Research Article

Stabilization of Networked Control Systems with Induced Delays and Actuator Saturation

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The problem of state feedback stabilization is studied for networked control systems (NCSs) subject to actuator saturation and network-induced delays. To facilitate the controller design, the NCSs are modeled as a class of discrete-time systems with bounded delays and input saturation. Based on Lyapunov-Krasovskii theory and free weighting matrix approach, the sufficient condition is derived in terms of linear matrix inequality for the asymptotic stability. Finally, the effectiveness of the developed control approach is proved through numerical examples.

1. Introduction

Control systems whose feedback paths are implemented by communication networks are called networked control systems (NCSs) [1, 2]. NCSs are widely used in many areas, such as industrial automation, intelligent vehicle systems, flight control, and mobile robotics [3]. In recent years, NCSs have received increasing attention due to the accelerated development of communication techniques, network techniques, and control methods. Network-induced delays and actuator saturation are often encountered in many practical systems including NCSs [4, 5]. If the time delays and the saturation problem are ignored in system analysis, the control performance of NCSs may deteriorate. Since the existence of time delays and actuator saturation often leads to the instability of NCSs, research on time delay systems with actuator saturation is a topic of great practical and theoretical importance. Time delays are often encountered in various practical engineering systems, such as chemical systems, power systems, and networked control systems [6–8]. In [9], a brief overview of time delay control was provided. The state estimation problem was considered for discrete-time linear systems with Markovian time delay in [10]. In [11], a linear controller was designed to realize chaos control and synchronization for Lorenz system with time-varying lags. Since the introduction of networks, almost all networked control systems have network-induced delays which can

make systems unstable. In the last decades, many researchers have studied stability analysis and controller design for stabilization of NCSs in the presence of network-induced delays. In [12], an approach was established to codesign an output feedback controller and a channel-access managing policy for networked control systems. In [13], stochastic optimal controllers were designed for networked control systems whose induced delay was longer than a sampling period. The stabilization problem was studied for a class of networked control systems in the discrete-time domain with random delays in [14]. The robust control problem was studied for a class of stochastic uncertain discrete-time delay systems with missing measurements in [15]. In [16], extensions to continuous NCSs were studied and a design scheme was proposed for the observer-based output feedback controller. In [17], the problem of integrated design of controller and communication sequences was addressed for networked control systems with simultaneous consideration of medium access limitations and network-induced delays, packet dropouts, and measurement quantization. Although NCSs with time delays were widely investigated, actuator saturation problems in NCSs were seldom considered in the existing research results which needs to be further studied.

In practical systems, due to some physical limitations, the saturation problem is inevitable. In recent years, saturated systems have become a popular research topic. In [18], adaptive output feedback control was studied for a kind of

FIGURE 1: NCS model.

uncertain system in the presence of saturation. The problem was studied for stabilizing a linear system with delayed and saturating feedback in [19]. In [20, 21], LMI-based methods were presented for regional stability and performance of linear antiwindup compensators for linear control systems. In [22], the design of guaranteed transient performance based attitude control was described for the near space vehicle (NSV) with control input saturation using backstepping method. A method was proposed for the analysis and control design of linear systems in the presence of actuator saturation and disturbances in [23]. In [24], robust control was studied for uncertain linear system subject to input saturation. Adaptive tracking control problem was investigated for uncertain nonlinear systems with input saturation and unknown external disturbances in [25]. In [26], an actuator robust fault tolerant control was proposed for ocean surface vessels with parametric uncertainties, unknown disturbances, and input saturation. During the last few years, problems with actuator saturation have been extended to singular systems [27], Markovian jump systems [28], decentralized control systems [29], and Hamilton systems [30]. The problem of adaptive control of linear discrete-time systems with actuator saturation and unknown parameters was investigated in [31]. In [32], a novel adaptive neural network control approach was presented for a class of uncertain discrete-time nonlinear strict-feedback systems with input saturation. However, time delays and saturation problems were seldom considered in NCSs simultaneously.

This work is motivated by the stabilization control of networked control systems with network-induced delays and actuator saturation. Based on Lyapunov-Krasovskii stability theory and free weighting matrix approach, stability analysis and a state feedback controller of NCSs are studied.

The organization of the paper is as follows. Section 2 describes networked control systems (NCSs) with network-induced delays and actuator saturation and details the problem formulation. Section 3 gives stability analysis and stabilization approach of NCSs. Numerical simulations are presented in Section 4 to demonstrate the effectiveness of the developed control method. The conclusion is given in Section 5.

2. Problem Formulation

In this section, system modeling of a kind of NCS with actuator saturation is considered. As illustrated in Figure 1 [3], NCS consists of four components: a plant, an actuator, a controller, and a sensor. A certain communication network connects these components.

In practical NCSs, state and input delays of control plant usually exist. Thus, the control plant of NCSs can be modeled as a class of discrete systems with state, input delays, and input saturation which can be described by

$$x(k+1) = Ax(k) + A_d x(k-d(k)) + B \text{sat}(u(k)) + B_h \text{sat}(u(k-h(k))), \quad (1)$$

where $x(k) \in \mathbb{R}^n$ is the state vector of the system and $u(k) \in \mathbb{R}^m$ is the control input signal. $d(k)$ is state delay with lower and upper bounds $0 \leq d_1 \leq d(k) \leq d_2$ and $h(k)$ is input delay which satisfies $0 \leq h_1 \leq h(k) \leq h_2$. A , A_d , B , and B_h are known real matrices with appropriate dimensions. The control input $\text{sat}(u(k))$ is required to meet the saturation levels where $\text{sat}(\cdot)$ is a saturation function vector defined as

$$\text{sat}(u) = [\text{sat}(u_1), \text{sat}(u_2), \dots, \text{sat}(u_m)]^T, \quad (2)$$

with $\text{sat}(u_j) = \text{sign}(u_j) \max\{u_{j,\max}, |u_j|\}$, $j \in [1, m]$, and a saturation boundary $u_{j,\max}$.

In the NCSs, the time delay is composed of two parts: τ^{ca} is generated from the controller to the actuator and τ^{sc} is produced from the sensor to the controller. Compared with these two parts of time delays, the calculation time of controller, actuator, and sensor is small enough so that it often can be ignored. To design the controller, the following assumptions are needed.

Assumption 1 (see [33]). The sensor is time driven. The controller and actuator are event driven.

Assumption 2 (see [33]). The data transmission is with a single packet and there are no packet dropouts.

In this paper, a state feedback controller is designed as $u(k) = 2Kx(k - \tau_{ca} - \tau_{sc})$. We combine τ_{ca} and τ_{sc} to $\tau(k)$. Then, the state feedback controller is rewritten as follows:

$$u(k) = 2Kx(k - \tau(k)), \quad (3)$$

where K is the state feedback controller gain and $\tau(k)$ is network-induced delay which is time varying and satisfies $0 \leq \tau_1 \leq \tau(k) \leq \tau_2$.

Under controller (3), define $\eta_1(k) = \text{sat}(2Kx(k - \tau(k))) - Kx(k - \tau(k))$ and $\eta_2(k) = \text{sat}(2Kx(k - \tau(k) - h(k))) - Kx(k - \tau(k) - h(k))$. Then, the closed-loop system can be written as follows:

$$\begin{aligned} x(k+1) = & Ax(k) + A_d x(k-d(k)) \\ & + BKx(k - \tau(k)) \\ & + B_h Kx(k - \tau(k) - h(k)) + B\eta_1(k) \\ & + B_h \eta_2(k). \end{aligned} \quad (4)$$

Before presenting the main results of this paper, we need to introduce the following lemmas which are crucial to our main results.

Lemma 3 (see [34]). *Given symmetric constant matrices S_{11} , S_{22} and constant matrix S_{12} of approximate dimensions, then $S_{22} < 0$ and $S_{11} - S_{12}S_{22}^{-1}S_{12}^T < 0$ holds if and only if*

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{bmatrix} < 0. \quad (5)$$

Lemma 4 (see [35]). *Considering $\eta_1(k)$ and $\eta_2(k)$ which are defined before, one has the following inequalities:*

$$\begin{aligned} \eta_1^T(k) \eta_1(k) &\leq x^T(k - \tau(k)) K^T K x(k - \tau(k)), \\ \eta_2^T(k) \eta_2(k) &\leq x^T(k - \tau(k) - h(k)) K^T K x(k - \tau(k) - h(k)). \end{aligned} \quad (6)$$

Proof. Let $\eta(k) = [\eta_1^T(k), \eta_2^T(k), \dots, \eta_m^T(k)]^T$ and $K = [K_1^T, K_2^T, \dots, K_m^T]^T$. According to (2) and (3), the following is yielded:

$$\begin{aligned} \text{sat}(2K_j x(k - \tau(k))) &= \text{sat}(u_j(k - \tau(k))) \\ &= \text{sign}(2K_j x(k - \tau(k))) \\ &\quad \cdot \max(u_{j\max}, |2K_j x(k - \tau(k))|). \end{aligned} \quad (7)$$

(1) If $2K_j x(k - \tau(k)) \geq u_{j\max} > 0$, we have

$$\begin{aligned} \text{sat}(2K_j x(k - \tau(k))) - K_j x(k - \tau(k)) &\leq 2K_j x(k - \tau(k)) - K_j x(k - \tau(k)) \\ &= K_j x(k - \tau(k)) \implies \\ |\eta_{1j}(k)| &= |\text{sat}(2K_j x(k - \tau(k))) - K_j x(k - \tau(k))| \end{aligned} \quad (8)$$

$$\begin{aligned} \eta_{1j}^2(k) &= (\text{sat}(2K_j x(k - \tau(k))) - K_j x(k - \tau(k)))^2 \\ &\leq (K_j x(k - \tau(k)))^2. \end{aligned}$$

Hence, one has

$$\begin{aligned} \eta_1^T(k) \eta_1(k) &= \sum_{j=1}^m \eta_{1j}^2(k) \leq \sum_{j=1}^m (K_j x(k - \tau(k)))^2 \\ &= x^T(k - \tau(k)) K^T K x(k - \tau(k)). \end{aligned} \quad (9)$$

(2) If $-u_{j\max} \leq 2K_j x(k - \tau(k)) \leq u_{j\max}$, we have

$$\begin{aligned} \eta_{1j}(k) &= \text{sat}(2K_j x(k - \tau(k))) - K_j x(k - \tau(k)) \\ &= K_j x(k - \tau(k)) \implies \\ \eta_{1j}^2(k) &= (\text{sat}(2K_j x(k - \tau(k))) - K_j x(k - \tau(k)))^2 \\ &= (K_j x(k - \tau(k)))^2. \end{aligned} \quad (10)$$

Hence, one has

$$\eta_1^T(k) \eta_1(k) = x^T(k - \tau(k)) K^T K x(k - \tau(k)). \quad (11)$$

(3) If $2K_j x(k - \tau(k)) \leq -u_{j\max} < 0$, this yields

$$\begin{aligned} \text{sat}(2K_j x(k - \tau(k))) - K_j x(k - \tau(k)) &\geq 2K_j x(k - \tau(k)) - K_j x(k - \tau(k)) \\ &= K_j x(k - \tau(k)), \\ \text{sat}(2K_j x(k - \tau(k))) - K_j x(k - \tau(k)) &< -K_j x(k - \tau(k)) \implies \\ |\eta_j(k)| &= |\text{sat}(2K_j x(k - \tau(k))) - K_j x(k - \tau(k))| \\ &\leq |-K_j x(k - \tau(k))| \implies \\ (\eta_j(k))^2 &= (\text{sat}(2K_j x(k - \tau(k))) - K_j x(k - \tau(k)))^2 \\ &\leq (K_j x(k - \tau(k)))^2. \end{aligned} \quad (12)$$

Hence, we have

$$\begin{aligned} \eta_1^T(k) \eta_1(k) &= \sum_{j=1}^m \eta_j^2(k) \leq \sum_{j=1}^m (K_j x(k - \tau(k)))^2 \\ &= x^T(k - \tau(k)) K^T K x(k - \tau(k)). \end{aligned} \quad (13)$$

Combining (9), (11), and (13), we have $\eta_1^T(k) \eta_1(k) \leq x^T(k - \tau(k)) K^T K x(k - \tau(k))$. Similarly, we can prove $\eta_2^T(k) \eta_2(k) \leq x^T(k - \tau(k) - h(k)) K^T K x(k - \tau(k) - h(k))$. This completes the proof. $\square$

Our main purpose of this paper is to design a state feedback controller for the networked control system (1) such that the closed-loop system (4) is asymptotically stable with bounded network-induced delays and actuator saturation.

3. Main Results

In this section, we obtained a sufficient condition for the asymptotical stability of system (1) and the synthesis of controller design will be established. The asymptotical stability criteria are given in Theorem 5.

Theorem 5. *The closed-loop NCS is asymptotically stable under controller (3) if there exist symmetric matrices $P > 0$, $Q_i > 0$, $S_i > 0$, $i = 1, 2, 3$, $X > 0$, $\bar{X} > 0$, $\bar{X} > 0$ and*

matrices N , M , T with appropriate dimensions such that the following matrix inequalities hold:

$$\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & \Xi_{15} & \Xi_{16} \\ * & \Xi_{22} & \Xi_{23} & \Xi_{24} & \Xi_{25} & \Xi_{26} \\ * & * & \Xi_{33} & \Xi_{34} & \Xi_{35} & \Xi_{36} \\ * & * & * & \Xi_{44} & \Xi_{45} & \Xi_{46} \\ * & * & * & * & \Xi_{55} & \Xi_{56} \\ * & * & * & * & * & \Xi_{66} \end{bmatrix} < 0, \quad (14)$$

$$\psi_1 = \begin{bmatrix} X & N \\ * & Q_1 \end{bmatrix} > 0, \quad (15)$$

$$\psi_2 = \begin{bmatrix} \bar{X} & M \\ * & Q_2 \end{bmatrix} > 0, \quad (16)$$

$$\psi_3 = \begin{bmatrix} \bar{X} & T \\ * & Q_3 \end{bmatrix} > 0, \quad (17)$$

where $*$ is used as an ellipse for terms induced by symmetry and

$$\begin{aligned} \Xi_{11} &= J + P(A - I)^T + (A - I)^T P \\ &\quad + (A - I)^T H(A - I) + N_1^T + N_1 + M_1^T \\ &\quad + M_1 + T_1^T + T_1 + d_2 X_{11} + \tau_2 \bar{X}_{11} \\ &\quad + (\tau_2 + h_2) \bar{X}_{11}, \\ \Xi_{12} &= PA_d + (A - I)^T HA_d - N_1 + N_2^T + d_2 X_{12} \\ &\quad + \tau_2 \bar{X}_{12} + (\tau_2 + h_2) \bar{X}_{12}, \\ \Xi_{13} &= PBK + (A - I)^T HBK - M_1 + M_2^T + d_2 X_{13} \\ &\quad + \tau_2 \bar{X}_{13} + (\tau_2 + h_2) \bar{X}_{13}, \\ \Xi_{14} &= PB_d K + (A - I)^T HB_d K - T_1 + T_2^T + d_2 X_{14} \\ &\quad + \tau_2 \bar{X}_{14} + (\tau_2 + h_2) \bar{X}_{14}, \\ \Xi_{15} &= PB + (A - I)^T HB + d_2 X_{15} + \tau_2 \bar{X}_{15} \\ &\quad + (\tau_2 + h_2) \bar{X}_{15}, \\ \Xi_{16} &= PB_d + (A - I)^T HB_d + d_2 X_{16} + \tau_2 \bar{X}_{16} \\ &\quad + (\tau_2 + h_2) \bar{X}_{16}, \\ \Xi_{22} &= A_d^T HA_d - S_1 - N_2^T - N_2 + d_2 X_{22} + \tau_2 \bar{X}_{22} \\ &\quad + (\tau_2 + h_2) \bar{X}_{22}, \\ \Xi_{23} &= A_d^T HBK + d_2 X_{23} + \tau_2 \bar{X}_{23} + (\tau_2 + h_2) \bar{X}_{23}, \\ \Xi_{24} &= A_d^T HB_d K + d_2 X_{24} + \tau_2 \bar{X}_{24} + (\tau_2 + h_2) \bar{X}_{24}, \\ \Xi_{25} &= A_d^T HB + d_2 X_{25} + \tau_2 \bar{X}_{25} + (\tau_2 + h_2) \bar{X}_{25}, \end{aligned}$$

$$\Xi_{26} = A_d^T HB_d + d_2 X_{26} + \tau_2 \bar{X}_{26} + (\tau_2 + h_2) \bar{X}_{26},$$

$$\begin{aligned} \Xi_{33} &= (BK)^T H(BK) - S_2 - M_2^T - M_2 + \alpha_1 K^T K \\ &\quad + d_2 X_{33} + \tau_2 \bar{X}_{33} + (\tau_2 + h_2) \bar{X}_{33}, \end{aligned}$$

$$\begin{aligned} \Xi_{34} &= (BK)^T H(B_d K) + d_2 X_{34} + \tau_2 \bar{X}_{34} \\ &\quad + (\tau_2 + h_2) \bar{X}_{34}, \end{aligned}$$

$$\Xi_{35} = (BK)^T HB + d_2 X_{35} + \tau_2 \bar{X}_{35} + (\tau_2 + h_2) \bar{X}_{35},$$

$$\Xi_{36} = (BK)^T HB_d + d_2 X_{36} + \tau_2 \bar{X}_{36} + (\tau_2 + h_2) \bar{X}_{36},$$

$$\begin{aligned} \Xi_{44} &= (B_d K)^T HB_d K - S_3 - T_2 - T_2^T + \alpha_2 K^T K \\ &\quad + d_2 X_{44} + \tau_2 \bar{X}_{44} + (\tau_2 + h_2) \bar{X}_{44}, \end{aligned}$$

$$\Xi_{45} = (B_d K)^T HB + d_2 X_{45} + \tau_2 \bar{X}_{45} + (\tau_2 + h_2) \bar{X}_{45},$$

$$\Xi_{46} = (B_d K)^T HB_d + d_2 X_{46} + \tau_2 \bar{X}_{46} + (\tau_2 + h_2) \bar{X}_{46},$$

$$\Xi_{55} = B^T HB - \alpha_1 I + d_2 X_{55} + \tau_2 \bar{X}_{55} + (\tau_2 + h_2) \bar{X}_{55},$$

$$\Xi_{56} = B^T HB_d + d_2 X_{56} + \tau_2 \bar{X}_{56} + (\tau_2 + h_2) \bar{X}_{56},$$

$$\begin{aligned} \Xi_{66} &= B_d^T HB_d - \alpha_2 I + d_2 X_{66} + \tau_2 \bar{X}_{66} \\ &\quad + (\tau_2 + h_2) \bar{X}_{66}, \end{aligned}$$

$$H = P + d_2 Q_1 + \tau_2 Q_2 + (\tau_2 + h_2) Q_3,$$

$$\begin{aligned} J &= (d_2 - d_1 + 1) S_1 + (\tau_2 - \tau_1 + 1) S_2 \\ &\quad + ((\tau_2 + h_2) - (\tau_1 + h_1) + 1) S_3, \end{aligned} \quad (18)$$

$$X = \begin{bmatrix} X_{11} & X_{12} & X_{13} & X_{14} & X_{15} & X_{16} \\ * & X_{22} & X_{23} & X_{24} & X_{25} & X_{26} \\ * & * & X_{33} & X_{34} & X_{35} & X_{36} \\ * & * & * & X_{44} & X_{45} & X_{46} \\ * & * & * & * & X_{55} & X_{56} \\ * & * & * & * & * & X_{66} \end{bmatrix}, \quad (19)$$

$$\bar{X} = \begin{bmatrix} \bar{X}_{11} & \bar{X}_{12} & \bar{X}_{13} & \bar{X}_{14} & \bar{X}_{15} & \bar{X}_{16} \\ * & \bar{X}_{22} & \bar{X}_{23} & \bar{X}_{24} & \bar{X}_{25} & \bar{X}_{26} \\ * & * & \bar{X}_{33} & \bar{X}_{34} & \bar{X}_{35} & \bar{X}_{36} \\ * & * & * & \bar{X}_{44} & \bar{X}_{45} & \bar{X}_{46} \\ * & * & * & * & \bar{X}_{55} & \bar{X}_{56} \\ * & * & * & * & * & \bar{X}_{66} \end{bmatrix}, \quad (20)$$

$$\bar{X} = \begin{bmatrix} \bar{X}_{11} & \bar{X}_{12} & \bar{X}_{13} & \bar{X}_{14} & \bar{X}_{15} & \bar{X}_{16} \\ * & \bar{X}_{22} & \bar{X}_{23} & \bar{X}_{24} & \bar{X}_{25} & \bar{X}_{26} \\ * & * & \bar{X}_{33} & \bar{X}_{34} & \bar{X}_{35} & \bar{X}_{36} \\ * & * & * & \bar{X}_{44} & \bar{X}_{45} & \bar{X}_{46} \\ * & * & * & * & \bar{X}_{55} & \bar{X}_{56} \\ * & * & * & * & * & \bar{X}_{66} \end{bmatrix}, \quad (21)$$

$$N = \begin{bmatrix} N_1^T & N_2^T & 0 & 0 & 0 & 0 \end{bmatrix}^T, \quad (22)$$

$$M = \begin{bmatrix} M_1^T & 0 & M_2^T & 0 & 0 & 0 \end{bmatrix}^T, \quad (23)$$

$$T = \begin{bmatrix} T_1^T & 0 & 0 & T_2^T & 0 & 0 \end{bmatrix}^T. \quad (24)$$

Proof. In order to analyze the closed-loop system stability, the Lyapunov-Krasovskii functional candidate is constructed as follows:

$$V(k) = V_1(k) + V_2(k) + V_3(k) + V_4(k) + V_5(k) + V_6(k) + V_7(k), \quad (25)$$

where

$$\begin{aligned} V_1(k) &= x^T(k) P x(k), \\ V_2(k) &= \sum_{\theta=-d_2+1}^0 \sum_{l=k-1+\theta}^{k-1} y^T(l) Q_1 y(l), \\ V_3(k) &= \sum_{\theta=-d_2+1}^{-d_1+1} \sum_{l=k-1+\theta}^{k-1} x^T(l) S_1 x(l), \\ V_4(k) &= \sum_{\theta=-\tau_2+1}^0 \sum_{l=k-1+\theta}^{k-1} y^T(l) Q_2 y(l), \\ V_5(k) &= \sum_{\theta=-\tau_2+1}^{-\tau_1+1} \sum_{l=k-1+\theta}^{k-1} x^T(l) S_2 x(l), \\ V_6(k) &= \sum_{\theta=-\tau_2-h_2+1}^0 \sum_{l=k-1+\theta}^{k-1} y^T(l) Q_3 y(l), \\ V_7(k) &= \sum_{\theta=-\tau_2-h_2+1}^{-\tau_1-h_2+1} \sum_{l=k-1+\theta}^{k-1} x^T(l) S_3 x(l), \\ y(l) &= x(l+1) - x(l). \end{aligned} \quad (26)$$

The time shift of $V(k)$ can be written as follows:

$$\begin{aligned} \Delta V(k) &= \sum_{i=1}^7 \Delta V_i(k), \\ \Delta V_1(k) &= 2x^T(k) P y(k) + y^T(k) P y(k), \\ \Delta V_2(k) &= d_2 y^T(k) Q_1 y(k) - \sum_{l=k-d_2}^{k-1} y^T(l) Q_1 y(l) \\ &\leq d_2 y^T(k) Q_1 y(k) - \sum_{l=k-d_2(k)}^{k-1} y^T(l) Q_1 y(l), \\ \Delta V_3(k) &= (d_2 - d_1 + 1) x^T(k) S_1 x(k) - \sum_{l=k-d_2}^{k-d_1} x^T(l) S_1 x(l) \end{aligned}$$

$$\leq (d_2 - d_1 + 1) x^T(k) S_1 x(k)$$

$$- x^T(k - d(k)) S_1 x(k - d(k)),$$

$$\Delta V_4(k) = \tau_2 y^T(k) Q_2 y(k) - \sum_{l=k-\tau_2}^{k-1} y^T(l) Q_2 y(l)$$

$$\leq \tau_2 y^T(k) Q_2 y(k) - \sum_{l=k-\tau(k)}^{k-1} y^T(l) Q_2 y(l),$$

$$\Delta V_5(k)$$

$$\leq (\tau_2 - \tau_1 + 1) x^T(k) S_2 x(k)$$

$$- x^T(k - \tau(k)) S_2 x(k - \tau(k)),$$

$$\Delta V_6(k)$$

$$\leq (\tau_2 + h_2) y^T(k) Q_3 y(k)$$

$$- \sum_{l=k-\tau(k)-h(k)}^{k-1} y^T(l) Q_3 y(l),$$

$$\Delta V_7(k)$$

$$\leq ((\tau_2 + h_2) - (\tau_1 + h_1) + 1) x^T(k) S_3 x(k)$$

$$- x^T(k - \tau(k) - h(k)) S_3 x(k - \tau(k) - h(k)). \quad (27)$$

From (27), we can conclude that

$$\begin{aligned} \Delta V(k) &\leq 2x^T(k) P y(k) + y^T(k) P y(k) + y^T(k) H y(k) \\ &\quad + x^T(k) J x(k) - \sum_{l=k-d(k)}^{k-1} y^T(l) Q_1 y(l) \\ &\quad - \sum_{l=k-\tau(k)}^{k-1} y^T(l) Q_2 y(l) \\ &\quad - \sum_{l=k-\tau(k)-h(k)}^{k-1} y^T(l) Q_3 y(l) \\ &\quad - x^T(k - d(k)) S_1 x(k - d(k)) \\ &\quad - x^T(k - \tau(k)) S_2 x(k - \tau(k)) \\ &\quad - x^T(k - \tau(k) - h(k)) S_2 x(k - \tau(k) - h(k)). \end{aligned} \quad (28)$$

For any matrices N_i , M_i , T_i , $i = 1, 2$, with appropriate dimensions, we have

$$\begin{aligned} &2 \left[x^T(k) N_1 + x^T(k - d(k)) N_2 \right] \\ &\cdot \left[x(k) - x(k - d(k)) - \sum_{l=k-d(k)}^{k-1} y(l) \right] = 0, \end{aligned}$$

$$\begin{aligned}
& 2 \left[x^T(k) M_1 + x^T(k - \tau(k)) M_2 \right] \\
& \cdot \left[x(k) - x(k - \tau(k)) - \sum_{l=k-\tau(k)}^{k-1} y(l) \right] = 0, \\
& 2 \left[x^T(k) T_1 + x^T(k - \tau(k) - h(k)) T_2 \right] \\
& \cdot \left[x(k) - x(k - \tau(k) - h(k)) - \sum_{l=k-\tau(k)-h(k)}^{k-1} y(l) \right] \\
& = 0.
\end{aligned} \tag{29}$$

Then, we denote matrix $X > 0$, $\bar{X} > 0$, $\tilde{X} > 0$ with appropriate dimensions which are described in Theorem 5. Let $\zeta_1(k) = [x^T(k) \ x^T(k - h(k)) \ x^T(k - \tau(k)) \ x^T(k - \tau(k) - d(k)) \ \eta_1^T(k) \ \eta_2^T(k)]^T$ and $\zeta_2(k, l) = [\zeta_1^T(k) \ y^T(l)]^T$; we can obtain the following:

$$\begin{aligned}
& d_2 \zeta_1^T(k) X \zeta_1(k) - \sum_{l=k-d(k)}^{k-1} \zeta_1^T(k) X \zeta_1(k) > 0, \\
& \tau_2 \zeta_1^T(k) \bar{X} \zeta_1(k) - \sum_{l=k-\tau(k)}^{k-1} \zeta_1^T(k) \bar{X} \zeta_1(k) > 0, \\
& (\tau_2 + h_2) \zeta_1^T(k) \tilde{X} \zeta_1(k) - \sum_{l=k-\tau(k)-h(k)}^{k-1} \zeta_1^T(k) \tilde{X} \zeta_1(k) \\
& > 0.
\end{aligned} \tag{30}$$

From Lemma 4, we have

$$\begin{aligned}
& \zeta_1^T(k) \begin{bmatrix} 0 & 0 & \alpha_1 K^T K & 0 & -\alpha_1 I & 0 \end{bmatrix} \zeta_1(k) > 0, \\
& \zeta_1^T(k) \begin{bmatrix} 0 & 0 & 0 & \alpha_2 K^T K & 0 & -\alpha_2 I \end{bmatrix} \zeta_1(k) > 0,
\end{aligned} \tag{31}$$

where α_1, α_2 are positive scalars.

Considering (28) (29), (30), and (31), we obtain the following:

$$\begin{aligned}
\Delta V(k) & < \zeta_1^T(k) \Xi \zeta_1(k) - \sum_{l=k-d(k)}^{k-1} \zeta_2^T(k, l) \psi_1 \zeta_2(k, l) \\
& - \sum_{l=k-\tau(k)}^{k-1} \zeta_2^T(k, l) \psi_2 \zeta_2(k, l) \\
& - \sum_{l=k-\tau(k)-h(k)}^{k-1} \zeta_2^T(k, l) \psi_3 \zeta_2(k, l).
\end{aligned} \tag{32}$$

It is clear that $\Delta V(k) < 0$ is true if the conditions in Theorem 5 are satisfied. The proof is completed. $\square$

Theorem 5 gives the sufficient condition for the stability of system (1). However, it should be noted that the inequalities in Theorem 5 are not LMIs and are not easy to deal with. Then, in the following, we present LMI-based criteria to design a state feedback controller for NCSs with induced delays and actuator saturation.

Theorem 6. For given nonzero scalars $\varepsilon_1, \varepsilon_2, \varepsilon_3$ and positive scalars α_1, α_2 , if there exist positive symmetric matrices $L, \bar{S}_i, X', \bar{X}, \tilde{X}, i = 1, 2, 3$, and matrices with appropriate dimensions $\bar{N}_1, \bar{N}_2, \bar{M}_1, \bar{M}_2, \bar{T}_1, \bar{T}_2, R$ satisfying the following LMIs:

$$\tilde{\Phi} < 0, \tag{33}$$

$$\begin{aligned}
\bar{\Psi}_1 &= \begin{bmatrix} X' & \bar{N} \\ * & \varepsilon_1 L \end{bmatrix} > 0, \\
\bar{\Psi}_2 &= \begin{bmatrix} \bar{X}' & \bar{M} \\ * & \varepsilon_2 L \end{bmatrix} > 0, \\
\bar{\Psi}_3 &= \begin{bmatrix} \tilde{X}' & \bar{T} \\ * & \varepsilon_3 L \end{bmatrix} > 0,
\end{aligned} \tag{34}$$

then a state feedback controller $u(k) = 2Kx(k - \tau(k)) = 2RL^{-1}x(k - \tau(k))$ can be obtained such that the closed-loop system is asymptotically stable, where

$$\tilde{\Phi} = \begin{bmatrix} \tilde{\Phi}_{11} & \tilde{\Phi}_{12} & \tilde{\Phi}_{13} & \tilde{\Phi}_{14} & \tilde{\Phi}_{15} & \tilde{\Phi}_{16} & L(A-I)^T & d_2 L(A-I)^T & \tau_2 L(A-I)^T & (\tau_2 + h_2) L(A-I)^T & 0 & 0 \\ * & \tilde{\Phi}_{22} & \tilde{\Phi}_{23} & \tilde{\Phi}_{24} & \tilde{\Phi}_{25} & \tilde{\Phi}_{26} & LA_d^T & d_2 LA_d^T & \tau_2 LA_d^T & (\tau_2 + h_2) LA_d^T & 0 & 0 \\ * & * & \tilde{\Phi}_{33} & \tilde{\Phi}_{34} & \tilde{\Phi}_{35} & \tilde{\Phi}_{36} & R^T B^T & d_2 R^T B^T & \tau_2 R^T B^T & (\tau_2 + h_2) R^T B^T & R^T & 0 \\ * & * & * & \tilde{\Phi}_{44} & \tilde{\Phi}_{45} & \tilde{\Phi}_{46} & R^T B_h^T & d_2 R^T B_h^T & \tau_2 R^T B_h^T & (\tau_2 + h_2) R^T B_h^T & 0 & R^T \\ * & * & * & * & \tilde{\Phi}_{55} & \tilde{\Phi}_{56} & LB^T & d_2 LB^T & \tau_2 LB^T & (\tau_2 + h_2) LB^T & 0 & 0 \\ * & * & * & * & * & \tilde{\Phi}_{66} & LB_h^T & d_2 LB_h^T & \tau_2 LB_h^T & (\tau_2 + h_2) LB_h^T & 0 & 0 \\ * & * & * & * & * & * & -L & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -h_2 \varepsilon_1^{-1} L & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -\tau_2 \varepsilon_2^{-1} L & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -(\tau_2 + h_2) \varepsilon_3^{-1} L & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & -\alpha_1^{-1} I & 0 \\ * & * & * & * & * & * & * & * & * & * & 0 & -\alpha_2^{-1} I \end{bmatrix},$$

$$\begin{aligned}\bar{\Phi}_{11} = & (d_2 - d_1 + 1)\bar{S}_1 + (\tau_2 - \tau_1 + 1)\bar{S}_2 + ((\tau_2 + h_2) - (\tau_1 + h_1) + 1)\bar{S}_3 + (A - I)L + L(A - I)^T + \bar{N}_1^T + \bar{N}_1 + \bar{M}_1^T \\ & + \bar{M}_1 + \bar{T}_1^T + \bar{T}_1 + h_2X'_{11} + \tau_2\bar{X}'_{11} + (\tau_2 + d_2)\bar{X}'_{11},\end{aligned}$$

$$\bar{\Phi}_{12} = A_dL - \bar{N}_1 + \bar{N}_2^T + d_2X'_{12} + \tau_2\bar{X}'_{12} + (\tau_2 + h_2)\bar{X}'_{12},$$

$$\bar{\Phi}_{13} = BR - \bar{M}_1 + \bar{M}_2^T + d_2X'_{13} + \tau_2\bar{X}'_{13} + (\tau_2 + h_2)\bar{X}'_{13},$$

$$\bar{\Phi}_{14} = B_hR - \bar{T}_1 + \bar{T}_2^T + d_2X'_{14} + \tau_2\bar{X}'_{14} + (\tau_2 + h_2)\bar{X}'_{14},$$

$$\bar{\Phi}_{15} = BL + d_2X'_{15} + \tau_2\bar{X}'_{15} + (\tau_2 + h_2)\bar{X}'_{15},$$

$$\bar{\Phi}_{16} = B_hL + d_2X'_{16} + \tau_2\bar{X}'_{16} + (\tau_2 + h_2)\bar{X}'_{16},$$

$$\bar{\Phi}_{22} = -\bar{S}_1 - \bar{N}_2^T - \bar{N}_2 + d_2X'_{22} + \tau_2\bar{X}'_{22} + (\tau_2 + h_2)\bar{X}'_{22},$$

$$\bar{\Phi}_{23} = d_2X'_{23} + \tau_2\bar{X}'_{23} + (\tau_2 + h_2)\bar{X}'_{23},$$

$$\bar{\Phi}_{24} = d_2X'_{24} + \tau_2\bar{X}'_{24} + (\tau_2 + h_2)\bar{X}'_{24},$$

$$\bar{\Phi}_{25} = d_2X'_{25} + \tau_2\bar{X}'_{25} + (\tau_2 + h_2)\bar{X}'_{25},$$

$$\bar{\Phi}_{26} = d_2X'_{26} + \tau_2\bar{X}'_{26} + (\tau_2 + h_2)\bar{X}'_{26},$$

$$\bar{\Phi}_{33} = -\bar{S}_2 - \bar{M}_2^T - \bar{M}_2 + d_2X'_{33} + \tau_2\bar{X}'_{33} + (\tau_2 + h_2)\bar{X}'_{33},$$

$$\bar{\Phi}_{34} = h_2X'_{34} + d_2\bar{X}'_{34} + (\tau_2 + h_2)\bar{X}'_{34},$$

$$\bar{\Phi}_{35} = h_2X'_{35} + d_2\bar{X}'_{35} + (\tau_2 + h_2)\bar{X}'_{35},$$

$$\bar{\Phi}_{36} = h_2X'_{36} + d_2\bar{X}'_{36} + (\tau_2 + h_2)\bar{X}'_{36},$$

$$\bar{\Phi}_{44} = -\bar{S}_3 - \bar{T}_2 - \bar{T}_2^T + d_2X'_{44} + \tau_2\bar{X}'_{44} + (\tau_2 + h_2)\bar{X}'_{44},$$

$$\bar{\Phi}_{45} = d_2X'_{45} + \tau_2\bar{X}'_{45} + (\tau_2 + h_2)\bar{X}'_{45},$$

$$\bar{\Phi}_{46} = d_2X'_{46} + \tau_2\bar{X}'_{46} + (\tau_2 + h_2)\bar{X}'_{46},$$

$$\bar{\Phi}_{55} = \alpha_1I - 2\alpha_1L + d_2X'_{55} + \tau_2\bar{X}'_{55} + (\tau_2 + h_2)\bar{X}'_{55},$$

$$\bar{\Phi}_{56} = h_2X'_{56} + \tau_2\bar{X}'_{56} + (\tau_2 + h_2)\bar{X}'_{56},$$

$$\bar{\Phi}_{66} = \alpha_2I - 2\alpha_2L + d_2X'_{66} + \tau_2\bar{X}'_{66} + (\tau_2 + h_2)\bar{X}'_{66},$$

$$X' = \begin{bmatrix} X'_{11} & X'_{12} & X'_{13} & X'_{14} & X'_{15} & X'_{16} \\ * & X'_{22} & X'_{23} & X'_{24} & X'_{25} & X'_{26} \\ * & * & X'_{33} & X'_{34} & X'_{35} & X'_{36} \\ * & * & * & X'_{44} & X'_{45} & X'_{46} \\ * & * & * & * & X'_{55} & X'_{56} \\ * & * & * & * & * & X'_{66} \end{bmatrix},$$

$$\bar{X}' = \begin{bmatrix} \bar{X}'_{11} & \bar{X}'_{12} & \bar{X}'_{13} & \bar{X}'_{14} & \bar{X}'_{15} & \bar{X}'_{16} \\ * & \bar{X}'_{22} & \bar{X}'_{23} & \bar{X}'_{24} & \bar{X}'_{25} & \bar{X}'_{26} \\ * & * & \bar{X}'_{33} & \bar{X}'_{34} & \bar{X}'_{35} & \bar{X}'_{36} \\ * & * & * & \bar{X}'_{44} & \bar{X}'_{45} & \bar{X}'_{46} \\ * & * & * & * & \bar{X}'_{55} & \bar{X}'_{56} \\ * & * & * & * & * & \bar{X}'_{66} \end{bmatrix},$$

$$\tilde{X}' = \begin{bmatrix} \bar{X}'_{11} & \bar{X}'_{12} & \bar{X}'_{13} & \bar{X}'_{14} & \bar{X}'_{15} & \bar{X}'_{16} \\ * & \bar{X}'_{22} & \bar{X}'_{23} & \bar{X}'_{24} & \bar{X}'_{25} & \bar{X}'_{26} \\ * & * & \bar{X}'_{33} & \bar{X}'_{34} & \bar{X}'_{35} & \bar{X}'_{36} \\ * & * & * & \bar{X}'_{44} & \bar{X}'_{45} & \bar{X}'_{46} \\ * & * & * & * & \bar{X}'_{55} & \bar{X}'_{56} \\ * & * & * & * & * & \bar{X}'_{66} \end{bmatrix},$$

$$\bar{N} = [\bar{N}_1^T \quad \bar{N}_2^T \quad 0 \quad 0 \quad 0 \quad 0]^T,$$

$$\bar{M} = [\bar{M}_1^T \quad 0 \quad \bar{M}_2^T \quad 0 \quad 0 \quad 0]^T,$$

$$\bar{T} = [\bar{T}_1^T \quad 0 \quad 0 \quad \bar{T}_2^T \quad 0 \quad 0]^T.$$

(35)

Proof. From Theorem 5, if the closed-loop system is asymptotically stable, (14) holds. By Lemma 3, (14) is equivalent to

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} & \Phi_{15} & \Phi_{16} & (A-I)^T P & d_2 (A-I)^T Q_1 & \tau_2 (A-I)^T Q_2 & (\tau_2 + h_2) (A-I)^T Q_3 \\ * & \Phi_{22} & \Phi_{23} & \Phi_{24} & \Phi_{25} & \Phi_{26} & A_d^T P & d_2 A_d^T Q_1 & \tau_2 A_d^T Q_2 & (\tau_2 + h_2) A_d^T Q_3 \\ * & * & \Phi_{33} & \Phi_{34} & \Phi_{35} & \Phi_{36} & (BK)^T P & d_2 (BK)^T Q_1 & \tau_2 (BK)^T Q_2 & (\tau_2 + h_2) (BK)^T Q_3 \\ * & * & * & \Phi_{44} & \Phi_{45} & \Phi_{46} & (B_h K)^T P & d_2 (B_h K)^T Q_1 & \tau_2 (B_h K)^T Q_2 & (\tau_2 + h_2) (B_h K)^T Q_3 \\ * & * & * & * & \Phi_{55} & \Phi_{56} & B^T P & d_2 B^T Q_1 & \tau_2 B^T Q_2 & (\tau_2 + h_2) B^T Q_3 \\ * & * & * & * & * & \Phi_{66} & B_h^T P & d_2 B_h^T Q_1 & \tau_2 B_h^T Q_2 & (\tau_2 + h_2) B_h^T Q_3 \\ * & * & * & * & * & * & -P & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -d_2 Q_1 & 0 & 0 \\ * & * & * & * & * & * & * & * & -\tau_2 Q_2 & 0 \\ * & * & * & * & * & * & * & * & * & -(\tau_2 + h_2) Q_3 \end{bmatrix} < 0, \quad (36)$$

where

$$\begin{aligned}
 \Phi_{11} &= J + P(A - I)^T + (A - I)^T P + N_1^T + N_1 + M_1^T \\
 &\quad + M_1 + T_1^T + T_1 + d_2 X_{11} + \tau_2 \bar{X}_{11} \\
 &\quad + (\tau_2 + d_2) \bar{X}_{11}, \\
 \Phi_{12} &= PA_d - N_1 + N_2^T + d_2 X_{12} + \tau_2 \bar{X}_{12} \\
 &\quad + (\tau_2 + h_2) \bar{X}_{12}, \\
 \Phi_{13} &= PBK + N_3^T - M_1 + M_2^T + d_2 X_{13} + \tau_2 \bar{X}_{13} \\
 &\quad + (\tau_2 + h_2) \bar{X}_{13}, \\
 \Phi_{14} &= PB_h K - T_1 + T_2^T + d_2 X_{14} + \tau_2 \bar{X}_{14} \\
 &\quad + (\tau_2 + h_2) \bar{X}_{14}, \\
 \Phi_{15} &= PB + d_2 X_{15} + \tau_2 \bar{X}_{15} + (\tau_2 + h_2) \bar{X}_{15}, \\
 \Phi_{16} &= PB_h + d_2 X_{16} + \tau_2 \bar{X}_{16} + (\tau_2 + h_2) \bar{X}_{16}, \\
 \Phi_{22} &= -S_1 - N_2^T - N_2 + d_2 X_{22} + \tau_2 \bar{X}_{22} \\
 &\quad + (\tau_2 + h_2) \bar{X}_{22}, \\
 \Phi_{23} &= d_2 X_{23} + \tau_2 \bar{X}_{23} + (\tau_2 + h_2) \bar{X}_{23}, \\
 \Phi_{24} &= d_2 X_{24} + \tau_2 \bar{X}_{24} + (\tau_2 + h_2) \bar{X}_{24}, \\
 \Phi_{25} &= d_2 X_{25} + \tau_2 \bar{X}_{25} + (\tau_2 + h_2) \bar{X}_{25}, \\
 \Phi_{26} &= d_2 X_{26} + \tau_2 \bar{X}_{26} + (\tau_2 + h_2) \bar{X}_{26},
 \end{aligned}$$

$$\begin{aligned}
 \Phi_{33} &= -S_2 - M_2^T - M_2 + \alpha_1 K^T K + d_2 X_{33} + \tau_2 \bar{X}_{33} \\
 &\quad + (\tau_2 + h_2) \bar{X}_{33}, \\
 \Phi_{34} &= d_2 X_{34} + \tau_2 \bar{X}_{34} + (\tau_2 + h_2) \bar{X}_{34}, \\
 \Phi_{35} &= d_2 X_{35} + \tau_2 \bar{X}_{35} + (\tau_2 + h_2) \bar{X}_{35}, \\
 \Phi_{36} &= d_2 X_{36} + \tau_2 \bar{X}_{36} + (\tau_2 + h_2) \bar{X}_{36}, \\
 \Phi_{44} &= -S_3 - T_2 - T_2^T + \alpha_2 K^T K + d_2 X_{44} + \tau_2 \bar{X}_{44} \\
 &\quad + (\tau_2 + h_2) \bar{X}_{44}, \\
 \Phi_{45} &= d_2 X_{45} + \tau_2 \bar{X}_{45} + (\tau_2 + h_2) \bar{X}_{45}, \\
 \Phi_{46} &= d_2 X_{46} + \tau_2 \bar{X}_{46} + (\tau_2 + h_2) \bar{X}_{46}, \\
 \Phi_{55} &= -\alpha_1 I + d_2 X_{55} + \tau_2 \bar{X}_{55} + (\tau_2 + h_2) \bar{X}_{55}, \\
 \Phi_{56} &= d_2 X_{56} + \tau_2 \bar{X}_{56} + (\tau_2 + h_2) \bar{X}_{56}, \\
 \Phi_{66} &= -\alpha_2 I + d_2 X_{66} + \tau_2 \bar{X}_{66} + (\tau_2 + h_2) \bar{X}_{66}.
 \end{aligned}$$

(37)

Then, pre- and postmultiply Φ by $\text{diag}\{P^{-1} \ P^{-1} \ P^{-1} \ P^{-1} \ P^{-1} \ P^{-1} \ P^{-1} \ Q_1^{-1} \ Q_2^{-1} \ Q_3^{-1}\}$ and pre- and postmultiply (17), (18), and (19) by $\text{diag}\{P^{-1} \ P^{-1} \ P^{-1} \ P^{-1} \ P^{-1} \ P^{-1} \ P^{-1}\}$ similarly. Let $P^{-1} = L$, $KP^{-1} = R$, $\bar{S}_i = P^{-1} S_i P^{-1}$, $i = 1, 2, 3$, $P^{-1} X P^{-1} = X'$, $P^{-1} \bar{X} P^{-1} = \bar{X}'$, $P^{-1} \bar{X} P^{-1} = \bar{X}'$, $P^{-1} N_{1,2} P^{-1} = \bar{N}_{1,2}$, $P^{-1} M_{1,2} P^{-1} = \bar{M}_{1,2}$, $P^{-1} T_{1,2} P^{-1} = \bar{T}_{1,2}$, $Q_i = \varepsilon_i L^{-1}$, $i = 1, 2, 3$. Then, we have

$$\begin{aligned}
 \bar{\Phi} &= \begin{bmatrix} \bar{\Phi}_{11} & \bar{\Phi}_{12} & \bar{\Phi}_{13} & \bar{\Phi}_{14} & \bar{\Phi}_{15} & \bar{\Phi}_{16} & L(A - I)^T & d_2 L(A - I)^T & \tau_2 L(A - I)^T & (\tau_2 + h_2) L(A - I)^T \\ * & \bar{\Phi}_{22} & \bar{\Phi}_{23} & \bar{\Phi}_{24} & \bar{\Phi}_{25} & \bar{\Phi}_{26} & LA_d^T & d_2 LA_d^T & \tau_2 LA_d^T & (\tau_2 + h_2) LA_d^T \\ * & * & \bar{\Phi}_{33} & \bar{\Phi}_{34} & \bar{\Phi}_{35} & \bar{\Phi}_{36} & R^T B^T & d_2 R^T B^T & \tau_2 R^T B^T & (\tau_2 + h_2) R^T B^T \\ * & * & * & \bar{\Phi}_{44} & \bar{\Phi}_{45} & \bar{\Phi}_{46} & R^T B_d^T & d_2 R^T B_d^T & \tau_2 R^T B_d^T & (\tau_2 + h_2) R^T B_d^T \\ * & * & * & * & \bar{\Phi}_{55} & \bar{\Phi}_{56} & LB^T & d_2 LB^T & \tau_2 LB^T & (\tau_2 + h_2) LB^T \\ * & * & * & * & * & \bar{\Phi}_{66} & LB_d^T & d_2 LB_d^T & \tau_2 LB_d^T & (\tau_2 + h_2) LB_d^T \\ * & * & * & * & * & * & -L & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -h_2 \varepsilon_1^{-1} L & 0 & 0 \\ * & * & * & * & * & * & * & * & -\tau_2 \varepsilon_2^{-1} L & 0 \\ * & * & * & * & * & * & * & * & * & -\tau_2 \varepsilon_2^{-1} L \end{bmatrix} < 0, \\
 \bar{\Psi}_1 &= \begin{bmatrix} X'_{11} & X'_{12} & X'_{13} & X'_{14} & X'_{15} & X'_{16} & \bar{N}_1 \\ * & X'_{22} & X'_{23} & X'_{24} & X'_{25} & X'_{26} & \bar{N}_2 \\ * & * & X'_{33} & X'_{34} & X'_{35} & X'_{36} & 0 \\ * & * & * & X'_{44} & X'_{45} & X'_{46} & 0 \\ * & * & * & * & X'_{55} & X'_{56} & 0 \\ * & * & * & * & * & X'_{66} & 0 \\ * & * & * & * & * & * & \varepsilon_1 L \end{bmatrix} > 0,
 \end{aligned}$$

$$\begin{aligned}
\bar{\psi}_2 &= \begin{bmatrix} \bar{X}'_{11} & \bar{X}'_{12} & \bar{X}'_{13} & \bar{X}'_{14} & \bar{X}'_{15} & \bar{X}'_{16} & \bar{M}_1 \\ * & \bar{X}'_{22} & \bar{X}'_{23} & \bar{X}'_{24} & \bar{X}'_{25} & \bar{X}'_{26} & 0 \\ * & * & \bar{X}'_{33} & \bar{X}'_{34} & \bar{X}'_{35} & \bar{X}'_{36} & \bar{M}_2 \\ * & * & * & \bar{X}'_{44} & \bar{X}'_{45} & \bar{X}'_{46} & 0 \\ * & * & * & * & \bar{X}'_{55} & \bar{X}'_{56} & 0 \\ * & * & * & * & * & \bar{X}'_{66} & 0 \\ * & * & * & * & * & * & \varepsilon_2 L \end{bmatrix} > 0, \\
\bar{\psi}_3 &= \begin{bmatrix} \bar{X}'_{11} & \bar{X}'_{12} & \bar{X}'_{13} & \bar{X}'_{14} & \bar{X}'_{15} & \bar{X}'_{16} & \bar{T}_1 \\ * & \bar{X}'_{22} & \bar{X}'_{23} & \bar{X}'_{24} & \bar{X}'_{25} & \bar{X}'_{26} & 0 \\ * & * & \bar{X}'_{33} & \bar{X}'_{34} & \bar{X}'_{35} & \bar{X}'_{36} & 0 \\ * & * & * & \bar{X}'_{44} & \bar{X}'_{45} & \bar{X}'_{46} & \bar{T}_2 \\ * & * & * & * & \bar{X}'_{55} & \bar{X}'_{56} & 0 \\ * & * & * & * & * & \bar{X}'_{66} & 0 \\ * & * & * & * & * & * & \varepsilon_3 L \end{bmatrix} > 0,
\end{aligned} \tag{38}$$

where

$$\begin{aligned}
\bar{\Phi}_{11} &= (d_2 - d_1 + 1)\bar{S}_1 + (\tau_2 - \tau_1 + 1)\bar{S}_2 \\
&\quad + ((\tau_2 + h_2) - (\tau_1 + h_1) + 1)\bar{S}_3 + (A - I)L \\
&\quad + L(A - 1)^T + \bar{N}_1^T + \bar{N}_1 + \bar{M}_1^T + \bar{M}_1 + \bar{T}_1^T \\
&\quad + \bar{T}_1 + d_2 X'_{11} + \tau_2 \bar{X}'_{11} + (\tau_2 + h_2)\bar{X}'_{11}, \\
\bar{\Phi}_{12} &= A_h L - \bar{N}_1 + \bar{N}_2^T + d_2 X'_{12} + \tau_2 \bar{X}'_{12} \\
&\quad + (\tau_2 + h_2)\bar{X}'_{12}, \\
\bar{\Phi}_{13} &= BR - \bar{M}_1 + \bar{M}_2^T + d_2 X'_{13} + \tau_2 \bar{X}'_{13} \\
&\quad + (\tau_2 + h_2)\bar{X}'_{13}, \\
\bar{\Phi}_{14} &= B_d R - \bar{T}_1 + \bar{T}_2^T + d_2 X'_{14} + \tau_2 \bar{X}'_{14} \\
&\quad + (\tau_2 + h_2)\bar{X}'_{14}, \\
\bar{\Phi}_{15} &= BL + d_2 X'_{15} + \tau_2 \bar{X}'_{15} + (\tau_2 + h_2)\bar{X}'_{15}, \\
\bar{\Phi}_{16} &= B_d L + d_2 X'_{16} + \tau_2 \bar{X}'_{16} + (\tau_2 + h_2)\bar{X}'_{16}, \\
\bar{\Phi}_{22} &= -\bar{S}_1 - \bar{N}_2^T - \bar{N}_2 + d_2 X'_{22} + \tau_2 \bar{X}'_{22} \\
&\quad + (\tau_2 + h_2)\bar{X}'_{22}, \\
\bar{\Phi}_{23} &= d_2 X'_{23} + \tau_2 \bar{X}'_{23} + (\tau_2 + h_2)\bar{X}'_{23},
\end{aligned}$$

$$\begin{aligned}
\bar{\Phi}_{24} &= d_2 X'_{24} + \tau_2 \bar{X}'_{24} + (\tau_2 + h_2)\bar{X}'_{24}, \\
\bar{\Phi}_{25} &= d_2 X'_{25} + \tau_2 \bar{X}'_{25} + (\tau_2 + h_2)\bar{X}'_{25}, \\
\bar{\Phi}_{26} &= d_2 X'_{26} + \tau_2 \bar{X}'_{26} + (\tau_2 + h_2)\bar{X}'_{26}, \\
\bar{\Phi}_{33} &= -\bar{S}_2 - \bar{M}_2^T - \bar{M}_2 + \alpha_1 L^T L + h_2 X'_{33} + \tau_2 \bar{X}'_{33} \\
&\quad + (\tau_2 + d_2)\bar{X}'_{33}, \\
\bar{\Phi}_{34} &= d_2 X'_{34} + \tau_2 \bar{X}'_{34} + (\tau_2 + h_2)\bar{X}'_{34}, \\
\bar{\Phi}_{35} &= d_2 X'_{35} + \tau_2 \bar{X}'_{35} + (\tau_2 + h_2)\bar{X}'_{35}, \\
\bar{\Phi}_{36} &= d_2 X'_{36} + \tau_2 \bar{X}'_{36} + (\tau_2 + h_2)\bar{X}'_{36}, \\
\bar{\Phi}_{44} &= -\bar{S}_3 - \bar{T}_2 - \bar{T}_2^T + \alpha_2 L^T L + d_2 X'_{44} + \tau_2 \bar{X}'_{44} \\
&\quad + (\tau_2 + h_2)\bar{X}'_{44}, \\
\bar{\Phi}_{45} &= d_2 X'_{45} + \tau_2 \bar{X}'_{45} + (\tau_2 + h_2)\bar{X}'_{45}, \\
\bar{\Phi}_{46} &= d_2 X'_{46} + \tau_2 \bar{X}'_{46} + (\tau_2 + h_2)\bar{X}'_{46}, \\
\bar{\Phi}_{55} &= -\alpha_1 L^T L + d_2 X'_{55} + \tau_2 \bar{X}'_{55} + (\tau_2 + h_2)\bar{X}'_{55}, \\
\bar{\Phi}_{56} &= h_2 X'_{56} + \tau_2 \bar{X}'_{56} + (\tau_2 + d_2)\bar{X}'_{56}, \\
\bar{\Phi}_{66} &= -\alpha_2 L^T L + d_2 X'_{66} + \tau_2 \bar{X}'_{66} + (\tau_2 + h_2)\bar{X}'_{66},
\end{aligned} \tag{39}$$

and X_{ij} , $\bar{X}_{ij}$, $\tilde{X}_{ij}$, $i, j = 1, \dots, 6$, are matrices with appropriate dimensions. During the proof, there exist nonlinear items $-\alpha_i L^T L$, $i = 1, 2$. Obviously, we can have $(I - L)^T(I - L) \geq 0$; then we obtain $I - 2L + L^T L \geq 0$. Thus, the nonlinear items satisfy the following formula:

$$\begin{aligned} -\alpha_1 L^T L &\leq \alpha_1 I - 2\alpha_1 L, \\ -\alpha_2 L^T L &\leq \alpha_2 I - 2\alpha_2 L. \end{aligned} \quad (40)$$

Then by Lemma 3, (33) can be obtained. Above all, if LMIs (33) and (34) hold, we can have $\Delta V(k) < 0$. The proof is completed. $\square$

4. Numerical Simulation

In this section, in order to demonstrate the validity of the proposed method, numerical examples are given to illustrate the effectiveness of the developed results.

Example 1. Consider the following system as

$$\begin{aligned} A &= \begin{bmatrix} 1 & 0 \\ 0.25 & 0.4 \end{bmatrix}, \\ A_d &= \begin{bmatrix} -0.2 & 0.1 \\ 0 & 0.1 \end{bmatrix}, \\ B &= \begin{bmatrix} 0.3 & 0.1 \\ 0.2 & 0.3 \end{bmatrix}, \\ B_h &= \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.4 \end{bmatrix}, \\ d_2 &= 4, \\ d_1 &= 1, \\ \tau_2 &= 4, \\ \tau_1 &= 1, \\ h_2 &= 3, \\ h_1 &= 1. \end{aligned} \quad (41)$$

Choosing $\alpha_1 = 0.5$, $\alpha_2 = 0.5$, $\varepsilon_1 = 2$, $\varepsilon_2 = 2$, $\varepsilon_3 = 2$, $u_{\max} = 0.1$ and using Matlab LMI toolbox to solve LMIs (33) and (34), we can obtain that the LMIs are feasible with

$$\begin{aligned} R &= \begin{bmatrix} -0.0033 & -0.0006 \\ -0.0006 & 0.0029 \end{bmatrix}, \\ P &= \begin{bmatrix} 17.8771 & 4.3888 \\ 4.3888 & 18.7636 \end{bmatrix}, \\ K &= \begin{bmatrix} -0.0610 & -0.0249 \\ 0.0029 & 0.0527 \end{bmatrix}. \end{aligned} \quad (42)$$

FIGURE 2: State trajectory of x .

FIGURE 3: Control signal u obtained by Theorem 6.

According to Theorem 6, a state feedback controller can be constructed as

$$\begin{aligned} u(k) &= 2Kx(k - \tau(k)) \\ &= \begin{bmatrix} -0.1220 & -0.0498 \\ 0.0057 & 0.1054 \end{bmatrix} x(k - \tau(k)). \end{aligned} \quad (43)$$

With an initial condition $x(0) = [1; -1]$, simulation results are shown as follows. The state trajectories of the closed-loop system are shown in Figure 2 and Figure 3 shows the control signal response.

It can be clearly seen that the close-loop system is stabilized under the controller obtained by Theorem 6 and the control signal satisfies the actuator saturation.

Example 2. Consider the following system as

$$\begin{aligned} A &= \begin{bmatrix} 0 & 0.1 & 0.1 \\ 0.1 & -0.2 & 0.1 \\ 0.1 & 0.1 & 0.1 \end{bmatrix}, \\ A_h &= \begin{bmatrix} 0 & 0.1 & 0 \\ -0.1 & -0.1 & 0.1 \\ 0.1 & 0.2 & 0.1 \end{bmatrix}, \end{aligned}$$

$$\begin{aligned}
 B &= \begin{bmatrix} 0.1 & 0.1 & 0 \\ 0.2 & 0.3 & -0.1 \\ 0 & 0.2 & -0.1 \end{bmatrix}, \\
 B_d &= \begin{bmatrix} 0.2 & 0.1 & -0.1 \\ 0.1 & 0.4 & 0 \\ -0.1 & 0.3 & -0.1 \end{bmatrix}, \\
 d_2 &= 4, \\
 d_1 &= 1, \\
 \tau_2 &= 4, \\
 \tau_1 &= 1, \\
 h_2 &= 3, \\
 h_1 &= 1.
 \end{aligned} \tag{44}$$

Choosing $\alpha_1 = 0.5$, $\alpha_2 = 0.5$, $\varepsilon_1 = 2$, $\varepsilon_2 = 2$, $\varepsilon_3 = 2$, $u_{\max} = 0.1$ and by using Matlab LMI toolbox, we have the following result:

$$\begin{aligned}
 R &= \begin{bmatrix} -0.0015 & -0.0023 & -0.0143 \\ -0.00023 & 0.0040 & 0.0030 \\ -0.0143 & 0.0030 & -0.0279 \end{bmatrix}, \\
 P &= \begin{bmatrix} 23.5668 & 9.2097 & -3.7498 \\ 9.2097 & 38.4496 & -6.0510 \\ -3.7498 & -6.0510 & 9.3252 \end{bmatrix}, \\
 K &= \begin{bmatrix} -0.0016 & -0.0145 & -0.1145 \\ -0.0283 & 0.1142 & 0.0121 \\ -0.2064 & 0.1501 & -0.2283 \end{bmatrix}.
 \end{aligned} \tag{45}$$

According to Theorem 6, a state feedback controller can be constructed as

$$\begin{aligned}
 u(k) &= 2Kx(k - \tau(k)) \\
 &= \begin{bmatrix} 0.0032 & -0.0290 & -0.2290 \\ -0.0566 & 0.02284 & 0.0241 \\ -0.4129 & 0.3003 & -0.4477 \end{bmatrix} x(k - \tau(k)).
 \end{aligned} \tag{46}$$

With an initial condition $x(0) = [-1; 1; 0]$, the corresponding results are displayed in Figures 4 and 5 which indicate that the closed-loop system can be stabilized and the saturation constraint is satisfied.

From the above simulation results, we can conclude that the developed state feedback stabilization is valid for NCSs subject to actuator saturation and induced delays.

5. Conclusion

In this paper, the stabilization problem has been developed for the NCSs with network-induced delays and actuator

FIGURE 4: State trajectory of x .

FIGURE 5: Control signal u obtained by Theorem 6.

saturation. State delay and input delay of control plant have been both considered. Lyapunov-Krasovskii theory and free weighting matrix method are used during the proof. Sufficient conditions in terms of LMIs are derived to ensure the existence of a state feedback controller which can stabilize the system. Numerical simulations are given to illustrate the necessity and the effectiveness of the proposed approach.

Competing Interests

The authors declare that they have no competing interests.

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