Research Article

Pricing Decisions of a Dual-Channel Closed-Loop Supply Chain under Uncertain Demand of Indirect Channel

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1. Introduction

As practice indicates, the recycle and reuse of waste products not only help enterprises to maximize resource utilization but also enable them to create profits [1] and enhance competitiveness [2]. Presently, quite a few enterprises, such as Xerox, Kodak, and HP, integrate reverse logistics into their strategies, making it a new source of cost reduction and competitive advantage. For example, Xerox achieved the increase in the recycle rate from 56% in 1996 to 89% in 2004 through its regeneration strategy. For its new products, 90% of the components are recycled, which allows Xerox to save 45%–60% on manufacturing costs and gain hundred-million dollars [3]. Compared to the EU and US, China lags behind in adopting such strategies, but some enterprises have already implemented innovative ideas. In the beginning of 2006, the first independently developed cartridge-automatic-remanufacturing production line in China, namely, the handling system of waste cartridges, began its operation in Shanghai [4]. This marked a substantial step for China in recycling of used products. Given the clear advantages of recycling economies and sustainable development, there is theoretical and practical significance of studying of related issues and, in particular, the closed-loop supply chains (CLSCs).

In practice, due to warranty, repair return, end-of-use return, and end-of-life (EOL) return, customers may return their products during and after the product life cycles [5]. In this paper, we mainly address the problem of remanufacturing the EOL products. The steps of remanufacturing the EOL products are collecting the EOL items, disassembling those items into their parts, cleaning, reprocessing and inspecting each part, and finally reassembling the parts to be used again [6].

In this paper, we will investigate three noncooperative games of different power structures between the manufacturer and the retailer, that is, Nash game (Model N), manufacturer-Stackelberg game (Model M), and retailer-Stackelberg game (Model R). In fact, industry provides numerous examples of different channel power structures.
For instance, manufacturers (e.g., Apple and Nike) play a more dominant role than their suppliers and their downstream members (e.g., third-party logistics providers and retailers) in some supply chains, that is, Apple and Nike, are usually considered to be the Stackelberg leader, while the other supply chain members are followers [7]. This kind of game model is most common to see in practice, and it also has been widely used in the supply chain literature [8–13]. Meanwhile, recent years have also seen a significant increase in the power of retailers, such as Wal-Mart, Carrefour, and Hudson’s Bay, and they have greater market power than other supply chain members in their respective supply chains. Moreover, it is not unusual to see in the business world that there has not been channel leader in the market; that is, no one dominates the market. In some supply chains, both the manufacturer and the retailer may be engaged in vertical game competition in a small or local market for selling private brands [14].

Early research has proved that different channel power structures have a substantial effect on the performance of the CLSC [8]. Hence, based on the above observations, this paper establishes the CLSC model in three different channel power structures and mainly discusses the following questions:

1. How do different channel power structures of CLSCs under uncertain demand of indirect channel influence the wholesale price, the retail price, the direct sale price, the acquisition price of the used product, the transfer price, and the channel performance?

2. Among the three types of channel power structures, which one is the best from the entire CLSC’s perspective?

3. How does customer acceptance of the direct channel influence the wholesale price, the retail price, the direct sale price, the acquisition price of the used product, the transfer price, and the channel performance?

Addressing the above important research questions highlights the research objectives and contributions of this paper. To the best of our knowledge, this is the first paper which specifically addresses these channel power structures in a dual-channel CLSC under uncertain demand of indirect channel.

The remainder of this paper is organized as follows. In the following section, we briefly review the relevant literature, and we introduce notation and assumptions of the modeling framework in Section 3. Section 4 describes three different game models in detail. In Section 5, three different models are presented. Numerical examples are provided in Section 6 to illustrate the results. Section 7 concludes and indicates some possible directions for future research.

2. Literature Review

Given the prevalence and importance of the pricing decision problem related to the CLSC with product remanufacturing, it has been studied extensively (Ferrer and Swaminathan [15, 16]; Choi et al. [8]; Chen and Chang [17]). There is a stream of research using the game theory that examines the competition of new and remanufactured products. This research has been inspired by Ferrer [18], who addresses the opportunity to market remanufactured and new products in a steady-state environment. Ferrer and Swaminathan [15] establish two-period and multiperiod competition models of new and remanufactured products, and they further investigate optimal pricing policies in monopoly and duopoly markets. Choi et al. [8] study the retail price, transfer price, and channel performance of different CLSCs under different types of channel leadership with a price-dependent demand. Li et al. [24] analyze remanufacturing and pricing decisions when supply and demand factors are uncertain. These papers assume that used products can be collected at a fixed price. However, in reality, most collecting agents regard buy-back prices as decision variables.

There is also a growing stream of the CLSC literature that investigates used product collecting issues under different reverse channel structures (Savaskan et al. [9]; Hong et al. [10]; Huang et al. [11]; Vercaene [19]; Cai et al. [20]). For example, Savaskan et al. [9] develop three collection channel modes, namely, the manufacturer collection, retailer collection, and third-party collection, to investigate manufacturer’s reverse channel choices. Hong et al. [10] study pricing decisions and collection channel choices with hybrid dual-channel collection in a single-period CLSC model. Unlike the present analysis, these papers do not address competition between collection channels. In contrast, they assume that the manufacturer and remanufacturer simultaneously collect used products from customers and compete in such collection activities. Huang et al. [11] investigate the competition between two collection channels, but their work does not consider patent licensing issues related to the remanufacturing process. Unlike the present paper, these studies are all based on definite market demand and do not consider market demand disturbances. However, in reality, the market demand is usually uncertain.

For a more complete review on the CLSC literature, readers can refer to Guide Jr. and Van Wassenhove [21], Souza [22], and Govindan et al. [23].

Pricing decisions related to the CLSC under uncertain environments have already been considered in the literature (Li et al. [24]; Vorasayan and Ryan [25]). Li et al. [24] investigate pricing and remanufacturing decisions in a stochastic environment where both demand and remanufacturing yield are random. Vorasayan and Ryan [25] model sales, returns, refurbishment, and resale processes in an open queueing network and investigate both optimal price and proportion of refurbished products.

Our work is different from Li et al. [24] and Vorasayan and Ryan [25] in the following aspects. (1) Both studies only consider the influence of the uncertain demand of the indirect sales channel (the traditional retail channel) on the pricing decision of the CLSC and ignore the influence of the direct sales channel on the retail channel when the two channels coexist. This research finds that even if the direct sales channel itself does not create profits, its existence can still help to control the retailer’s price and prompt the retailer to sell more products to avoid double marginalization, thus
increasing the total profits from channel sales. Therefore, there is theoretical and practical significance of studying the influence of the direct sales channel on the retail channel as well as the entire CLSC. (2) This paper investigates the recovery pricing of the CLSC and joint pricing of recycled products in three different channel power structures, while the above studies are only focused on a single channel power structure. However, different structures can produce different effects on the pricing decisions of the CLSC, and the structures considered in this paper are quite common in the real world.

3. Model Assumptions and Notation

3.1. Description of Relevant Parameters. We consider a dual-channel CLSC system consisting of a manufacturer and a retailer. The manufacturer sells both new and remanufactured products to the retailer, as well as to the end customers directly. Customers may use either the retail channel (the traditional channel) or the direct channel (online channel) to purchase the products. To induce collection, we assume that the manufacturer subcontracts the collection activity to the retailer.

The interrelation between the manufacturer and the retailer and relevant decision variables is shown in Figure 1.

The main parameters used in this paper are the following. $C_r$ represents the unit cost of the remanufactured product, $C_m$ denotes the unit cost of the new product, $w$ represents the wholesale price of the manufacturer, $P_t$ represents the retail price of the retailer, $P_d$ represents the direct sale price, $P_c$ denotes the acquisition price of the used product, $P_m$ represents the transfer price paid by the manufacturer to the retailer when collecting used products, $Q_d$ represents the order quantity of the direct channel, $Q_t$ represents the order quantity of the indirect channel, $C_s$ denotes the unit sale cost of the retailer, $C_b$ represents the unit shortage cost of the retailer, and $S$ denotes the unit residual value of the surplus products that are not sold.

Further, $I_t^{ij}$ and $I_r^{ij}$ represent the profit and expected profit of supply chain member $j$, respectively. Here, $i = N, M, R$ represents the Nash game, the manufacturer-Stackelberg game, and the retailer-Stackelberg game, respectively, and $j = m, r$ represents the manufacturer and retailer, respectively.

3.2. Assumptions. For convenience, this paper has simplified the complex situation and made some assumptions based on reality and logic.

Assumption 1. The unit production cost $C_r$ of the remanufactured product is lower than the unit production cost $C_m$ of the new product, namely, $C_r < C_m$. We also assume that the unit production cost of all remanufactured products is the same [9–11].

Let $\Delta$ denote the unit cost saved by remanufacturing: $\Delta = C_m - C_r$.

Assumption 2. Provided that the manufacturer only produces a single-brand product, the new product and the remanufactured product are sold at the same price [8–11].

This assumption is consistent with works by Choi et al. [8], Savaskan et al. [9], Huang et al. [11], Kumar Jena and Sarmah [5], and Hong et al. [13]: we suppose that there is no difference between the new and remanufactured products, so the retailer can sell them in the same market at the same price. In the real world, this assumption is reasonable for some products, for example, single-use cameras and refillable cylinders [6, 26].

Assumption 3. The market demand is linear: $d_i(P_d, P_s) = \theta \phi - P_d + \alpha P_s$, and $d_t(P_d, P_s) = (1 - \theta) \phi - P_s + \alpha P_d$ [12], where $d_i(P_d, P_s)$ represents the market demand of the direct channel, $d_t(P_d, P_s)$ denotes the market demand of the indirect channel, $\phi$ represents the potential market demand, $\theta$ $(0 < \theta < 1)$ represents customer acceptance of the direct channel, and $\alpha$ $(0 < \alpha < 1)$ denotes the substitution effect between the two channels.

In this paper, to maintain analytical tractability, we assume that the demand of the direct channel is decreasing with the direct sale price $P_d$ and increasing with the retail price $P_s$, while the demand of the indirect channel is decreasing with the retail price $P_s$ and increasing with the direct sale price $P_d$. The downward linear demand function is widely used in the operations literature (e.g., [12, 27]); it can enable us to develop a first-cut analysis of the CLSC decision of the manufacturer; the generalizability of the results to nonlinear demand functions is a question of future research.

Assumption 4. The manufacturer has the make-to-order manufacturing model and does not bear any losses caused by uncertainty in market demand. The market demand facing the retailer is uncertain, and we adopt the additive uncertainty model to describe the random market demand of the retailer, where we denote it by $D_t = d_t(P_d, P_s) + \varepsilon$. Here, $\varepsilon$ represents the random disturbance term with the probability density $f(x)$ and distribution function $F(x)$.
Assumption 5. In the indirect channel, the retailer takes all the risks caused by uncertain market demand. That is, the retailer bears the shortage cost $C_s$ if supply is insufficient and sells the surplus product at $S$ if supply is excessive. $S$ is determined by the market, and generally $S < P_r$.

Assumption 6. The total market recovery of the used product is $Q_b = bP_b^k$ ($b > 0, k > 1$), where $b$ is the conversion constant and $k$ is the market price elasticity.

Assumption 7. All recycled used products can be used for remanufacturing, the processing cost is zero, and the sales cost of the manufacturer is zero as well.

Assumption 8. In the production, the manufacturer first remanufactures, the processing cost is zero, and the sales of the used product is $P_k > 0$. The manufacturer uses recycled used products and then uses the new raw materials to produce a certain amount of new products.

At the same time, remanufactured products cannot meet the market demand, and the manufacturer must produce a certain amount of new products [8–10].

4. System Description

In this paper, we consider three decentralized decision models with different channel power structures, namely, Nash game between the manufacturer and retailer (Model N), retailer-Stackelberg game (Model R), and manufacturer-Stackelberg game (Model M).

We propose three different game models mainly dependent on the different channel power between the retailer and the manufacturer. That is, when the retailer has significantly larger power, it is the retailer-Stackelberg game. Likewise, when the manufacturer has significantly larger power, it is the manufacturer-Stackelberg game, and when the power difference between the manufacturer and the retailer is minor, it is the Nash game.

Kraljic matrix gives detailed explanation about the source of power in the supply chain and validates these three models in this paper. In the work of [28], Kraljic introduces a comprehensive portfolio approach as a tool for professional purchasers. The first matrix focuses on the products classifying problem on the basis of two dimensions: profit impact and supply risk, and the second matrix, which is closely related to our work, focuses on the relative power position of the buyer and the supplier. They list some important factors that weigh the bargaining power of the buyer and its supplier, such as supplier's capacity utilization, supplier's break-even stability, uniqueness of the supplier's products, and the market share of the buyer. After weighing the power, on items where the company (buyer) plays a dominant market role and the suppliers' strength is rated medium or low, the buyer obtains larger power. Otherwise, on items where the company's (buyer's) role in the supply market is secondary and the suppliers are strong, the supplier obtains larger power. Caniêls and Gelderman [29, 30] extend the work of Kraljic with a perspective of power and independence. They conclude that mutual dependence and power are closely related concepts. The buyer's dependence on the supplier is a source of power for the supplier and vice versa. Dabhilkar et al. [31] and Mediavilla et al. [32] investigate the Kraljic matrix with similar perspective.

Therefore, in our model, if the retailer depends on the manufacturer more than the manufacturer depends on the retailer, it is Model M; otherwise, it is Model R, and if the mutual independence is approximately equal, it is model N.

The rules of these games for three possible power balance scenarios are as follows.

Nash Game (Model N). The manufacturer chooses its wholesale price $w$, direct sale price $P_d$, and transfer price $P_m$ conditional on the retailer's retail price $P_r$, and the acquisition price of used product $P_r$. The retailer determines the retail price $P_r$ and the acquisition price of used product $P_r$ conditional on the manufacturer's wholesale price $w$, direct channel price $P_d$, and transfer price $P_m$.

Manufacturer-Stackelberg Game (Model M). The manufacturer determines its wholesale price $w$, direct sale price $P_d$, and transfer price $P_m$ using the retailer's response functions. The retailer chooses the retail price $P_r$ and the acquisition price of used product $P_r$ so as to maximize its expected profit given the manufacturer's wholesale price $w$, direct sale price $P_d$, and transfer price $P_m$.

Retailer-Stackelberg Game (Model R). The retailer chooses its retail price $P_r$ and the acquisition price of used product $P_r$ using the manufacturer's response functions. The manufacturer then determines its wholesale price $w$, direct sale price $P_d$, and transfer price $P_m$ so as to maximize its expected profit given the retailer's retail price $P_r$ and the acquisition price of used product $P_r$. These rules are illustrated in Figures 2–4, respectively.

5. The Model

Based on the assumptions stated above, the profit functions of the retailer and manufacturer are as follows:

\[
\Pi_r = (P_r - C_s) \min(D_s, Q_s) + (P_m - P_r) bP_r^k - wQ_s \\
+ \left[ Q_s - \min(D_s, Q_s) \right] S \\
- \left[ D_s - \min(D_s, Q_s) \right] C_b,
\]  
(1)

\[
\Pi_m = (w - C_m) Q_s + (\Delta - P_m) bP_r^k + (P_d - C_m) Q_d.
\]

Let $Q_s = d_2(P_d, P_r) + z$, where $z$ is the inventory factor. Then we have

\[
\Pi_r = (P_r - C_s) \left[ d_2(P_d, P_r) + \min(\epsilon, z) \right] \\
+ (P_m - P_r) bP_r^k + \left[ z - \min(\epsilon, z) \right] S \\
- \left[ z - \min(\epsilon, z) \right] C_b,
\]  
(2)

\[
\Pi_m = (w - C_m) \left( d_2(P_d, P_r) + z \right) + (\Delta - P_m) bP_r^k \\
+ (P_d - C_m) d_1(P_d, P_r).
\]  
(3)
(i) Manufacturer sets $w, P_d,$ and $P_m$

(i) Retailer sets $P_s$ and $Pr$

1st

Customer

1st

(i) Retailer determines $P_l$ and $Pr$

2nd

(i) Manufacturer sets $w, P_l,$ and $P_m$

Customer

Based on (3), the expected profit of the manufacturer is

$$
\Gamma_m = (w - C_m) (d_2 (P_d, P_s) + z) + (\Delta - P_m) b P_r^k + (P_d - C_m) d_1 (P_d, P_s).$$

(5)

Based on (4) and (5), we state the following proposition.

**Proposition 9.** (1) If $2 (P_s + C_b - S - C_s) f(z) < (1 - F(z))^2$, (4) is strictly jointly concave in $P_s, P_r,$ and $z$. (2) Equation (5) is strictly jointly concave in $w, P_d,$ and $P_m$.

**Proof.** Taking the second-order partial derivatives of (4) with respect to $P_s, P_r,$ and $z$, we have the Hessian matrix

$$
H = \begin{bmatrix}
-2 & 0 & 1 - F(z) \\
0 & -b (1 + k) P_r^{k-1} & 0 \\
1 - F(z) & 0 & (S + C_s - P_s - C_b) f(z)
\end{bmatrix}.
$$

(6)

Since $\partial^2 \Gamma_r / \partial P_s^2 = -2 < 0,$

$$
\begin{bmatrix}
-2 & 0 \\
0 & -b (1 + k) P_r^{k-1}
\end{bmatrix} = 2b (1 + k) P_r^{k-1} > 0.
$$

(7)

Equation (4) is strictly jointly concave in $P_s$ and $P_r$. Similarly, we have

$$
H' = \begin{bmatrix}
-2 & 0 & 1 - F(z) \\
0 & -b (1 + k) P_r^{k-1} & 0 \\
1 - F(z) & 0 & (S + C_s - P_s - C_b) f(z)
\end{bmatrix}.
$$

(8)
Only when \( b(1 + k)(S + C_s - P_s - C_b) f(z) - (1 - F(z))^2 \) < 0, that is, \( 2(P_s + C_b - S - C_s) f(z) < (1 - F(z))^2 \), the Hessian matrix will be negative definite.

Hence, for \( 2(P_s + C_b - S - C_s) f(z) < (1 - F(z))^2 \), (4) is strictly jointly concave in \( P_s, P_r, \) and \( z \).

With a similar manner, we can easily prove that (5) is strictly jointly concave in \( w, P_d, \) and \( m \).

Similarly, in the other two game models, we can easily prove that (4) is strictly jointly concave in \( P_s, P_r, \) and \( z \), and (5) is strictly jointly concave in \( w, P_d, \) and \( m \) (for proof, see Appendix). \( \square \)

5.1. Model N: Nash Game between the Manufacturer and Retailer. The Nash game between the manufacturer and retailer is a market game with no leader. In the market where neither manufacturer nor retailer is a leader, suppose that the manufacturer and retailer have simultaneous actions, and the manufacturer decides the wholesale price \( w \), direct channel price \( P_d \), and transfer price \( P_m \), and the retailer decides the retail price \( P_r \), acquisition price of used product \( P_s \), and inventory factor \( z \).

Take the derivatives with respect to \( P_s, P_r, \) and \( z \) of (4), respectively. Then the first-order conditions are as follows:

\[
\begin{align*}
\frac{\partial}{\partial P_s} (4) &= -w + (1 - \theta) \phi + \alpha P_d + C_s + \int_{-\infty}^{z} xf(x) \, dx = 0, \\
\frac{\partial}{\partial P_r} (4) &= \frac{bk \Delta}{P_r} - b P_r^k = 0, \\
\frac{\partial}{\partial z} (4) &= 2 \phi + 2 \alpha P_d = 0.
\end{align*}
\]

Let \( P_m = P_s + m, P_s = w + n, m, \) and \( n \) denote the retailer's marginal profit ratio from recycle and sales, respectively.

By substituting \( P_m = P_s + m \) and \( P_s = w + n \) into (5) and taking the derivatives with respect to \( w, P_d, \) and \( P_m \), respectively, we obtain the first-order conditions:

\[
\begin{align*}
(1 - \theta) \phi + (1 - \alpha) C_m - P_s - w + z + 2\alpha P_d &= 0, \\
\alpha (w - C_m) + \theta \phi - 2P_d + C_m &= 0, \\
kb (\Delta - P_m) P_r^{k-1} - b P_r^k &= 0.
\end{align*}
\]

Solving (9)–(14) simultaneously, we can derive that

\[
\begin{align*}
P_s^{N^*} &= \frac{w^{N^*} + (1 - \theta) \phi + \alpha P_d^{N^*} + C_s + \int_{-\infty}^{z} xf(x) \, dx + z [1 - F(z)]}{2}, \\
P_r^{N^*} &= \frac{bk \Delta^{N^*}}{1 + k}, \\
P_m^{N^*} &= \frac{k \Delta^{N^*}}{k}, \\
w^{N^*} &= \frac{(1 - \theta + \alpha \theta) \phi + (1 - \alpha^2) C_m - (1 - \alpha^2) P_s^{N^*} + z}{1 - \alpha^2}, \\
P_d^{N^*} &= \frac{[\theta + \alpha (1 - \theta) \phi + (1 - \alpha^2) C_m + \alpha z]}{2 (1 - \alpha^2)}.
\end{align*}
\]

Again solving \( P_s^{N^*}, w^{N^*}, \) and \( P_d^{N^*} \) simultaneously, one can determine the analytical solutions for the retail price, wholesale price, and direct sale price. Solving \( P_s^{N^*}, w^{N^*}, \) and \( P_m^{N^*} \) simultaneously, one can determine the analytical solutions for the acquisition price and transfer price.

Then, by substituting each analytical solution into (4) and (5), we can obtain the expected profits of the manufacturer and retailer.

Due to the fact that the proposed model is a nonlinear programming model, which is difficult to solve, we will present numerical experiments to illustrate the analytical solutions and give some managerial implications in Section 6.

From the first-order conditions above, the following proposition can be derived.

**Proposition 10.** In the Nash game between the manufacturer and retailer, the retailer's marginal profit ratio from recycled used products is only related to the market price elasticity \( k \), being a decreasing function of \( k \). The transfer price \( P_m \) paid by the manufacturer to the retailer is an increasing function of \( \Delta \) and a decreasing function of the market price elasticity \( k \).

**Proof.** From (10), \( (P_m^* - P_s)/P_m^* = 1/(1 + k) \). Let \( r = 1/(1 + k) \); then \( r \) is the marginal profit ratio of the retailer from recycling of used products. Obviously, \( r \) is a decreasing function of \( k \).

From (10) and (14), \( P_m^* = (1 + k) \Delta/(2 + k) \); then we have \( \partial P_m^*/\partial k = \Delta/(2 + k)^2 \). Hence, \( P_m^* \) is an increasing function of \( \Delta \) and a decreasing function of the market price elasticity \( k \).

Proposition 10 indicates that for larger market price elasticity \( k \) (namely, when customers are more sensitive to the acquisition price of used product), raising acquisition price will increase the amount of recycle and thus retailer's marginal profit ratio of recycling used products will also increase. Therefore, when customers are more sensitive to the acquisition price of used product, the retailer should raise acquisition price so as to increase the amount of recycle and thus make more profit in recycling used products. Otherwise, it is better for the retailer to set a lower acquisition price. \( \square \)

5.2. Model R: Retailer-Stackelberg Game. In the retailer-Stackelberg game, assume that the manufacturer and retailer play a two-stage dynamic game, where the game order is as follows.

1. The retailer first decides the retail price \( P_r \) and the acquisition price of used product \( P_s \).
2. Based on the prices set by the retailer, the manufacturer decides its wholesale price \( w \), the sales price \( P_d \) in the direct channel, and the transfer price \( P_m \).

In the retailer-Stackelberg game, the retailer can set the retail price \( P_r \), the acquisition price of used product \( P_s \), and the inventory factor \( z \) to maximize \( \Gamma^R \) based on the manufacturer's reaction function.

Hence, the retailer solves

\[
\begin{align*}
\max & \quad \Gamma^R \big( P_s, P_r, z \big) \\
\text{s.t.} & \quad (1 - \theta) \phi + (1 - \alpha) C_m - P_s - w + z + 2\alpha P_d = 0
\end{align*}
\]
\[
\alpha (w - C_m + P_s) + \theta \phi - 2P_d + C_m = 0
\]

\[P_s > w > C_m.\]

Solving (12)–(14) simultaneously, we can derive that
\[
\begin{align*}
R^* &= \frac{1}{1+k} (1 - \alpha^2) C_m + \alpha z + \int_{-\infty}^{z} x f(x) dx + z [1 - F(z)]
\end{align*}
\]

Due to the fact that the proposed model is a nonlinear programming model, which is difficult to solve, we will present numerical experiments to illustrate the analytical solutions and give some managerial implications in Section 6.

6. Example Analysis

In the previous section, we have analyzed relevant issues of the dual-channel CLSC in three different market structures. The model structure is quite complex, being a nonlinear programming problem, since it involves both certain demand of the direct sales channel and uncertain demand of the indirect channel. This makes it difficult to obtain explicit solutions and provide a comparative analysis of the results of each model. Accordingly, we plan to work out optimal solutions for each decision variable using numerical simulations, then find optimal expected profits for each model, and provide a comparative analysis of the results to make corresponding conclusions.

6.1. Parameter Assignment. In order to better study the influence of the direct channel on the indirect channel, we plan to analyze the changing trend of each decision variable and expected profits as functions of customer acceptance of the direct channel \( \theta \). We suppose that \( \theta \) varies from 0.1 to 0.6 in increments of 0.1. In the indirect channel, the distribution of the random factor \( \epsilon \) needs to be considered due to uncertain market demand. For simplicity, we assume that the random factor \( \epsilon \) follows the uniform distribution \( U(0,2) \). The assignment of other parameters is shown in Table 1.

6.2. Analysis. Using the parameters in Table 1 in each decision model and the expected profit functions of the manufacturer and retailer, we can obtain the optimal solution for each decision variable and optimal expected profits. See Table 2. We illustrate the influence of \( \theta \) on each decision variable and expected profits in Figures 5–10 based on the data in Table 2. We reach the following conclusions.

(1) Figure 5 shows that the wholesale price decreases when \( \theta \) increases in all three models, where \( w^{M^{*}} \) remains the largest in each case. For 0.1 \( \leq \theta < 0.291 \), \( w^{N^{*}} > w^{R^{*}} \); for 0.291 \( < \theta \leq 0.6 \), \( w^{N^{*}} < w^{R^{*}} \); for \( \theta = 0.291 \), \( w^{N^{*}} = w^{R^{*}} \).

This can be explained as the manufacturer can take advantage of its control over the channel and decide the
Table 1: Parameters of the model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$C_m$</th>
<th>$C_r$</th>
<th>$C_s$</th>
<th>$C_b$</th>
<th>$\alpha$</th>
<th>$k$</th>
<th>$b$</th>
<th>$S$</th>
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</thead>
<tbody>
<tr>
<td>Value</td>
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<td>5</td>
<td>1</td>
<td>5</td>
<td>0.3</td>
<td>2</td>
<td>0.5</td>
<td>5</td>
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</table>

wholesale price preferentially in the manufacturer-Stackelberg game. In contrast, the retailer, as a follower, can only accept the manufacturer’s price. In this case, the price set by the manufacturer is usually high. In the retailer-Stackelberg game, the retailer can take advantage of its control over the channel and forces the manufacturer to lower the wholesale price. In this case, the wholesale price is lower than that in the manufacturer-Stackelberg game. In the Nash game between the manufacturer and retailer, no one is in a dominant position, and there is fierce competition. Under these circumstances, the wholesale price is a little lower than that in the manufacturer-Stackelberg game and a little higher than that in the retailer-Stackelberg game. However, in the dual-channel CLSC, the wholesale price in the indirect sales channel is influenced by the direct channel. Except for the manufacturer-Stackelberg game, the wholesale prices in the other two channel power structures present different relationships with $\theta$.

(2) The retail price decreases when $\theta$ increases in all three models, and $P_{MN}^R > P_{MS}^M > P_{NS}^N$ always holds.
Table 2: Numerical simulation results for each model with changing $\theta$.

<table>
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<tr>
<th>Model</th>
<th>$\theta$</th>
<th>$w^*$</th>
<th>$P_i^*$</th>
<th>$P_d^*$</th>
<th>$P_m^*$</th>
<th>$\Gamma_m^*$</th>
<th>$\Gamma_r^*$</th>
<th>$\Gamma^*$</th>
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<td></td>
<td>0.1</td>
<td>45.224</td>
<td>72.943</td>
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<td>5.000</td>
<td>7.500</td>
<td>877.771</td>
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<td>67.403</td>
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<td>5.000</td>
<td>7.500</td>
<td>817.433</td>
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<td>7.500</td>
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<td>413.292</td>
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<td>56.306</td>
<td>39.497</td>
<td>5.000</td>
<td>7.500</td>
<td>843.868</td>
<td>293.215</td>
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<td>50.746</td>
<td>43.335</td>
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<td>195.414</td>
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<td>6.667</td>
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<td>409.331</td>
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<td>31.763</td>
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Note: $\Gamma^* = \Gamma_m^* + \Gamma_r^*$ in the chart denotes the total expected profit of the entire CLSC system.

Figure 9: The influence of changing $\theta$ on retailer’s expected profit.

Figure 10: The influence of changing $\theta$ on total expected profit of the supply chain.

Figure 6 implies that the direct and indirect channels have substitution effect in any channel power structure. Customer acceptance of the indirect channel decreases when $\theta$ increases. Considering its own interest, the retailer has to lower the retail price to make more profits. From this perspective, the existence of the online direct channel brings benefits to customers, and customers can get their desired products at a relatively low price through any channel.

The ordinal relationship of the wholesale price in the three different channel power structures is determined by the two parties’ positions in the game. In the retailer-Stackelberg game, the retailer can not only take advantage over its control of the market to force the manufacturer to lower the wholesale price, but also force customers to accept a higher retail price. Hence, it is logical that the retail price in the retailer-Stackelberg game is the highest. In the manufacturer-Stackelberg game, the wholesale price is the highest, which can lead to a higher retail price. In the Nash game between the manufacturer and retailer, neither party plays a dominant role, leading to a lower retail price compared to the other.
two cases. Thus, the no-leader market competition is more favorable for customers.

(3) The direct sale price increases when $\theta$ increases in all three models, and $P_d^R > P_d^N > P_d^M$ always holds.

From Figure 7, we find that there is positive correlation between the direct sale price and customer acceptance of the direct channel $\theta$. With $\theta$ increasing, customers are more willing to buy products through the online channel, and normally the increase in demand leads to an increase in the sales price.

The direct sale price in the three different channel power structures is also influenced by the control power of both parties. In the retailer-Stackelberg game, the wholesale price is relatively low, and the manufacturer increases the direct sale price to make more profits. In the manufacturer-Stackelberg game, since the wholesale price is the highest, the manufacturer makes more profits from the indirect channel and lowers the direct sale price to increase the sales through the direct channel. It is logical that the direct sale price in this case is the lowest. The data in Table 2 shows that there is little difference between the direct sale price in the Nash game and the direct sale price in the manufacturer-Stackelberg game. Thus, the price curves in the two cases are essentially coincident in Figure 7.

(4) The expected profit of the manufacturer is dependent on $\theta$ in all three game models, namely, $\Gamma_m^M > \Gamma_m^N > \Gamma_m^R$ when $0.1 \leq \theta < 0.220$, $\Gamma_m^M > \Gamma_m^R > \Gamma_m^N$ when $0.220 \leq \theta < 0.282$, and $\Gamma_m^R > \Gamma_m^M > \Gamma_m^N$ when $0.282 \leq \theta \leq 0.6$.

Figure 8 implies that the expected profit of the manufacturer in the three different game models shows different ordinal relationships with $\theta$. In the Nash game between the manufacturer and retailer and in the manufacturer-Stackelberg game, the expected profit of the manufacturer shows both a decreasing and increasing trend when $\theta$ increases. In the retailer-Stackelberg game, the expected profit of the manufacturer increases when $\theta$ increases. This can be explained as the retailer price is relatively high in the retailer-Stackelberg game, and more customers turn to buy products through the direct channel, which increases the profit of the direct channel and makes up for the manufacturer's loss in the indirect channel. Thus, adding the direct channel is favorable for the manufacturer. Figure 8 also implies that when customer acceptance of the direct channel $\theta$ is low ($\theta < 0.282$), the manufacturer has a direct incentive to become a leader.

Of course, the introduction of the direct channel unavoidably leads to the conflict with the indirect channel. To balance the relationship between the two channels, there should be a good coordination mechanism between the manufacturer and retailer. Due to limited space, this paper does not discuss coordination issues of the two channels in further detail.

(5) The retailer's expected profits decrease when $\theta$ increases in all three models, and $\Gamma_r^R > \Gamma_r^N > \Gamma_r^M$ always holds.

Therefore, customer acceptance of the direct channel directly influences the retailer's expected profits. Sales in the indirect channel decrease when $\theta$ increase, leading to the decrease in the retailer's expected profits.

Besides, in Figure 9, we can clearly see that the retailer makes the largest expected profit in the R Model followed by the N Model and then the M Model, which is mainly due to the two parties' position in the game. In the retailer-Stackelberg game, when $\theta$ is constant, the retail price is the highest, the wholesale price is relatively low, and also the retailer's profits from recycled used products are maximized (see Table 2). The joint effect of the above factors causes the retailer's expected profit to be maximized in the retailer-Stackelberg game. In the manufacturer-Stackelberg game, when $\theta$ is constant, the wholesale price is the highest, the retail price is relatively low, and also the retailer's profits from recycled used products are minimized (see Table 2). The joint effect of the above factors causes the retailer's expected profits to be minimized in the manufacturer-Stackelberg game. Hence, the retailer has a direct incentive to become a leader.

(6) The total expected profit of the dual-channel CLSC shows first decreasing and then increasing trend when $\theta$ increases in all three models, and $\Gamma^R > \Gamma^N > \Gamma^M$ always holds.

The total expected profit shown in Figure 10 is the superposition of the results of Figures 8 and 9. From Figure 10, we find that the total expected profit can be maximized in the retailer-Stackelberg game with constant $\theta$. Thus, for the entire CLSC, the channel power structure in which the retailer is the leader is optimal. In this channel power structure, the whole system can realize maximum profits.

From Table 2, we can easily come to the following conclusions.

(7) The optimal transfer price $P_m^*$ and the retailer's optimal acquisition price of used product $P_r^*$ are not influenced by $\theta$.

This can be explained as the reverse channel is a relatively independent channel compared to the two forward channels. The optimal acquisition price of used product $P_r^*$ and the optimal transfer price $P_m^*$ are only influenced by the two parties' positions in the competition and not influenced by $\theta$.

(8) One has $P_m^r > P_m^N > P_m^M$ and $P_r^N > P_r^M = P_r^R$.

This indicates that the optimal transfer price is the highest in the retailer-Stackelberg game and is the lowest in the manufacturer-Stackelberg game. The retailer's optimal acquisition prices are equal in the two Stackelberg cases and higher in the Nash case. This can be explained as follows. (a) In the retailer-Stackelberg game, the retailer can take advantage of its control over the channel to force the manufacturer to increase the transfer price. In the manufacturer-Stackelberg game, the manufacturer can take advantage of its control over the channel to force the retailer to accept a lower transfer price. In the Nash game between the manufacturer and retailer, since no one dominates the market, the game result makes the transfer price higher than that in the manufacturer-Stackelberg game and lower than that in the retailer-Stackelberg game. (b) In recycling of used products, the retailer's acquisition price of used product is influenced not only by the manufacturer's transfer price, but also by its position in the game. In the retailer-Stackelberg game, the retailer, on the one hand, forces the manufacturer to raise the transfer price and, on the other hand, forces customers
to accept a lower acquisition price of used product. In the manufacturer-Stackelberg game, the manufacturer forces the retailer to accept a lower transfer price. To get more profits from used products recycling, the retailer raises the acquisition price of used products instead. Thus, we have $P_{r}^{M} > P_{r}^{R}$. In the Nash game between the manufacturer and retailer, the game result makes the retailer’s acquisition price of used product higher than that in the other two cases. From $Q_{0} = b p_{r}^{k}$, we find that the amount of recycle in this case is the largest. Therefore, the no-leader market structure can be regarded as an ideal market structure from the perspective of resource recycling and environmental protection.

7. Conclusion

Within the framework of the game theory, this paper establishes three related models based on the Nash game between the manufacturer and retailer, the manufacturer-Stackelberg game, and the retailer-Stackelberg game. We mainly analyze pricing decision issues of the dual-channel CLSC under uncertain demand of the indirect channel. A numerical study is carried out to illustrate the three proposed models.

The results reveal that under uncertain demand of the indirect channel (1) the wholesale price, retail price, and manufacturer’s expected profits decrease when $\theta$ increases in all three models. (2) The direct sale price increases with $\theta$ in all three models, where $P_{d}^{R} > P_{d}^{N} > P_{d}^{M}$. In model $R$, the manufacturer’s expected profit also increases with $\theta$. (3) In model $N$ and model $M$, the manufacturer’s expected profit and the total expected profit of the CLSC show first a decreasing and then increasing trend when $\theta$ increases. (4) The optimal transfer price $P_{m}^{*}$ and the optimal acquisition price of used product $P_{r}^{*}$ are not influenced by $\theta$; moreover, $P_{m}^{R} > P_{m}^{N} > P_{m}^{M}$, and $P_{r}^{N} > P_{r}^{M} = P_{r}^{R}$.

The limitations of this study lie in the following aspects. First, the paper only discusses pricing decision issues of the dual-channel CLSC under uncertain demand of the indirect sales channel and does not study the coordination of the members of the supply chain. Second, this study is based on the symmetrical information condition of the two players, but actually asymmetry of information between the parties is more common in reality, and pricing decision issues of the CLSC in this case are worth studying as well. Third, in this paper, we only consider a dual-channel CLSC system consisting of a manufacturer and a retailer. Future research can relax this assumption; for example, it is an interesting research direction to extend our model to consider a CLSC system consisting of a manufacturer and two competitive retailers. We can assume that two retailers competitively collect used products from the market and investigate the effects of the two retailers different competitive behaviors—Collusion, Bertrand, and Stackelberg—on the optimal decisions of the manufacturer and two retailers.

Appendix

In the retailer-Stackelberg game model, the proof of the concavity of (4) with respect to $P_{s}, P_{r}$, and $z$ is as follows.

Solving (12)–(14) simultaneously, we can derive that

$$
P_{m} = \Delta - \frac{P_{r}}{k}, $$

$$
1 – \frac{z}{1 - a^{2}} - \frac{\theta - 1 - a \theta}{1 - a^{2}} \phi, \tag{A.1}
$$

$$
P_{d} = \frac{a + \theta - a \theta}{2 (1 - a^{2})} + \frac{a z}{2 (1 - a^{2})} + \frac{C_{m}}{2}.
$$

Substituting $P_{m}, w, \phi$, and $P_{d}$ into (4) and taking the second-order partial derivatives of (4) with respect to $P_{s}, P_{r}$, and $z$, we have the Hessian matrix

$$
H = \begin{bmatrix}
-4 & 0 & 1 - F(z) \\
0 & P_{r}^{k-2} \left[ \Delta b k (k - 1) - (b + k)(1 + k) \right] & 0 \\
1 - F(z) & 0 & (S + C_{s} - P_{s} - C_{b}) f(z) - \frac{2}{1 - a^{2}}
\end{bmatrix} \tag{A.2}
$$

Since $\frac{\partial^{2} \Gamma}{\partial P_{s}^{2}} = -4 < 0$,
The Hessian matrix will be negative definite if \( \Delta b_k(k-1)-(b+k)(1+k) < 0 \) and \( 4(S+C_s-P_s-C_b)f(z) + [1-F(z)]^2 < 8/(1-a^2) \). Hence, (4) is strictly jointly concave in \( P_r, P_s, \) and \( z \), if \( \Delta b_k(k-1)-(b+k)(1+k) < 0 \) and \( 4(S+C_s-P_s-C_b)f(z) + [1-F(z)]^2 < 8/(1-a^2) \). Similarly, in the manufacturer-Stackelberg game model, we can easily prove that (5) is strictly jointly concave in \( w, P_r, \) and \( P_m \).

**Competing Interests**

The authors declare no conflict of interests.

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**References**


