Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2016, Article ID 6318737, 14 pages
http://dx.doi.org/10.1155/2016/6318737

Research Article

Supply Chain Coordination with Variable Backorder, Inspections, and Discount Policy for Fixed Lifetime Products

Biswajit Sarkar

Department of Industrial & Management Engineering, Hanyang University, Ansan, Gyeonggi-do 426 791, Republic of Korea

Correspondence should be addressed to Biswajit Sarkar; bsbiswajitsarkar@gmail.com

Received 16 May 2015; Accepted 7 July 2015

Academic Editor: Ben T. Nohara

This paper illustrates a channel coordination and quantity discounts between a vendor and a buyer with single-setup multi-delivery (SSMD) strategy to reduce the joint total cost among supply chain players. The benefit of the coordination between a buyer and a vendor is considered as the vendor requests to the buyer for changing the ordering quantity such that the vendor can be benefited from lower inventory costs. After accepting the buyer's condition, the vendor compensates the buyer for his increased inventory cost and gives consent for additional savings by offering a quantity discount. The centralized decision making is examined for the effect of this strategy with the presence of backorder for buyer and inspection cost for the vendor. The quantity discount strategy, with the presence of variable backorder and inspections, can allow more savings for all players of supply chain. Some numerical examples, sensitivity analysis, and graphical representations are given to illustrate more savings from existing literature and comparisons between the several demand values.

1. Introduction

Nowadays, most of the industries can no longer compete solely as individual entities in this changing marketing environment. SCM (supply chain management) contains combination of buyer and suppliers, distributors and retailers, manufacturers and distributors, or many different forms of customers. Integrated supply chain management has been recognized as a core competitive strategy as the production industries provide products and services to their customers more cheaper and faster than their competitors. The main goal of SCM is to optimize the whole system cost by maintaining the cooperation between two different entities, because it lowers the joint total expected cost of the system and increases their revenues which are from the competitive advantages of SCM. Therefore, in modern marketing system, the cooperation between buyer and vendor plays an important role in an integrated vendor-buyer supply chain model. Recently, Cárdenas-Barrón and Sana [1] developed an interesting problem of channel coordination on single-supplier single-manufacturer model with backorder for buyer. Cárdenas-Barrón and Porter [2] discussed a better and simple algorithm for a supply chain model with preprocessing raw materials. Wee et al. [3] developed an integrated inventory model with a different approach of optimization rather than the classical one. Lin [4] discussed a channel coordination policy for two-stage supply chain considering quantity discounts and imperfect production. He used a solution procedure, free from algorithm based on convex optimization, to obtain the optimum solution. Ryu et al. [5] derived an excellent fractal echelon approach to maintain inventory in a supply chain network. They introduced fractal-based vendor management inventory where some responsibilities for the buyers are taken by the vendor. Wang et al. [6] developed the decentralized and centralized supply chain problem considering finite production rate for the supplier regarding pricing and lot sizing decision. They used all units discount policy and a franchise fee for the coordinated supply chain. Zissis et al. [7] derived a rational pair manufacturer and retailer where there is some information asymmetry which disturbs the coordination policy. The quantity discount is offered by the manufacturer to the retailer. They considered analytical derivation for the model. Saha and Goyal [8] developed supply chain coordination contracts with inventory level and retail price-dependent demand.

Goyal [9] was the first author who initiated the joint optimization concept for vendor and buyer. Banerjee [10] extended this concept for the vendor with a finite production rate to order for a buyer on a lot-for-lot basis where the operating cost of the manufacturer and buyer is considered. He assumed that the shipment can only be made when the production has been completed. Goyal [11] relaxed Banerjee's [10] lot-for-lot assumption and extended vendor's economic production quantity as an integer multiple of buyer's purchasing quantity by assuming the SSMD policy. He numerically proved that the SSMD policy is better than the SSSD (single-setup single-delivery) policy. Luo [12] extended the buyer-vendor coordination model with credit period incentives. Duan et al. [13] extended Luo's [12] model with quality discount incentive for fixed lifetime products. Cárdenas-Barrón [14] derived the explanation of different inventory models with two types of backorders costs using analytic geometry and algebra.

When products are transported to buyer from vendor, it is very important to inspect all product's quality. Many researchers considered integrated-inventory models, but they did not assume inspection policies for imperfect production. Duffuaa and Khan [15] explained an optimal repeat inspection policy with several classifications. Wee and Chung [16] wrote a note on the economic lot size of the integrated vendor-buyer inventory system derived without derivatives by considering inspection costs for the vendor. Hsieh and Liu [17] assumed an investment for quality improvement and they considered inspection strategies for imperfect items in a single-supplier single-manufacturer model. They proposed four noncooperative game models with several degrees of information available about the player's inspection sampling rates as well as an investment for quality improvement. Khan et al. [18] developed an inventory model with imperfect quality items and inspection errors as Type I and Type II errors. J.-T. Hsu and L.-F. Hsu [19] discussed an inventory model with imperfect quality items, inspection errors, backorders, and sales returns. Aust et al. [20] wrote a note on Hsieh and Liu's [17] model with higher profit.

In modern marketing system, a practical problem is deterioration or perishability of raw materials or finished products. Many products have limited lifetime, after which it may deteriorate or it cannot be used. Therefore, the management system at each stage in supply chain from production to consumers can lead to high system costs. Liu and Shi [21] classified product's perishability and deterioration into two categories as decay model and finite lifetime model. Moreover, products with finite lifetime model can be separated into common or fixed lifetime models and random finite lifetime models. Basically, items like fresh products, canned fruits, and medicines with common finite lifetime are considered in the inventory literature as perishable items (Liu and Lian [22]) if they are not used within their lifetimes. The random finite lifetime can be treated as random variable which follows some probability distributions such as Erlang or exponential. Liu and Lian [22], Sana [23], and Sarkar [24] discussed fixed lifetime of products for single-stage inventory systems. Sarkar [25] explained different probabilistic deterioration for the single-vendor single-buyer model with algebraic method. Priyan and Uthayakumar [26] described trade-credit financing in a single-vendor single-buyer integrated model with cost reduction policy. They considered backorder price-discount policy when receiving quantity is uncertain. Pahl and Voß [27] surveyed about deteriorating and lifetime constraints in production and supply chain planning.

This study considers a supply chain coordination model with backorder for the buyer, inspection cost for the vendor, and a coordination strategy between them. The vendor requests the buyer to alter his ordering quantity for reducing inventory costs of the vendor and as a policy, the vendor must compensate the buyer for his increased inventory costs and allow some additional savings by offering a quantity discount as incentives. The vendor considers three different types of inspection costs (see, e.g., Wee and Chung [16]) to ensure the quality for their products. The system cannot consider any defective item during production and if it arises, it is immediately discarded from the system. The model considered only that type of product which has fixed lifetime (see, e.g., Sana [23]). This model considers a variable backorder rate for the buyer. The centralized decision-making model with effect of coordination between vendor and buyer is considered under the presence of buyer's backorder cost and vendor's inspection cost for fixed lifetime products. See Table 1 for the contribution of different authors.

The paper is organized as follows. Notation and assumptions are given in Section 2. The mathematical model is formulated in Section 3. Some numerical examples are given to illustrate the model in Section 4. Finally, we discuss the concluding remarks in Section 5.

2. Notation and Assumptions

The model uses the following notation to develop this model.

Decision variables are as follows:

Q: buyer's economic order quantity (units),

z: vendor's order multiple in absence of any coordination,

y: vendor's order multiple under coordination,

k: backorder rate,

λ: buyer's order multiple under coordination and λQ as the buyer's new order quantity.

Parameters are as follows:

D: demand per year (units/year),

A: buyer's ordering cost per order (\$/order),

h_b: buyer's holding cost per unit per year (\$/unit/year),

π_b: buyer's unit backorder cost (\$/unit backorder item),

$B(\lambda)$: discount per unit dollar to the buyer (\$/unit dollar) by the vendor if the buyer places the order λQ each time,

R: lifetime of products (year),

Table 1: Comparison between contributions of different authors.

Author(s)	Quantity discount	Lot size	Backorder	Inspection policy	Shipment number
Cárdenas-Barrón and Sana [1]		√	√		
Cárdenas Barrón and Porter [2]		√			√
Wee et al. [3]			√	√	
Lin [4]	√	√		√	√
Ryu et al. [5]		√			
Wang et al. [6]		√			√
Zissis et al. [7]		√			√
Saha and Goyal [8]		√			
Goyal [9]		√			
Banerjee [10]		√			
Goyal [11]		√			√
Luo [12]		√			√
Duan et al. [13]	√	√			√
Cárdenas Barrón [14]		√	√		√
Duffuaa and Khan [15]				√	
Wee and Chung [16]		√	√	√	√
Hsieh and Liu [17]					
Khan et al. [18]		√		√	
J.-T. Hsu and L.-F. Hsu [19]		√	√	√	
Aust et al. [20]	√			√	
Liu and Shi [21]		√			√
Liu and Lian [22]	√				√
Sana [23]			√		
Sarkar [24]	√	√			
Sarkar [25]		√			√
Priyan and Uthayakumar [26]		√	√		√
Pahl and Voß [27]	√				
This paper	√	√	√	√	√

P: production rate per year ($P > D$) (units/year),

h_v: vendor's holding cost per unit per year (\$/unit/year),

S: vendor's setup cost per setup (\$/setup),

C_{i0}: vendor's variable inspection cost per delivery (\$/delivery),

C_{ivv}: vendor's unit inspection cost (\$/unit item inspected),

C_{ivf}: vendor's fixed inspection cost per production lot (\$/production lot).

The following assumptions are considered to develop this model.

(1) The supply chain coordination in single-vendor and single-buyer model is considered for a single type of item. The model assumes that when buyer orders products from vendor, he requests buyer to change the order quantity by a fraction of present order quantity and then the vendor offers a quantity discount if the buyer accepts its request to change the order quantity.

(2) Production and demand rate are constant. The backorder rate for buyer is assumed as a decision variable.

(3) Before the vendor's shipment, the final inspection is implemented. There are three steps of inspections such that there will be no defective product and if it arises, it will be discarded from the system.

(4) The numbers of vendor's order multiple with and without coordination are integer numbers.

(5) The vendor and the buyer have complete knowledge of information of each other. There is no asymmetry of information.

(6) The system does not consider wait-in-process items and defective items.

(7) The replenishment is instantaneous and the lead time is negligible.

3. Mathematical Model

In absence of any coordination, the model considers a single-setup-multi-delivery (SSMD) policy (see, e.g., Goyal [11]) where buyer's order quantity is produced at single stage, but it is delivered to the vendor by equal shipment in multiple deliveries after a fixed time interval. Each delivery is designed in just-in-time (JIT) way which indicates that each delivery

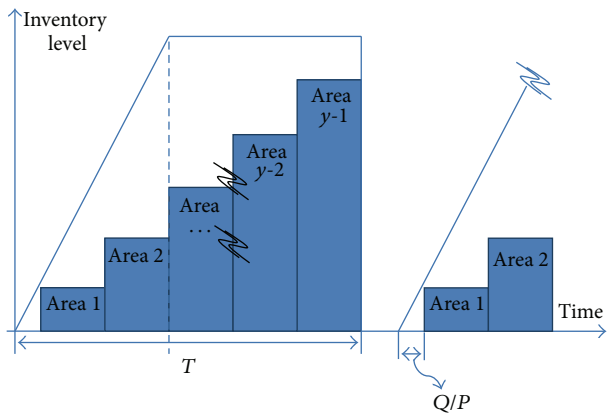

FIGURE 1: Inventory positions for the vendor.

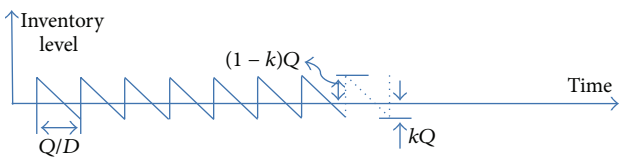

FIGURE 2: Inventory positions for the buyer with backorder.

arrives at the time when all items from the previous delivery have just been depleted. The shortages are considered for the buyer. During shortages, the backorder rate is k. The inspection of items is completed before sending to buyer. This model does not consider the production of the defective items during the production; if this arises, it is discarded from the system. Figures 1 and 2 represent the buyer's and the vendor' inventory versus time.

3.1. Buyer Cost in Presence of Backorder. Consider the following.

Ordering cost $= AD/Q$.

Holding cost $= h_b(1-k)^2Q/2$.

Backorder cost $= k^2Q\pi_b/2$.

Therefore, the buyer's cost function is

$$\begin{aligned} TC_b &= \frac{AD}{Q} + \frac{h_b(1-k)^2 Q}{2} + \frac{k^2 Q\pi_b}{2} \\ &= k^2\left(\frac{Q\pi_b}{2} + \frac{h_b Q}{2}\right) - h_b kQ + \frac{AD}{Q} + \frac{h_b Q}{2}. \end{aligned} \tag{1}$$

The concept of parabola is used here (see, e.g., [14]). If $f(x) = ax^2 + bx + c$ is an equation of a parabola, then it is known to all that the function has the global minimum $c - b^2/4a$ at $x = -b/2a$ if and only if $a > 0$. This function has the global maximum $c - b^2/4a$ at $x = -b/2a$ if and only if $a < 0$. Therefore, the value of the backorder is

$$k = \frac{h_b}{\pi_b + h_b}. \tag{2}$$

Thus, the minimum cost with respect to k is

$$TC_b(Q) = \frac{AD}{Q} + \frac{h_b Q\pi_b}{2(\pi_b + h_b)}. \tag{3}$$

An algebraic procedure (see, e.g., Sarkar [25]) has been employed to obtain the minimum cost with respect to the ordering quantity and the backorder:

$$TC_b = \sqrt{\frac{2ADh_b\pi_b}{\pi_b + h_b}}, \tag{4}$$

where the optimum ordering quantity is given by

$$Q = \sqrt{\frac{2AD(\pi_b + h_b)}{h_b\pi_b}} \tag{5}$$

and the fixed interval is given by

$$t = \frac{Q}{D} = \sqrt{\frac{2A(\pi_b + h_b)}{h_b\pi_b D}}. \tag{6}$$

3.2. Vendor's Cost in Presence of Inspection for Defective Items. Consider the following.

Setup cost $= DS/zQ$.

Holding cost $= (h_vQ/2)[(z-1)(1-D/P) + D/P]$.

Inspection cost $= DC_{io}/Q + DC_{ivv} + (D/zQ)C_{ivf}$.

Therefore, the vendor's cost in presence of inspection cost is

$$\begin{aligned} TC_v(z) &= \frac{DS}{zQ} + \frac{h_vQ}{2}\left[(z-1)\left(1-\frac{D}{P}\right) + \frac{D}{P}\right] \\ &\quad + \frac{D}{zQ}\left(zC_{io} + zQC_{ivv} + C_{ivf}\right). \end{aligned} \tag{7}$$

Now, the cost is simplified as

$$\begin{aligned} TC_v(z) &= \frac{DS}{zQ} + \frac{h_vQ}{2}\left[(z-1)\left(1-\frac{D}{P}\right) + \frac{D}{P}\right] \\ &\quad + \frac{DC_{io}}{Q} + DC_{ivv} + \frac{D}{zQ}C_{ivf} \\ &= \frac{DS + DC_{ivf}}{zQ} + \frac{h_vzQ(1-D/P)}{2} \\ &\quad + \frac{h_vQ}{2}\left(\frac{2D}{P} - 1\right) + \frac{DC_{io}}{Q} + DC_{ivv} \\ &= a_1z + \frac{a_2}{z} + a_3 \end{aligned} \tag{8}$$

(see Appendix A for the values of a_1, a_2, and a_3).

Therefore, without coordination vendor's cost can be written as

$$\begin{aligned} \text{Min} \quad & TC_v(z) = a_1z + \frac{a_2}{z} + a_3 \\ \text{subject to} \quad & zt \le R \\ & z \ge 1. \end{aligned} \tag{9}$$

The optimum value of z is given by

$$z = \sqrt{\frac{a_2}{a_1}} = \sqrt{\frac{h_b \pi_b \left(S + C_{ivf}\right)}{A h_v \left(\pi_b + h_b\right)\left(1 - D/P\right)}}. \tag{10}$$

If z_1 is the optimum value of z, then it is found that

$$z_1 = \max\left\{\left\lceil \sqrt{\frac{h_b \pi_b \left(S + C_{ivf}\right)}{A h_v \left(\pi_b + h_b\right)\left(1 - D/P\right)}} \right\rceil, 1\right\} \geq 1. \tag{11}$$

The optimal cost for the vendor with respect to z is

$$\begin{aligned} TC_v &= 2\sqrt{a_1 a_2} + a_3 \\ &= \sqrt{2Dh_v\left(1 - \frac{D}{P}\right)\left(S + C_{ivf}\right)} \\ &\quad + h_v\left(\frac{2D}{P} - 1\right)\sqrt{\frac{AD\left(\pi_b + h_b\right)}{2h_b\pi_b}} \\ &\quad + C_{io}\sqrt{\frac{h_b\pi_b D}{2A\left(\pi_b + h_b\right)}} + DC_{ivv}. \end{aligned} \tag{12}$$

Substituting the value of the interval t, one can obtain

$$zt \leq R \Longrightarrow z\sqrt{\frac{2A\left(\pi_b + h_b\right)}{h_b\pi_b D}} \leq R \Longrightarrow z \leq \frac{R}{\sqrt{\phi}}, \quad \text{where } \phi = \frac{2A\left(\pi_b + h_b\right)}{h_b\pi_b D}. \tag{13}$$

It is assumed that $z_2 = [R/\sqrt{\phi}]$. Now $R^2 \geq \phi$ implies $z_2 \geq 1$. As TC_v is convex and if $z_1 \leq z_2$, then $z = z_1$; else $z = z_2$.

Therefore, if $R^2 \geq \phi$, $z^* = \min\{z_1, z_2\}$. Thus, if $R^2 \geq \phi$, then

$$z^* = \min\left[\sqrt{\frac{h_b\pi_b\left(S + C_{ivf}\right)}{A h_v\left(\pi_b + h_b\right)\left(1 - D/P\right)}}, \left\lceil \frac{R}{\sqrt{\phi}} \right\rceil\right], \tag{14}$$

where $\lceil x \rceil$ is the least integer greater than or equal to x.

3.3. Mathematical Model with Decentralized Decision Making. The paper discusses two-part coordination contract in a decentralized decision making. For the coordination system, there is a strategy that the vendor requests the buyer to change his current order size by a factor λ (>0) and offers the buyer a quantity discount by a discount factor $B(\lambda)$ and the buyer accepts the offer. Thus, the vendor's and buyer's new order quantities are $y\lambda Q$ and λQ.

The vendor's total cost is as follows:

$$\begin{aligned} \widetilde{TC}_v(y,\lambda) &= \frac{DS}{y\lambda Q} + \frac{h_v\lambda Q}{2}\left[(y-1)\left(1 - \frac{D}{P}\right) + \frac{D}{P}\right] \\ &\quad + \frac{D}{y\lambda Q}\left(yC_{io} + y\lambda Q C_{ivv} + C_{ivf}\right) \\ &\quad + DB(\lambda). \end{aligned} \tag{15}$$

The problem with coordination can be formulated as

$$\begin{aligned} \text{Min} \quad & \widetilde{TC}_v(y,\lambda) \\ &= \frac{DS}{y\lambda Q} \\ &\quad + \frac{h_v\lambda Q}{2}\left[(y-1)\left(1 - \frac{D}{P}\right) + \frac{D}{P}\right] \\ &\quad + \frac{D}{y\lambda Q}\left(yC_{io} + y\lambda Q C_{ivv} + C_{ivf}\right) \\ &\quad + DB(\lambda) \\ \text{subject to} \quad & y\lambda t \leq R \\ & \frac{AD}{\lambda Q} + \frac{h_b(1-k)^2\lambda Q}{2} + \frac{k^2\lambda Q\pi_b}{2} \\ &\quad - \sqrt{2ADh_b\left(\frac{\pi_b}{\pi_b + h_b}\right)} \leq DB(\lambda) \\ & y \geq 1. \end{aligned} \tag{16}$$

In the above constraint, the term $DB(\lambda)$, in the right side of (16), represents the compensation of the vendor to the buyer, the constraints represent the items, which are not overdue before they are used by the buyer, and the buyer's cost under coordination cannot exceed that in absence of any coordination.

Theorem 1. *The optimum cost under coordination is always less than or equal to the optimum cost without coordination; that is,*

$$\widetilde{TC}_v(y^*, \lambda^*) \leq TC_v(z^*). \tag{17}$$

Proof. From the second constraint of (16), the term $DB(\lambda)$ gives the smallest value if the second constraint forms an equation which indicates the minimization of vendor's cost $\widetilde{TC}_v(y)$; that is,

$$\begin{aligned} &\frac{AD}{rQ} + \frac{h_b(1-k)^2\lambda Q}{2} + \frac{k^2\lambda Q\pi_b}{2} - \sqrt{\frac{2ADh_b\pi_b}{\pi_b + h_b}} \\ &\quad = DB(\lambda) \end{aligned} \tag{18}$$

which gives

$$\begin{aligned} B(\lambda) = \frac{1}{D}\Bigg[&\frac{AD}{\lambda Q} + \frac{h_b(1-k)^2 rQ}{2} + \frac{k^2 rQ\pi_b}{2} \\ &- \sqrt{\frac{2ADh_b\pi_b}{\pi_b + h_b}}\Bigg]. \end{aligned} \tag{19}$$

If $\lambda = 1$ (i.e., after the request of vendor, buyer's order size is Q and that of vendor's lot size is yQ), then the discount factor is

$$B(1) = \frac{1}{C_d D}\left[\frac{AD}{Q} + \frac{h_b(1-k)^2 Q}{2} + \frac{k^2 Q\pi_b}{2} - \sqrt{\frac{2ADh_b\pi_b}{\pi_b + h_b}}\right]. \tag{20}$$

We substitute the value of optimum k and Q from the noncoordination Section 3.1, and then

$$B(1) = \frac{1}{D}\left[\frac{AD}{\sqrt{2AD(\pi_b + h_b)/h_b\pi_b}} + \frac{h_b\pi_b}{2(\pi_b + h_b)}\sqrt{\frac{2AD(\pi_b + h_b)}{h_b\pi_b}} - \sqrt{\frac{2AD\pi_b h_b}{\pi_b + h_b}}\right]. \tag{21}$$

Simplifying this expression, one has

$$B(1) = \frac{1}{D}\left[-\sqrt{\frac{2ADh_b\pi_b}{(\pi_b + h_b)}} + \sqrt{\frac{2ADh_b\pi_b}{(\pi_b + h_b)}}\right] = 0. \tag{22}$$

Therefore, if $\lambda = 1$, (16) is reduced to (9) which implies (9) is a special case of (16). Hence the inequality in (17) holds. This completes the proof. □

By the theorem, it can be easily seen that after receiving offer from the vendor, the buyer and the vendor both are gainer. Therefore, they can follow the coordination policy for a long time for their own profit.

As the vendor offers a fraction of lot size, thus, we can use the value of $B(\lambda)$ in (19). Thus, the cost in (15) is modified as

$$\begin{aligned}\widetilde{TC}_v(y,\lambda) = {} & \frac{DS}{y\lambda Q} + \frac{h_v\lambda Q}{2}\left[(y-1)\left(1-\frac{D}{P}\right) + \frac{D}{P}\right] \\ & + \frac{D}{y\lambda Q}\left(yC_{io} + y\lambda QC_{ivv} + C_{ivf}\right) \\ & + \frac{AD}{\lambda Q} + \frac{h_b(1-k)^2\lambda Q}{2} + \frac{k^2\lambda Q\pi_b}{2} \\ & - \sqrt{\frac{2ADh_b\pi_b}{\pi_b + h_b}}.\end{aligned} \tag{23}$$

The cost equation can be simplified as

$$\begin{aligned}\widetilde{TC}_v(y,\lambda) = {} & \lambda\left[\frac{h_v Q}{2}\left\{(y-1)\left(1-\frac{D}{P}\right) + \frac{D}{P}\right\} + \frac{h_b(1-k)^2 Q}{2} + \frac{k^2 Q\pi_b}{2}\right] + \frac{1}{\lambda}\left[\frac{DS}{yQ} + \frac{AD}{Q} + \frac{DC_{io}}{Q} + \frac{DC_{ivf}}{yQ}\right] \\ & + DC_{ivv} - \sqrt{\frac{2ADh_b\pi_b}{\pi_b + h_b}} = a_4\lambda + \frac{a_5}{\lambda} + a_6\end{aligned} \tag{24}$$

(see Appendix B for the values of a_4, a_5, and a_6).

With the help of similar algebraical procedure, it is found that

$$\lambda = \sqrt{\frac{a_5}{a_4}} = \frac{1}{Q} \cdot \sqrt{\frac{2\left[D\left(S + C_{ivf}\right)/y + D\left(A + C_{io}\right)\right]}{h_v\{(y-1)(1-D/P) + D/P\} + h_b(1-k)^2 + k^2\pi_b}}. \tag{25}$$

Substitute (25) and t into $y\lambda t \le R$, which gives

$$\begin{aligned}& y^2\left[\frac{S + C_{ivf}}{y} + (A + C_{io})\right] \le R^2\left[\frac{h_v D}{2}\left\{\left(1-\frac{D}{P}\right)y - 1 + \frac{2D}{P}\right\} + \frac{h_b\pi_b D}{2(\pi_b + h_b)}\right].\end{aligned}$$

Suppose

$$\begin{aligned}\psi(y) = {} & R^2\left[\frac{h_v D}{2}\left\{\left(1-\frac{D}{P}\right)y - 1 + \frac{2D}{P}\right\} + \frac{h_b\pi_b D}{2(\pi_b + h_b)}\right] - y^2\left[\frac{S + C_{ivf}}{y} + (A + C_{io})\right] \\ = {} & -y^2(A + C_{io}) + y\left[\frac{R^2 h_v D}{2}\left(1-\frac{D}{P}\right) - \left(S + C_{ivf}\right)\right] + R^2\left\{\frac{h_v D}{2}\left(\frac{2D}{P} - 1\right) + \frac{h_b\pi_b D}{2(\pi_b + h_b)}\right\}.\end{aligned} \tag{26}$$

Therefore, the first constraint of (16) is of the form $\psi(y) \ge 0$.

Now substituting the value of λ and t into $\widetilde{TC}_v(y)$,

$$\begin{aligned}\widetilde{TC}_v(y) &= 2\sqrt{a_4 a_5} + a_6 \\ \text{subject to} \quad & \psi(y) \ge 0 \\ & y \ge 1.\end{aligned} \tag{27}$$

As a_6 is constant with respect to y and square root of a function Q is strictly increasing if $Q \ge 0$, therefore, $\widetilde{TC}_v(y)$ from (27) can be written as

$$\widetilde{\widetilde{TC}}_v(y) = \left[h_v\left\{(y-1)\left(1-\frac{D}{P}\right) + \frac{D}{P}\right\} + \frac{h_b\pi_b}{(\pi_b + h_b)}\right] \cdot \left[\frac{DS + DC_{ivf}}{y} + AD + DC_{io}\right]$$

$$= y\left[h_v D\left(1-\frac{D}{P}\right)(A+C_{io})\right] + \frac{\left[D\left(S+C_{ivf}\right)\{h_v(2D/P-1)+h_b\pi_b/(\pi_b+h_b)\}\right]}{y} + h_v D\left(1-\frac{D}{P}\right)\left(S+C_{ivf}\right) + h_v D\left(\frac{2D}{P}-1\right) \cdot (A+C_{io}) + \frac{h_b\pi_b D(A+C_{io})}{(\pi_b+h_b)}. \tag{28}$$

Thus, (27) can be written as

$$\widetilde{\widetilde{TC}}_v(y) = a_7 y + \frac{a_8}{y} + a_9 \quad \text{subject to} \quad \psi(y) \geq 0, \quad y \geq 1 \tag{29}$$

(see Appendix C for the values of a_7, a_8, and a_9).

It is known to all that (29) is a nonlinear program. We now discuss the property of (29) in order to minimize it. We know that $\widetilde{\widetilde{TC}}_v(y)$ is a convex function if

$$h_v\left(\frac{2D}{P}-1\right) > \frac{h_b\pi_b}{\pi_b+h_b} \tag{30}$$

and $\psi(y)$ is strictly concave as $(A+C_{io}) > 0$ (from the theory of parabola, we know that $\psi(y)$ has the global minimum solution if the coefficient of y^2 is less than zero and as $A + C_{io} > 0$ and there is a negative sign, $\psi(y)$ is strictly concave).

Theorem 2. *If $h_v(2D/P-1) > h_b\pi_b/(\pi_b+h_b)$, then the optimal value of y is*

$$y_1^* = \left\lceil \sqrt{\frac{\left(S+C_{ivf}\right)\left[h_v(2D/P-1)+h_b\pi_b/(\pi_b+h_b)\right]}{h_v(1-D/P)(A+C_{io})}} \right\rceil; \tag{31}$$

otherwise $y_1^* = 1$.

Proof. From the cost function, one has

$$\widetilde{\widetilde{TC}}_v(y) = a_7 y + \frac{a_8}{y} + a_9. \tag{32}$$

Using the necessary condition of optimization procedure, one can obtain the optimal value of y as

$$y_1^* = \left\lceil \sqrt{\frac{a_8}{a_7}} \right\rceil = \left\lceil \sqrt{\frac{h_v(2D/P-1)\left(DS+DC_{ivf}\right)+h_b\pi_b\left(DS+DC_{ivf}\right)/(\pi_b+h_b)}{h_v(1-D/P)(AD+DC_{io})}} \right\rceil. \tag{33}$$

By our assumption $P > D$, thus the denominator term is positive but the numerator term is positive or negative.

Now the numerator part is positive if

$$h_v\left(\frac{2D}{P}-1\right)\left(DS+DC_{ivf}\right) > \frac{h_b\pi_b\left(DS+DC_{ivf}\right)}{(\pi_b+h_b)} \Longrightarrow h_v\left(\frac{2D}{P}-1\right) > \frac{h_b\pi_b}{\pi_b+h_b}; \quad \text{that is, if } h_v\left(\frac{2D}{P}-1\right) > \frac{h_b\pi_b}{\pi_b+h_b} \text{ holds,} \tag{34}$$

then y_1^*

$$= \left\lceil \sqrt{\frac{\left(S+C_{ivf}\right)\left[h_v(2D/P-1)+h_b\pi_b/(\pi_b+h_b)\right]}{h_v(1-D/P)(A+C_{io})}} \right\rceil.$$

Otherwise $y_1^* = 1$. This completes the proof. □

Now we discuss the properties of $\psi(y)$

$$\psi(y) = -(y)^2(A+C_{io}) + y\left[\frac{R^2 h_v D(1-D/P)}{2} - \left(S+C_{ivf}\right)\right] + R^2\left\{\frac{h_v D(2D/P-1)}{2} + \frac{h_b\pi_b D}{2(\pi_b+h_b)}\right\} = a_{10}(y)^2 + a_{11}y + a_{12} \tag{35}$$

(see Appendix D for the values of a_{10}, a_{11}, and a_{12}).

On the basis of $y \geq 1$, we obtain the following.

(1) If $y_2^{(1)^*} < 1$, then $\psi(y) < 0$ for all $y \geq 1$.

(2) If $y_2^{(2)^*} \geq 1$, then $\psi(y) \geq 0$ for $\lceil y_2^{(2)^*} \rceil \leq y \leq y_2^{(1)^*}$.

(3) If $y_2^{(2)^*} < 1$ and $y_2^{(1)^*} \geq 1$, then on the basis of y which is positive integer

$\psi(y) \geq 0$ for $1 \leq y \leq y_2^{(1)^*}$. Let us consider $y_2^{(1)^*}$ and $y_2^{(2)^*}$ as solutions of the equation $\psi(y) = 0$. Therefore based on this, the following illustrations can be found.

(i) If $a_{11}^2 - 4a_{10}a_{12} < 0$ or $a_{11}^2 - 4a_{10}a_{12} \geq 0$ and $y_2^{(1)^*} < 1$, then $\psi(y) < 0$ for all $y \geq 1$.

(ii) If $a_{11}^2 - 4a_{10}a_{12} \geq 0$ and $y_2^{(1)^*} \geq 1$, then

if $y_2^{(2)^*} \geq 1$, $\psi(y) \geq 0$ for all $\lceil y_2^{(2)^*} \rceil \leq y \leq y_2^{(1)^*}$, and
if $y_2^{(2)^*} < 1$ and $y_2^{(1)^*} \geq 1$, then $\psi(y) \geq 0$ for $1 \leq y \leq y_2^{(1)^*}$.

Note 1. If (i) holds, the first constraint of (16) does not hold for any $y \geq 1$. In that case the problem is meaningless. If (ii) holds, the first constraint of (16) holds for $\lceil y_2^{(2)^*} \rceil \leq y \leq y_2^{(1)^*}$.

Theorem 3. *If $h_v(2D/P-1) > h_b\pi_b/(\pi_b+h_b)$ and $y_2^{(1)^*} \geq 1$ hold, then the following conditions are obtained. (i) If $1 \leq y_1^* \leq y_2^{(1)^*}$ holds, then $y^* = y_1^*$.*

(ii) If $y_1^* > y_2^{(1)^*}$ holds, then $y^* = y_2^{(1)^*}$.

Proof. From the 1st condition of the theorem, $\widetilde{\widetilde{TC}}_v(y)$ is convex function and by Theorem 2, y_1^* is the minimum value of the cost function $\widetilde{\widetilde{TC}}_v(y)$ for $y \geq 1$. Thus, if $1 \leq y_1^* \leq y_2^{(1)^*}$ holds, then it is found that $y^* = y_1^*$ and if $y_1^* > y_2^{(1)^*}$ holds, then it is obtained as $y^* = y_2^{(1)^*}$. It is to be noted that $\widetilde{\widetilde{TC}}_v(y)$ is decreasing on the interval; hence the second condition of the theorem holds easily.

This completes the proof. □

Theorem 4. *If $h_v(2D/P-1) > h_b\pi_b/(\pi_b+h_b)$ holds, then the buyer's order multiple is greater than 1; that is, $\lambda^*(y^*) > 1$.*

Proof. From the optimal value of λ, one can obtain

$$\begin{aligned}\lambda^*(y^*) &= \frac{1}{Q}\sqrt{\frac{2\left[D\left(S+C_{ivf}\right)/y + D\left(A+C_{io}\right)\right]}{h_v\left\{(y-1)(1-D/P)+D/P\right\} + h_b(1-k)^2 + k^2\pi_b}} \\ &= \sqrt{\frac{h_b\pi_b\left\{\left(\left(S+C_{ivf}\right)/y\right) + \left(A+C_{io}\right)\right\}}{h_v\left(\pi_b+h_b\right)(2D/P-1) + h_b\pi_b + h_v\left(\pi_b+h_b\right)y(1-D/P)}}.\end{aligned} \tag{36}$$

(i) If $y^* = y_1^* = 1$, then

$$\lambda^* = \sqrt{\frac{S+C_{ivf}+A+C_{io}}{h_vD\left(\pi_b+h_b\right)/Ph_b\pi_b + 1}} > 1 \quad \text{as } \left(S+C_{ivf}+A+C_{io}\right) > \left(\frac{h_vD\left(\pi_b+h_b\right)}{Ph_b\pi_b}+1\right). \tag{37}$$

(ii) From (33), we obtain a simplified form of y_1^* as

$$y_1^* = \left\lceil \sqrt{\frac{\left(S+C_{ivf}\right)\left\{h_v(2D/P-1)\left(\pi_b+h_b\right)+h_b\pi_b\right\}}{h_v(1-D/P)\left(A+C_{io}\right)\left(\pi_b+h_b\right)}} \right\rceil. \tag{38}$$

If $y^* = y_1^*$ and $h_v(2D/P-1) > h_b\pi_b/(\pi_b+h_b)$, then

$$y^* = y_1^* = \left\lceil \sqrt{\frac{\left(S+C_{ivf}\right)\left\{h_v(2D/P-1)\left(\pi_b+h_b\right)+h_b\pi_b\right\}}{h_v(1-D/P)\left(A+C_{io}\right)\left(\pi_b+h_b\right)}} \right\rceil. \tag{39}$$

Now we show $\lambda^*(y^*)|_{y^*=y_1^*} > 1$.

The expression is as follows:

$$\begin{aligned}&\frac{h_b\pi_b\left\{\left(S+C_{ivf}\right)/y + A + C_{io}\right\}}{h_v\left(\pi_b+h_b\right)(2D/P-1) + h_b\pi_b + h_v\left(\pi_b+h_b\right)y(1-D/P)} \\ &= \frac{h_b\pi_b\sqrt{\left(A+C_{io}\right)/x_2}\left\{\sqrt{\left(A+C_{io}\right)x_2}+\sqrt{x_1}\right\}}{\sqrt{x_2/\left(A+C_{io}\right)}\left\{\sqrt{\left(A+C_{io}\right)x_2}+\sqrt{x_1}\right\}} \\ &= h_b\pi_b\left(\frac{A+C_{io}}{x_2}\right) > 1 \quad \text{if } h_b\pi_b\left(A+C_{io}\right) > x_2\end{aligned} \tag{40}$$

(see Appendix E for the simplifications and values of x_1 and x_2).

(iii) If $y^* = y_2^{(1)^*}$, then $y_1^* > y_2^{(1)^*}$. Now $r^*(y^*)|_{y=y_1^*} > 1$ and thus, $\lambda^*(y^*)$ is a decreasing function; hence, $\lambda^*(y^*)|_{y_2^{(1)^*}} \geq \lambda|_{y_1^*} > 1$.

Therefore, from the above three conditions, we obtain $r^* > 1$. That completes the proof. □

3.4. Mathematical Model for Centralized Decision. This section considers a centralized system optimization problem with a single decision maker. This centralized model explains whether the decentralized contract, described in previous section, can optimize the supply chain and achieve a win-win outcome. Therefore, it can be obtained if there is only one decision maker within the supply chain coordination for both the buyer and vendor, and then the problem, to reduce the total cost of the system, can be defined as follows. For that, let buyer's order quantity be Q_1; thus, for vendor, it is yQ_1. For buyer, the backorder rate is k_c. Therefore, the problem can be formulated as

$$\begin{aligned}\text{Min} \quad & TC\left(y, Q_1, k_c\right) \\ &= \frac{AD}{Q_1} + \frac{h_b\left(1-k_c\right)^2 Q_1}{2} + \frac{k_c^2 Q_1 \pi_b}{2} \\ &\quad + \frac{DS}{yQ_1} \\ &\quad + \frac{h_v Q_1}{2}\left[(y-1)\left(1-\frac{D}{P}\right)+\frac{D}{P}\right] \\ &\quad + \frac{D}{yQ_1}\left(yC_{io} + yQ_1C_{ivv} + C_{ivf}\right) \\ \text{subject to} \quad & \frac{yQ_1}{D} \leq R \\ & y \geq 1.\end{aligned} \tag{41}$$

The cost equation can be written as

$$\begin{aligned} TC(y,Q_1,k_c) &= \frac{AD}{Q_1} + \frac{h_b(1-k_c)^2 Q_1}{2} + \frac{k_c^2 Q_1 \pi_b}{2} \\ &\quad + \frac{DS}{yQ_1} \\ &\quad + \frac{h_v Q_1}{2}\left[(y-1)\left(1-\frac{D}{P}\right)+\frac{D}{P}\right] \\ &\quad + \frac{D}{yQ_1}\left(yC_{io} + yQ_1 C_{ivv} + C_{ivf}\right) \\ &= k_c^2\left(\frac{Q_1\pi_b}{2} + \frac{h_b Q_1}{2}\right) - h_b k_c Q_1 \\ &\quad + \frac{AD}{Q_1} + \frac{h_b Q_1}{2} + \frac{DS}{yQ_1} \\ &\quad + \frac{h_v Q_1}{2}\left[(y-1)\left(1-\frac{D}{P}\right)+\frac{D}{P}\right] \\ &\quad + \frac{D}{yQ_1}\left(yC_{io} + yQ_1 C_{ivv} + C_{ivf}\right). \end{aligned} \tag{42}$$

Using the necessary conditions of the multivariable optimization problem, the optimal backorder is obtained as

$$k_c = \frac{h_b}{\pi_b + h_b}. \tag{43}$$

Substituting the value of k_c in the expression of $TC(y, Q_1, k_c)$, one has

$$\begin{aligned} &TC(y,Q_1) \\ &\quad = \frac{DS}{yQ_1} + \frac{h_v Q_1}{2}\left[(y-1)\left(1-\frac{D}{P}\right)+\frac{D}{P}\right] + \frac{DC_{io}}{Q_1} \\ &\qquad + DC_{ivv} + \frac{DC_{ivf}}{yQ_1} + \frac{DA}{Q_1} + \frac{h_b Q_1 \pi_b}{2(\pi_b + h_b)} \\ &\quad = Q_1\left[\frac{h_v}{2}\left\{(y-1)\left(1-\frac{D}{P}\right)+\frac{D}{P}\right\} + \frac{h_b\pi_b}{2(\pi_b+h_b)}\right] \\ &\qquad + \frac{1}{Q_1}\left[\frac{DS}{y} + DC_{io} + \frac{DC_{ivf}}{y} + DA\right] + DC_{ivv}. \end{aligned} \tag{44}$$

Thus, the optimal order quantity is obtained as

$$Q_1^* = \sqrt{\frac{D(S+C_{ivf})/y + D(A+C_{io})}{(h_v/2)\{y(1-D/P)+(2D/P-1)\} + \{h_b\pi_b/2(\pi_b+h_b)\}}}. \tag{45}$$

Substituting this value in $E[TC(y, Q_1)]$, it is found as

$$TC(y) = \sqrt{2\left\{\frac{D(S+C_{ivf})}{y} + D(A+C_{io})\right\}\left[h_v\left\{y\left(1-\frac{D}{P}\right)+\left(\frac{2D}{P}-1\right)\right\} + \frac{h_b\pi_b}{(\pi_b+h_b)}\right]} + DC_{ivv} \tag{46}$$

$$\begin{aligned} &\text{subject to} \quad \frac{y^2(A+C_{io}) + (S+C_{ivf})y}{(h_v D/2)\{y(1-D/P)+(2D/P-1)\} + h_b\pi_b D/2(\pi_b+h_b)} \le R^2 \\ &\qquad y \ge 1. \end{aligned} \tag{47}$$

The cost expression $TC(y)$, as the last term is constant with respect to y and the square root of $f(x)$ is an increasing function x if $x \ge 0$, can be written as

$$\begin{aligned} \widetilde{TC}(y) &= \left[\frac{D(S+C_{ivf})}{y} + D(A+C_{io})\right]\left[h_v\left\{y\left(1-\frac{D}{P}\right)+\left(\frac{2D}{P}-1\right)\right\} + \frac{h_b\pi_b}{(\pi_b+h_b)}\right] \\ \text{subject to} \quad &\psi(y) \ge 0 \\ &y \ge 1. \end{aligned} \tag{48}$$

It is similar to (29). Hence, the value is the same for y^*. By (27) and (47), we have

$$TC(y^*) = \widetilde{TC}_v(y^*) + \sqrt{\frac{2ADh_b\pi_b}{\pi_b + h_b}}, \tag{49}$$

where the second term in the right hand side indicates the buyer's exact cost under coordination contract with the vendor. Again from (5), (25), and (45), it is clear that the buyer's optimal quantity r^*Q^* is equal to the optimal order quantity Q_1^* under system optimization and for vendor's optimal order quantity is equal in these cases; that is, $y^*\lambda^*Q^* = y^*Q_1^*$.

Table 2: Computational results with $D = 10{,}000$ units/year.

A	h_v	h_b	$r^*(y^*)$	$d(r^*)$	SIP_b (%)	SIP_{v1} (%)	SIP_{v2} (%)	SIP_s (%)
100	2	5	1.18303	0.00433537	15.7801	13.3032	26.6065	14.4362
100	3	5	1.16289	0.00349305	21.7968	14.6644	29.3288	17.533
100	4	5	1.14915	0.00296376	26.4994	15.2994	30.5988	19.3989
100	5	5	1.1389	0.00259332	30.4357	15.6533	31.3065	20.6739
100	7	10	1.15258	0.00424273	25.2699	15.3221	30.6443	19.0771
100	5	6	1.14667	0.00312683	27.4144	15.4398	30.8795	19.7541
100	5	7	1.15333	0.00364709	25.0082	15.2142	30.4284	18.9188
100	5	8	1.15913	0.00415334	23.0275	14.9815	29.963	18.1529
100	5	9	1.16426	0.00464541	21.3569	14.7447	29.4895	17.4453

4. Numerical Examples

Example 1. This model considers the same numerical data from Duan et al. [13] to obtain the savings of vendor and buyer and vendor's savings if he does not share the savings with the buyer and the system savings in percentage. We consider the numerical data as $D = 10{,}000$ units/year, $\alpha = 0.5$, $h_b = \$5$/unit/year, $h_v = \$2$/unit/year, $R = 0.25$ year, $A = \$100$/order, and $S = \$300$/setup, from Duan et al. [13] and we assume $P = 20{,}000$/year, $C_{io} = \$1$/delivery, $C_{ivv} = \$0.02$/unit inspection, $C_{ivf} = \$0.2$/production lot, and $\pi_b = \$75$/unit backorder. The results are summarized in Table 2 to show the increasing value of savings percentage. $\mathrm{SIP}_b(\%)$, $\mathrm{SIP}_{v1}(\%)$, $\mathrm{SIP}_{v2}(\%)$, and $\mathrm{SIP}_s(\%)$ indicate the savings of buyer, the savings of vendor, the savings of vendor if he does not share the savings with buyer, and the savings of the whole system. The calculations of $\mathrm{SIP}_b(\%)$ can be done by using the formula $100\alpha(\widetilde{TC}_v(y^*) - TC_v(z^*))/TC_b(z^*)$, the calculation of $\mathrm{SIP}_{v1}(\%)$ can be found by calculating $100(1-\alpha)(\widetilde{TC}_v(y^*) - TC_v(z^*))/TC_v(z^*)$, the calculation of $\mathrm{SIP}_{v2}(\%)$ can be done by using $100(\widetilde{TC}_v(y^*) - TC_v(z^*))/TC_v(z^*)$, and the calculation of $\mathrm{SIP}_s(\%)$ can be found by calculating $100(\widetilde{TC}_v(y^*) - TC_v(z^*))/(TC_b(z^*) + TC_v(z^*))$.

Insight 1. If the holding cost for buyer is fixed and for vendor it is increasing, then all savings are increasing. The interesting observation is that the increasing value of vendor's holding cost implies the increasing value of vendor's savings with supply chain's savings also whereas if we fixed the vendor holding cost, the savings for buyer, vendor, and the whole supply chains are decreasing. This is the major insight for SSMD policy. If we use single-setup single-delivery policy (SSSD), we would get the reverse one. Thus, the industry managers can decide which policy is most suitable for them based on the study.

Insight 2. The value of the buyer's order multiple under coordination is always more than one; that is, the buyer always agrees with vendor's requests only when the saving percentages increase for all. When the buyer's holding cost is fixed and the vendor's holding cost is increasing, the value of buyer's order multiple under coordination is decreasing. But if we consider the reverse case regarding holding cost, the value of buyer's order multiple under coordination is increasing.

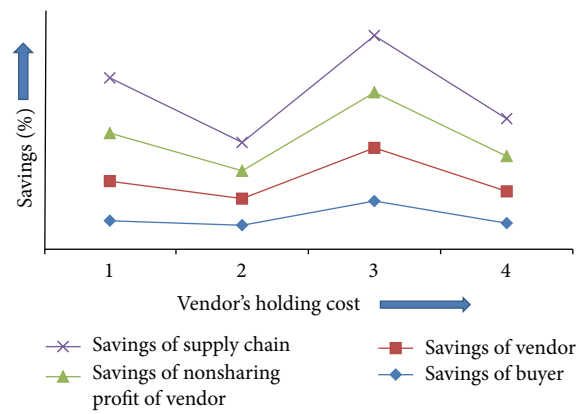

Figure 3: Savings for Example 1 with increasing vendor's holding cost.

Insight 3. The discount per unit dollar to the buyer from the vendor is very less and the discount decreases if the holding cost increases for the vendor. Similar observations are found if vendor's holding cost is fixed and we increase buyer's holding cost whereas this is just the reverse case for the value of buyer's order multiple. It increases with the increasing value of buyer's holding cost when vendor's holding cost is fixed and it decreases when the vendor's holding cost increases by taking constant buyer's holding cost. See Figures 3 and 4 for savings comparison between them.

Example 2. In this example, the value of demand is changed to $D = 9{,}000$ units/year and the optimum result will be considered in Table 3.

Insight 4. When the demand is reduced to 9000 units/year, we find similar results as in Example 1, but the most important observation is that the savings for all increase from the previous observation. The rest observations are similar to Example 1. See Figures 5 and 6 for savings comparison between them.

Example 3. In this example, the value of demand is changed to $D = 8{,}000$ units/year and the optimum result will be considered in Table 4.

Insight 5. When the demand reduces more, we find the maximum savings for buyer, vendor, and supply chain by comparing with other two examples.

Table 3: Computational results with $D = 9{,}000$ units/year.

A	h_v	h_b	$r^*(y^*)$	$d(r^*)$	SIP_b (%)	SIP_{v1} (%)	SIP_{v2} (%)	SIP_s (%)
100	2	5	1.21199	0.00598349	16.3708	13.5031	27.0062	14.7993
100	3	5	1.20728	0.00574296	22.2455	14.7532	29.5064	17.7408
100	4	5	1.20903	0.00583203	26.784	15.3317	30.6634	19.5008
100	5	5	1.21449	0.00611312	30.5387	15.6488	31.2977	20.6937
100	7	10	1.20809	0.00793559	25.6026	15.358	30.7161	19.1992
100	5	6	1.20996	0.00640098	27.6607	15.4603	30.9205	19.8345
100	5	7	1.20793	0.00675051	25.3506	15.2575	30.515	19.0497
100	5	8	1.20726	0.00713054	23.4383	15.0464	30.0928	18.3274
100	5	9	1.20737	0.0075251	21.8183	14.8306	29.6612	17.6583

Table 4: Computational results with $D = 8{,}000$ units/year.

A	h_v	h_b	$r^*(y^*)$	$d(r^*)$	SIP_b (%)	SIP_{v1} (%)	SIP_{v2} (%)	SIP_s (%)
100	2	5	1.2435	0.00816159	16.8543	13.6528	27.3057	15.0856
100	3	5	1.25742	0.00901988	22.5309	14.7868	29.5737	17.8554
100	4	5	1.27945	0.0104472	26.8262	15.291	30.5819	19.4789
100	5	5	1.3073	0.0123643	30.2945	15.5456	31.0912	20.5473
100	7	10	1.27261	0.0137134	25.7176	15.3264	30.6529	19.2066
100	5	6	1.28509	0.0117849	27.6439	15.4026	30.8051	19.7827
100	5	7	1.27126	0.0115781	25.4801	15.2346	30.4691	19.0682
100	5	8	1.26214	0.0115725	23.6688	15.0524	30.1048	18.4019
100	5	9	1.25589	0.0116847	22.122	14.862	29.724	17.7794

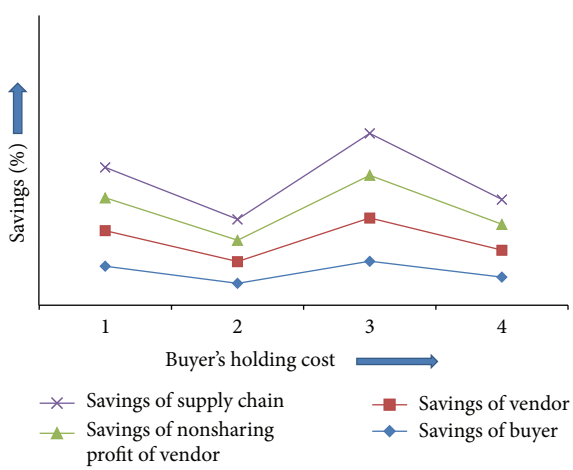

Figure 4: Savings for Example 1 with increasing buyer's holding cost.

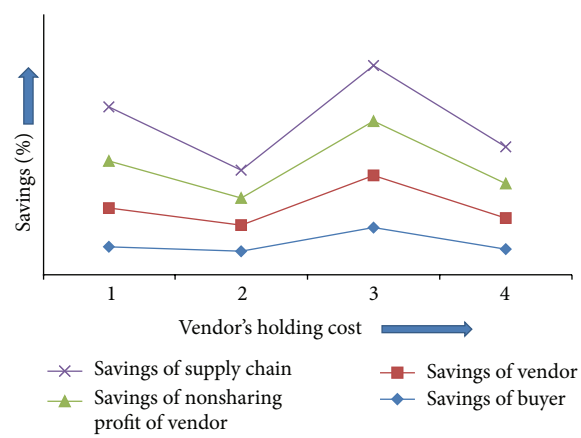

Figure 5: Savings for Example 2 with increasing vendor's holding cost.

Insight 6. The important findings are that when both holding costs are the same for buyer and vendor, then savings for buyer and savings for the supply chain are more for Example 2, but savings for vendor (in both cases SIP_{v1} and SIP_{v1}) are more in Example 1.

Insight 7. If we increase the vendor's holding cost by taking constant holding cost for buyer, the savings of buyer are more for all three values of demand whereas when we fix vendor's holding cost and increase buyer's holding cost, it increases with less percentage for all three different values demand. See Figures 7 and 8 for savings comparison between them.

5. Concluding Remarks

This model considered a supply chain coordination model with variable backorder cost for the buyer and three different types of inspection costs used by the vendor. The products were with fixed lifetime. The model was derived analytically to obtain the optimal strategy with the presence of coordination policy. A noble approach was employed to obtain closed-form solutions. The buyer's ordering quantity was obtained larger at cooperation against the noncooperation ($\lambda > 1$) with respect to some conditions related to holding and backordering cost. Three different values of demand

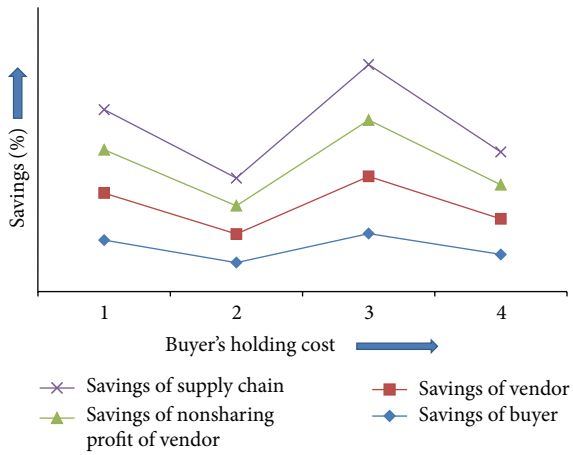

FIGURE 6: Savings for Example 2 with increasing buyer's holding cost.

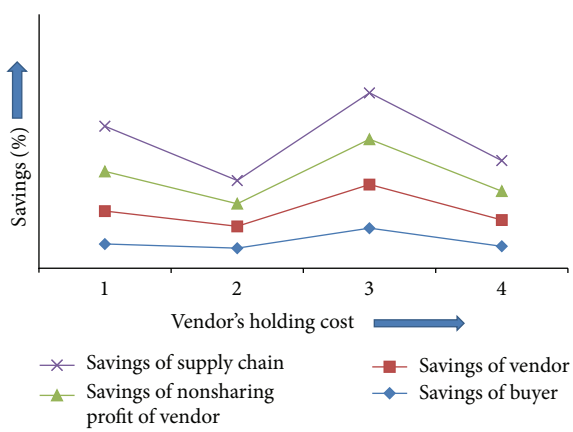

FIGURE 7: Savings for Example 3 with increasing vendor's holding cost.

are considered to study numerical studies and comparative managerial insights were given between them. From the numerical results, it was found that this model gave more savings than Duan et al. [13] for each value of demand. Within the three types of demand values, it was found that demand, in Example 3, gave more savings than others. Therefore, managers of the industry sectors chose this strategy when they found any one of these amounts of demand patterns for their products. The main limitation of this model is to consider zero-lead time. Thus, this model can be extended by incorporating non-zero lead time. A fruitful research would be the controllable lead time with some discrete lead time crashing costs. In this model, the demand is assumed as constant which can be considered in fuzzy sense, and then it will be another extension of the model in uncertain environment. If there is any information asymmetry, then based on asymmetry condition, revenue sharing or delay-in-payments policy can be considered for a further extension of this model. If the model will assume about multiple products, then that will be a good extension of this model.

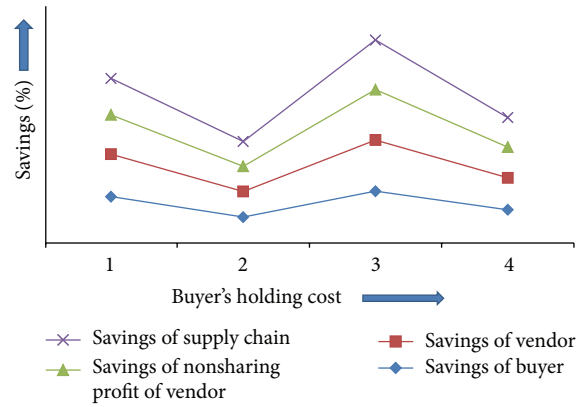

FIGURE 8: Savings for Example 3 with increasing buyer's holding cost.

Appendices

A. The Values of a_1, a_2, and a_3

Consider the following:

$$\begin{aligned} a_1 &= h_v\left(1-\frac{D}{P}\right)\sqrt{\frac{AD\left(\pi_b+h_b\right)}{2h_b\pi_b}} \\ a_2 &= \frac{DS+DC_{ivf}}{Q} = \left(S+C_{ivf}\right)\sqrt{\frac{h_b\pi_b D}{2A\left(\pi_b+h_b\right)}} \\ a_3 &= h_v\left(\frac{2D}{P}-1\right)\sqrt{\frac{AD\left(\pi_b+h_b\right)}{2h_b\pi_b}} \\ &\quad + C_{io}\sqrt{\frac{h_b\pi_b D}{2A\left(\pi_b+h_b\right)}} + DC_{ivv}. \end{aligned} \tag{A.1}$$

B. The Values of a_4, a_5, and a_6

Consider the following:

$$\begin{aligned} a_4 &= \frac{h_v Q}{2}\left\{(y-1)\left(1-\frac{D}{P}\right)+\frac{D}{P}\right\} + \frac{h_b(1-k)^2 Q}{2} \\ &\quad + \frac{k^2 Q\pi_b}{2} \\ a_5 &= \frac{DS+DC_{ivf}}{yQ} + \frac{AD+DC_{io}}{Q} \\ &= \frac{D}{Q}\left[\frac{S+C_{ivf}}{y} + A + C_{io}\right] \\ a_6 &= DC_{ivv} - \sqrt{\frac{2ADh_b\pi_b}{\pi_b+h_b}}. \end{aligned} \tag{B.1}$$

C. The Values of a_7, a_8, and a_9

Consider the following:

$$\begin{aligned} a_7 &= \left[h_v D\left(1-\frac{D}{P}\right)(A+C_{io})\right] \\ a_8 &= \left[D\left(S+C_{ivf}\right)\left\{h_v\left(\frac{2D}{P}-1\right)+\frac{h_b\pi_b}{(\pi_b+h_b)}\right\}\right] \\ a_9 &= h_v D\left(1-\frac{D}{P}\right)\left(S+C_{ivf}\right) \\ &\quad + h_v D\left(\frac{2D}{P}-1\right)(A+C_{io}) \\ &\quad + \frac{h_b\pi_b D(A+C_{io})}{(\pi_b+h_b)}. \end{aligned} \tag{C.1}$$

D. The Values of a_{10}, a_{11}, and a_{12}

Consider the following:

$$\begin{aligned} a_{10} &= -(A+C_{io}) \\ a_{11} &= \left[\frac{R^2 h_v D(1-D/P)}{2}-\left(S+C_{ivf}\right)\right] \\ a_{12} &= R^2\left\{\frac{h_v D(2D/P-1)}{2}+\frac{h_b\pi_b D}{2(\pi_b+h_b)}\right\}. \end{aligned} \tag{D.1}$$

Let us consider that solution of $\psi(y) = 0$ is $y_2^{(1)^*}$ and $y_2^{(2)^*}$ where:

$$\begin{aligned} y_2^{(1)^*} &= \frac{-a_{11}+\sqrt{a_{11}^2-4a_{10}a_{12}}}{2a_{10}} \\ y_2^{(2)^*} &= \frac{-a_{11}-\sqrt{a_{11}^2-4a_{10}a_{12}}}{2a_{10}}. \end{aligned} \tag{D.2}$$

From the property of quadratic equation, we know if $a_{11}^2 - 4a_{10}a_{12} >, =,$ or $< 0, \psi(y)$ has real, equal, or imaginary solutions,

where

$$\begin{aligned} a_{11}^2 - 4a_{10}a_{12} &= \left\{\frac{R^2 h_v D(1-D/P)}{2}-\left(S+C_{ivf}\right)\right\}^2 \\ &\quad + 4(A+C_{io}) \\ &\quad \cdot R^2\left\{\frac{h_v D(2D/P-1)}{2}+\frac{h_b\pi_b D}{2(\pi_b+h_b)}\right\}. \end{aligned} \tag{D.3}$$

E. The Values of x_1, x_2, and Some Simplifications

For simplification, let us symbolize x_1 and x_2 as follows:

$$\begin{aligned} x_1 &= h_v(\pi_b+h_b)\left(S+C_{ivf}\right)\left(1-\frac{D}{P}\right) \\ x_2 &= h_v(\pi_b+h_b)\left(\frac{2D}{P}-1\right)+h_b\pi_b. \end{aligned} \tag{E.1}$$

From the denominator part, we obtain

$$\begin{aligned} &h_v(\pi_b+h_b)\left(\frac{2D}{P}-1\right)+h_b\pi_b+h_v(\pi_b+h_b) \\ &\quad \cdot y\left(1-\frac{D}{P}\right) = x_2+h_v(\pi_b+h_b)\left(1-\frac{D}{P}\right) \\ &\quad \cdot\sqrt{\frac{x_2\left(S+C_{ivf}\right)}{h_v(1-D/P)(A+C_{io})(\pi_b+h_b)}} \\ &\quad = \sqrt{\frac{x_2}{A+C_{io}}}\left[\sqrt{x_2(A+C_{io})}+\sqrt{x_1}\right]. \end{aligned} \tag{E.2}$$

From the numerator part

$$\begin{aligned} &h_b\pi_b\left\{\frac{S+C_{ivf}}{y}+A+C_{io}\right\} = h_b\pi_b\left\{(A+C_{io})\right. \\ &\quad + \left(S+C_{ivf}\right) \\ &\quad \left.\cdot\sqrt{\frac{h_v(1-D/P)(A+C_{io})(\pi_b+h_b)}{\left(S+C_{ivf}\right)x_2}}\right\} \\ &\quad = h_b\pi_b\sqrt{\frac{A+C_{io}}{x_2}}\left\{\sqrt{(A+C_{io})x_2}+\sqrt{x_1}\right\}. \end{aligned} \tag{E.3}$$

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

Acknowledgment

This work was supported by the research fund of Hanyang University (HY-2015-N, Project no. 201500000000971) for new faculty members.

References

[1] L. E. Cárdenas-Barrón and S. S. Sana, "A production-inventory model for a two-echelon supply chain when demand is dependent on sales teams' initiatives," *International Journal of Production Economics*, vol. 155, pp. 249–258, 2014.

[2] L. E. Cárdenas Barrón and J. D. Porter, "Supply chain models for an assembly system with preprocessing of raw materials: a simple and better algorithm," *Applied Mathematical Modelling*, vol. 37, no. 14-15, pp. 7883–7887, 2013.

[3] H.-M. Wee, W.-T. Wang, and L. E. C. Barrón, "An alternative analysis and solution procedure for the EPQ model with rework process at a single-stage manufacturing system with planned backorders," *Computers & Industrial Engineering*, vol. 64, no. 2, pp. 748–755, 2013.

[4] T.-Y. Lin, "Coordination policy for a two-stage supply chain considering quantity discounts and overlapped delivery with imperfect quality," *Computers & Industrial Engineering*, vol. 66, no. 1, pp. 53–62, 2013.

[5] K. Ryu, I. Moon, S. Oh, and M. Jung, "A fractal echelon approach for inventory management in supply chain networks," *International Journal of Production Economics*, vol. 143, no. 2, pp. 316–326, 2013.

[6] C. Wang, R. Huang, and Q. Wei, "Integrated pricing and lot-sizing decision in a two-echelon supply chain with a finite production rate," *International Journal of Production Economics*, vol. 161, pp. 44–53, 2015.

[7] D. Zissis, G. Ioannou, and A. Burnetas, "Supply chain coordination under discrete information asymmetries and quantity discounts," *Omega*, vol. 53, pp. 21–29, 2015.

[8] S. Saha and S. K. Goyal, "Supply chain coordination contracts with inventory level and retail price dependent demand," *International Journal of Production Economics*, vol. 161, pp. 140–152, 2015.

[9] S. K. Goyal, "An integrated inventory model for a single supplier-single customer problem," *International Journal of Production Research*, vol. 15, no. 1, pp. 107–111, 1977.

[10] A. Banerjee, "A joint economic-lot-size model for purchaser and vendor," *Decision Sciences*, vol. 17, no. 3, pp. 292–311, 1986.

[11] S. K. Goyal, "A joint economic-lot-size model for purchaser and vendor: a comment," *Decision Sciences*, vol. 19, no. 1, pp. 236–241, 1988.

[12] J. Luo, "Buyer-vendor inventory coordination with credit period incentives," *International Journal of Production Economics*, vol. 108, no. 1-2, pp. 143–152, 2007.

[13] Y. Duan, J. Luo, and J. Huo, "Buyer-vendor inventory coordination with quantity discount incentive for fixed lifetime product," *International Journal of Production Economics*, vol. 128, no. 1, pp. 351–357, 2010.

[14] L. E. Cárdenas Barrón, "The derivation of EOQ/EPQ inventory models with two backorders costs using analytic geometry and algebra," *Applied Mathematical Modelling*, vol. 35, no. 5, pp. 2394–2407, 2011.

[15] S. O. Duffuaa and M. Khan, "An optimal repeat inspection plan with several classifications," *Journal of the Operational Research Society*, vol. 53, no. 9, pp. 1016–1026, 2002.

[16] H. M. Wee and C. J. Chung, "A note on the economic lot size of the integrated vendor-buyer inventory system derived without derivatives," *European Journal of Operational Research*, vol. 177, no. 2, pp. 1289–1293, 2006.

[17] C.-C. Hsieh and Y.-T. Liu, "Quality investment and inspection policy in a supplier-manufacturer supply chain," *European Journal of Operational Research*, vol. 202, no. 3, pp. 717–729, 2010.

[18] M. Khan, M. Y. Jaber, and M. Bonney, "An economic order quantity (EOQ) for items with imperfect quality and inspection errors," *International Journal of Production Economics*, vol. 133, no. 1, pp. 113–118, 2011.

[19] J.-T. Hsu and L.-F. Hsu, "An EOQ model with imperfect quality items, inspection errors, shortage backordering, and sales returns," *International Journal of Production Economics*, vol. 143, no. 1, pp. 162–170, 2013.

[20] G. Aust, I. Bräuer, and U. Buscher, "A note on 'quality investment and inspection policy in a supplier-manufacturer supply chain'," *European Journal of Operational Research*, vol. 234, no. 3, pp. 910–915, 2014.

[21] L. Liu and D.-H. Shi, "An (s, S) model for inventory with exponential lifetimes and renewal demands," *Naval Research Logistics*, vol. 46, no. 1, pp. 39–56, 1999.

[22] L. Liu and Z. Lian, "(s, S) model for inventory with fixed lifetime," *Operations Research*, vol. 47, no. 1, pp. 130–158, 1999.

[23] S. S. Sana, "Optimal selling price and lotsize with time varying deterioration and partial backlogging," *Applied Mathematics and Computation*, vol. 217, no. 1, pp. 185–194, 2010.

[24] B. Sarkar, "An EOQ model with delay in payments and time varying deterioration rate," *Mathematical and Computer Modelling*, vol. 55, no. 3-4, pp. 367–377, 2012.

[25] B. Sarkar, "A production-inventory model with probabilistic deterioration in two-echelon supply chain management," *Applied Mathematical Modelling*, vol. 37, no. 5, pp. 3138–3151, 2013.

[26] S. Priyan and R. Uthayakumar, "Trade credit financing in the vendor-buyer inventory system with ordering cost reduction, transportation cost and backorder price discount when the received quantity is uncertain," *Journal of Manufacturing Systems*, vol. 33, no. 4, pp. 654–674, 2014.

[27] J. Pahl and S. Voß, "Integrating deterioration and lifetime constraints in production and supply chain planning: a survey," *European Journal of Operational Research*, vol. 238, no. 3, pp. 654–674, 2014.