/cgi-bin/atm/ATM [29]. In our experiments, the EEG recording chb01_31.edf is used to demonstrate the superiority of our approach. To quantify the difference between the estimate results and the original data, MSE and MCC are used. MSE = \( \sum_{g=1}^{G} \| \bar{X} - \bar{X}/GNC \|_F^2/G \) measures the average of the squares of the errors. G is the number of the experiments. MCC = \( \sum_{g=1}^{G} \text{vec}(X)^T \text{vec}(\bar{X})/G \| X \|_F \text{vec}(\bar{X})/G \) is equivalent to the structural similarity index, which measures the similarity of two signals [30].

The parameter settings of CS-TSPN are as follows: the second-order difference matrix is chosen as the analysis operator; the sampling matrix is the Gaussian matrix; the number of compressive measurements is denoted by rate = \( M/N \); \( q \) and \( p \) are variables, which are selected from 0.1 to 1.0.

**Algorithm 1:** Compressive sensing recovery via \( l_1 \) norm and Schatten-\( p \) norm regularization.
0.5 by step 0.1, respectively. In order to save computational complexity, the inner iteration numbers $t$ and $c$ are set to 1. In addition, the sparse dictionary of SOMP is Daubechies wavelets.

Figures 3–5 display the values of MSE, MCC, and CPU time of the different approaches for compressive sensing recovery of multichannel signals at different sensing rates. We can see that SCLR-I and SCLR-A can obtain better results than BSBL, which is reported to be the best candidate for EEG signal recovery based on sparse synthesis model. This conclusion has been demonstrated in [15]. LQSP outperforms the other ones in accuracy. The speed of LQSP is faster than SCLR-I and the same as SCLR-A. The greedy algorithms SOMP and SGAP are much faster than the rest; their accuracy is much worse and not acceptable. Therefore, it is demonstrated that our proposed LQSP approach is a better candidate for multichannel EEG signal recovery than the other methods.

It is very important to choose the proper $q$ and $p$ for LQSP. We analyze the influence of variables $q$ and $p$ on the signal recovery results. Table 1 lists all the optimal values of $q$ and $p$ at each sensing rate, from which we can find that the optimal $q$ and $p$ values are different in each case. Figure 6 shows the MCC versus $q$ value with rate $= 0.3$ and $p = 0.5$. Figure 7 shows the MCC versus $p$ value with rate $= 0.3$ and $q = 0.4$. From Figures 5 and 6, the selections of $p$ and $q$ values are crucial to our proposed approach.
Figure 7: MCC versus $p$ value with rate $= 0.3$ measurements and $q = 0.4$.

Table 1: The optimal values of variables $p$ and $q$ at each sensing rate.

<table>
<thead>
<tr>
<th>Value</th>
<th>Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.1</td>
</tr>
<tr>
<td>$p$</td>
<td>0.1</td>
</tr>
<tr>
<td>$q$</td>
<td>0.1</td>
</tr>
</tbody>
</table>

5. Conclusion
In this paper, we have presented a new approach toward CS recovery of multichannel EEG signals by exploiting cosparsity and low-rank property simultaneously in a unified manner. $l_q$ norm and schatten-$p$ norm are used as the nonconvex surrogate functions for the $l_0$ norm and matrix rank, respectively, and ADMM is applied to efficiently solve the resulting nonconvex optimization problem. Experiments have shown that LQSP can achieve the superior performance compared with other competitive reconstruction algorithms with the same amount of measurements.

Competing Interests
The authors declare that they have no competing interests.

Acknowledgments
This work was supported by the Natural Science Fund for Colleges and Universities in Jiangsu Province, Grants no. 16KJB520014 and no. 14KJB520012, the Jiangsu Key Laboratory of Image and Video Understanding for Social Safety (Nanjing University of Science and Technology), Grant no. 3091601407, the National Natural Science Foundation of China, no. 61375121, and the Doctoral Scientific Research Foundation of Jinling Institute of Technology, no. Jit-b-201508.

References


Submit your manuscripts at
http://www.hindawi.com