Research Article

Fractional-Order Terminal Sliding-Mode Control for Buck DC/DC Converter

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In recent years, the combination of fractional calculus (FC) and sliding-mode control (SMC) has been gaining more and more interests due to fusion characteristics of SMC and FC. This paper presents the fractional-order terminal sliding-mode control (FTSMC) which has a new fractional-order sliding surface and assures the finite time convergence of the output voltage error to the equilibrium point during the load changes. TSMC is a special case of FTSMC. Through mathematical analysis, the system can reach the sliding-mode surface in finite time. The theoretical considerations have been verified by numerical simulations. And a Buck DC/DC converter application is presented and compared to illustrate the effectiveness of the proposed method. It is shown that the novel fractional terminal sliding-mode control exhibits considerable improvement in terms of a faster output voltage response during load changes.

1. Introduction

The concept of fractional calculus as an extension of ordinary calculus can stretch back to 1695. In the letter to L'Hospital, Leibniz proposed the possibility of generalizing the operation of half-order derivative [1]. Though fractional calculus has a long history, only in recent years have the applications of fractional calculus to physics and engineering become an important aspect of modern technology. In comparison with the classical elementary calculus, the main advantage of fractional calculus is that it can provide an elegant description for the memory and hereditary properties of various real objects. The list of the applied fields of fractional calculus has been ever growing and includes the electrode-electrolyte polarization, viscoelastic fluids, chaotic systems, and power converters [2–6]. Most striking of all, the fractional differentials and integrals are applied to the control theory, when the controller or the controlled system is described by fractional calculus [7–11]. In 1996, Oustaloup et al. developed the first fractional-order controller which is the

so-called CRONE [12]. Then some fractional-order control strategies are proposed one after another, such as fractional-order PID controllers [13], fractional-order sliding-mode controllers [14–16], fractional-order optimal controllers [17], and fractional-order adaptive controllers [18, 19].

DC/DC power converters, which work in switch mode, are applied in a wide variety of applications, including DC motor drives, active filters, computers, power supplies, and medical instrumentation. The Buck converter is one of the simplest but most useful power converters that can convert a DC input to a DC output at a lower voltage. There are some other basic DC/DC converters, namely, the Boost, Buck-Boost, Cuk, Zeta, and Sepic. Each of these converters consists of the passive power switch, the active power switch, and the storage elements. The main objective of most closed-loop feedback controlled DC/DC converters is to ensure that the converter operates with fast dynamical response, small steady-state output error, and low overshoot, while maintaining high efficiency and low noise emission in terms of rejection of input voltage changes, parameter

uncertainties, and load variations [20]. The designing of high performance control strategy is always a challenge, because DC/DC converters are inherently time-varying variable-structure nonlinear systems. Nonlinear control strategies [21–23] are better candidates in DC/DC converters than other linear ones. Among the nonlinear control strategies, the sliding-mode control (SMC) has been well known due to its fast dynamic response, robustness to disturbances, guaranteed stability, parameter variations, and simplicity in implementation. Moreover, compared with other nonlinear control strategies, the SMC method is easy to implement and has a high degree of flexibility in the designing process.

The sliding-mode control is an effective robust control strategy with the feature of switching the control law to force the state trajectories of the system from the initial states onto some predefined sliding surface which exhibit desired dynamics. When in the sliding mode, the closed-loop response becomes totally insensitive to both internal parameter uncertainties and external disturbances. Compared with the SMC, the terminal sliding-mode control (TSMC) has some superior properties, such as the state of the system which converges in finite time and the higher steady-state tracking accuracy. Near the equilibrium point the rate of the convergence is being speeded up, so the TSMC is more suitable for high precision control. In recent years, the combination of fractional-order control (FOC) and sliding-mode control (SMC) has been gaining more and more interests from the systems control community. An introductory work on the application in fractional-order sliding mode is reported in [24]; the double integrator is introduced as the plant under control. A fuzzy fractional-order sliding-mode controller is adopted to control nonlinear systems in [25]. In [26], fractional-order sliding-mode control strategies for power electronic Buck converters are presented, where pulse-width modulation (PWM) sets the basis for the regulation of switched mode converters. In [27], authors collect different methods to apply FOC in SMC through the use of fractional-order surfaces which are fractional-order PID or fractional-order PI and propose a direct Boolean control strategy in order to avoid using PWM.

In this paper, we focus on the introduction of the FOC in TSMC for switching systems. The fractional-order terminal sliding-mode control (FTSMC) method has been adopted for controlling the Buck converter. The idea of such method is used to design a novel nonlinear sliding surface function which is a fusion of characteristics of TSMC and FOC. Based on the Lyapunov stability theory, a fractional-order sliding-mode control law is derived to assure finite time convergence of the output voltage error to the equilibrium point. Through mathematical analysis, we obtain the finite time t_s^{FTSMC} of FTSMC. It is shown that TSMC is a special case of FTSMC. The theoretical considerations have been verified by numerical simulations and a Buck converter application is given to show the superiority of the proposed strategy. The fractional terminal sliding-mode control exhibits considerable improvement in terms of a faster output voltage response during load changes. FTSMC can overcome the influence which disturbances bring about to control system and guarantee that DC/DC converter keeps good dynamic

and steady performances. The output can follow the given well, and the disturbances almost do not affect the output.

The rest of this paper is organized as follows. In Section 2, some basic concept and definitions of the fractional calculus are introduced. Then the basic principle of the Buck converter in CCM is introduced. In Section 3, the integer-order TSMC is briefly reviewed. In Section 4, FTSMC is described for the Buck converter. In Section 5, numerical simulation results are presented to verify the theoretical considerations and an application to Buck converter is given to show the superiority of the proposed strategy. Finally, some concluding remarks of this paper are drawn in Section 6.

2. Preliminaries

In this section, firstly, some basic concept and definitions of fractional calculus are introduced. Afterwards, a brief introduction to the basic principle of the Buck converter in CCM is presented.

2.1. The Basis of Fractional Calculus. In fractional calculus, the operator ${}_a D_t^\alpha$ is the differintegral operator which takes both integrals and derivatives in one single expression. It denotes generalization of integrals and derivatives to arbitrary order and can be defined as follows:

$${}_a D_t^\alpha = \begin{cases} \frac{d^\alpha}{dt^\alpha}, & \text{Re}[\alpha] > 0 \\ 1, & \text{Re}[\alpha] = 0 \\ \int_a^t (d\tau)^{-\alpha}, & \text{Re}[\alpha] < 0, \end{cases} \quad (1)$$

where $\alpha \in \mathbb{R}$ can be an arbitrary real number, which denotes the order of the operation, and a and t are the lower and upper limits.

Definition 1. In the fractional calculus, the Gamma function proposed by Euler is defined by the integral

$$\Gamma(z) = \int_0^\infty e^{-t} t^{z-1} dt, \quad (2)$$

which satisfies $\text{Re}(z) > 0$, and converges in the right half of the complex plane.

Several reputed definitions for fractional derivatives are put forward including Riemann-Liouville definition, Grunwald-Letnikov definition, Caputo definition, Weyl definition, and Marchaud definition. Among them, being the best-known one, Riemann-Liouville definition is precisely studied.

Definition 2 (see [7]). Riemann-Liouville definition of the α th-order fractional derivative operator ${}_0 D_t^\alpha$ is given by

$${}_0 D_t^\alpha f(t) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \int_0^t (t-\tau)^{-\alpha} f(\tau) d\tau, \quad (3)$$

where $0 < \alpha < 1$.

FIGURE 1: (a) Buck converter; (b) the switch S is turned on; (c) the switch S is turned off.

Definition 3 (see [7]). Riemann-Liouville definition of the α th-order fractional derivative operator ${}_0D_t^{-\alpha}$ is defined by

$${}_0D_t^{-\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} f(\tau) d\tau. \quad (4)$$

According to this definition, the derivative of a time function with $f(t) = t^\beta$, $\beta > -1$, is evaluated as

$${}_0D_t^\alpha t^\beta = \frac{\Gamma(\alpha+1)}{\Gamma(\alpha-\beta+1)} t^{\beta-\alpha}. \quad (5)$$

Definition 4 (see [7]). If the fractional derivative ${}_0D_t^\alpha f(t)$ ($k-1 < \alpha < k$) and a function $f(t)$ are integrable, then

$${}_aD_t^{-\alpha} ({}_0D_t^{-\alpha} f(t)) = f(t) - \sum_{j=1}^k \left[{}_0D_t^{\alpha-j} f(t) \right]_{t=a} \frac{(t-a)^{\alpha-j}}{\Gamma(\mu-j+1)}. \quad (6)$$

2.2. Brief Introduction of the Buck Converter in CCM. The Buck converter, sometimes called a step-down power stage, is a popular nonisolated power stage topology. A simplified schematic of the Buck converter is shown in Figure 1(a).

The basic operation of the Buck converter can be regarded as piecewise switched dynamical systems, involving toggling among a set of linear or nonlinear circuit topologies. In continuous conduction mode (CCM), there are two switching modes. (1) When the switch (labeled as S in Figure 1(b)) is turned on, the inductor current passes through the switch, and the diode is reverse-biased with the inductor current i_L ramping up. (2) When the switch (labeled as S in Figure 1(c)) is turned off, the inductor maintains current flow in the same direction so that the diode is forward-biased. And this causes the inductor current to ramp down. The process repeats cyclically. For the operation described above, we have two state equations as follows.

When S is turned on,

$$\begin{bmatrix} \frac{di_L(t)}{dt} \\ \frac{dv_o(t)}{dt} \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{CR} \end{bmatrix} \begin{bmatrix} i_L(t) \\ v_o(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} v_{in}(t). \quad (7)$$

When S is turned off,

$$\begin{bmatrix} \frac{di_L(t)}{dt} \\ \frac{dv_o(t)}{dt} \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{L} \\ -\frac{1}{C} & -\frac{1}{CR} \end{bmatrix} \begin{bmatrix} i_L(t) \\ v_o(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} v_{in}(t). \quad (8)$$

Combining (7) and (8) gives

$$\begin{bmatrix} \frac{di_L(t)}{dt} \\ \frac{dv_o(t)}{dt} \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{CR} \end{bmatrix} \begin{bmatrix} i_L(t) \\ v_o(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} uv_{in}(t), \quad (9)$$

where u is the control input. The switch is in the ON state for $u = 1$ and in the OFF state for $u = 0$.

Define the output voltage error x_1 as follows:

$$x_1 = v_o - V_{ref}, \quad (10)$$

where V_{ref} is the reference value of the output voltage. By taking the time derivative of (10), x_2 which is the rate of change of voltage error can be expressed as

$$x_2 = \dot{x}_1 = \dot{v}_o. \quad (11)$$

Then the dynamics of x_2 can be given as

$$\dot{x}_2 = \frac{1}{LC} (uV_{in} - V_{ref} - x_1) - \frac{x_2}{RC}. \quad (12)$$

So, it can be concluded that

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{LC} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ uV_{in} - V_{ref} \end{bmatrix} \frac{1}{LC}. \quad (13)$$

3. The Integer-Order Terminal Sliding-Mode Control for the Buck Converter

The conventional TSM concept can be described as

$$s = \alpha x_1^\beta + x_2, \quad (14)$$

where α is a designed positive constant, $\beta = q/p$, and p and q are both positive odd integers which satisfy the following condition: $0 < q/p < 1$. When the system is in the terminal sliding mode ($s = 0$), we can get its dynamics from (14):

$$\dot{x}_1 = -\alpha x_1^\beta. \quad (15)$$

Rewriting (15) yields

$$dt = -\frac{1}{\alpha x_1^\beta} dx. \quad (16)$$

Given any initial state $x_1(0) \neq 0$ and $x_1(t_s) = 0$ which is the terminal attractor of the system. Taking integral of both sides of (16), the finite time is t_s obtained by

$$s = \alpha x_1^\beta + {}_{t_0}D_t^{\mu-1} x_2. \quad (17)$$

This means that state x_1 can converge to zero in a finite time. In other words, the term “terminal” referred to equilibrium zero and can be reached in finite time and it is stable:

$$\begin{aligned} \dot{s} &= \alpha \beta x_1^{\beta-1} \dot{x}_1 + \dot{x}_2 \\ &= \alpha \beta x_1^{\beta-1} x_2 + \frac{1}{LC} (uV_{in} - V_{ref} - x_1) - \frac{x_2}{RC} = 0. \end{aligned} \quad (18)$$

The equivalent control can be expressed as

$$u_{eq} = \left(\frac{x_2}{RC} - \alpha \beta x_1^{\beta-1} x_2 \right) \frac{LC}{V_{in}} + \frac{(V_{ref} + x_1)}{V_{in}}. \quad (19)$$

4. The Fractional-Order Terminal Sliding-Mode Control Method for the Buck Converter

Theorem 5. In order to introduce the so-called fractional-order terminal sliding-mode model, a fractional-order sliding surface function can be defined as

$$s = \alpha x_1^\beta + {}_{t_0}D_t^{\mu-1} x_2, \quad (20)$$

where $\mu \in (0, 1]$, α is a designed positive constant, $\beta = q/p$, and p and q are both positive odd integers which satisfy the following condition: $0 < q/p < 1$. For Buck converter with FTSMC, if the control is designed as

$$\begin{aligned} u &= \left[\frac{x_2}{RC} - D^{1-\mu} (\alpha \beta x_1^{\beta-1} x_2) \right] \frac{LC}{V_{in}} + \frac{(V_{ref} + x_1)}{V_{in}} \\ &\quad - K \operatorname{sign}(s) - Ks, \end{aligned} \quad (21)$$

where $0 < \beta < 1$, $k > 0$, $0 < \mu < 1$, then states x_1 and x_2 will converge to zero in finite time. Furthermore, the system will be in the terminal sliding mode.

Proof. Select the Lyapunov function as

$$V = \frac{1}{2} s^2. \quad (22)$$

Taking the derivative of (22),

$$\dot{V} = s\dot{s} = s(\alpha \beta x_1^{\beta-1} \dot{x}_1 + D^{\mu-1} \dot{x}_2). \quad (23)$$

Substituting (12) into (23) results in

$$\begin{aligned} \dot{V} &= s \alpha \beta x_1^{\beta-1} x_2 \\ &\quad + s D^{\mu-1} \left[\frac{1}{LC} (uV_{in} - V_{ref} - x_1) - \frac{x_2}{RC} \right]. \end{aligned} \quad (24)$$

Substituting (21) into (24) results in

$$\dot{V} = s(-K \operatorname{sign}(s) - Ks) = -K(|s| + s^2). \quad (25)$$

Because the controller parameter K is positive, we can find $V > 0$ and $\dot{V} < 0$. In other words, the controlled system satisfies the reaching condition.

When the system reaches the sliding surface ($s = 0$), it is in the terminal sliding mode. Its dynamics can be determined by the following equation:

$${}_0D_{t_s}^{\mu-1} \dot{x}_1 = -\alpha {}_0x_1^\beta. \quad (26)$$

Taking the concept of fractional integral and derivative operators into account, one obtains

$${}_0D_{t_s}^{1-\mu} ({}_0D_{t_s}^{\mu-1} \dot{x}_1) = -\alpha {}_0D_{t_s}^{1-\mu} x_1^\beta. \quad (27)$$

From (6), we can get

$$\begin{aligned} \dot{x}_1(t) - [{}_0D_t^{\mu-2} \dot{x}_1(t)]_{t=0} \frac{(t-t_r)^{\mu-2}}{\Gamma(\mu-1)} \\ = -\alpha {}_0D_{t_s}^{1-\mu} x_1^\beta(t), \end{aligned} \quad (28)$$

where $[{}_0D_t^{\mu-2} \dot{x}_1(t)]_{t=0}((t-t_r)^{\mu-2}/\Gamma(\mu-1)) = 0$. So, it can be concluded that

$$\dot{x}_1(t) = -\alpha {}_0D_{t_s}^{1-\mu} x_1^\beta(t). \quad (29)$$

According to Definition 3, we have

$$D^{1-\mu} x_1^\beta(t) = \frac{\Gamma(\beta+1)}{\Gamma(\beta+\mu)} x_1^{\beta+\mu-1}(t). \quad (30)$$

Substituting (30) into (29) results in

$$\dot{x}_1(t) = -\frac{\alpha \Gamma(\beta+1)}{\Gamma(\beta+\mu)} x_1^{\beta+\mu-1}(t). \quad (31)$$

Equation (31) can be rewritten as

$$dx_1(t) = -\frac{\alpha \Gamma(\beta+1)}{\Gamma(\beta+\mu)} x_1^{\beta+\mu-1}(t) dt. \quad (32)$$

Taking integral of both sides of (32),

$$\int_0^{t_s} x_1^{1-\beta-\mu}(t) dx_1(t) = -\frac{\alpha \Gamma(\beta+1)}{\Gamma(\beta+\mu)} \int_0^{t_s} dt. \quad (33)$$

The finite time t_s is obtained by

$$t_s^{\text{FTSMC}} = \frac{\Gamma(\beta+\mu) x(0)^{2-\mu-\beta}}{\alpha \Gamma(\beta+1) (2-\mu-\beta)}. \quad (34)$$

FIGURE 2: Basic structure of an FTMSC Buck converter.

TABLE 1: SMC parameters.

Description	α	β	μ
SMC	100	—	—
TSMC	100	0.6	—
FTSMC1	100	0.6	0.9
FTSMC2	100	0.6	0.8
FTSMC3	100	0.6	0.7
FTSMC4	100	0.6	0.6

Therefore, it can be concluded that system trajectories can reach the equilibrium point in finite time. When $\mu = 1$, it is obvious that t_s^{TSMC} is equivalent to t_s^{FTSMC} . It means that the finite time taken to attain to the equilibrium point of the FTSMC system is the same as the one of the TSMC system as given in (17). $\square$

5. Simulation Results

The block diagram of the DC/DC Buck converter with the proposed FTSMC method is depicted in Figure 2. In order to show the performance of the proposed methods, the DC/DC Buck converter system has been tested by simulations. Simulations are carried out using Matlab/Simulink. Parameters of Buck converter are given in Abbreviations.

In other literatures, the influences of parameters α and β have been studied. This paper focuses on the influence of the order parameter μ on the control effect. Table 1 shows the parameters of the controller.

As shown in Figure 3, when $\alpha = 100$ and $\beta = 0.6$, the value range of μ is between 0 and 1. $\mu - T_s$ curve is not a monotonic function. When the value of μ is about 0.8, T_s get the minimum value. The parameter μ increases the degree of freedom of the original terminal sliding-mode controller. The sliding-mode surface is selected by the original 2 parameters (α, β) and becomes the three parameters (α, β, μ).

Figure 4 shows the simulated start-up and transient responses of the output voltage obtained by FTSMC strategies with different μ values. It is interesting to note that the output

FIGURE 3: T_s responses due to the changes in μ . FIGURE 4: Output voltage responses due to the changes in μ .

voltage responses become faster with decreasing the value of μ . Therefore, the value of μ is chosen to make compromise between start-up and transient responses of the converter. With the decrease of μ , the output voltage response is gradually reduced. When $\mu = 0.7$, the overshoot of the system appears. The reason is that, with the decrease of μ , the integral weight is gradually increasing; that is, the cumulative effect on the time scale increases and then affects the system overshoot, which also verifies the influence of μ on T_s as shown in Figure 3. When $\mu = 0.6$, the overshoot of the system exceeds 20%.

In order to compare the control effect of TSMC, FTSMC, and SMC, select $\alpha = 100$, $\beta = 0.6$, and $\mu = 0.8$. As shown in Figure 5, the response time of the system with FTSMC is less than others. At $t = 0.15$ s, the load resistance is changed from 2Ω to 8Ω . Because the load is lighter, the output current will be reduced. Then the output voltage has a short step-up. It can be seen that the output voltage returns faster to 5 V in FTSMC.

FIGURE 5: Output voltage responses due to the step changes in R from $2\ \Omega$ to $8\ \Omega$ at $t = 0.15$ s.

FIGURE 6: Output voltage responses due to the step changes in R from $8\ \Omega$ to $2\ \Omega$ at $t = 0.15$ s.

Figure 6 shows the output voltage V_o responses due to the step changes in R from $8\ \Omega$ to $2\ \Omega$. We can find the same results as above. The control effect of FTSMC is better.

It is obvious that the FTSMC method acts faster than others in correcting the output voltage when a step change takes place in V_{in} . On the other hand, as shown in Figure 7, it is clear that SMC and TSMC result in a slight steady-state error. By contrast, the FTSMC method has the lesser steady-state error.

FIGURE 7: Output voltage responses due to the step changes in V_{in} from 10 V to 9 V at $t = 0.15$ s.

6. Conclusions

This paper presents the fractional-order terminal sliding-mode control (FTSMC) which has a new fractional-order surface and assures the finite time convergence of the output voltage error to the equilibrium point during the load changes. It introduces a novel tuning parameter, the order μ , which allows the adjustment of the system output accordingly. Through mathematical analysis, we obtain the finite time of fractional-order terminal sliding-mode control. It is shown that when the order μ of FTSMC is one, the finite time taken to attain to the equilibrium point of the FTSMC system is the same as the one of the TSMC system. In other words, it means that TSMC is a special case of FTSMC. In addition, simulation results for a Buck converter are given to show the good performance of the FTSMC in comparison with the use of the traditional TSMC and SMC. The novel fractional terminal sliding-mode control exhibits considerable improvement in terms of a faster output voltage response during load changes.

Abbreviations

Specifications of Buck Converter

- V_{in} : Input voltage (10 V)
- C : Capacitance ($1000\ \mu\text{F}$)
- L : Inductance (1 mH)
- R_{min} : Minimum load resistance ($2\ \Omega$)
- R_{max} : Maximum load resistance ($8\ \Omega$)
- V_o : Desired output voltage (5 V).

Competing Interests

The authors declare that they have no competing interests.

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