Corrigendum

Corrigendum to “On Interval-Valued Hesitant Fuzzy Soft Sets”

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1. Introduction

We point out that assertions (3) and (4) of Theorem 36 in the paper titled “On Interval-Valued Hesitant Fuzzy Soft Sets” [1] are not true in general. We verify that the corresponding assertions in Zhang et al. [1] are incorrect by a counterexample. Finally, we introduce reasonable definitions to improve the results.

The soft set theory, proposed by Molodtsov [2], can be used as a general mathematical tool for dealing with uncertainty. Maji et al. [3] presented the concept of fuzzy soft set which is based on a combination of the fuzzy set and soft set models. Later on, Maji et al. [4] defined some operations on soft sets and showed that the distributive law of soft sets is varied. Ali et al. [5] pointed out that the distributive law of soft sets is not true in general. This implies that the distributive law of fuzzy soft sets is not true. In this paper, we show that assertions (3) and (4) of Theorem 36 proposed by Zhang et al. [1] are incorrect by a counterexample and we reconfirm Theorem 36 (3) and (4) using the generalized distributive law of interval-valued hesitant fuzzy soft sets.

For the general terminologies in this paper, please refer to [1–4, 6, 7].

Definition 1. Let $a = [a^L, a^U] = \{x \mid a^L \leq x \leq a^U\}$; then a is called an interval number. In particular, a is a real number, if $a^L = a^U$.

Definition 2. Let $a = [a^L, a^U]$, and let $b = [b^L, b^U]$, and $l_a = a^U - a^L$ and $l_b = b^U - b^L$; then the degree of possibility of $a \geq b$

is defined as

$$p(a \geq b) = \max \left\{ 1 - \max \left(\frac{b^U - a^L}{l_a + l_b}, 0 \right), 0 \right\}. \quad (1)$$

Similarly, the degree of possibility of $b \geq a$ is defined as

$$p(b \geq a) = \max \left\{ 1 - \max \left(\frac{a^U - b^L}{l_a + l_b}, 0 \right), 0 \right\}. \quad (2)$$

Definition 3. Suppose that U is an initial universe set, E is a set of parameters, $P(U)$ is the power set of U , and $A \subset E$. A pair (F, A) is called soft set over U , where F is a mapping given by $F: A \rightarrow P(U)$.

Definition 4. A pair (F, A) is called a fuzzy soft set over U , if $A \subseteq E$ and $\tilde{F}: A \rightarrow F(U)$, where $F(U)$ is the set of all fuzzy subsets of U .

Definition 5. Let X be a fixed set, and let $\text{Int}[0, 1]$ be the set of all closed subintervals of $[0, 1]$. An interval-valued hesitant fuzzy set (IVHFS, for short) $\tilde{A}$ on X is defined as

$$\tilde{A} = \{ \langle x, h_{\tilde{A}}(x) \rangle \mid x \in X \}, \quad (3)$$

where $h_{\tilde{A}}(x): X \rightarrow \text{Int}[0, 1]$ denotes all possible interval-valued membership degrees of the element $x \in X$ to $\tilde{A}$.

For convenience, we call $h_{\tilde{A}}(x)$ an interval-valued hesitant fuzzy element (IVHFE, for short). The set of all interval-valued hesitant fuzzy sets on U is denoted by $\text{IVHF}(U)$. We can note that an IVHFS $\tilde{A}$ can be seen as an interval-valued

fuzzy set if there is only one element in $h_{\tilde{A}}(x)$, which indicates that interval-valued fuzzy sets are a special type of IVHFSs.

Definition 6. Let h_1 and h_2 be two IVHFSs; then one has the following:

- (1) $h_1 \cup h_2 = \{[\gamma_1^- \vee \gamma_2^-, \gamma_1^+ \vee \gamma_2^+] \mid \gamma_1 \in h_1, \gamma_2 \in h_2\}$,
- (2) $h_1 \cap h_2 = \{[\gamma_1^- \wedge \gamma_2^-, \gamma_1^+ \wedge \gamma_2^+] \mid \gamma_1 \in h_1, \gamma_2 \in h_2\}$.

Definition 7. Let (U, E) be a soft universe and $A \subseteq E$. A pair $\mathfrak{S} = (\tilde{F}, A)$ is called an interval-valued hesitant fuzzy soft set over U , where $\tilde{F}$ is a mapping given by $\tilde{F} : A \rightarrow \text{IVHF}(U)$.

An interval-valued hesitant fuzzy soft set is a parameterized family of interval-valued hesitant fuzzy subsets of U . That is to say, $\tilde{F}(e)$ is an interval-valued hesitant fuzzy subset in U , $\forall e \in A$. Following the standard notations, $\tilde{F}(e)$ can be written as

$$\tilde{F}(e) = \{ \langle x, \tilde{F}(e)(x) \rangle : x \in U \}. \quad (4)$$

Sometimes we write $\tilde{F}$ as $(\tilde{F}, E)$. If $A \subseteq E$, we can also have an interval-valued hesitant fuzzy soft set $(\tilde{F}, A)$.

Definition 8. Let U be an initial universe and let E be a set of parameters. Supposing that $A, B \subseteq E$ and $(\tilde{F}, A)$ and $(\tilde{G}, B)$ are two interval-valued hesitant fuzzy soft sets, one says that $(\tilde{F}, A)$ is an interval-valued hesitant fuzzy soft subset of $(\tilde{G}, B)$ if and only if

- (1) $A \subseteq B$,
- (2) $\gamma_1^{\sigma(k)} \leq \gamma_2^{\sigma(k)}$,

where, for all $e \in A$, $x \in U$, $\gamma_1^{\sigma(k)}$ and $\gamma_2^{\sigma(k)}$ stand for the k th largest interval number in the IVHFEs $\tilde{F}(e)(x)$ and $\tilde{G}(e)(x)$, respectively.

In this case, we write $(\tilde{F}, A) \sqsubseteq (\tilde{G}, B)$. $(\tilde{F}, A)$ is said to be an interval-valued hesitant fuzzy soft super set of $(\tilde{G}, B)$ if $(\tilde{G}, B)$ is an interval-valued hesitant fuzzy soft subset of $(\tilde{F}, A)$. We denote it by $(\tilde{F}, A) \supseteq (\tilde{G}, B)$.

Definition 9. Let $(\tilde{F}, A)$ and $(\tilde{G}, B)$ be two interval-valued hesitant fuzzy soft sets. Now $(\tilde{F}, A)$ and $(\tilde{G}, B)$ are said to be interval-valued hesitant fuzzy soft equal if and only if

- (1) $(\tilde{F}, A) \sqsubseteq (\tilde{G}, B)$,
- (2) $(\tilde{G}, B) \sqsubseteq (\tilde{F}, A)$,

which can be denoted by $(\tilde{F}, A) = (\tilde{G}, B)$.

Definition 10. Let $(\tilde{F}, A)$ and $(\tilde{G}, B)$ be two interval-valued hesitant fuzzy soft sets over U . The “ $(\tilde{F}, A)$ AND $(\tilde{G}, B)$,” denoted by $(\tilde{F}, A) \wedge (\tilde{G}, B)$, is defined by

$$(\tilde{F}, A) \wedge (\tilde{G}, B) = (\tilde{H}, A \times B), \quad (5)$$

where, for all $(\alpha, \beta) \in A \times B$,

$$\begin{aligned} \tilde{H}(\alpha, \beta) &= \{ \langle x, \tilde{H}(\alpha, \beta)(x) \rangle : x \in U \} \\ &= \{ \langle x, \tilde{F}(\alpha)(x) \cap \tilde{G}(\beta)(x) \rangle : x \in U \}. \end{aligned} \quad (6)$$

Definition 11. Let $(\tilde{F}, A)$ and $(\tilde{G}, B)$ be two interval-valued hesitant fuzzy soft sets over U . The “ $(\tilde{F}, A)$ OR $(\tilde{G}, B)$,” denoted by $(\tilde{F}, A) \vee (\tilde{G}, B)$, is defined by

$$(\tilde{F}, A) \vee (\tilde{G}, B) = (\tilde{I}, A \times B), \quad (7)$$

where, for all $(\alpha, \beta) \in A \times B$,

$$\begin{aligned} \tilde{I}(\alpha, \beta) &= \{ \langle x, \tilde{I}(\alpha, \beta)(x) \rangle : x \in U \} \\ &= \{ \langle x, \tilde{F}(\alpha)(x) \cup \tilde{G}(\beta)(x) \rangle : x \in U \}. \end{aligned} \quad (8)$$

2. Counterexample

We begin this section with Theorem 12 below, originally proposed as Theorem 36 in Zhang et al. [1] and provide a counterexample to show that assertions (3) and (4) are not true.

Theorem 12 (see [1]). *Let $(\tilde{F}, A)$, $(\tilde{G}, B)$, and $(\tilde{H}, C)$ be three interval-valued hesitant fuzzy soft sets over U . Then one has the following:*

- (1) $(\tilde{F}, A) \wedge ((\tilde{G}, B) \wedge (\tilde{H}, C)) = ((\tilde{F}, A) \wedge (\tilde{G}, B)) \wedge (\tilde{H}, C)$,
- (2) $(\tilde{F}, A) \vee ((\tilde{G}, B) \vee (\tilde{H}, C)) = ((\tilde{F}, A) \vee (\tilde{G}, B)) \vee (\tilde{H}, C)$,
- (3) $(\tilde{F}, A) \wedge ((\tilde{G}, B) \vee (\tilde{H}, C)) = ((\tilde{F}, A) \wedge (\tilde{G}, B)) \vee ((\tilde{F}, A) \wedge (\tilde{H}, C))$,
- (4) $(\tilde{F}, A) \vee ((\tilde{G}, B) \wedge (\tilde{H}, C)) = ((\tilde{F}, A) \vee (\tilde{G}, B)) \wedge ((\tilde{F}, A) \vee (\tilde{H}, C))$.

The following example shows that assertions (3) and (4) of Theorem 12 above are not true in general.

Example 13. Let $U = \{x_1, x_2\}$ be a set of two houses and let $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$ be a set of parameters, which stand for expensive, beautiful, cheap, size, location, and in the green surroundings, respectively. Consider A, B , and C to be subsets of E , where $A = \{e_1, e_2\} = \{\text{expensive, beautiful}\}$, $B = \{e_3, e_4\} = \{\text{cheap, size}\}$, and $C = \{e_5, e_6\} = \{\text{location, in the green surroundings}\}$. Suppose that $(\tilde{F}, A)$, $(\tilde{G}, B)$, and $(\tilde{H}, C)$ are three interval-valued hesitant fuzzy soft sets defined by

$$\begin{aligned} \tilde{F}(e_1) &= \{ \langle x_1, \{[0.2, 0.4], [0.5, 0.7]\} \rangle, \\ &\quad \langle x_2, \{[0.4, 0.8], [0.5, 0.8]\} \rangle \}, \\ \tilde{F}(e_2) &= \{ \langle x_1, \{[0.3, 0.5], [0.6, 0.7]\} \rangle, \\ &\quad \langle x_2, \{[0.1, 0.3], [0.5, 0.9]\} \rangle \}, \\ \tilde{G}(e_3) &= \{ \langle x_1, \{[0.4, 0.6], [0.8, 0.9]\} \rangle, \\ &\quad \langle x_2, \{[0.2, 0.3], [0.4, 0.5]\} \rangle \}, \end{aligned}$$

$$\begin{aligned}
\tilde{G}(e_4) &= \{ \langle x_1, \{[0.6, 0.9], [0.5, 0.7]\} \rangle, \\
&\quad \langle x_2, \{[0.4, 0.6], [0.7, 0.8]\} \rangle \}, \\
\tilde{H}(e_5) &= \{ \langle x_1, \{[0.1, 0.3], [0.4, 0.6]\} \rangle, \\
&\quad \langle x_2, \{[0.3, 0.6], [0.6, 0.8]\} \rangle \}, \\
\tilde{H}(e_6) &= \{ \langle x_1, \{[0.4, 0.5], [0.6, 0.7]\} \rangle, \\
&\quad \langle x_2, \{[0.7, 0.8], [0.7, 0.9]\} \rangle \}.
\end{aligned} \tag{9}$$

By Definitions 10 and 11, the interval-valued hesitant fuzzy soft set $(\tilde{F}, A) \wedge ((\tilde{G}, B) \vee (\tilde{H}, C))$ has the parameter set $A \times (B \times C)$ and interval-valued hesitant fuzzy soft set $((\tilde{F}, A) \wedge (\tilde{G}, B)) \vee ((\tilde{F}, A) \wedge (\tilde{H}, C))$ has a set of parameters as $(A \times B) \times (A \times C)$. But we can not find any notion which ensures $A \times (B \times C) = (A \times B) \times (A \times C)$. Hence Theorem 12 above is not true.

3. Main Results

Definition 14. Let U be an initial universe and let E be a set of parameters. For subsets A and B of E , let $(\tilde{F}, A)$ and $(\tilde{G}, B)$ be interval-valued hesitant fuzzy soft sets. One says that $(\tilde{F}, A)$ is a generalized interval-valued hesitant fuzzy soft subset of $(\tilde{G}, B)$, denoted by $(\tilde{F}, A) \tilde{\subseteq}_g (\tilde{G}, B)$, if, for every $\alpha \in A$, there exists $\beta \in B$ such that $\gamma_1^{\sigma(k)} \leq \gamma_2^{\sigma(k)}$, where, for all $\alpha \in A$, $\beta \in B$, $x \in U$, $\gamma_1^{\sigma(k)}$ and $\gamma_2^{\sigma(k)}$ stand for the k th largest interval number in the IVHFEs $\tilde{F}(\alpha)(x)$ and $\tilde{G}(\beta)(x)$, respectively.

Example 15. Let $U = \{x_1, x_2, x_3\}$ be an initial universe and let $E = \{e_1, e_2, e_3\}$ be a set of parameters. With $A = \{e_1, e_2\}$ and $B = \{e_1, e_3\}$, let $(\tilde{F}, A)$ and $(\tilde{G}, B)$ be interval-valued hesitant fuzzy soft sets defined by

$$\begin{aligned}
\tilde{F}(e_1) &= \{ \langle x_1, \{[0.1, 0.4], [0.3, 0.7], [0.3, 0.8]\} \rangle, \\
&\quad \langle x_2, \{[0.4, 0.5], [0.5, 0.6]\} \rangle, \\
&\quad \langle x_3, \{[0.2, 0.4], [0.4, 0.5]\} \rangle \}, \\
\tilde{F}(e_2) &= \{ \langle x_1, \{[0.2, 0.5], [0.3, 0.6]\} \rangle, \\
&\quad \langle x_2, \{[0.3, 0.5], [0.4, 0.7], [0.6, 0.8]\} \rangle, \\
&\quad \langle x_3, \{[0.3, 0.9], [0.5, 0.8]\} \rangle \}, \\
\tilde{G}(e_1) &= \{ \langle x_1, \{[0.3, 0.5], [0.4, 0.8]\} \rangle, \\
&\quad \langle x_2, \{[0.3, 0.7], [0.4, 0.8]\} \rangle, \\
&\quad \langle x_3, \{[0.3, 0.5], [0.3, 0.7], [0.5, 0.8]\} \rangle \}, \\
\tilde{G}(e_3) &= \{ \langle x_1, \{[0.3, 0.5], [0.5, 0.6], [0.8, 1.0]\} \rangle, \\
&\quad \langle x_2, \{[0.55, 0.6], [0.7, 0.9]\} \rangle, \\
&\quad \langle x_3, \{[0.3, 0.7], [0.8, 0.85], [0.9, 1.0]\} \rangle \}.
\end{aligned} \tag{10}$$

Then $(\tilde{F}, A) \tilde{\subseteq}_g (\tilde{G}, B)$.

Definition 16. Let $(\tilde{F}, A)$ and $(\tilde{G}, B)$ be interval-valued hesitant fuzzy soft sets. One says that $(\tilde{F}, A)$ and $(\tilde{G}, B)$ are generalized interval-valued hesitant fuzzy soft set equal, denoted by $(\tilde{F}, A) \equiv (\tilde{G}, B)$, if $(\tilde{F}, A) \tilde{\subseteq}_g (\tilde{G}, B)$ and $(\tilde{G}, B) \tilde{\subseteq}_g (\tilde{F}, A)$.

Theorem 17 is the corrected version of assertions (3) and (4) of Theorem 12, originally written as Theorem 36 of Zhang et al. [1].

Theorem 17. Let $(\tilde{F}, A)$, $(\tilde{G}, B)$, and $(\tilde{H}, C)$ be three interval-valued hesitant fuzzy soft sets over U . Then one has the following:

- (3) $(\tilde{F}, A) \wedge ((\tilde{G}, B) \vee (\tilde{H}, C)) \equiv ((\tilde{F}, A) \wedge (\tilde{G}, B)) \vee ((\tilde{F}, A) \wedge (\tilde{H}, C))$,
- (4) $(\tilde{F}, A) \vee ((\tilde{G}, B) \wedge (\tilde{H}, C)) \equiv ((\tilde{F}, A) \vee (\tilde{G}, B)) \wedge ((\tilde{F}, A) \vee (\tilde{H}, C))$.

Proof. For any $\alpha \in A$, $\beta \in B$, and $\gamma \in C$, we have $\tilde{F}(\alpha) \cap (\tilde{G}(\beta) \cup \tilde{H}(\gamma)) = (\tilde{F}(\alpha) \cap \tilde{G}(\beta)) \cup (\tilde{F}(\alpha) \cap \tilde{H}(\gamma))$. Hence conclusion (3) is valid. Similarly, we can prove assertion (4). $\square$

4. Conclusions

Zhang et al. [1] introduced interval-valued hesitant fuzzy soft set based on interval-valued hesitant fuzzy set and proposed several theorems and some operations on an interval-valued hesitant fuzzy soft set. However, we pointed out that assertion (3) and (4) of Theorem 36 [1] are not true. Using the notions of a generalized interval-valued hesitant fuzzy soft subset and fuzzy soft equal, Theorem 36 (3) and (4) of [1] is proposed and proven to be true.

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