Research Article

H_∞ Controller Design for Asynchronous Hybrid Systems with Multiple Delays

Xiaoqiang Sun and Weijie Mao

Institute of Cyber-Systems and Control, Zhejiang University, Yuquan Campus, Hangzhou, Zhejiang 310027, China

Correspondence should be addressed to Weijie Mao; wjmao@iipc.zju.edu.cn

Received 17 December 2015; Revised 27 June 2016; Accepted 11 July 2016

Academic Editor: Masoud Hajarian

Copyright © 2016 X. Sun and W. Mao. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Solutions for the H_∞ synthesis problems of asynchronous hybrid systems with input-output delays are proposed. The continuous-time lifting approach of sampled-data systems is extended to a hybrid system with multiple delays, and some feasible formulas to calculate the operators of the equivalent discrete-time (DT) system are given. Different from the existing methods derived from symplectic pair theory or by state augmentation, a Lyapunov-Krasovskii functional to solve the synthesis problem is explicitly constructed. The delay-dependent stability conditions we obtained can be described in terms of nonstrict linear matrix inequalities (LMIs), which are much more convenient to be solved by LMI tools.

1. Introduction

Networked Control Systems (NCSs) as shown in Figure 1 are controlled systems containing several distributed plants which are connected through a communication network. For NCSs, one has to check the robustness of a control law with respect to the additional dynamics introduced by the communication networks [1–5].

NCSs are widely applied, such as mobile sensor networks, remote surgery, automated highway systems, and unmanned aerial vehicles [6, 7]. Time delay is encountered in many dynamic systems such as chemical or process control systems and NCSs and often results in poor performance which even can lead to instability [8]. This paper focuses on the influence of transmission delay and asynchronous samplings of NCSs.

In the last decades, input-delay approach became popular in NCSs, which is based on time-independent Lyapunov-Krasovskii functionals or Lyapunov-Razumikhin functions to analyze systems and design controller under uncertain sampling delay with known upper bound on the sampling intervals [9–11].

Meanwhile, robust stability for sampled-data systems with delays has attracted extensive attention during recent years [12–14]. Among them, most works are devoted to discuss continuous-time systems with delay based on algebraic

Riccati equation (ARE) such as [15] or the LMI approach derived via Lyapunov-Krasovskii functionals [16, 17]. Many control problems cannot be dealt with easily when adopting these methods. Lyapunov-Krasovskii functionals are also adopted to analyze switched systems with delay and asynchronous switching [18, 19]. However, in these papers, input delay and switching delay are considered separately.

With regard to sampled-data systems, lifting technique is a crucial method to transform a continuous-time system without delay to a discrete-time one. Most papers adopt this technique to transform multirate sampled-data linear continuous-time systems to corresponding discrete-time-invariant systems [20, 21]. Then, the DT system is analyzed according to symplectic pair theory. However, symplectic pair theory is very complicated to analyze and is difficult to acquire a controller via guaranteed cost control method, such as H_∞ control, H_2 , and LQG, because, according to the theory, it needs to compute controllers and disturbance attenuation alternately under the condition that eigenvalues of corresponding matrices S_l and S_r must be on $\partial\mathbb{D}$. And the system we discuss in this paper is with time delay; some examples have shown that although eigenvalues of corresponding matrix are less than 1, the DT system with delays still cannot be stable even with very small delays [22].

where t_k is a strictly increasing sequence of numbers, presenting the instants when the control input is updated; then,

$$\begin{aligned} A_1 &= \begin{bmatrix} A_c & B_{2c} \\ 0 & 0 \end{bmatrix}, \\ B_1 &= \begin{bmatrix} B_{1c} \\ 0 \end{bmatrix}, \\ C_1 &= [C_{1c} \quad D_{12c}], \\ A_2(t_k) &= \begin{bmatrix} I & 0 \\ 0 & I - H^{\tau_2(k)} \end{bmatrix}, \\ B_2(t_k) &= \begin{bmatrix} 0 \\ H^{\tau_2(k)} \end{bmatrix}, \\ C_2(t_k) &= [S^{\tau_1(i)} C_{2c} \quad S^{\tau_1(i)} D_{22c}]. \end{aligned} \quad (4)$$

Sampler S with period T_1 is supposed to be an ideal sampling process with bounded time delays; sampling delay is denoted as $\tau_1(k)$ at every sampling instant; then,

$$y = S^{\tau_1(k)} y_c, \quad k \in Z_+, \quad (5)$$

where

$$S^{\tau_1(k)} := \text{diag} \{v_1(k), v_2(k), \dots, v_q(k)\}. \quad (6)$$

The sampling delay is supposed to satisfy

$$0 < \tilde{T}_m \leq \tau_1(k) \leq \tilde{T}_M, \quad (7)$$

and element of $v_i(k)$ is

$$v_i(k) := \begin{cases} 1, & \text{if } k = T_k + \phi, \quad 0 < \phi \leq T_{k+1} - T_k \\ 0, & \phi > T_{k+1} - T_k \end{cases} \quad (8)$$

$\forall i = 1, 2, \dots, q$, j is an integer, and ϕ is the time skew of sampling.

Similarly, holder H with period T_2 is a zero-order hold with time delay, and input delay is $\tau_2(k)$ at every updating instant; that is,

$$u_c = H^{\tau_2(k)} u, \quad k \in Z_+, \quad (9)$$

where

$$H^{\tau_2(k)} := \text{diag} \{h_1(k), h_2(k), \dots, h_m(k)\}. \quad (10)$$

The transmission delay is supposed to satisfy

$$0 < \bar{T}_m \leq \tau_2(k) \leq \bar{T}_M, \quad (11)$$

and elements of $h_i(k)$ satisfy

$$h_i(k) := \begin{cases} 1, & \text{if } k = T_k + \varphi, \quad 0 < \varphi \leq T_{k+1} - T_k \\ 0, & \varphi > T_{k+1} - T_k \end{cases} \quad (12)$$

$\forall i = 1, 2, \dots, m$, j is an integer, and φ denotes the transmission skew of actuator.

The whole delay of system satisfies

$$\mathcal{T}_m = \tilde{T}_m + \bar{T}_m \leq \tau_1(k) + \tau_2(k) \leq \tilde{T}_M + \bar{T}_M = \mathcal{T}_M. \quad (13)$$

The following lemmas will be used in this paper.

Lemma 1 (see [31]). Assume $\beta > 0$. Then β^2 is an eigenvalue of $D_{11}D_{11}^T$ iff $\det[\underline{Q}_{11}(h)] = 0$ ($\underline{Q}_{11}(h)$ will be defined in the following section).

Lemma 2 (Schur complement). Given constant matrices Ω_1 , Ω_2 , and Ω_3 where $\Omega_1 = \Omega_1^T$ and $0 < \Omega_2 = \Omega_2^T$ then $\Omega_1 + \Omega_3^T \Omega_2^{-1} \Omega_3 < 0$ if and only if

$$\begin{aligned} \begin{bmatrix} \Omega_1 & \Omega_3^T \\ \Omega_3 & -\Omega_2 \end{bmatrix} &< 0 \text{ or} \\ \begin{bmatrix} -\Omega_2 & \Omega_3 \\ \Omega_3^T & \Omega_1 \end{bmatrix} &< 0. \end{aligned} \quad (14)$$

Lemma 3 (see [34]). Assume that $a \in \mathbb{R}^{n_a}$, $b \in \mathbb{R}^{n_b}$, and $N \in \mathbb{R}^{n_a \times n_b}$. Then, for any matrices $X \in \mathbb{R}^{n_a \times n_b}$, $Y \in \mathbb{R}^{n_a \times n_b}$, and $Z \in \mathbb{R}^{n_b \times n_b}$ satisfying

$$\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} \geq 0, \quad (15)$$

the following holds:

$$-2a^T N b \leq \begin{bmatrix} a \\ b \end{bmatrix}^T \begin{bmatrix} X & Y - N \\ Y^T - N^T & Z \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}. \quad (16)$$

3. Model Transformation

In this paper, t_k is scheduled to be uniform distribution after optimization to make a trade-off between discretization time for better performance and smaller delay bounds. Due to the fact that the sampler and holder both with delays, thus input time and time slot for discretization need not be simultaneous, and sampling and holding need not be synchronous either.

Defining $\theta_k := t_{k+1} - t_k$, if t_k , $k = 1, 2, \dots$, are equally separated, that is, $\theta_k = h$, $k = 1, 2, \dots$, consider the lifting operator $W: L_2[0, \infty) \rightarrow \ell(L_2(0, h])$, and the hybrid system G can be transformed to the following DT system $\hat{G}$, shown in Figure 3.

Let $\hat{w} = Ww$ and $\hat{z} = Wz$; then,

$$\hat{G}: \begin{cases} \hat{x}_{k+1} = \hat{A}_k \hat{x}_k + \hat{B}_{1k} \hat{w}_k + \hat{B}_{2k} \hat{u}_{k, \tau_2}, \\ \hat{z}_k = \hat{C}_{1k} \hat{x}_k + \hat{D}_{11k} \hat{w}_k + \hat{D}_{12k} \hat{u}_{k, \tau_2}, \\ \hat{y}_k = \hat{C}_{2k} \hat{x}_{k, \tau_1}, \\ \hat{x}(k) = \varphi(k), \quad k = -\mathcal{T}_M, -\mathcal{T}_M + 1, \dots, 0, \end{cases} \quad (17)$$

FIGURE 3: The process of lifting system G .

where $\hat{x}_k := \tilde{x}(t_k)$, $\hat{u}_{k,\tau_2} := u(t_k - \tau_2(k))$, $\hat{z}_k := z(t_k)$, $\hat{w}_k := w(t_k)$, $\hat{y}_k := \tilde{y}(t_k)$, $\hat{x}_{k,\tau_1} := \tilde{x}(t_k - \tau_1(k))$, and

$$\begin{aligned}\hat{A}_k &= e^{hA_1} A_2(t_k), \\ \hat{B}_{1k} &= \int_0^h e^{(h-\eta)A_1} B_1 w(\eta) d\eta, \\ \hat{B}_{2k} &= e^{hA_1} B_2(t_k), \\ \hat{C}_{2k} &= C_2(t_k), \\ (\hat{C}_{1k}x)(t) &= C_1 e^{(t-t_k)A_1} A_2(t_k) x(t), \\ (\hat{D}_{11k}w)(t) &= C_1 \int_{t_k}^t e^{(\eta-t_k)A_1} B_1 w(\eta) d\eta, \\ (\hat{D}_{12k}u)(t) &= C_1 e^{(t-t_k)A_1} B_2(t_k) u(t).\end{aligned}\quad (18)$$

Here $t \in (t_k, t_{k+1}]$. Since W is an isometric isomorphism, with any controller in place, the induced norm of the lifted system $\hat{G}$ is the same as that of the original system.

The lifted system $\hat{G}$ belongs to an infinite input-output space, so traditional methods cannot be applied. The following lemma gives a proper way to transform it to an equivalent DT system; then the traditional methods can be applied.

Lemma 4 (see [32]). *Suppose that $\|\hat{D}_{11k}\|_{l_2 \rightarrow l_2} < \gamma$ holds for all $k \geq 0$; then $\hat{G}$ can be transformed to system G_d as follows:*

$$G_d : \begin{cases} x_{k+1} = A_k x_k + B_{1k} w_k + B_{2k} u(t_k - \tau_2(k)), \\ z_k = C_{1k} x_k + D_{12k} u(t_k - \tau_2(k)), \\ y_k = C_{2k} x(t_k - \tau_1(k)), \end{cases} \quad (19)$$

where (A_k, B_{2k}) is stabilizable and (A_k, C_{2k}) is detectable; $x_k \in \mathbb{R}^n$, $u(t_k - \tau_2(k)) \in \mathbb{R}^m$, $z_k \in \mathbb{R}^p$, and $y_k \in \mathbb{R}^q$; $\tau_1(k)$ is sampling delay and $\tau_2(k)$ is input delay at the k th time instant, and

$$\begin{aligned}A_k &= \hat{A}_k + \hat{B}_{1k} \hat{D}_{11k}^T \hat{L}_k \hat{C}_{1k}, \\ B_{2k} &= \hat{B}_{2k} + \hat{B}_{1k} \hat{D}_{11k}^T \hat{L}_k \hat{D}_{12k}, \\ L_k &= \left(\gamma^2 I - \hat{D}_{11k} \hat{D}_{11k}^T \right)^{-1}, \\ C_{2k} &= \hat{C}_{2k}.\end{aligned}\quad (20)$$

B_{1k} , C_{1k} , and D_{12k} satisfy

$$\begin{aligned}B_{1k} B_{1k}^T &= \gamma \hat{B}_{1k} \left(\gamma^2 I - \hat{D}_{11k} \hat{D}_{11k}^T \right)^{-1} \hat{B}_{1k}^T, \\ \begin{bmatrix} C_{1k}^T \\ D_{12k}^T \end{bmatrix} [C_{1k} \ D_{12k}] &= \gamma \begin{bmatrix} \hat{C}_{1k}^T \\ \hat{D}_{12k}^T \end{bmatrix} \hat{L}_k [\hat{C}_{1k} \ \hat{D}_{12k}],\end{aligned}\quad (21)$$

and then, the following are equivalent:

- (i) The controller K stabilizes the DT system $\hat{G}$, and the closed-loop system satisfies $\|\hat{g}\|_\infty < \gamma$.
- (ii) The controller K stabilizes the equivalent system G_d , and the closed-loop system satisfies $\|g_d\|_\infty < \gamma$.

Finally, we obtain system G_d , which can be analyzed by traditional method. And there is an example in [31] that shows that direct sampled-data design requires a sampling rate which is 10 times slower than digital implementation via step-invariant transformation and thus can let us schedule sampling time easily.

4. Static Output-Feedback Robust H_∞ Control

In this section, a sufficient condition for the existence of a delay-dependent output-feedback H_∞ controller which guarantees the stability of system (1) and meets H_∞ performance is discussed.

The static output-feedback stabilization problem is to design the following controller,

$$u(t_k - \tau_2(k)) = \mathcal{K}_k y(t_k - \tau_2(k)), \quad (22)$$

for discrete-time system G_d , where $\mathcal{K}_k$ is an appropriately dimensioned matrix to be determined. If $C_{2c} = I_{q \times n}$ and $D_{22c} = 0_{q \times m}$, the output-feedback controller turns to be a static state-feedback one; the proposed method in this paper is also valid and the corresponding results will be clean and concise. However, this is not what we care about in this paper as the state-feedback problem can be solved more easily than output-feedback case.

By connecting the controller (22) with system G_d , we can obtain the following closed-loop system of G_d :

$$\begin{aligned}x_{k+1} &= \bar{A}_k x_k + \bar{B}_{1k} w_k + \bar{E}_k x_d, \\ z_k &= \bar{C}_{1k} x_k + \bar{F}_k x_d,\end{aligned}\quad (23)$$

where $x_d := x(t_k - \tau_2(k) - \tau_1(k))$, and

$$\begin{aligned}\bar{E}_k &= \bar{B}_{2k} \mathcal{K}_k \bar{C}_{2k}, \\ \bar{F}_k &= \bar{D}_{12k} \mathcal{K}_k \bar{C}_{2k}.\end{aligned}\quad (24)$$

Denote $d(k) := \tau_2(k) + \tau_1(k)$; the bounds of delays also satisfy (13).

The following theorem presents a sufficient condition for the existence of such H_∞ controller that can stabilize system G .

Theorem 5. For a given disturbance attenuation γ , system G with a static output-feedback controller (22) can be stabilized if there exist appropriate matrices $\mathcal{K}_k$, $\mathcal{P}_k = \mathcal{P}_k^T > 0$, $\mathcal{Q}_k = \mathcal{Q}_k^T > 0$, $\mathcal{X}_k = \mathcal{X}_k^T > 0$, $\mathcal{Y}_k$, and $\mathcal{Z}_k$, satisfying the following inequalities:

$$\begin{bmatrix} -\mathcal{P}_k^{-1} & 0 & 0 & \bar{A}_k & \bar{B}_{2k}\mathcal{K}_k\bar{C}_{2k} & \bar{B}_{1k} \\ * & -\mathcal{T}_M^{-1}\mathcal{X}_k^{-1} & 0 & \bar{A}_k - I & \bar{B}_{2k}\mathcal{K}_k\bar{C}_{2k} & \bar{B}_{1k} \\ * & * & -I_z & \bar{C}_{1k} & \bar{D}_{12k}\mathcal{K}_k\bar{C}_{2k} & 0 \\ * & * & * & \mathcal{Y}_k & -\mathcal{Z}_k & 0 \\ * & * & * & * & -\mathcal{Q}_k & 0 \\ * & * & * & * & * & -\gamma^2 I_w \end{bmatrix} \quad (25)$$

< 0 ,

$$\begin{bmatrix} \mathcal{X}_k & \mathcal{Y}_k \\ \mathcal{Y}_k^T & \mathcal{X}_k \end{bmatrix} > 0, \quad (26)$$

where I , I_z , and I_w are appropriate identity matrices:

$$\begin{aligned} \mathcal{Y}_k &:= -\mathcal{P}_k + \mathcal{Y}_k + \mathcal{Y}_k^T + \mathcal{T}_M \mathcal{X}_k \\ &+ (\mathcal{T}_M - \mathcal{T}_m + 1) \mathcal{Q}_k, \end{aligned} \quad (27)$$

$$\begin{aligned} \bar{A}_k &= (A_d + F_1) \bar{A}_2(t_k), \\ \bar{B}_{2k} &= (B_{2d} + F_2) \bar{B}_2(t_k), \\ \bar{C}_{1k} &= C_1 \bar{A}_2(t_k), \\ \bar{C}_{2k} &= \bar{C}_2(t_k), \\ \bar{D}_{12k} &= C_1 \bar{B}_2(t_k), \end{aligned} \quad (28)$$

where $A_d = B_{2d} = R_{11}$, $F = [F_1 \ F_2] = [(Q_{11}^{-1})^T \ 0] M^T R$, B_{1k} , and C_1 , respectively, satisfy the following:

$$\bar{B}_{1k} \bar{B}_{1k}^T = \gamma \underline{Q}_{21} \underline{Q}_{11}^{-1}, \quad (29)$$

$$\begin{aligned} &[C_1 \ C_1]^T [C_1 \ C_1] \\ &= \gamma^{-1} \left(R^T M \begin{bmatrix} \underline{Q}_{11}^{-1} & 0 \\ 0 & 0 \end{bmatrix} N - R^T L + T_{22}^T T_{12} \right), \end{aligned} \quad (30)$$

and the other matrices in the operators can be obtained by the following matrices exponential computing or partitioning some blocks on the left-hand side of the following matrices into 2×2 block matrices:

$$\begin{aligned} \begin{bmatrix} \underline{P} & \underline{M} & \underline{L} \\ 0 & \underline{Q} & \underline{N} \\ 0 & 0 & \underline{R} \end{bmatrix} &= \exp \left\{ h \begin{bmatrix} -\underline{A}^T & \underline{U} & 0 \\ 0 & \underline{S} & \underline{V} \\ 0 & 0 & \underline{A} \end{bmatrix} \right\}, \\ \begin{bmatrix} T_{11} & T_{12} \\ 0 & T_{22} \end{bmatrix} &= \exp \left\{ h \begin{bmatrix} -\underline{A}^T & \underline{U} \\ 0 & \underline{A} \end{bmatrix} \right\}, \end{aligned} \quad (31)$$

where

$$\underline{A} = \begin{bmatrix} \bar{A}_1 & 0 \\ 0 & \bar{A}_1 \end{bmatrix}, \quad (32)$$

$$\underline{U} = [\bar{C}_1 \ \bar{C}_1]^T [\bar{C}_1 \ \bar{C}_1], \quad (33)$$

$$\underline{S} = \begin{bmatrix} -\bar{A}_1^T & -\bar{C}_1^T \bar{C}_1 \\ \frac{\bar{B}_1 \bar{B}_1^T}{\gamma^2} & \bar{A}_1 \end{bmatrix}, \quad (34)$$

$$\underline{V} = [\bar{C}_1 \ 0]^T [\bar{C}_1 \ \bar{C}_1]. \quad (35)$$

Proof. To calculate the operators $\bar{B}_{1k}$, $\bar{C}_{1k}$, and $\bar{D}_{12k}$, we can extend methods in [12, 32]. The derivation is standard on the ground of matrix exponential computing and adjoint operators; as the process is very trivial, we omit it here and only give the results we have derived.

The final DT system G_d is obtained by symmetric factorization of two expressions of (29) and (30); all that remains is to synthesize a controller for G_d .

Since we have

$$x(k-d(k)) = x(k) - \sum_{m=k-d(k)}^{k-1} \eta(m), \quad (36)$$

where $\eta(m) = x(m+1) - x(m) = (\bar{A}_m - I)x(m) + \bar{B}_{1m}w(m) + \bar{E}_m x(m-d(m))$, then the closed system of G_d can be transformed to

$$\begin{aligned} x(k+1) &= (\bar{A}_k + \bar{E}_k) x(k) + \bar{B}_{1k} w(k) \\ &- \bar{E}_k \sum_{m=k-d(k)}^{k-1} \eta(m). \end{aligned} \quad (37)$$

Choose a Lyapunov-Krasovskii functional candidate as follows:

$$V(k) = V_1(k) + V_2(k) + V_3(k) + V_4(k),$$

$$V_1(k) = x^T(k) \mathcal{P}_k x(k),$$

$$V_2(k) = \sum_{i=k-d(k)}^{k-1} x^T(i) \mathcal{Q}_k x(i), \quad (38)$$

$$V_3(k) = \sum_{j=-\mathcal{T}_M+1}^{-\mathcal{T}_m+1} \sum_{i=k+j-1}^{k-1} x^T(i) \mathcal{Q}_k x(i),$$

$$V_4(k) = \sum_{i=-\mathcal{T}_M}^{-1} \sum_{m=k+i}^{k-1} \eta^T(m) \mathcal{Z}_k \eta(m),$$

where $\mathcal{P}_k$, $\mathcal{Q}_k$, and $\mathcal{Z}_k$ are positive definite matrices to be determined.

Define $\Delta V(k) = V(k+1) - V(k)$; then along the solution of (19) we get

$$\begin{aligned}
 \Delta V_1(k) &= x^T(k+1) \mathcal{P}_k x(k+1) - x^T(k) \mathcal{P}_k x(k) \\
 &= x^T(k) \left[(\bar{A}_k + \bar{E}_k)^T \mathcal{P}_k (\bar{A}_k + \bar{E}_k) - \mathcal{P}_k \right] x(k) \\
 &\quad + \sum_{m=k-d(k)}^{k-1} \left[-2x^T(k) (\bar{A}_k + \bar{E}_k)^T \mathcal{P}_k \bar{E}_k \eta(m) \right] \\
 &\quad + \sum_{m=k-d(k)}^{k-1} \left[-2w^T(k) \bar{B}_{1k}^T \mathcal{P}_k \bar{E}_k \eta(m) \right] \\
 &\quad + w^T(k) \bar{B}_{1k}^T \mathcal{P}_k \bar{B}_{1k} w(k) \\
 &\quad + 2x^T(k) (\bar{A}_k + \bar{E}_k)^T \mathcal{P}_k \bar{B}_{1k} w(k) \\
 &\quad + \left[\bar{E}_k \sum_{m=k-d(k)}^{k-1} \eta(m) \right]^T \mathcal{P}_k \left[\bar{E}_k \sum_{m=k-d(k)}^{k-1} \eta(m) \right]. \tag{39}
 \end{aligned}$$

According to Lemma 3, the following inequalities can be obtained:

$$\begin{aligned}
 &\sum_{m=k-d(k)}^{k-1} \left[-2x^T(k) (\bar{A}_k + \bar{E}_k)^T \mathcal{P}_k \bar{E}_k \eta(m) \right] \\
 &\leq \mathcal{T}_M x^T(k) \mathcal{X}_k x(k) \\
 &\quad + 2x^T(k) \left[\mathcal{Y}_k - (\bar{A}_k + \bar{E}_k)^T \mathcal{P}_k \bar{E}_k \right] \sum_{m=k-d(k)}^{k-1} \eta(m) \\
 &\quad + \sum_{m=k-\mathcal{T}_M}^{k-1} \eta^T(m) \mathcal{Z}_k \eta(m). \tag{40}
 \end{aligned}$$

In addition, we have

$$\begin{aligned}
 \Delta V_2(k) &= x^T(k) \mathcal{Q}_k x(k) \\
 &\quad - x^T(k-d(k)) \mathcal{Q}_k x(k-d(k)) \\
 &\quad + \sum_{i=k-d(k+1)+1}^{k-1} x^T(i) \mathcal{Q}_k x(i) \\
 &\quad - \sum_{i=k-d(k)+1}^{k-1} x^T(i) \mathcal{Q}_k x(i). \tag{41}
 \end{aligned}$$

Note that

$$\begin{aligned}
 &\sum_{i=k-d(k+1)+1}^{k-1} x^T(i) \mathcal{Q}_k x(i) \\
 &= \sum_{i=k-\mathcal{T}_M+1}^{k-1} x^T(i) \mathcal{Q}_k x(i) \\
 &\quad + \sum_{i=k-d(k+1)+1}^{k-\mathcal{T}_M} x^T(i) \mathcal{Q}_k x(i) \\
 &\leq \sum_{i=k-d(k)+1}^{k-1} x^T(i) \mathcal{Q}_k x(i) + \sum_{i=k-\mathcal{T}_M+1}^{k-\mathcal{T}_M} x^T(i) \mathcal{Q}_k x(i); \tag{42}
 \end{aligned}$$

therefore, we have

$$\begin{aligned}
 \Delta V_2(k) &\leq x^T(k) \mathcal{Q}_k x(k) \\
 &\quad - x^T(k-d(k)) \mathcal{Q}_k x(k-d(k)) \\
 &\quad + \sum_{i=k-\mathcal{T}_M+1}^{k-\mathcal{T}_M} x^T(i) \mathcal{Q}_k x(i). \tag{43}
 \end{aligned}$$

Similarly, we can have

$$\begin{aligned}
 \Delta V_3(k) &= (\mathcal{T}_M - \mathcal{T}_m) x^T(k) \mathcal{Q}_k x(k) \\
 &\quad - \sum_{i=k-\mathcal{T}_M+1}^{k-\mathcal{T}_M} x^T(i) \mathcal{Q}_k x(i). \tag{44}
 \end{aligned}$$

Finally,

$$\begin{aligned}
 \Delta V_4(k) &= \mathcal{T}_M \eta^T(k) \mathcal{Z}_k \eta(k) \\
 &\quad - \sum_{m=k-\mathcal{T}_M}^{k-1} \eta^T(m) \mathcal{Z}_k \eta(m). \tag{45}
 \end{aligned}$$

Then, combining (39)–(41) and (43)–(45), we have

$$\Delta V(k) = \Delta V_1(k) + \Delta V_2(k) + \Delta V_3(k) + \Delta V_4(k). \tag{46}$$

In order to obtain robustly stable system G_d with a disturbance attenuation γ , it is required that the associated Hamiltonian function satisfies

$$\begin{aligned}
 &H(x(k), x(k-d(k)), w(k)) \\
 &= \Delta V(k) + z^T(k) z(k) - \gamma w^T(k) w(k) < 0. \tag{47}
 \end{aligned}$$

Then, from (19) and (47), we can obtain

$$H(x(k), x(k-d(k)), w(k)) \leq \lambda^T(k) \Omega \lambda(k) < 0, \tag{48}$$

where $\lambda(k) = [x^T(k) \ x^T(k-d(k)) \ w^T(k)]^T$, and

$$\begin{aligned} \Omega = & \begin{bmatrix} \bar{A}_k^T \mathcal{P}_k \bar{A}_k & \bar{A}_k^T \mathcal{P}_k \bar{E}_k & \bar{A}_k^T \mathcal{P}_k \bar{B}_{1k} \\ * & \bar{E}_k^T \mathcal{P}_k \bar{E}_k & \bar{E}_k^T \mathcal{P}_k \bar{B}_{1k} \\ * & * & \bar{B}_{1k}^T \mathcal{P}_k \bar{B}_{1k} \end{bmatrix} \\ & + \begin{bmatrix} Y_k & -\mathcal{Y}_k & 0 \\ * & -\mathcal{Q}_k & 0 \\ * & * & -\gamma^2 I_w \end{bmatrix} \\ & + [\bar{C}_{1k} \ \bar{F}_k \ 0]^T [\bar{C}_{1k} \ \bar{F}_k \ 0] \\ & + \mathcal{T}_M [\bar{A}_k - I \ \bar{E}_k \ \bar{B}_{1k}]^T \mathcal{X}_k \\ & \cdot [\bar{A}_k - I \ \bar{E}_k \ \bar{B}_{1k}]. \end{aligned} \quad (49)$$

By Schur complement, the following inequality is obtained:

$$\begin{bmatrix} -\mathcal{P}_k & 0 & 0 & \mathcal{P}_k \bar{A}_k & \mathcal{P}_k \bar{E}_k & \mathcal{P}_k \bar{B}_{1k} \\ * & -\mathcal{T}_M^{-1} \mathcal{X}_k & 0 & \mathcal{X}_k (\bar{A}_k - I) & \mathcal{X}_k \bar{E}_k & \mathcal{X}_k \bar{B}_{1k} \\ * & * & -I_z & \bar{C}_{1k} & \bar{F}_k & 0 \\ * & * & * & Y_k & -\mathcal{Y}_k & 0 \\ * & * & * & * & -\mathcal{Q}_k & 0 \\ * & * & * & * & * & -\gamma^2 I_w \end{bmatrix} < 0, \quad (50)$$

where Y_k as (27) shows that $\mathcal{X}_k$, $\mathcal{Y}_k$, and $\mathcal{Z}_k$ satisfy inequality (26). A congruence transformation to (50) by pre- and postmultiplying $\text{diag}\{\mathcal{P}_k^{-1}, \mathcal{X}_k^{-1}, I, I, I, I\}$ together with the substitution of the matrices defined as (24) leads to (25). Therefore, we can conclude from the H_∞ stability theory, to a given disturbance attenuation γ , that conditions (25)–(35) guarantee the closed system of G_d to be robustly stable if all time-varying delays $d(k)$ satisfy $\mathcal{T}_m \leq d(k) \leq \mathcal{T}_M$. Due to the fact that the lifting process is an isometric isomorphism and the transformation from $\widehat{G}$ to G_d is equivalent, the hybrid system G can equivalently be stabilized by controller (22) under the same disturbance attenuation level. $\square$

Remark 6. The obtained condition in Theorem 5 is less conservative due to the proper Lyapunov functional we choose. First, more detailed information (the minimum and the maximum delay bounds) is included in the Lyapunov-Krasovskii functional candidate; that is, the result is delay-dependent. Second, the constructed Lyapunov-Krasovskii functional candidate is sampling-dependent; that is, the tuning matrices P_k , Q_k , and Z_k are time-varying, which enlarges the feasible domain and thus leads to less conservative results.

Remark 7. Due to the existence of inverse matrices of $\mathcal{P}_k$ and $\mathcal{X}_k$, (25) is not a strict LMI and cannot be directly solved by the LMI tools. However, notice that the matrices and their inverse matrices appear in pairs; we can solve the inequality by iteration on LMI according to cone complementarity linearization algorithm (CCLA) [22].

If the delay of the system is constant, the minimum and the maximum delay bounds become identical; that is, $\mathcal{T}_M = \mathcal{T}_m = d$, and Theorem 5 is specialised to the following corollary.

Corollary 8. The closed system of G_d with $d(k) \equiv d$ is robustly stable with a given disturbance attenuation γ , if there exist $\mathcal{P}_k = \mathcal{P}_k^T > 0$, $\mathcal{Q}_k = \mathcal{Q}_k^T > 0$, $\mathcal{X}_k = \mathcal{X}_k^T > 0$, $\mathcal{X}_k$, and $\mathcal{Y}_k$, satisfying (26) and

$$\begin{bmatrix} -\mathcal{P}_k & 0 & 0 & \mathcal{P}_k \bar{A}_k & \mathcal{P}_k \bar{E}_k & \mathcal{P}_k \bar{B}_{1k} \\ * & -d^{-1} \mathcal{X}_k & 0 & \mathcal{X}_k (\bar{A}_k - I) & \mathcal{X}_k \bar{E}_k & \mathcal{X}_k \bar{B}_{1k} \\ * & * & -I_z & \bar{C}_{1k} & \bar{F}_k & 0 \\ * & * & * & Y_k & -\mathcal{Y}_k & 0 \\ * & * & * & * & -\mathcal{Q}_k & 0 \\ * & * & * & * & * & -\gamma^2 I_w \end{bmatrix} < 0; \quad (51)$$

here I , I_z , and I_w are appropriate identity matrices, and $\bar{A}_k$, $\bar{B}_{1k}$, $\bar{B}_{2k}$, $\bar{C}_{1k}$, $\bar{C}_{2k}$, and $\bar{D}_{12k}$ are defined in (28), (29), and

$$Y_k := -\mathcal{P}_k + \mathcal{Y}_k + \mathcal{Y}_k^T + d\mathcal{X}_k + \mathcal{Q}_k. \quad (52)$$

5. Dynamic Output-Feedback Robust H_∞ Control

In this section, dynamic output-feedback controller is adopted; that is, we choose the following controller:

$$\begin{aligned} \tilde{x}_{k+1} &= A_c \tilde{x}_k + B_c y_{k,\tau_2}, \\ u_{k,\tau_2} &= C_c \tilde{x}_k + D_c y_{k,\tau_2}, \\ \tilde{x}_k &= 0, \quad k \leq 0, \end{aligned} \quad (53)$$

where $y_{k,\tau_2} := y(t_k - \tau_2(k))$, $\tilde{x}_k := \tilde{x}(t_k)$, $\tilde{x}_k \in \mathfrak{R}^r$ ($1 \leq r < n$), and A_c , B_c , C_c , and D_c are appropriately dimensioned matrices to be determined.

By defining $\xi_k := [x_k^T \ \tilde{x}_k^T]^T$, combining controller (53) with system G_d , the closed-loop system of G_d can be given by

$$\begin{aligned} \xi_{k+1} &= \mathcal{A}_k \xi_k + \mathcal{B}_k \widehat{w}_k + \mathcal{E}_k K \xi_d, \\ z_k &= \mathcal{C}_k \xi_k + \mathcal{F}_k K \xi_d, \end{aligned} \quad (54)$$

where $\xi_d := \xi(t_k - \tau_2(k) - \tau_1(i))$, I_n is an identity matrix with appropriate dimension, and

$$\begin{aligned}\mathcal{A}_k &:= \begin{bmatrix} \bar{A}_k & \bar{B}_{2k}C_c \\ 0 & A_c \end{bmatrix}, \\ \mathcal{B}_k &:= \begin{bmatrix} \bar{B}_{1k} \\ 0 \end{bmatrix}, \\ \mathcal{C}_k &:= \begin{bmatrix} \bar{B}_{2k}D_c\bar{C}_{2k} \\ B_c\bar{C}_{2k} \end{bmatrix}, \\ \mathcal{E}_k &:= [\bar{C}_{1k} \quad \bar{D}_{12k}C_c], \\ \mathcal{F}_k &:= \bar{D}_{12k}D_c\bar{C}_{2k}, \\ K &:= [I_n \quad 0],\end{aligned}\quad (55)$$

where $\bar{A}_k$, $\bar{B}_{1k}$, $\bar{B}_{2k}$, $\bar{C}_{1k}$, $\bar{C}_{2k}$, and $\bar{D}_{12k}$ are defined as (28) and (29). Similar to Theorem 5, we can obtain the following theorem.

Theorem 9. For a given disturbance attenuation γ , system G with a dynamic output-feedback controller (53) can be stabilized if there exist appropriate matrices A_c , B_c , C_c , and D_c , and $0 < \mathcal{P}_k = \mathcal{P}_k^T \in \mathfrak{R}^{(n+r) \times (n+r)}$, $0 < \mathcal{Q}_k = \mathcal{Q}_k^T \in \mathfrak{R}^{(n+r) \times (n+r)}$, $\mathcal{X}_k = \mathcal{X}_k^T > 0$, $\mathcal{Y}_k$, $\mathcal{Z}_k$, satisfying the following inequalities:

$$\begin{bmatrix} -\mathcal{P}_k^{-1} & 0 & 0 & \mathcal{A}_k & \mathcal{E}_k & \mathcal{B}_k \\ * & -\mathcal{T}_M^{-1}\mathcal{X}_k^{-1} & 0 & K(\mathcal{A}_k - I) & K\mathcal{E}_k & K\mathcal{B}_k \\ * & * & -I_z & \mathcal{C}_k & \mathcal{F}_k & 0 \\ * & * & * & \mathcal{Y}_k & -\mathcal{Z}_k & 0 \\ * & * & * & * & \mathcal{Q}_k & 0 \\ * & * & * & * & * & -\gamma^2 I_w \end{bmatrix} < 0, \quad (56)$$

$$\begin{bmatrix} \mathcal{X}_k & \mathcal{Y}_k \\ \mathcal{Y}_k^T & \mathcal{X}_k \end{bmatrix} > 0,$$

where I , I_z , and I_w are appropriate identity matrices with appropriate dimensions; $\mathcal{A}_k$, $\mathcal{B}_k$, $\mathcal{C}_k$, $\mathcal{D}_k$, $\mathcal{E}_k$, $\mathcal{F}_k$, and K are defined as (55), and

$$\begin{aligned}\mathcal{Y}_k &:= -\mathcal{P}_k + \mathcal{Y}_k K + K^T \mathcal{Y}_k^T + \mathcal{T}_M \mathcal{X}_k \\ &+ (\mathcal{T}_M - \mathcal{T}_m + 1) K^T \mathcal{Q}_k K.\end{aligned}\quad (57)$$

Proof. From (36) we have

$$\xi(k - d(k)) = \xi(k) - \sum_{m=k-d(k)}^{k-1} \eta(m), \quad (58)$$

where $\eta(m) = \xi(m+1) - \xi(m) = (\mathcal{A}(m) - I)\eta(m) + \mathcal{B}w(m) + \mathcal{E}(m)K\xi(m - d(m))$.

To the closed system of G_d with dynamic output-feedback controller (53), because there is no delay in the state of dynamic controller, we can construct mixed descriptor Lyapunov-Krasovskii functionals as follows:

$$\begin{aligned}W(k) &= W_1(k) + W_2(k) + W_3(k) + W_4(k), \\ W_1(k) &= \xi^T(k) \mathcal{P}_k \xi(k), \\ W_2(k) &= \sum_{i=k-d(k)}^{k-1} \xi^T(i) K^T \mathcal{Q}_k K \xi(i), \\ W_3(k) &= \sum_{j=-d_M+2}^{-d_M+1} \sum_{i=k+j-1}^{k-1} \xi^T(i) K^T \mathcal{Q}_k K \xi(i), \\ W_4(k) &= \sum_{i=-d_M}^{-1} \sum_{m=k+i}^{k-1} \eta^T(m) K^T \mathcal{Z}_k K \eta(m).\end{aligned}\quad (59)$$

Then, the proof of Theorem 9 can be carried out as that of Theorem 5 and hence omitted here. $\square$

Remark 10. From Theorems 5 and 9, we can notice that theories we proposed are different from the current methods, which are difficult to calculate γ and feedback gains alternatively under the condition that eigenvalues of corresponding matrices of the closed system are on the unit circle, especially to dynamic feedback-output control system; the method based on eigenvalues is more difficult and hard. However, applying methods we proposed, γ can be assigned before calculation, and all parameters in the LMIs can be solved clearly and conveniently.

6. Numerical Example

In this section, a numerical example is presented to illustrate the effectiveness and applicability of the proposed methods. As an example, we construct a hybrid system G described by the following matrices:

$$\begin{aligned}A_c &= \begin{bmatrix} -0.5485 & 1.0812 \\ 0.3041 & -2.6803 \end{bmatrix}, \\ B_{1c} &= \begin{bmatrix} 1.3908 & -1.1711 \\ 0.0364 & 0.5731 \end{bmatrix}, \\ B_{2c} &= \begin{bmatrix} 1.3572 & -1.7605 \\ 0.3329 & 0.0048 \end{bmatrix}, \\ C_{1c} &= [0.2 \quad 0], \\ C_{2c} &= [1 \quad 0], \\ D_{12c} &= [0.8 \quad -0.5], \\ D_{22c} &= [-0.6 \quad 0.5].\end{aligned}\quad (60)$$

FIGURE 4: The states response to static output-feedback control.

According to Lemma 2, we can calculate $\beta = 0.0639$. The output-sampling matrices with period 3 are chosen to be as follows:

$$\begin{aligned} S(1) &= 1, \\ S(2) &= 0, \\ S(3) &= 0. \end{aligned} \quad (61)$$

The input-holding mechanism is assumed with period 2 and can be described as follows:

$$\begin{aligned} H(1) &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \\ H(2) &= \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}. \end{aligned} \quad (62)$$

The period for discretization is chosen to be 0.3 s. w is a Gaussian white noise with amplitude of 0.1. Bounds of sampling and holding delays are chosen to be $[0, 1]$ and $[1, 2]$, respectively, which are much larger than those of most current literatures.

Disturbance attenuation level is given as $\gamma = 0.6$; however, the levels obtained by current methods are all larger than 1. Then, we simulate the system with two control strategies, respectively; the results are shown below.

Case 1 (static output-feedback robust H_∞ control). Initial status is specified as $x_0 = [30 \ 20 \ 0 \ 0]^T$.

From the figures, we can see that the system can be stabilized by static output-feedback H_∞ controller.

Case 2 (dynamic output-feedback robust H_∞ control). Here, initial status also is $x_0 = [30 \ 20 \ 0 \ 0]^T$. From the figures, we can see that the system can be stabilized by dynamic output-feedback H_∞ controller, and comparing Figures 4 and 5 with Figures 6 and 7, respectively, we can see that the performance of the system is improved by offering more parameters of controller.

FIGURE 5: The static output-feedback control.

FIGURE 6: The states response to dynamic output-feedback control.

FIGURE 7: The dynamic output-feedback control.

7. Conclusion

Motivated by a desire to provide a better solution for NCSs with delays, a computable solution to the asynchronous multirate hybrid H_∞ synthesis problem is addressed. At the beginning, the traditional continuous-time lifting method is extended to the hybrid system with random delays. Due to asynchronism, the sampling time should be scheduled to be equal for discretization. Next, the lifted DT system that lies in infinite input-output space is transformed to an equivalent DT system, which belongs to a finite space. To solve the operators in the equivalent DT system, a series of computable formulas are provided for the first time according to adjoint

operator theory. Then, different from any existing approach, a Lyapunov-Krasovskii functionals candidate is proposed to deal with the H_∞ synthesis problem of the equivalent DT system. The results based on this method are delay-dependent and can be solved by LMI tools. Compared with the current methods, this method is more convenient to calculate the controller, especially in dynamic feedback case. Finally, two examples exhibit the validity of the two theories, respectively.

Competing Interests

The authors declare that they have no competing interests.

Acknowledgments

This work was supported by the National Natural Science Foundation of China (61473252 and 61633019) and the Zhejiang Provincial Natural Science Foundation of China (LR12F03002).

References

- [1] B. Tavassoli, "Stability of nonlinear networked control systems over multiple communication links with asynchronous sampling," *IEEE Transactions on Automatic Control*, vol. 59, no. 2, pp. 511–515, 2014.
- [2] L.-S. Hu, T. Bai, P. Shi, and Z. Wu, "Sampled-data control of networked linear control systems," *Automatica*, vol. 43, no. 5, pp. 903–911, 2007.
- [3] M. B. Cloosterman, N. van de Wouw, W. P. Heemels, and H. Nijmeijer, "Stability of networked control systems with uncertain time-varying delays," *IEEE Transactions on Automatic Control*, vol. 54, no. 7, pp. 1575–1580, 2009.
- [4] F. W. Yang, Z. D. Wang, Y. S. Hung, and M. Gani, " H_∞ control for networked systems with random communication delays," *IEEE Transactions on Automatic Control*, vol. 51, no. 3, pp. 511–518, 2006.
- [5] W. Chen and L. Qiu, "Stabilization of networked control systems with multirate sampling," *Automatica*, vol. 49, no. 6, pp. 1528–1537, 2013.
- [6] J. P. Hespanha, P. Naghshtabrizi, and Y. Xu, "A survey of recent results in networked control systems," *Proceedings of the IEEE*, vol. 95, no. 1, pp. 138–162, 2007.
- [7] T. C. Yang, "Networked control system: a brief survey," *IEE Proceedings: Control Theory & Applications*, vol. 153, no. 4, pp. 403–412, 2006.
- [8] Z.-H. Guan, C. X. Yang, and J. Huang, "Stabilization of networked control systems with short or long random delays: a new multirate method," *International Journal of Robust and Nonlinear Control*, vol. 20, no. 16, pp. 1802–1816, 2010.
- [9] E. Fridman, A. Seuret, and J.-P. Richard, "Robust sampled-data stabilization of linear systems: an input delay approach," *Automatica*, vol. 40, no. 8, pp. 1441–1446, 2004.
- [10] H. Gao, T. Chen, and J. Lam, "A new delay system approach to network-based control," *Automatica*, vol. 44, no. 1, pp. 39–52, 2008.
- [11] H. Gao and T. Chen, "Network-based H_∞ output tracking control," *IEEE Transactions on Automatic Control*, vol. 53, no. 3, pp. 655–667, 2008.
- [12] X. Zhao, Y. Yao, F. He, and F. W. Meng, " H_∞ control for multirate sampled-data systems with time-delay," in *Proceedings of the 7th International Conference on Control, Automation, Robotics and Vision*, pp. 904–909, Singapore, December 2002.
- [13] A. Seuret, "Stability analysis of networked control systems with asynchronous sampling and input delay," in *Proceedings of the American Control Conference (ACC '11)*, pp. 533–538, IEEE, San Francisco, Calif, USA, July 2011.
- [14] V. Suplin, E. Fridman, and U. Shaked, " H_∞ sampled data control of systems with time-delays," *International Journal of Control*, vol. 82, no. 2, pp. 298–309, 2009.
- [15] P. Colaneri, R. Scattolini, and N. Schiavoni, "LQG optimal control of multirate sampled-data systems," *IEEE Transactions on Automatic Control*, vol. 37, no. 5, pp. 675–682, 1992.
- [16] X. F. Jiang, S. M. Fei, and C. B. Feng, "Delay-dependent feedback control for linear time-delay systems with input delay," *Acta Automatica Sinica*, vol. 27, no. 1, pp. 108–114, 2001.
- [17] W. Q. Sun, K. M. Nagpal, and P. P. Khargonekar, " H_∞ control and filtering for sampled-data systems," *IEEE Transactions on Automatic Control*, vol. 38, no. 8, pp. 1162–1175, 1993.
- [18] Y.-E. Wang, X.-M. Sun, and B.-W. Wu, "Lyapunov-Krasovskii functionals for switched nonlinear input delay systems under asynchronous switching," *Automatica*, vol. 61, pp. 126–133, 2015.
- [19] Y.-E. Wang, J. Zhao, and B. Jiang, "Stabilization of a class of switched linear neutral systems under asynchronous switching," *IEEE Transactions on Automatic Control*, vol. 58, no. 8, pp. 2114–2119, 2013.
- [20] B. Bamieh, J. B. Pearson, B. A. Francis, and A. Tannenbaum, "A lifting technique for linear periodic systems with applications to sampled-data control," *Systems & Control Letters*, vol. 17, no. 2, pp. 79–88, 1991.
- [21] D. G. Meyer, "Cost translation and a lifting approach to the multirate LQG problem," *IEEE Transactions on Automatic Control*, vol. 37, no. 9, pp. 1411–1415, 1992.
- [22] Y. Xiao and X. Y. Du, "Theorem and algorithm of asymptotic stability test for time-variant discrete system," *Journal of Northern Jiaotong University*, vol. 22, no. 6, 1998.
- [23] V. S. Ritzchev and G. F. Franklin, "A stability criterion for asynchronous multirate linear systems," *IEEE Transactions on Automatic Control*, vol. 34, no. 5, pp. 529–535, 1989.
- [24] M. F. S¸agfors, H. T. Toivonen, and B. Lennartson, " H_∞ control of multirate sampled-data systems: a state-space approach," *Automatica*, vol. 34, no. 4, pp. 415–428, 1998.
- [25] M. F. S¸agfors and H. T. Toivonen, " H_∞ and LQG control of asynchronous sampled-data systems," *Automatica*, vol. 33, no. 9, pp. 1663–1668, 1997.
- [26] H. Shu and T. W. Chen, "State-space approach to discrete-time H_2 -optimal control with a causality constraint," *Systems & Control Letters*, vol. 26, no. 1, pp. 69–77, 1995.
- [27] X. Sun and W. Mao, " H_∞ controller design for csynchronous multirate sampled-data systems," *Mathematical Problems in Engineering*, vol. 2014, Article ID 757282, 12 pages, 2014.
- [28] G. E. Dullerud and S. Lall, "A new approach for analysis and synthesis of time-varying systems," *IEEE Transactions on Automatic Control*, vol. 44, no. 8, pp. 1486–1497, 1999.
- [29] Y. F. Guan and T. S. Li, "Study on the stability of linear time-varying discrete system," *Computing Technology and Automation*, vol. 23, no. 3, pp. 5–9, 2004.
- [30] J. Zhang, X.-S. Xin, and H.-B. Xu, "Output feedback control of a class of linear time-varying systems," *Acta Automatica Sinica*, vol. 40, no. 2, pp. 373–378, 2014.

- [31] T. W. Chen, *Optimal Sampled-Data Control Systems*, Springer, London, UK, 1995.
- [32] S. Lall and G. Dullerud, "An LMI solution to the robust synthesis problem for multi-rate sampled-data systems," *Automatica*, vol. 37, no. 12, pp. 1909–1922, 2001.
- [33] L. Guo and W.-H. Chen, "Output feedback H_∞ control for a class of uncertain nonlinear discrete-time delay systems," *Transactions of the Institute of Measurement and Control*, vol. 25, no. 2, pp. 107–121, 2003.
- [34] H. Gao, J. Lam, C. Wang, and Y. Wang, "Delay-dependent output-feedback stabilisation of discrete-time systems with time-varying state delay," *IEE Proceedings-Control Theory and Applications*, vol. 151, no. 6, pp. 691–698, 2004.
- [35] E. Fridman, U. Shaked, and V. Suplin, "Input/output delay approach to robust sampled-data H1 control," *Systems & Control Letters*, vol. 54, pp. 271–282, 2005.

Hindawi

Submit your manuscripts at
<http://www.hindawi.com>