Research Article

Tracking Control Based on Control Allocation with an Innovative Control Effector Aircraft Application

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This paper proposes a control allocation method for the tracking control problem of a class of morphing aircraft with special actuators which are different from the conventional actuation surfaces. This design of actuators can bring about some potential advantages to the flight vehicles; however, due to the integral constraints, the desired control cannot be performed accurately; therefore, it leads to undesirable tracking errors, so influencing the performance of the system. Because the system could be control allocated, based on the designed cost function that describes the tracking errors, the cuckoo search algorithm (CSA) is introduced to search for the optimum solution within the calculated actuator execution commands that are equivalent to the desired commands. Several improvement measures are proposed for boosting the efficiency of the CSA and ensuring reasonable solutions. Simulation results show that the proposed control allocation method is necessary and effective, and the improvement measures are helpful in obtaining the optimum solution.

1. Introduction

Morphing aircraft are the vehicles that can achieve superior performance under heterogeneous flight conditions via virtual geometry change. They exhibit definite advantages of being able to accomplish multiple types of missions and perform drastic maneuvers not possible with conventional flight vehicles [1].

A class of morphing aircraft using refreshing control effector devices is introduced in [2]. Without the requirement for conventional actuation surfaces such as ailerons or rudders, the so-called Innovative Control Effector (ICE) aircraft utilizes distributed arrays of hundreds of shape-change devices to generate control energy for stabilization and maneuver control. Each actuator (effector) can be completely turned “on” to supply full control energy or “off” to supply no control energy. The control system configures the working states of all the actuators to achieve the desired control effort.

This novel design of actuators is able to bring about some advantages such as flexibility and reduced redundancy. However, it also makes trouble for the aerospace vehicle flight control to some extent. Because each actuator has only two states (open or closed), the integrated actuators cannot generate an arbitrary control energy like rudders within saturation constraints. For example, the magnitude of the deflection angle of one controlled rudder is able to be an appropriate rational number such as 0.2°, 0.6°, or 0.8°, while one actuator in distributed arrays cannot supply two-tenths, six-tenths, or eight-tenths of full control energies. Additionally, the number of the activated actuators must be an integer in reality. Therefore, a desired control command might not be faultlessly achieved by the novel actuators even if it does not overrun the saturation constraints; that is, actuator execution error between desired control command and actual control input often occurs. There have been many studies [3–10] for the tracking problem of ICE aircraft; many of them demonstrate that the actuator execution error can influence the tracking performance of the control system. For this problem, some studies [3–5] weaken the integer constraints on the number of activated actuators, but their designed control commands are hard to be accurately actualized by the actuators; some studies [6, 7] investigate the control allocation methods with a view to realizable actuator execution commands, but their designed
methods cannot ensure suitable tracking performances; some studies [8–10] propose adaptive control methods and obtain certain achievements, but they make no further attempt to decrease the actuator execution error. In this paper, aiming at the actuator execution error, we propose a control allocation method for the tracking control of ICE aircraft. It should be noted that, due to the special characteristic of the actuators, the existing control allocation methods introduced in [11, 12] might not be applicable for ICE aircraft. To obtain such a control allocation method, some pivotal issues such as the analysis of actuator execution error and heuristic ideas for improving the optimization algorithm are studied.

This paper is organized as follows. Section 2 introduces the model of ICE morphing aircraft and formulates the problem in tracking control. Section 3 expatiates on the proposed control allocation method. Then, the simulation results are presented and discussed in Section 4. Finally, some conclusions are drawn in Section 5.

2. Problem Formulation

2.1. The Description of ICE Morphing Aircraft. The ICE aircraft model [2] is derived from Lockheed Martin Tactical Aircraft systems, and its wing span is depicted in Figure 1. The distributed shape-change device arrays are chosen to compose the effector suite which includes four arrays on each wing: three of them are located on the upper surface and one is located on the lower surface. There are totally 156 individual devices, 78 per wing on the entire suite of the actuator arrays. The upper-surface leading-edge (ULE) array consists of 10 actuators, the lower-surface trailing-edge (LTE) array and upper-surface trailing-edge (UTE) array both consist of 22 actuators, and the upper-surface wingtip (UTip) array consists of 24 actuators. The aircraft control system turns each actuator “on” or “off” to generate the required control signal. Numerical values of the actuator control signals are either 1 or 0.

Consider the linearized lateral-directional dynamic model of ICE aircraft

\[ \dot{x}(t) = Ax(t) + Bu(t), \]  

where \( x \in \mathbb{R}^4 \) and \( u \in \mathbb{R}^4 \) represent the state vector and control input vector, respectively, and they are defined as follows:

\[ x = [V \ p \ r \ \phi]^T, \]
\[ u = [u_1 \ u_2 \ u_3 \ u_4]^T, \]

where \( V, p, r, \) and \( \phi \) denote the body-axis lateral velocity, roll rate, yaw rate, and roll angle, respectively, and we assume that all of them are measurable; \( u_1, u_2, u_3, \) and \( u_4 \) denote the control inputs generated in arrays of UTE, LTE, ULE, and UTip on both wings, respectively. In this paper, drawing lessons from the assumptions in [3, 4], we consider the following control signal implementation cases: (i) the actuators on the left wing execute the negative \( u_i \) and those on the right wing execute the positive \( u_i \) and (ii) there is no difference between actuators within each array in terms of generating control energies and they are treated homogeneously; that is, only the totality of activated actuators in each array is cared about in our design. Let \( N_i, i = 1, 2, 3, 4 \), represent the number of deployed actuators in \( i \)th array on each wing and \( \sigma_i \) represent the control energy supplied by one activated actuator in the \( i \)th array. Then, \( u_i \) can be described as \( u_i = \sigma_i n_i \), where \( n_i \) represents the number of activated actuators in the \( i \)th array.

According to the actual situation of the ICE aircraft investigated in this paper, the following assumption [4, 5] is introduced.

**Assumption 1.** Matrix \( B \) is singular; that is, \( \text{rank}(B) = k < 4 \).

From Assumption 1, it is known that system (1) could be control allocated. Under this assumption, the matrix \( B \) can be factorized as \( B = B_nB_m, \text{rank}(B_n) = k \). Let \( v(t) = B_nu(t) \), where \( B_n \) is the control effectiveness matrix describing the relationship between \( u \) and \( v \) and has a full row rank; then, system (1) can be described as

\[ \dot{x}(t) = Ax(t) + B_nv(t). \]  

2.2. Nominal Feedback Control Law. The control objective is to deploy an appropriate number of actuators in order to track a desirable reference model which can be described as follows:

\[ \dot{x}_m(t) = A_mx_m(t) + B_mc(t), \]

where \( c(t) \in \mathbb{R}^4 \) is a reference input command. Assume that the system \((A, B)\) is controllable, and then the following desired state feedback control can be designed:

\[ v(t) = -K_1x(t) + K_2c(t), \]

where \( K_1, K_2 \in \mathbb{R}^{k \times 4} \) are both designed gain matrices. If the following conditions are met:

\[ A - B_nK_1 = A_n, \]
\[ B_nK_2 = B_m, \]
then the closed-loop system can be rewritten as
\[ \dot{x}(t) = A_m x(t) + B_m c(t). \] (7)

Let \( e \) denote the tracking error. From (4) and (7), the following dynamical system can be obtained:
\[ \dot{e}(t) = \dot{x}(t) - \dot{x}_m(t) = A_m e(t). \] (8)

It is easy to know that \( e \) is asymptotically stable provided that \( A_m \) is a Hurwitz matrix.

### 2.3. Actuator Execution Error

After obtaining the desired \( v \), we need to deploy the numbers of activated actuators \( n_i \) to achieve the desired virtual command. Let \( v_a \) represent the actual control effect; \( v_a \) can be described as
\[ v_a = B_n (\sigma \otimes n), \] (9)
where \( n = [n_1, n_2, n_3, n_4]^T \) denotes the actuator input vector, \( \sigma = [\sigma_1, \sigma_2, \sigma_3, \sigma_4]^T \), and \( \otimes \) denotes Hadamard multiplication operator. Assume that \( n_i \) lies in a compact set \( \Omega_i \):
\[ \Omega_i = \{ n_i \leq n_i \leq \bar{n}_i, n_i \in Z \}. \] (10)

When no actuator failure occurs, \( \Omega_i \) can be described as
\[ \Omega_i = \{ n_i \leq n_i \leq \bar{n}_i, n_i \in Z \}. \] Obviously, if we can find an appropriate \( n \) which can make \( v_a \) and \( v \) equal, then \( n \) can be considered as implementation of actuators. However, for the reason that \( n_i \) is limited to an integer, it is almost impossible to find such a suitable \( n \). That means an execution error between \( v \) and \( v_a \) almost always occurs. Let \( \Delta v = v - v_a \) denote the execution error. From (3), (4), and (6), the following equation can be obtained:
\[ \dot{e}(t) = A_m e(t) - B_m \Delta v = A_m (e(t) - A_m^{-1} B_m \Delta v). \] (11)

It can be found that \( \Delta v \) make the tracking error asymptotically converge to \( A_m^{-1} B_m \Delta v \) instead of zero. Due to the relation that \( \| A_m^{-1} B_m \Delta v \|_2 \leq \| A_m^{-1} B_m \|_F \| \Delta v \|_2 \), where \( \| \cdot \|_2 \) and \( \| \cdot \|_F \) denote the 2-norm and Frobenius norm, respectively, it can be concluded that the upper bound of the tracking error, that is, the system tracking performance, is related to the magnitude of \( \Delta v \). Let \( \sigma_{ij} \) represent the full control energy which can be supplied by the \( i \)th actuator in the \( j \)th array. In many studies, it is suggested to make a proper choice of the parameters \( \sigma_{ij} \) for improving the system tracking performance. However, for a fabricated ICE aircraft, the parameters \( \sigma_{ij} \) are determinate and cannot be optionally changed at certain flight conditions. Therefore, changing the parameters \( \sigma_{ij} \) should be avoided as far as possible. Based on Assumption 1, we consider trying to work out a control allocation method of \( n \) for improving the system tracking performance.

### 3. Control Allocation Method Design

#### 3.1. Objective of the Control Allocation Method

Due to the integral requirement of the vector \( n \), the study of developing control allocation method is able to come down to an integer programming problem to some extent. Generally, an integral programming problem is an NP-hard problem. Some classical methods, such as branch-bound method and cutting plane method, can be utilized to solve the simple linear integral programming problem. However, the computational complexities of those methods are growing with the expansion of the problem scale. Considering the practical problems studied in this paper, such as plenty of constraints and the requirement of the control cycle, we think that the abovementioned classical methods might be inefficient. Moreover, it is not necessary to pursue the optimal solution insistently.

In many recent researches on the integral programming problems, some metaheuristic algorithms are proposed and satisfying results are achieved [13–16]. Therefore, in this paper, we intend to propose a metaheuristic algorithm with amelioration for solving the problem. Let \( J = \| A_m B_m \Delta v \|_2 = \| A_m B_m (v - B_n (\sigma \otimes n)) \|_2 \); the primary objective of control allocation method is to search for an integer vector \( n \) that minimizes \( J \).

#### 3.2. Cuckoo Search Algorithm

In this paper, CSA is introduced for working out the control allocation method on account of its better searching ability [17]. CSA utilizes Lévy flight probability distribution to achieve diversification. Some preliminary researches show that it is highly promising and could outperform some existing algorithms [17, 18].

The following three idealized rules [18] are used to describe the CSA:

1. Each cuckoo lays an egg once and deposits it in a randomly selected nest.
2. The best nests with superior quality of eggs are kept until the next generation.
3. The number of available host nests is specified, and the host owner detects the egg laid by a cuckoo with a probability of \( p_a \in [0, 1] \).

In CSA, for generating a new solution \( r_k^{g+1} \) for cuckoo \( k \), a Lévy flight is carried out:
\[ r_k^{g+1} = r_k^g + \alpha \odot \text{Levy}(\lambda), \] (12)
where \( \alpha > 0 \) is the step size and it is relevant to the scale of the problem of interest. The Lévy flight provides a random walk while the random step length is obtained from a Lévy distribution:
\[ \text{Levy} \sim s = t^{-\lambda}. \] (13)
It has an infinite variance with an infinite mean.

In addition, according to the abovementioned 3rd rule, in the standard process of CSA, after performing the Lévy flight, some methods are introduced to construct new solutions instead of the selected worst nests [17, 18].

#### 3.3. Attentions and Targeted Amelioration

According to the practical features of the problem studied in this paper,
we would like to highlight the following attentions and incorporate targeted amelioration into the CSA for solving the control allocation problem with greater efficiency.

3.3.1. The Quality of Initial Population. For an optimization algorithm, the quality of initial population greatly influences the quality of the final solution. Superior quality of initial population is helpful in obtaining better solution. For the problem investigated in this paper, before generating the initial population, we solve the following equation:

\[ B_n (\mathbf{\sigma} \otimes \mathbf{\xi}) = \mathbf{v}. \]  

(14)

From Assumption 1, we can conclude that the number of the solutions of (14) is infinite and the solutions can be expressed as

\[ \mathbf{\xi}_j = \mathbf{\sigma}^* \otimes (B_n^T \mathbf{v} + h_1 \mathbf{\eta}_1 + h_2 \mathbf{\eta}_2 + \cdots + h_{K-k} \mathbf{\eta}_{K-k}), \]  

(15)

where \( \mathbf{\sigma}^* = [\sigma_1^{-1} \sigma_2^{-1} \sigma_3^{-1} \sigma_4^{-1}]^T; \mathbf{\eta}_1, \mathbf{\eta}_2, \ldots, \mathbf{\eta}_{K-k} \) are linear independent vectors; \( h_1, h_2, \ldots, h_{K-k} \) are coefficients; \( (\cdot)^* \) denotes pseudoinverse. It is probable that \( \mathbf{\xi}_j \) is not an integral vector. Let \( \mathbf{\bar{\xi}}_j = \text{round}(\mathbf{\xi}_j) \), where \text{round}(\cdot) denotes the rounding approximation operation; then \( \mathbf{\bar{\xi}}_j \) is an integral vector and can be used as one egg of the initial population. It should be noted that \( \mathbf{\bar{\xi}}_j \) is not always the optimal solution even if \( \mathbf{\xi}_j \) can result in a minimum \( j \). Nevertheless, such developed initial population is more likely to have high quality in comparison with those generated at random.

3.3.2. The Boundedness of the Solutions. In Section 2.3, it has been mentioned that \( n_i \) is restricted to the set \( \Omega_i \). For an actuator input vector \( \mathbf{n} \), we consider it reasonable if \( n_i \in \Omega_i \). Therefore, we must guarantee that the solutions of new generation obtained via Lévy flight are reasonable. For that reason, according to (10), we take the following measure when a new solution \( \mathbf{n}_k = [n_1^k n_2^k n_3^k n_4^k]^T \) is generated:

\[ \mathbf{n}_k = \begin{cases} n_i^k, & \text{if } n_i^k \in \Omega_i, \\ \bar{n}_i, & \text{if } n_i^k > \bar{n}_i, \\ \bar{n}_i, & \text{if } n_i^k < \bar{n}_i. \end{cases} \]  

(16)

Then, the vector \( \mathbf{n}_k^* = [n_1^k n_2^k n_3^k n_4^k]^T \) is certain to be reasonable. Thus, when a new generated solution \( \mathbf{n}_k \) is found to be unreasonable, we utilize (16) to remake it and replace it with the vector \( \mathbf{n}_k^* \).

3.3.3. Actuator Failures. During the flight, failures may occur in each of the actuators. The possible actuator failures are able to influence the sizes of the set \( \Omega_i \). Therefore, notice that we update the set \( \Omega_i \) with the condition of the actuator failures in the solving process.

4. Numerical Simulation

In this section, the performance of the proposed control allocation method applied to the linearized lateral-directional dynamic model of ICE aircraft shown in (1) is verified via simulations. The state and input matrices are given as follows [3]:

\[
A = \begin{bmatrix}
-0.0134 & 48.5474 & -632.3724 & 32.0756 \\
-0.0199 & -0.1209 & 0.1628 & 0 \\
-0.0024 & -0.0526 & -0.0252 & 0 \\
0 & 1 & 0.0768 & 0 \\
\end{bmatrix},
\]

(17)

\[
B = \begin{bmatrix}
0 & 0 & 0 & 0 \\
-0.0431 & 0.0476 & -0.0401 & -0.0308 \\
-0.0076 & -0.0023 & -0.0022 & 0.0297 \\
0 & 0 & 0 & 0 \\
\end{bmatrix},
\]

(18)

The same reference model and reference input command [3] are used as follows:

\[
A_m = \begin{bmatrix}
-0.0134 & 48.5474 & -632.3724 & 32.0756 \\
0.5386 & -1.7746 & -23.8313 & -4.8526 \\
0.0664 & 0.6431 & -11.2476 & 0.1192 \\
0 & 1 & 0.0768 & 0 \\
\end{bmatrix},
\]

\[
B_m = [0 1 0 0]^T,
\]

\[
c(t) = \begin{cases} 0.4, & 10i \leq t \leq 10i + 5, \ i = 0, 1, 2, \ldots, \\ -0.4, & \text{others}. \end{cases}
\]

Since \( \text{rank}(B) = 2 \), the designed parameters are chosen as [6]

\[
B_r = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
\end{bmatrix},
\]

(19)

\[
K_1 = \begin{bmatrix}
9.9 & 63.6 & -1278.8 & -11.9 \\
-2.8 & 92.3 & -653.8 & 91.2 \\
\end{bmatrix},
\]

\[
K_2 = \begin{bmatrix}
-4.9903 \\
16.4898 \\
\end{bmatrix},
\]

and \( \mathbf{\sigma} \) is chosen to be \( \mathbf{\sigma} = [0.65, 0.65, 0.6, 0.7]^T \). The parameter settings for CSA are given in Table 1.

The simulation time and fixed-step size are 20 seconds and 0.02 seconds, respectively; this is equivalent to executing 1000 control allocation calculations.

Scenario 1. In this scenario, to illustrate the superiority of the proposed control allocation method, the direct rounding approximation method referred to in [9] is introduced for comparison; this method directly rounds the original actuator execution \( \mathbf{\xi} \) to the nearest integer vector without further optimization. Figure 2 illustrates the responses of the system states obtained by using the proposed control allocation method, direct rounding approximation method, and reference input command, respectively. It can be observed that
The proposed control allocation strategy

Rounding without control allocation strategy

Reference

Table 1: Parameter settings for CSA.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi$</td>
<td>10</td>
<td>Number of nests</td>
</tr>
<tr>
<td>$g_{\text{max}}$</td>
<td>20</td>
<td>Maximum number of iterations</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>$[0.25$ 0.25 0.25 0.25$]^T$</td>
<td>Step size</td>
</tr>
<tr>
<td>$p_a$</td>
<td>0.25</td>
<td>Mutation probability value</td>
</tr>
</tbody>
</table>

Figure 2: The state responses of the closed-loop system.

In addition, it should be noted that the elapsed time of the simulation test of the proposed control allocation method is about 10 seconds with an Intel Core i7-3770 CPU @3.40 GHz; that is, a single optimization step takes up about 0.01 seconds on average. This computing speed is acceptable to a flight vehicle control system.

Scenario 2. In this scenario, the high efficiency of the proposed generation method for the initial population is demonstrated. For the purpose of comparison, the generation method introduced in [6] is introduced. The focal point of
The proposed control allocation strategy

The comparative generation method is to distribute the eggs of the initial population as widely as possible within the feasible region. The two generation methods are synchronously utilized to create initial population under the same conditions in two simulation experiments. In the first simulation, $\zeta$ is changed to 100; ten random sampling points are selected and thereupon ten initial populations are determined. Figure 5 illustrates the magnitude values of the cost function $J$ of those. It can be observed that the qualities of the initial population under the proposed generation method are much higher than those under the comparative generation method. In the second simulation, $\zeta$ is still set to 10; a hundred random

### Table 2: The original and final actuator execution commands of some data points.

<table>
<thead>
<tr>
<th>Original actuator execution commands</th>
<th>Final commands after direct rounding</th>
<th>$J$</th>
<th>Final commands after the proposed control allocation</th>
<th>$J$</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0.7016; -0.8011; 0.7201; 0.5457]</td>
<td>[1; -1; 1; 1]</td>
<td>0.3876</td>
<td>[13; 10; -2; 4]</td>
<td>0.0036</td>
</tr>
<tr>
<td>[1.1116; -1.0822; 1.0614; 0.3732]</td>
<td>[1; -1; 1; 0]</td>
<td>0.3217</td>
<td>[8; 10; 7; 3]</td>
<td>0.0113</td>
</tr>
<tr>
<td>[0.1814; -0.1902; 0.1778; 0.0895]</td>
<td>[0; 0; 0; 0]</td>
<td>0.0872</td>
<td>[6; 0; -6; 1]</td>
<td>0.0077</td>
</tr>
<tr>
<td>[8.3053; -7.2498; 7.4224; -0.3540]</td>
<td>[8; -7; 7; 0]</td>
<td>0.2898</td>
<td>[13; -3; 6; 1]</td>
<td>0.0042</td>
</tr>
<tr>
<td>[6.1142; -4.9641; 5.2801; -1.3986]</td>
<td>[6; -5; 5; -1]</td>
<td>0.3509</td>
<td>[5; -8; 3; -2]</td>
<td>0.0063</td>
</tr>
<tr>
<td>[3.1138; -2.2609; 2.5571; -1.5275]</td>
<td>[3; -2; 3; -2]</td>
<td>0.4187</td>
<td>[5; -8; 10; -2]</td>
<td>0.0142</td>
</tr>
</tbody>
</table>
The proposed control allocation strategy
Rounding without control allocation strategy

Figure 4: The magnitude values of the cost function in flight.

The proposed generation method
Comparative generation method

Figure 5: The magnitude values of the cost function of the initial population in ten random sampling points.

Figure 6: The magnitude values of the cost function of the final solutions in a hundred random sampling points.

5. Conclusions

In this paper, a novel control allocation method was proposed for the tracking control problem of ICE morphing aircraft which adopts distributed shape-change effector arrays as actuators. The main conclusions are acquired as follows:

1. The influence of the actuator execution error on the system tracking performance was analyzed, and a cost function was proposed for describing it. Following that, an improved CSA was introduced to search for the optimum solution that minimizes the cost function.

2. According to the features of the research subject and the specific problem, on the basis of the CSA, some targeted measures, such as the improvement in generating initial population and guarantee for reasonable solutions, were proposed.

3. The simulation results show that the proposed control allocation method is effective and able to improve the system tracking performance. Moreover, the computing speed is appropriate for the control requirement of the flight vehicle. Therefore, the proposed method is available for the tracking control of ICE morphing aircraft and valuable for engineering applications.

4. The simulation results show that improving the quality of the initial population is helpful in obtaining the optimum solution. Future research will discuss other improvement measures for boosting the efficiency of the optimization algorithm.

Competing Interests

The authors declare that there are no competing interests regarding the publication of this paper.
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