Research Article

Supply Chain Coordination with Variable Backorder, Inspections, and Discount Policy for Fixed Lifetime Products

Biswajit Sarkar

Department of Industrial & Management Engineering, Hanyang University, Ansan, Gyeonggi-do 426 791, Republic of Korea

Correspondence should be addressed to Biswajit Sarkar; bsbiswajitsarkar@gmail.com

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This paper illustrates a channel coordination and quantity discounts between a vendor and a buyer with single-setup multi-delivery (SSMD) strategy to reduce the joint total cost among supply chain players. The benefit of the coordination between a buyer and a vendor is considered as the vendor requests to the buyer for changing the ordering quantity such that the vendor can be benefited from lower inventory costs. After accepting the buyer’s condition, the vendor compensates the buyer for his increased inventory cost and gives consent for additional savings by offering a quantity discount. The centralized decision making is examined for the effect of this strategy with the presence of backorder for buyer and inspection cost for the vendor. The quantity discount strategy, with the presence of variable backorder and inspections, can allow more savings for all players of supply chain. Some numerical examples, sensitivity analysis, and graphical representations are given to illustrate more savings from existing literature and comparisons between the several demand values.

1. Introduction

Nowadays, most of the industries can no longer compete solely as individual entities in this changing marketing environment. SCM (supply chain management) contains combination of buyer and suppliers, distributors and retailers, manufacturers and distributors, or many different forms of customers. Integrated supply chain management has been recognized as a core competitive strategy as the production industries provide products and services to their customers more cheaper and faster than their competitors. The main goal of SCM is to optimize the whole system cost by maintaining the cooperation between two different entities, because it lowers the joint total expected cost of the system and increases their revenues which are from the competitive advantages of SCM. Therefore, in modern marketing system, the cooperation between buyer and vendor plays an important role in an integrated vendor-buyer supply chain model. Recently, Cárdenas-Barrón and Sana [1] developed an interesting problem of channel coordination on single-supplier single-manufacturer model with backorder for buyer. Cárdenas-Barrón and Porter [2] discussed a better and simple algorithm for a supply chain model with preprocessing raw materials. Wee et al. [3] developed an integrated inventory model with a different approach of optimization rather than the classical one. Lin [4] discussed a channel coordination policy for two-stage supply chain considering quantity discounts and imperfect production. He used a solution procedure, free from algorithm based on convex optimization, to obtain the optimum solution. Ryu et al. [5] derived an excellent fractal echelon approach to maintain inventory in a supply chain network. They introduced fractal-based vendor management inventory where some responsibilities for the buyers are taken by the vendor. Wang et al. [6] developed the decentralized and centralized supply chain problem considering finite production rate for the supplier regarding pricing and lot sizing decision. They used all units discount policy and a franchise fee for the coordinated supply chain. Zissis et al. [7] derived a rational pair manufacturer and retailer where there is some information asymmetry which disturbs the coordination policy. The quantity discount is offered by the manufacturer to the retailer. They considered analytical derivation for the model. Saha and Goyal [8] developed supply chain coordination contracts with inventory level and retail price-dependent demand.
Goyal [9] was the first author who initiated the joint optimization concept for vendor and buyer. Banerjee [10] extended this concept for the vendor with a finite production rate to order for a buyer on a lot-for-lot basis where the operating cost of the manufacturer and buyer is considered. He assumed that the shipment can only be made when the production has been completed. Goyal [11] relaxed Banerjee’s [10] lot-for-lot assumption and extended vendor’s economic production quantity as an integer multiple of buyer’s purchasing quantity by assuming the SSMD policy. He numerically proved that the SSMD policy is better than the SSSD (single-setup single-delivery) policy. Luo [12] extended the buyer-vendor coordination model with credit period incentives. Duan et al. [13] extended Luo’s [12] model with quality discount incentive for fixed lifetime products. Cárdenas-Barrón [14] derived the explanation of different inventory models with two types of backorders costs using analytic geometry and algebra.

When products are transported to buyer from vendor, it is very important to inspect all product’s quality. Many researchers considered integrated-inventory models, but they did not assume inspection policies for imperfect production. Duffuaa and Khan [15] explained an optimal repeat inspection policy with several classifications. Wei and Chung [16] wrote a note on the economic lot size of the integrated vendor-buyer inventory system derived without derivatives by considering inspection costs for the vendor. Hsieh and Liu [17] assumed an investment for quality improvement and they considered inspection strategies for imperfect items in a single-supplier single-manufacturer model. They proposed four noncooperative game models with several degrees of information available about the player’s inspection sampling rates as well as an investment for quality improvement. Khan et al. [18] developed an inventory model with imperfect quality items and inspection errors as Type I and Type II errors. J.-T. Hsu and L.-F. Hsu [19] discussed an inventory model with imperfect quality items, inspection errors, backorders, and sales returns. Aust et al. [20] wrote a note on Hsieh and Liu’s [17] model with higher profit.

In modern marketing system, a practical problem is deterioration or perishability of raw materials or finished products. Many products have limited lifetime, after which it may deteriorate or it cannot be used. Therefore, the management system at each stage in supply chain from production to consumers can lead to high system costs. Liu and Shi [21] classified product’s perishability and deterioration into two categories as decay model and finite lifetime model. Moreover, products with finite lifetime model can be separated into common or fixed lifetime models and random finite lifetime models. Basically, items like fresh products, canned fruits, and medicines with common finite lifetime are considered in the inventory literature as perishable items (Liu and Lian [22]) if they are not used within their lifetimes. The random finite lifetime can be treated as random variable which follows some probability distributions such as Erlang or exponential. Liu and Lian [22], Sana [23], and Sarkar [24] discussed fixed lifetime of products for single-stage inventory systems. Sarkar [25] explained different probabilistic deterioration for the single-vendor single-buyer model with algebraic method. Priyan and Uthayakumar [26] described trade-credit financing in a single-vendor single-buyer integrated model with cost reduction policy. They considered backorder price-discount policy when receiving quantity is uncertain. Pahl and Voß [27] surveyed about deteriorating and lifetime constraints in production and supply chain planning.

This study considers a supply chain coordination model with backorder for the buyer, inspection cost for the vendor, and a coordination strategy between them. The vendor requests the buyer to alter his ordering quantity for reducing inventory costs of the vendor and as a policy, the vendor must compensate the buyer for his increased inventory costs and allow some additional savings by offering a quantity discount as incentives. The vendor considers three different types of inspection costs (see, e.g., Wee and Chung [16]) to ensure the quality for their products. The system cannot consider any defective item during production and if it arises, it is immediately discarded from the system. The model considered only that type of product which has fixed lifetime (see, e.g., Sana [23]). This model considers a variable backorder rate for the buyer. The centralized decision-making model with effect of coordination between vendor and buyer is considered under the presence of buyer’s backorder cost and vendor’s inspection cost for fixed lifetime products. See Table 1 for the contribution of different authors.

The paper is organized as follows. Notation and assumptions are given in Section 2. The mathematical model is formulated in Section 3. Some numerical examples are given to illustrate the model in Section 4. Finally, we discuss the concluding remarks in Section 5.

2. Notation and Assumptions

The model uses the following notation to develop this model.

**Decision variables** are as follows:

- \( Q \): buyer’s economic order quantity (units),
- \( Z \): vendor’s order multiple in absence of any coordination,
- \( Y \): vendor’s order multiple under coordination,
- \( k \): backorder rate,
- \( \lambda \): buyer’s order multiple under coordination and \( \lambda Q \) as the buyer’s new order quantity.

**Parameters** are as follows:

- \( D \): demand per year (units/year),
- \( A \): buyer’s ordering cost per order ($/order),
- \( h_b \): buyer’s holding cost per unit per year ($/unit/year),
- \( p_b \): buyer’s unit backorder cost ($/unit backorder item),
- \( B(\lambda) \): discount per unit dollar to the buyer ($/unit dollar) by the vendor if the buyer places the order \( \lambda Q \) each time,
- \( R \): lifetime of products (year),
Table 1: Comparison between contributions of different authors.

<table>
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<tr>
<th>Author(s)</th>
<th>Quantity discount</th>
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$P$: production rate per year ($P > D$) (units/year),

$h_v$: vendor’s holding cost per unit per year ($/unit/ year),

$S$: vendor’s setup cost per setup ($/setup),

$C_{iv}$: vendor’s variable inspection cost per delivery ($$/delivery),

$C_{ivp}$: vendor’s unit inspection cost ($$/unit item inspected),

$C_{ixf}$: vendor’s fixed inspection cost per production lot ($$/production lot).

The following assumptions are considered to develop this model.

1. The supply chain coordination in single-vendor and single-buyer model is considered for a single type of item. The model assumes that when buyer orders products from vendor, he requests buyer to change the order quantity by a fraction of present order quantity and then the vendor offers a quantity discount if the buyer accepts its request to change the order quantity.

2. Production and demand rate are constant. The backorder rate for buyer is assumed as a decision variable.

3. Before the vendor’s shipment, the final inspection is implemented. There are three steps of inspections such that there will be no defective product and if it arises, it will be discarded from the system.

4. The numbers of vendor’s order multiple with and without coordination are integer numbers.

5. The vendor and the buyer have complete knowledge of information of each other. There is no asymmetry of information.

6. The system does not consider wait-in-process items and defective items.

7. The replenishment is instantaneous and the lead time is negligible.

3. Mathematical Model

In absence of any coordination, the model considers a single-setup-multi-delivery (SSMD) policy (see, e.g., Goyal [11]) where buyer’s order quantity is produced at single stage, but it is delivered to the vendor by equal shipment in multiple deliveries after a fixed time interval. Each delivery is designed in just-in-time (JIT) way which indicates that each delivery
arrives at the time when all items from the previous delivery have just been depleted. The shortages are considered for the buyer. During shortages, the backorder rate is $k$. The inspection of items is completed before sending to buyer. This model does not consider the production of the defective items during the production; if this arises, it is discarded from the system. Figures 1 and 2 represent the buyer’s and the vendor’ inventory versus time.

3.1. Buyer Cost in Presence of Backorder. Consider the following.

Ordering cost = $AD/Q$.
Holding cost = $h_b(1-k)^2Q/2$.
Backorder cost = $k^2Q\pi_b/2$.

Therefore, the buyer’s cost function is

$$TC_b = \frac{AD}{Q} + \frac{h_b(1-k)^2Q}{2} + \frac{k^2Q\pi_b}{2}$$

Thus, the minimum cost with respect to $k$ is

$$TC_b(Q) = \frac{AD}{Q} + \frac{h_bQ\pi_b}{2(\pi_b + h_b)}.$$  \hspace{1cm} (3)

An algebraic procedure (see, e.g., Sarkar [25]) has been employed to obtain the minimum cost with respect to the ordering quantity and the backorder:

$$TC_b = \sqrt{\frac{2ADh_b\pi_b}{\pi_b + h_b}},$$ \hspace{1cm} (4)

where the optimum ordering quantity is given by

$$Q = \sqrt{\frac{2AD(\pi_b + h_b)}{h_b\pi_b}}.$$ \hspace{1cm} (5)

and the fixed interval is given by

$$t = \frac{Q}{D} = \sqrt{\frac{2A(\pi_b + h_b)}{h_b\pi_b D}}.$$ \hspace{1cm} (6)

3.2. Vendor’s Cost in Presence of Inspection for Defective Items. Consider the following.

Setup cost = $DS/zQ$.
Holding cost = $(h_v/Q)[(z-1)(1-D/P) + D/P]$.
Inspection cost = $DC_{io}/Q + DC_{iv} + (D/zQ)C_{ivf}$.

Therefore, the vendor’s cost in presence of inspection cost is

$$TC_v(z) = \frac{DS}{zQ} + \frac{h_vQ}{2} \left[ (z-1)\left(1 - \frac{D}{P}\right) + \frac{D}{P} \right]$$

$$+ \frac{D}{zQ}\left(zC_{io} + zQ C_{iv} + C_{ivf}\right).$$ \hspace{1cm} (7)

Now, the cost is simplified as

$$TC_v(z) = \frac{DS}{zQ} + \frac{h_vQ}{2} \left[ (z-1)\left(1 - \frac{D}{P}\right) + \frac{D}{P} \right]$$

$$+ \frac{DC_{io}}{Q} + DC_{iv} + \frac{D}{zQ} C_{ivf}$$

$$= \frac{DS + DC_{ivf}}{zQ} + \frac{h_vQ(1-D/P)}{2}$$

$$+ \frac{h_vQ}{2} \left( \frac{D}{P} - 1 \right) + \frac{DC_{io}}{Q} + DC_{iv}$$

$$= a_1z + a_2 \frac{z}{z} + a_3.$$ \hspace{1cm} (8)

(see Appendix A for the values of $a_1$, $a_2$, and $a_3$).

Therefore, without coordination vendor’s cost can be written as

$$\text{Min } TC_v(z) = a_1z + a_2 \frac{z}{z} + a_3$$

subject to $zt \leq R$

$$z \geq 1.$$ \hspace{1cm} (9)
The optimum value of \( z \) is given by
\[
z = \sqrt{\frac{a_2}{a_1}} = \sqrt{\frac{h_b \pi_b (S + C_{ivf})}{A h_v (\pi_b + h_b) (1 - D/P)}}. \tag{10}
\]

If \( z_1 \) is the optimum value of \( z \), then it is found that
\[
z_1 = \max \left\{ \left[ \frac{h_b \pi_b (S + C_{ivf})}{A h_v (\pi_b + h_b) (1 - D/P)} \right], 1 \right\} \geq 1. \tag{11}
\]

The optimal cost for the vendor with respect to \( z \) is
\[
TC_v = 2 \sqrt{a_1 a_2} + a_3
\]
\[
= 2D h_v \left( 1 - \frac{D}{P} \right) (S + C_{ivf}) + h_b \left( \frac{2D}{P} - 1 \right) \sqrt{AD (\pi_b + h_b)} + C_{io} \sqrt{\frac{h_b \pi_b D}{2A (\pi_b + h_b)}} + DC_{ivf}.
\]

Substituting the value of the interval \( t \), one can obtain
\[
z t \leq R \Rightarrow z = \sqrt{\frac{2A (\pi_b + h_b)}{h_b \pi_b D}} \leq R \Rightarrow z \leq \frac{R}{\sqrt{\phi}}.
\]
where \( \phi = \frac{2A (\pi_b + h_b)}{h_b \pi_b D} \).

It is assumed that \( z_2 = \lceil R/\sqrt{\phi} \rceil \). Now \( R^2 \geq \phi \) implies \( z_2 \geq 1 \). As \( TC_v \) is convex and if \( z_1 \leq z_2 \), then \( z = z_1 \); else \( z = z_2 \).

Therefore, if \( R^2 \geq \phi \), \( z^* = \min \{z_1, z_2\} \). Thus, if \( R^2 \geq \phi \), then
\[
z^* = \min \left[ \frac{h_b \pi_b (S + C_{ivf})}{A h_v (\pi_b + h_b) (1 - D/P)} \right] \left( \frac{R}{\sqrt{\phi}} \right). \tag{14}
\]

where \( \lceil x \rceil \) is the least integer greater than or equal to \( x \).


The paper discusses two-part coordination contract in a decentralized decision making. For the coordination system, there is a strategy that the vendor requests the buyer to change his current order size by a factor \( \lambda \) (>0) and offers the buyer a quantity discount by a discount factor \( B(\lambda) \) and the buyer accepts the offer. Thus, the vendor's and buyer's new order quantities are \( yQ \lambda \) and \( Q \).

The vendor's total cost is as follows:
\[
\overline{TC}_v(y, \lambda) = \frac{DS}{y \lambda Q} + h_b \frac{\lambda Q}{2} \left[ (y - 1) \left( 1 - \frac{D}{P} \right) + \frac{D}{P} \right]
\]
\[
+ \frac{D}{y \lambda Q} \left( y C_{iv} + y \lambda Q C_{ivf} + C_{ivf} \right)
\]
\[
+ DB(\lambda).
\]

The problem with coordination can be formulated as
\[
\begin{align*}
\text{Min} & \quad \overline{TC}_v(y, \lambda) \\
& \quad = \frac{DS}{y \lambda Q} + h_b \frac{\lambda Q}{2} \left[ (y - 1) \left( 1 - \frac{D}{P} \right) + \frac{D}{P} \right] \\
& \quad + \frac{D}{y \lambda Q} \left( y C_{iv} + y \lambda Q C_{ivf} + C_{ivf} \right) \\
& \quad + DB(\lambda) \tag{16}
\end{align*}
\]

subject to \( y \lambda t \leq R \)
\[
AD + \frac{h_b (1 - k)^2 \lambda Q}{2} + \frac{k^2 \lambda Q \pi_b}{2} - 2AD h_b \frac{\pi_b}{\pi_b + h_b} \leq DB(\lambda) \]
\[
y \geq 1.
\]

In the above constraint, the term \( DB(\lambda) \), in the right side of (16), represents the compensation of the vendor to the buyer, the constraints represent the items, which are not overdue before they are used by the buyer, and the buyer's cost under coordination cannot exceed that in absence of any coordination.

**Theorem 1.** The optimum cost under coordination is always less than or equal to the optimum cost without coordination; that is,
\[
\overline{TC}_v(y^*, \lambda^*) \leq TC_v(z^*). \tag{17}
\]

**Proof.** From the second constraint of (16), the term \( DB(\lambda) \) gives the smallest value if the second constraint forms an equation which indicates the minimization of vendor's cost \( \overline{TC}_v(y) \); that is,
\[
\begin{align*}
& \quad = \frac{AD}{rQ} + \frac{h_b (1 - k)^2 \lambda Q}{2} + \frac{k^2 \lambda Q \pi_b}{2} - 2AD h_b \frac{\pi_b}{\pi_b + h_b} \tag{18} \\
& \quad = DB(\lambda)
\end{align*}
\]

which gives
\[
B(\lambda) = \frac{1}{D} \left[ \frac{AD}{\lambda Q} + \frac{h_b (1 - k)^2 rQ}{2} + \frac{k^2 rQ \pi_b}{2} \right] - \sqrt{\frac{2AD h_b \pi_b}{\pi_b + h_b}}. \tag{19}
\]
If \( \lambda = 1 \) (i.e., after the request of vendor, buyer's order size is \( Q \) and that of vendor's lot size is \( yQ \)), then the discount factor is

\[
B(1) = \frac{1}{C_{i}D} \left[ \frac{AD}{Q} + \frac{h_{b}(1-k)^{2}Q}{2} + \frac{k^{2}Q\pi_{b}}{2} \right]
\]

(20)

\[
- \frac{2ADh_{b}\pi_{b}}{\pi_{b} + h_{b}}.
\]

We substitute the value of optimum \( k \) and \( Q \) from the noncoordination Section 3.1, and then

\[
B(1) = \frac{1}{D} \left[ \frac{AD}{\sqrt{2AD(\pi_{b} + h_{b})/h_{b}\pi_{b}}} + \frac{h_{b}\pi_{b}}{2(\pi_{b} + h_{b})} \right]
\]

(21)

\[
+ \frac{2AD(\pi_{b} + h_{b})}{h_{b}\pi_{b}} - \frac{2AD\pi_{b}h_{b}}{\pi_{b} + h_{b}} \right] .
\]

Simplifying this expression, one has

\[
B(1) = \frac{1}{D} \left[ - \frac{2ADh_{b}\pi_{b}}{\pi_{b} + h_{b}} + \frac{2ADh_{b}\pi_{b}}{\pi_{b} + h_{b}} \right] = 0.
\]

(22)

Therefore, if \( \lambda = 1 \), (16) is reduced to (9) which implies (9) is a special case of (16). Hence the inequality in (17) holds. This completes the proof. \( \square \)

By the theorem, it can be easily seen that after receiving offer from the vendor, the buyer and the vendor both are gainer. Therefore, they can follow the coordination policy for a long time for their own profit.

As the vendor offers a fraction of lot size, thus, we can use the value of \( B(\lambda) \) in (19). Thus, the cost in (15) is modified as

\[
\overline{\text{TC}}_{v}(y, \lambda) = \frac{DS}{y\lambda Q} + \frac{h_{b}\lambda Q}{2} \left[ (y-1) \left( 1 - \frac{D}{P} \right) + \frac{D}{P} \right]
\]

\[
+ \frac{D}{y\lambda Q} \left( yC_{iw} + y\lambda QC_{inv} + C_{ivf} \right)
\]

\[
+ \frac{AD}{\lambda Q} + \frac{h_{b}(1-k)^{2}\lambda Q}{2} + \frac{k^{2}\lambda Q\pi_{b}}{2}
\]

\[
- \frac{2ADh_{b}\pi_{b}}{\pi_{b} + h_{b}}.
\]

(23)

The cost equation can be simplified as

\[
\overline{\text{TC}}_{v}(y, \lambda) = \lambda \left[ \frac{h_{b}Q}{2} \left( (y-1) \left( 1 - \frac{D}{P} \right) + \frac{D}{P} \right) \right]
\]

\[
+ \frac{h_{b}(1-k)^{2}Q}{2} + \frac{k^{2}Q\pi_{b}}{2} \right] + \frac{1}{\lambda} \left[ \frac{DS}{yQ} + \frac{AD}{Q} \right]
\]

\[
+ DC_{iw} \cdot \frac{DC_{ivf}}{yQ} \right] + DC_{ivf} - \sqrt{2ADh_{b}\pi_{b}}\pi_{b} + h_{b} = a_{4}\lambda
\]

\[
+ \frac{a_{4}}{\lambda} + a_{6}
\]

(24)

(see Appendix B for the values of \( a_{4}, a_{5}, \) and \( a_{6} \)).

With the help of similar algebraical procedure, it is found that

\[
\lambda = \frac{a_{5}}{a_{4}} = \frac{1}{Q}
\]

(25)

\[
\left( \frac{2[D(S + C_{ivf})/y + D(A + C_{iw})]}{h_{b}[(y-1)(1 - D/P) + D/P] + h_{b}(1-k) + k^{2}\pi_{b}} \right).
\]

Substitute (25) and \( t \) into \( y\lambda t \leq R \), which gives

\[
y^{2} \left[ \frac{S + C_{ivf}}{y} + (A + C_{iw}) \right]
\]

\[
\leq R^{2} \left[ \left( \frac{1 - D}{P} \right) y - 1 + \frac{2D}{P} \right]
\]

\[
+ \frac{h_{b}D}{2} \left\{ \left( \frac{1 - D}{P} \right) y - 1 + \frac{2D}{P} \right\}
\]

\[
+ \frac{h_{b}\pi_{b}D}{2(\pi_{b} + h_{b})} \right].
\]

(26)

Suppose \( \psi(y) = R^{2} \left[ \frac{h_{b}D}{2} \left\{ \left( \frac{1 - D}{P} \right) y - 1 + \frac{2D}{P} \right\}
\]

\[
+ \frac{h_{b}\pi_{b}D}{2(\pi_{b} + h_{b})} \right] - y^{2} \left[ \frac{S + C_{ivf}}{y} + (A + C_{iw}) \right]
\]

\[
= -y^{2} (A + C_{iw}) + y \left[ R^{2}h_{b}D \left( \frac{1 - D}{P} \right) \right]
\]

\[
- (S + C_{ivf}) \right] + R^{2} \left[ \frac{h_{b}D}{2} \left( \frac{2D}{P} - 1 \right)
\]

\[
+ \frac{h_{b}\pi_{b}D}{2(\pi_{b} + h_{b})} \right].
\]

Therefore, the first constraint of (16) is of the form \( \psi(y) \geq 0 \).

Now substituting the value of \( \lambda \) and \( t \) into \( \overline{\text{TC}}_{v}(y) \),

\[
\overline{\text{TC}}_{v}(y) = 2\sqrt{\overline{\text{TC}}_{v}} + a_{6}
\]

subject to \( \psi(y) \geq 0 \) \( \text{for} \quad y \geq 1. \)

As \( a_{6} \) is constant with respect to \( y \) and square root of a function \( Q \) is strictly increasing if \( Q \geq 0 \), therefore, \( \overline{\text{TC}}_{v}(y) \) from (27) can be written as

\[
\overline{\text{TC}}_{v}(y) = \left[ h_{b} \left\{ (y-1) \left( 1 - \frac{D}{P} \right) + \frac{D}{P} \right\} + \frac{h_{b}\pi_{b}}{\pi_{b} + h_{b}} \right]
\]

\[
\left[ \frac{DS + DC_{ivf}}{yQ} + AD + DC_{iw} \right]
\]
\[
= y \left[ h_v \left( 1 - \frac{D}{P} \right) (A + C_{io}) \right] + \frac{h_v (2D/P - 1) + h_v \pi_b/(\pi_b + h_b)}{h_v \left( 1 - \frac{D}{P} \right) (A + C_{io}) + \frac{h_v \pi_b D (A + C_{io})}{(\pi_b + h_b)}.
\]

Thus, (27) can be written as
\[
\overline{TC}_v(y) = \alpha y + \frac{a_0}{y} + \alpha_0
\]
subject to \( \psi(y) \geq 0 \)
\[
y \geq 1
\]
(see Appendix C for the values of \( \alpha_0, \alpha_0, \) and \( \alpha_0 \)).

It is known to all that (29) is a nonlinear program. We now discuss the property of (29) in order to minimize it. We know that \( \overline{TC}_v(y) \) is a convex function if
\[
h_v \left( \frac{2D}{P} - 1 \right) > \frac{h_v \pi_b}{\pi_b + h_b}
\]
(30)

By our assumption \( P > D \), thus the denominator term is positive but the numerator term is positive or negative.
Now the numerator part is positive if
\[
\Rightarrow h_v \left( \frac{2D}{P} - 1 \right) > \frac{h_v \pi_b}{\pi_b + h_b};
\]
that is, if \( h_v \left( \frac{2D}{P} - 1 \right) > \frac{h_v \pi_b}{\pi_b + h_b} \) holds,
then \( y_1^* \)
\[
= \sqrt{\frac{(S + C_{io}) [h_v (2D/P - 1) + h_v \pi_b/(\pi_b + h_b)]}{h_v (1 - D/P) (A + C_{io})}}.
\]
Otherwise \( y_1^* = 1 \). This completes the proof.

\[
\psi(y) = -(A + C_{io}) + y \left[ \frac{R^2 h_v D (1 - D/P) - (S + C_{io})}{2} \right] + R^2 \left[ \frac{h_v (2D/P - 1) + h_v \pi_b D}{2 (\pi_b + h_b)} \right]
\]
(35)

Now we discuss the properties of \( \psi(y) \)
\[
\psi(y) \geq 0 \quad \text{for} \quad 1 \leq y \leq y^{(1)}.
\]
(see Appendix D for the values of \( a_{00}, a_{01}, \) and \( a_{12} \)).

On the basis of \( y \geq 1 \), we obtain the following.

(1) If \( y^{(1)} < 1 \), then \( \psi(y) < 0 \) for all \( y \geq 1 \).
(2) If \( y^{(2)} > 1 \), then \( \psi(y) \geq 0 \) for \( \left[ y^{(2)} \right] \leq y \leq y^{(1)} \).
(3) If \( y^{(2)} < 1 \) and \( y^{(1)} > 1 \), then on the basis of \( y \) which is positive integer

\[
\psi(y) \geq 0 \quad \text{for} \quad 1 \leq y \leq y^{(1)}.
\]

Let us consider \( y^{(1)}_2 \) and \( y^{(2)}_2 \) as solutions of the equation \( \psi(y) = 0 \). Therefore based on this, the following illustrations can be found.
(i) If \( a_{11}^2 - 4a_{10}a_{12} < 0 \) or \( a_{11}^2 - 4a_{10}a_{12} \geq 0 \) and \( y_{2}^{(1)*} < 1 \), then \( \psi(y) < 0 \) for all \( y \geq 1 \).

(ii) If \( a_{11}^2 - 4a_{10}a_{12} \geq 0 \) and \( y_{2}^{(1)*} \geq 1 \), then

\[
\psi(y) \geq 0 \quad \text{for} \quad 1 \leq y \leq y_{2}^{(1)*}.
\]

Note 1. If (i) holds, the first constraint of (16) does not hold for any \( y \geq 1 \). In that case the problem is meaningless. If (ii) holds, the first constraint of (16) holds for \( \{y_{2}^{(1)*} \leq y \leq y_{2}^{(1)*} \} \).

Theorem 3. If \( h_{r}(2D/P - 1) > h_{y}\pi_{b}/(\pi_{b} + h_{b}) \) and \( y_{2}^{(1)*} \geq 1 \) hold, then the following conditions are obtained. (i) If \( 1 \leq y_{1}^{*} \leq y_{2}^{(1)*} \) holds, then \( y^{*} = y_{1}^{*} \).

(ii) If \( y_{1}^{*} > y_{2}^{(1)*} \) holds, then \( y^{*} = y_{2}^{(1)*} \).

Proof. From the 1st condition of the theorem, \( \overline{TC}(y) \) is convex function and by Theorem 2, \( y_{1}^{*} \) is the minimum value of the cost function \( \overline{TC}(y) \) for \( y \geq 1 \). Thus, if \( 1 \leq y^{*} \leq y_{2}^{(1)*} \) holds, then it is found that \( y^{*} = y_{1}^{*} \) and if \( y_{1}^{*} > y_{2}^{(1)*} \) holds, then it is obtained as \( y^{*} = y_{2}^{(1)*} \). It is to be noted that \( \overline{TC}(y) \) is decreasing on the interval; hence the second condition of the theorem holds easily.

This completes the proof.

Theorem 4. If \( h_{r}(2D/P - 1) > h_{y}\pi_{b}/(\pi_{b} + h_{b}) \) holds, then the buyer's order multiple is greater than 1; that is, \( \lambda^{*}(y^{*}) > 1 \).

Proof. From the optimal value of \( \lambda^{*} \), one can obtain

\[
\lambda^{*}(y^{*}) = \frac{1}{Q} \sqrt{\frac{2[D(S + C_{inv})/y + D(A + C_{io})]}{h_{r}(y - 1)(1 - D/P) + h_{b}(1 - k)^{2} + k^{2}\pi_{b}}}
\]

\[
= \frac{h_{y}\pi_{b}[(S + C_{inv})/y] + (A + C_{io})}{h_{r}(\pi_{b} + h_{b})(2D/P - 1) + h_{y}\pi_{b} + h_{r}(\pi_{b} + h_{b})} y(1 - D/P)
\]

(i) If \( y^{*} = y_{1}^{*} = 1 \), then

\[
\lambda^{*} = \frac{S + C_{inv} + A + C_{io}}{h_{r}D(\pi_{b} + h_{b})/Ph_{y}\pi_{b} + 1} > 1.
\]

\( \text{as} \quad (S + C_{inv} + A + C_{io}) > \left( \frac{h_{r}D(\pi_{b} + h_{b})}{Ph_{y}\pi_{b}} + 1 \right). \)

(ii) From (33), we obtain a simplified form of \( y_{1}^{*} \) as

\[
y_{1}^{*} = \sqrt{\frac{(S + C_{inv})}[h_{r}(2D/P - 1)(\pi_{b} + h_{b}) + h_{y}\pi_{b}]}{h_{r}(1 - D/P)(A + C_{io})(\pi_{b} + h_{b})}. \]

If \( y^{*} = y_{1}^{*} \) and \( h_{r}(2D/P - 1) > h_{y}\pi_{b}/(\pi_{b} + h_{b}) \), then

\[
y^{*} = y_{1}^{*} = \sqrt{\frac{(S + C_{inv})[h_{r}(2D/P - 1)(\pi_{b} + h_{b}) + h_{y}\pi_{b}]}{h_{r}(1 - D/P)(A + C_{io})(\pi_{b} + h_{b})}}.
\]

Now we show \( \lambda^{*}(y^{*}) \) to \( y_{1}^{*} > y_{2}^{(1)*} \).

The expression is as follows:

\[
\frac{h_{b}\pi_{b}[(S + C_{inv})/y + A + C_{io}]}{h_{r}(\pi_{b} + h_{b})(2D/P - 1) + h_{y}\pi_{b} + h_{r}(\pi_{b} + h_{b})} y(1 - D/P)
\]

\[
= \frac{h_{b}\pi_{b}[(A + C_{io})/x_{2}] + \sqrt{(A + C_{io})/x_{2} + \sqrt{x_{1}}}}{\sqrt{x_{2}[(A + C_{io})/x_{2} + \sqrt{x_{1}}]}},
\]

\[
= \frac{h_{b}\pi_{b}(A + C_{io})/x_{2}}{\sqrt{(A + C_{io})/x_{2} + \sqrt{x_{1}}}} > 1 \quad \text{if} \quad h_{b}\pi_{b}(A + C_{io}) > x_{2}
\]

(see Appendix E for the simplifications and values of \( x_{1} \) and \( x_{2} \)).

(iii) If \( y^{*} = y_{2}^{(1)*} \), then \( y_{1}^{*} > y_{2}^{(1)*} \). Now \( r^{*}(y^{*}) \mid_{y^{*}=y_{2}^{(1)*}} > 1 \) and thus, \( \lambda^{*}(y^{*}) \) is a decreasing function; hence, \( \lambda^{*}(y^{*}) \mid_{y^{*}=y_{2}^{(1)*}} > \lambda^{*}(y_{1}^{*}) > 1 \).

Therefore, from the above three conditions, we obtain \( r^{*} > 1 \). That completes the proof.

3.4. Mathematical Model for Centralized Decision. This section considers a centralized system optimization problem with a single decision maker. This centralized model explains whether the decentralized contract, described in previous section, can optimize the supply chain and achieve a win-win outcome. Therefore, it can be obtained if there is only one decision maker within the supply chain coordination for both the buyer and vendor, and then the problem, to reduce the total cost of the system, can be defined as follows. For that, let buyer's order quantity be \( Q_{1} \); thus, for vendor, it is \( yQ_{1} \). For buyer, the backorder rate is \( k_{r} \). Therefore, the problem can be formulated as

\[
\begin{align*}
\text{Min} \quad TC(y, Q_{1}, k_{r}) &= \frac{AD}{Q_{1}} + \frac{h_{y}(1 - k)^{2}Q_{1}}{2} + 2 + \frac{k^{2}Q_{1}\pi_{b}}{2} \\
&\quad + \frac{h_{b}Q_{1}}{2} \left( y - 1 \right) \left( 1 - \frac{D}{P} + \frac{D}{P} \right) \\
&\quad + \frac{D}{yQ_{1}} \left( yC_{io} + yQ_{1}C_{inv} + C_{inv} \right) \\
\text{subject to} \quad & \frac{yQ_{1}}{D} \leq R \\
& y \geq 1.
\end{align*}
\]
The cost equation can be written as

\[ TC(y, Q_1, k_c) = \frac{AD}{Q_1} + h_b \left( \frac{(1 - k_c)^2 Q_1}{2} + \frac{k_c^2 Q_1 \pi_b}{2} \right) + \frac{DS}{yQ_1} + \frac{h_b Q_1}{2} \left[ (y - 1) \left( 1 - \frac{D}{P} \right) + \frac{D}{P} \right] + \frac{D}{yQ_1} \left( yC_{io} + yQ_1 C_{inv} + C_{inv} \right) \]

\[ = k_c^2 \left( \frac{Q_1 \pi_b}{2} + \frac{h_b Q_1}{2} \right) - h_b k_c Q_1 + \frac{AD}{Q_1} + \frac{h_b Q_1}{2} + \frac{DS}{yQ_1} + \frac{h_b Q_1}{2} \left[ (y - 1) \left( 1 - \frac{D}{P} \right) + \frac{D}{P} \right] + \frac{D}{yQ_1} \left( yC_{io} + yQ_1 C_{inv} + C_{inv} \right). \]  

(42)

Using the necessary conditions of the multivariable optimization problem, the optimal backorder is obtained as

\[ k_c = \frac{h_b}{\pi_b + h_b}. \]  

(43)

Substituting the value of \( k_c \) in the expression of \( TC(y, Q_1, k_c) \), one has

\[ TC(y, Q_1) = \frac{DS}{yQ_1} + \frac{h_b Q_1}{2} \left[ (y - 1) \left( 1 - \frac{D}{P} \right) + \frac{D}{P} \right] + \frac{DC_{io}}{Q_1} \cdot \frac{DS}{yQ_1} + \frac{DC_{ivf}}{yQ_1} + \frac{DA}{Q_1} + \frac{h_b Q_1 \pi_b}{2 (\pi_b + h_b)} \]  

(44)

Thus, the optimal order quantity is obtained as

\[ Q_i^* = \sqrt{\frac{D (S + C_{ivf}) / y + D (A + C_{io})}{(h_b \pi_b) \left( (h_b / 2) \left( y (1 - D / P) + (2D / P - 1) \right) + h_b \pi_b D / 2 (\pi_b + h_b) \right)}}. \]  

(45)

Substituting this value in \( E[TC(y, Q_1)] \), it is found as

\[ \overline{TC}(y) = \left[ \frac{D (S + C_{ivf})}{y} + D (A + C_{io}) \right] \left[ h_v \left( y \left( 1 - \frac{D}{P} \right) + \frac{2D}{P - 1} \right) \right] + \frac{h_b \pi_b}{(\pi_b + h_b)} + DC_{ivf} \]  

subject to \( y^2 (A + C_{io}) + (S + C_{ivf}) y \)

\[ \leq R^2 \]

(47)

subject to \( y \geq 1 \).

The cost expression \( TC(y) \), as the last term is constant with respect to \( y \) and the square root of \( f(x) \) is an increasing function \( x \) if \( x \geq 0 \), can be written as

\[ \overline{TC}(y) = \left[ \frac{D (S + C_{ivf})}{y} + D (A + C_{io}) \right] \left[ h_v \left( y \left( 1 - \frac{D}{P} \right) + \frac{2D}{P - 1} \right) \right] + \frac{h_b \pi_b}{(\pi_b + h_b)} \]  

subject to \( \psi(y) \geq 0 \)

\[ y \geq 1. \]

It is similar to (29). Hence, the value is the same for \( y^* \). By (27) and (47), we have

\[ TC(y^*) = \overline{TC}(y^*) + \sqrt{\frac{2ADh_b \pi_b}{\pi_b + h_b}}. \]  

(49)

where the second term in the right hand side indicates the buyer’s exact cost under coordination contract with the vendor. Again from (5), (25), and (45), it is clear that the buyer’s optimal quantity \( r^* Q^* \) is equal to the optimal order quantity \( Q_i^* \) under system optimization and for vendor’s optimal order quantity is equal in these cases; that is, \( y^* \lambda^* Q^* = y^* Q_i^* \).
### 4. Numerical Examples

**Example 1.** This model considers the same numerical data from Duan et al. [13] to obtain the savings of vendor and buyer and vendor's savings if he does not share the savings with the buyer and the system savings in percentage. We consider the numerical data as $D = 10,000$ units/year, $\alpha = 0.5$, $h_b = $5/unit/year, $h_c = $2/unit/year, $R = 0.25$ year, $\alpha = $100/order, and $S = $300/setup, from Duan et al. [13] and we assume $P = 20,000$/year, $C_{iv}$ = $1/delivery, $C_{iv} = $0.02/unit inspection, $C_{iv}$ = $0.22/production lot, and $n_b = $75/unit backorder. The results are summarized in Table 2 to show the increasing value of savings percentage. $SIP_i$, $SIP_{i+1}$, and $SIP_{i+2}$ indicate the savings of buyer, the savings of vendor, and the savings of vendor if he does not share the savings with buyer, and the savings of the whole system. The calculations of $SIP_i(\%)$ can be done by using the formula $100\alpha(TC_i(y^*) - TC_i(z^*)) / TC_b(z^*)$, the calculation of $SIP_{i+1}(\%)$ can be found by calculating $100(1 - \alpha)(TC_i(y^*) - TC_i(z^*)) / TC_c(z^*)$, the calculation of $SIP_{i+2}(\%)$ can be done by using $100(\overline{TC_i}(y^*) - TC_i(z^*) / TC_c(z^*)$, and the calculation of $SIP_{i+3}(\%)$ can be found by calculating $100(\overline{TC_i}(y^*) - TC_i(z^*) / (TC_b(z^*) + TC_c(z^*)$).

**Insight 1.** If the holding cost for buyer is fixed and for vendor it is increasing, then all savings are increasing. The interesting observation is that the increasing value of vendor's holding cost implies the increasing value of vendor's savings with supply chain's savings as well as if we fixed the vendor holding cost, the savings for buyer, vendor, and the whole supply chains are decreasing. This is the major insight for SSMD policy. If we use single-setup single-delivery policy (SSSD), we would get the reverse one. Thus, the industry managers can decide which policy is most suitable for them based on the study.

**Insight 2.** The value of the buyer's order multiple under coordination is always more than one; that is, the buyer always agrees with vendor's requests only when the savings percentages increase for all. When the buyer's holding cost is fixed and the vendor's holding cost is increasing, the value of buyer's order multiple under coordination is decreasing. But if we consider the reverse case regarding holding cost, the value of buyer's order multiple under coordination is increasing.

**Table 2: Computational results with $D = 10,000$ units/year.**

<table>
<thead>
<tr>
<th>$A$</th>
<th>$h_b$</th>
<th>$h_c$</th>
<th>$r^<em>(y^</em>)$</th>
<th>$d(r^*)$</th>
<th>$SIP_b(%)$</th>
<th>$SIP_{i+1}(%)$</th>
<th>$SIP_{i+2}(%)$</th>
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</tr>
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</tbody>
</table>

**Figure 3:** Savings for Example 1 with increasing vendor's holding cost.

**Insight 3.** The discount per unit dollar to the buyer from the vendor is very less and the discount decreases if the holding cost increases for the vendor. Similar observations are found if vendor's holding cost is fixed and we increase buyer's holding cost whereas this is just the reverse case for the value of buyer's order multiple. It increases with the increasing value of buyer's holding cost when vendor's holding cost is fixed and it decreases when the vendor's holding cost increases by taking constant buyer's holding cost. See Figures 3 and 4 for savings comparison between them.

**Example 2.** In this example, the value of demand is changed to $D = 9,000$ units/year and the optimum result will be considered in Table 3.

**Insight 4.** When the demand is reduced to 9000 units/year, we find similar results as in Example 1, but the most important observation is that the savings for all increase from the previous observation. The rest observations are similar to Example 1. See Figures 5 and 6 for savings comparison between them.

**Example 3.** In this example, the value of demand is changed to $D = 8,000$ units/year and the optimum result will be considered in Table 4.

**Insight 5.** When the demand reduces more, we find the maximum savings for buyer, vendor, and supply chain by comparing with other two examples.
Table 3: Computational results with $D = 9,000$ units/year.

<table>
<thead>
<tr>
<th>$A$</th>
<th>$h_v$</th>
<th>$h_b$</th>
<th>$r^<em>(y^</em>)$</th>
<th>$d(r^*)$</th>
<th>SIP$_b$ (%)</th>
<th>SIP$_{cl}$ (%)</th>
<th>SIP$_{vl}$ (%)</th>
<th>SIP$_s$ (%)</th>
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Figure 4: Savings for Example 1 with increasing buyer’s holding cost.

**Insight 6.** The important findings are that when both holding costs are the same for buyer and vendor, then savings for buyer and savings for the supply chain are more for Example 2, but savings for vendor (in both cases SIP$_{cl}$ and SIP$_{vl}$) are more in Example 1.

**Insight 7.** If we increase the vendor’s holding cost by taking constant holding cost for buyer, the savings of buyer are more for all three values of demand whereas when we fix vendor’s holding cost and increase buyer’s holding cost, it increases with less percentage for all three different values demand. See Figures 7 and 8 for savings comparison between them.

Table 4: Computational results with $D = 8,000$ units/year.

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<tr>
<th>$A$</th>
<th>$h_v$</th>
<th>$h_b$</th>
<th>$r^<em>(y^</em>)$</th>
<th>$d(r^*)$</th>
<th>SIP$_b$ (%)</th>
<th>SIP$_{cl}$ (%)</th>
<th>SIP$_{vl}$ (%)</th>
<th>SIP$_s$ (%)</th>
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</table>

Figure 5: Savings for Example 2 with increasing vendor’s holding cost.

**5. Concluding Remarks**

This model considered a supply chain coordination model with variable backorder cost for the buyer and three different types of inspection costs used by the vendor. The products were with fixed lifetime. The model was derived analytically to obtain the optimal strategy with the presence of coordination policy. A noble approach was employed to obtain closed-form solutions. The buyer’s ordering quantity was obtained larger at cooperation against the noncooperation ($\lambda > 1$) with respect to some conditions related to holding and backordering cost. Three different values of demand...
are considered to study numerical studies and comparative managerial insights were given between them. From the numerical results, it was found that this model gave more savings than Duan et al. [13] for each value of demand. Within the three types of demand values, it was found that demand, in Example 3, gave more savings than others. Therefore, managers of the industry sectors chose this strategy when they found any one of these amounts of demand patterns for their products. The main limitation of this model is to consider zero-lead time. Thus, this model can be extended by incorporating non-zero lead time. A fruitful research would be the controllable lead time with some discrete lead time crashing costs. In this model, the demand is assumed as constant which can be considered in fuzzy sense, and then it will be another extension of the model in uncertain environment. If there is any information asymmetry, then based on asymmetry condition, revenue sharing or delay-in-payments policy can be considered for a further extension of this model. If the model will assume about multiple products, then that will be a good extension of this model.

Appendices

A. The Values of \( a_1, a_2, \) and \( a_3 \)

Consider the following:

\[
a_1 = h_v \left( 1 - \frac{D}{P} \right) \sqrt{\frac{AD (\pi_b + h_b)}{2h_b \pi_b}} \]

\[
a_2 = \frac{DS + DC_{ivf}}{Q} = \left( S + C_{ivf} \right) \sqrt{\frac{h_b \pi_b D}{2A (\pi_b + h_b)}} \] \hspace{1cm} (A.1)

\[
a_3 = h_v \left( \frac{2D}{P} - 1 \right) \sqrt{\frac{AD (\pi_b + h_b)}{2h_b \pi_b}} \]

\[
+ C_{io} \sqrt{\frac{h_b \pi_b D}{2A (\pi_b + h_b)} + DC_{ivv}}
\]

B. The Values of \( a_4, a_5, \) and \( a_6 \)

Consider the following:

\[
a_4 = \frac{h_v Q}{2} \left\{ (y - 1) \left( 1 - \frac{D}{P} \right) + \frac{D}{P} \right\} + h_b (1 - k)^2 \frac{Q}{2} \]

\[
+ \frac{k^2 Q \pi_b}{2}
\]

\[
a_5 = \frac{DS + DC_{ivf}}{yQ} + \frac{AD + DC_{iv}}{Q} \] \hspace{1cm} (B.1)

\[
= \frac{D}{Q} \left[ \frac{S + C_{ivf}}{y} + A + C_{io} \right]
\]

\[
a_6 = DC_{ivv} - \sqrt{\frac{2ADh_b \pi_b}{\pi_b + h_b}}
\]
C. The Values of $a_7$, $a_8$, and $a_9$

Consider the following:

\[ a_7 = \left[ h_v D \left( 1 - \frac{D}{P} \right) (A + C_{io}) \right] \]

\[ a_8 = \left[ \frac{D (S + C_{irf})}{2} \left( \frac{2D}{P} - 1 \right) + \frac{h_v \pi b}{(\pi b + h_b)} \right] \]

\[ a_9 = h_v D \left( 1 - \frac{D}{P} \right) (S + C_{irf}) + h_v \left( \frac{2D}{P} - 1 \right) (A + C_{io}) \]

+ \frac{h_v \pi b D (A + C_{io})}{(\pi b + h_b)} \]

D. The Values of $a_{10}$, $a_{11}$, and $a_{12}$

Consider the following:

\[ a_{10} = - (A + C_{io}) \]

\[ a_{11} = \left[ \frac{R^2 h_v D (1 - D/P)}{2} - (S + C_{irf}) \right] \]

\[ a_{12} = R^2 \left\{ \frac{h_v D (2D/P - 1)}{2} + \frac{h_v \pi b D}{2(\pi b + h_b)} \right\} \]

Let us consider that solution of $\psi(y) = 0$ is $y_2^{(1)*}$ and $y_2^{(2)*}$ where:

\[ y_2^{(1)*} = \frac{-a_{11} + \sqrt{a_{11}^2 - 4a_{10}a_{12}}}{2a_{10}} \]

\[ y_2^{(2)*} = \frac{-a_{11} - \sqrt{a_{11}^2 - 4a_{10}a_{12}}}{2a_{10}} \]

E. The Values of $x_1$, $x_2$, and Some Simplifications

For simplification, let us symbolize $x_1$ and $x_2$ as follows:

\[ x_1 = h_v (\pi_b + h_b) \left( S + C_{irf} \right) \left( 1 - \frac{D}{P} \right) \]

\[ x_2 = h_v (\pi_b + h_b) \left( \frac{2D}{P} - 1 \right) + h_v \pi b \]

From the denominator part, we obtain

\[ h_v (\pi_b + h_b) \left( \frac{2D}{P} - 1 \right) + h_v \pi b + h_v (\pi_b + h_b) \]

\[ \cdot y \left( 1 - \frac{D}{P} \right) = x_2 + h_v (\pi_b + h_b) \left( 1 - \frac{D}{P} \right) \]

\[ \cdot \sqrt{x_2 \left( S + C_{irf} \right)} \]

\[ \sqrt{h_v (1 - D/P) (A + C_{io}) (\pi_b + h_b)} \]

\[ = \sqrt{x_2} \left[ \sqrt{(A + C_{io})} + \sqrt{x_1} \right] \]

From the numerator part

\[ h_v \pi b \left\{ \frac{S + C_{irf}}{y} + A + C_{io} \right\} = h_v \pi b \left\{ (A + C_{io}) \right. \]

\[ + (S + C_{irf}) \]

\[ \cdot \sqrt{h_v (1 - D/P) (A + C_{io}) (\pi_b + h_b)} \]

\[ \cdot \left[ \frac{S + C_{irf}}{x_2} \right] \]

\[ = h_v \pi b \left\{ \sqrt{(A + C_{io})} x_2 + \sqrt{x_1} \right\} \]

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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