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Research Article

Robust Stability Criteria for T-S Fuzzy Systems with Time-Varying Delays via Nonquadratic Lyapunov-Krasovskii Functional Approach

Sung Hyun Kim

School of Electrical Engineering, University of Ulsan, Daehak-ro 93, Nam-Gu, Ulsan 680-749, Republic of Korea

Correspondence should be addressed to Sung Hyun Kim; ie222.kim@gmail.com

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This paper tackles the issue of stability analysis for uncertain T-S fuzzy systems with interval time-varying delays, especially based on the nonquadratic Lyapunov-Krasovskii functional (NLKF). To this end, this paper first provides a less conservative relaxation technique and then derives a relaxed robust stability criterion that enhances the interactions among delayed fuzzy subsystems. The effectiveness of our method is verified by two examples.

1. Introduction

Over the past few decades, Takagi-Sugeno (T-S) fuzzy model has attracted great attention since it can systematically represent nonlinear systems via a kind of interpolation method that connects smoothly some local linear systems based on fuzzy weighting functions [1]. In particular, the T-S fuzzy model has the advantage that it allows the well-established linear system theory to be applied to the analysis and synthesis of nonlinear systems. For this reason, the T-S fuzzy model has been a popular choice not only in consumer products but also in industrial processes (refer to [2] and references therein).

As well-known, time-delay phenomena are ubiquitous in practical engineering systems such as aircraft systems, biological systems, and chemical engineering system [3–5]. Recently, thus, the research on nonlinear systems with state delays has been an important issue in the stability analysis of T-S fuzzy systems. In the literature, there are two major research trends to deal with such systems: one focuses on decreasing computational burdens required to solve a set of conditions from the Lyapunov-Krasovskii functional (LKF) approach, and the other focuses on improving the solvability of delay-dependent stability conditions despite significant computational efforts. Strictly speaking, the first trend is mainly based on Jensen's inequality approach [6–11] and the second one is based on the free-weighing matrix approach [12–16].

Recently, it is recognized that the common quadratic Lyapunov function approach leads to overconservative performance for a large number of fuzzy rules [17, 18]. For this reason, it is essential to tackle the issue of stability analysis in the light of the nonquadratic Lyapunov-Krasovskii functional (NLKF) [19–23]. However, to our best knowledge, up to now, little progress has been made toward using NLKFs for the stability analysis. Motivated by the above concern, this paper proposes a relaxed stability criterion for uncertain T-S fuzzy systems with interval time-varying delays, especially obtained by the NLKF approach. To this end, this paper offers a proper relaxation method that can enhance the interactions among delayed fuzzy subsystems. Further, it is worth noticing that Jensen's inequality, given in [24], is applicable only to the case where the internal matrix is constant, that is, to the case where the common quadratic Lyapunov-Krasovskii functional (CQLKF) is employed. Thus, this paper focuses more on exploring the second trend in the direction of reducing the conservatism that stems from the CQLKF approach, without resorting to any delay-decomposition method. In this sense, this paper provides two examples numerically to show the effectiveness of our method.

The rest of the paper is organized as follows. Section 2 gives a mathematical description of the system considered here and presents a useful lemma. Section 3 presents the main result of this paper. Furthermore, through numerical examples, Section 4 shows the verification of our results. Finally, Section 5 makes the concluding remarks.

Notation. Throughout this paper, standard notions will be adopted. The notations $X \geq Y$ and $X > Y$ mean that $X - Y$ is positive semidefinite and positive definite, respectively. In symmetric block matrices, $(*)$ is used as an ellipsis for terms that are induced by symmetry. For a square matrix $\mathcal{X}$, $\mathrm{He}(\mathcal{X})$ denotes $\mathcal{X} + \mathcal{X}^T$, where $\mathcal{X}^T$ is the transpose of $\mathcal{X}$. The natation $\mathrm{Conv}(\cdot)$ denotes the convex hull; $\mathrm{col}(v_1, v_2, \ldots, v_n) = \begin{bmatrix} v_1^T & v_2^T & \cdots & v_n^T \end{bmatrix}^T$ for any vector v_i; $\mathrm{diag}(\mathscr{A}, \mathscr{B})$ denotes a diagonal matrix with diagonal entries $\mathscr{A}$ and $\mathscr{B}$; and $\mathbb{N}_r^+ = \{1, 2, \ldots, r\}$. For any matrix $\mathcal{S}_i$ or $\mathcal{S}_{ij}$,

$$\begin{aligned} \left[\mathcal{S}_i\right]_r &= \left[\mathcal{S}_1 \ \ \mathcal{S}_2 \ \ \cdots \ \ \mathcal{S}_r\right], \\ \left[\mathcal{S}_{ij}\right]_{r\times r} &= \left[\left[\mathcal{S}_{1i}\right]_r^T \ \ \left[\mathcal{S}_{2i}\right]_r^T \ \ \cdots \ \ \left[\mathcal{S}_{ri}\right]_r^T\right]^T. \end{aligned} \tag{1}$$

All matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operation.

2. System Description and Preliminaries

Consider the following uncertain T-S fuzzy system, which represents a class of nonlinear systems: for $i \in \mathbb{N}_r^+$,

Plant Rule i. IF $\eta_1(t)$ is $\mathscr{F}_{i1}$ and $\cdots$ $\eta_s(t)$ is $\mathscr{F}_{is}$, THEN

$$\begin{aligned} \dot{x}(t) &= A_i x(t) + A_{d,i} x(t - d(t)) + E_i p(t), \\ q(t) &= G_i x(t) + G_{d,i} x(t - d(t)), \\ x(t) &= \psi(t), \\ & \qquad t \in [-d_2, 0], \end{aligned} \tag{2}$$

where $x(t) \in \mathbb{R}^{n_x}$ and $x(t - d(t)) \in \mathbb{R}^{n_x}$ denote the state and the delayed state, respectively; the initial condition $\psi(t)$ is a continuously differentiable vector-valued function; $\mathscr{F}_{ij}$ denotes a fuzzy set; $\eta_i(t)$ denotes the ith premise variable; and r denotes the number of IF-THEN rules. In (2), $p(t) \in \mathbb{R}^{n_p}$ and $q(t) \in \mathbb{R}^{n_q}$ are used to describe the structured feedback uncertainty such that $p(t) = \Delta(t) q(t)$ and $\Delta^T(t)\Delta(t) \leq I \in \mathbb{R}^{n_q \times n_q}$. Further, the state delay $d(t)$ is assumed to be unknown and time-varying with known bounds as follows: $d_1 \leq d(t) \leq d_2$, where d_1 and d_2 are constant. Then, the overall T-S fuzzy model is inferred as follows:

$$\begin{aligned} \dot{x}(t) &= A(\Theta_t) x(t) + A_d(\Theta_t) x(t - d(t)) \\ &\quad + E(\Theta_t) p(t), \\ q(t) &= G(\Theta_t) x(t) + G_d(\Theta_t) x(t - d(t)), \end{aligned} \tag{3}$$

where $A(\Theta_t) = \sum_{i=1}^r \theta_i A_i$, $A_d(\Theta_t) = \sum_{i=1}^r \theta_i A_{d,i}$, $E(\Theta_t) = \sum_{i=1}^r \theta_i E_i$, $G(\Theta_t) = \sum_{i=1}^r \theta_i G_i$, and $G_d(\Theta_t) = \sum_{i=1}^r \theta_i G_{d,i}$ in which θ_i $(=\theta_i(\eta(t))$ denotes the normalized fuzzy weighting function for the ith rule; $\eta(t) = \mathrm{col}(\eta_1(x(t)), \ldots, \eta_s(x(t)))$ denotes the premise variable vector; and $\Theta_t = \mathrm{col}(\theta_1, \ldots, \theta_r)$ belongs to

$$\mathbb{S}_\Theta = \left\{\mathrm{col}(\theta_1, \ldots, \theta_r) \mid \sum_{i=1}^r \theta_i = 1,\ \alpha_i \leq \theta_i \leq \beta_i,\ \forall i \in \mathbb{N}_r^+\right\}. \tag{4}$$

Assumption 1. The fuzzy weighting functions θ_i are differentiable and $\dot{\Theta}_t = \mathrm{col}(\dot{\theta}_1, \ldots, \dot{\theta}_r)$ belongs to

$$\mathbb{S}_{\dot{\Theta}} = \left\{\mathrm{col}(\dot{\theta}_1, \ldots, \dot{\theta}_r) \mid \sum_{i=1}^r \dot{\theta}_i = 0,\ \varrho_{1,i} \leq \dot{\theta}_i \leq \varrho_{2,i},\ \forall i \in \mathbb{N}_r^+\right\}. \tag{5}$$

To simplify the notations, we use $\theta_i^{d_1} = \theta_i(\eta(t - d_1))$ and $\theta_i^{d_2} = \theta_i(\eta(t - d_2))$. And, for later convenience, we define $\overline{x}(t) = \mathrm{col}(x(t), x(t - d_1), x(t - d(t)), x(t - d_2))$, $\eta(t) = \mathrm{col}(\overline{x}(t), p(t)) \in \mathbb{R}^{n_\eta}$, and $n_\eta = 4n_x + n_p$. And we use some block entry matrices $\mathbf{e}_i$ $(i = 1, 2, \ldots, 5)$ such that $x(t) = \mathbf{e}_1 \eta(t)$, $x(t - d_1) = \mathbf{e}_2 \eta(t)$, $x(t - d(t)) = \mathbf{e}_3 \eta(t)$, $x(t - d_2) = \mathbf{e}_4 \eta(t)$, and $p(t) = \mathbf{e}_5 \eta(t)$, which implies $\overline{x}(t) = \mathbf{e}_{14} \eta(t)$ by defining $\mathbf{e}_{14}^T = \begin{bmatrix} \mathbf{e}_1^T & \cdots & \mathbf{e}_4^T \end{bmatrix}$. Then, (3) becomes

$$\begin{aligned} \dot{x}(t) &= \Phi_t \eta(t), \\ q(t) &= \Psi_t \eta(t), \end{aligned} \tag{6}$$

where $\Phi_t = A(\Theta_t)\mathbf{e}_1 + A_d(\Theta_t)\mathbf{e}_3 + E(\Theta_t)\mathbf{e}_5$ and $\Psi_t = G(\Theta_t)\mathbf{e}_1 + G_d(\Theta_t)\mathbf{e}_3$.

Lemma 2. *Let $\Theta_t \in \mathbb{S}_\Theta$ be satisfied. Then, the following condition holds:*

$$\begin{aligned} 0 &> \mathscr{M} \\ &= \mathscr{M}_0 + \sum_{i=1}^r \theta_i \mathrm{He}(\mathscr{M}_i) + \sum_{i=1}^r \theta_i^2 \mathscr{M}_{ii} \\ &\quad + \sum_{i=1}^r \left(\sum_{j=i+1}^r \theta_i \theta_j \mathscr{M}_{ij} + \sum_{j=1}^{i-1} \theta_i \theta_j \mathscr{M}_{ji}^T\right) \end{aligned} \tag{7}$$

if there are all decision variables such that

$$\begin{aligned} 0 &> \mathscr{L} = \left[\begin{array}{c|c} \mathscr{L}_0 & [\mathscr{L}_i]_r \\ \hline (*) & [\mathscr{L}_{ij}]_{rr} \end{array}\right], \\ 0 &< X_i + X_i^T, \\ & \qquad \forall i \in \mathbb{N}_r^+, \end{aligned} \tag{8}$$

where $\mathscr{L}_0 = \mathscr{M}_0 + \mathrm{He}(S_0 - \sum_{i=1}^r \alpha_i \beta_i X_i)$, $\mathscr{L}_i = \mathscr{M}_i + S_i - S_0 + (\alpha_i + \beta_i) X_i$, $\mathscr{L}_{ii} = \mathscr{M}_{ii} + \mathrm{He}(-S_i - X_i)$, and $\mathscr{L}_{ij} = \mathscr{M}_{ij} - S_i - S_j$.

Lemma 3. *Let $\dot{\Theta}_t \in \mathbb{S}_{\dot{\Theta}}$ be satisfied. Then, the following condition holds:*

$$0 > \Omega + \sum_{i=1}^{r} \dot{\theta}_i \mathcal{P}_i \tag{9}$$

if there are all decision variables such that

$$\begin{aligned} & 0 > \Omega + \sum_{i=1}^{r} \varrho_{\ell_i,i} \left(\mathcal{P}_i + \mathcal{Q}\right), \\ & \qquad \forall \ell \in \mathbb{L} = \left\{(\ell_1, \ldots, \ell_r) \mid \ell_i \in \mathbb{N}_2^+,\ i \in \mathbb{N}_r^+\right\}. \end{aligned} \tag{10}$$

Proof. In view of $\dot{\Theta}_t \in \mathbb{S}_{\dot{\Theta}}$, we can get

$$\begin{aligned} \dot{\theta}_i &= \sum_{\ell_i=1}^{2} \lambda_{\ell_i}(t)\, \varrho_{\ell_i,i}, \\ 0 &= \sum_{i=1}^{r} \dot{\theta}_i \mathcal{N}, \end{aligned} \tag{11}$$

where coefficients λ_{ℓ_i} are all positive and sum to one and $\mathcal{N}$ is a constant slack variable. Then, (9) leads to

$$0 > \Omega + \sum_{i=1}^{r} \left(\sum_{\ell_i=1}^{2} \lambda_{\ell_i}(t)\, \varrho_{\ell_i,i} \right) \left(\mathcal{P}_i + \mathcal{N}\right), \tag{12}$$

which holds if (10) holds because $\sum_{\ell_i=1}^{2} \lambda_{\ell_i}(t) \varrho_{\ell_i,i}(\mathcal{P}_i + \mathcal{N}) \in \mathrm{Conv}(\varrho_{\ell_i,i}(\mathcal{P}_i + \mathcal{N}))$, where ℓ_i denotes the ith element of $\ell \in \mathbb{L}$. □

3. Θ_t-Dependent Stability Criterion

Based on a nonquadratic Lyapunov-Krasovskii functional (NLKF), this section provides a less conservative stability criterion. To this end, we first choose an NLKF of the following form:

$$\begin{aligned} V(t) &= V_1(t) + V_2(t) + V_3(t), \\ V_1(t) &= x^T(t) P(\Theta_t) x(t), \\ V_2(t) &= \int_{t-d_1}^{t} x^T(\alpha) Q_1(\Theta_\alpha) x(\alpha)\, d\alpha \\ &\quad + \int_{t-d_2}^{t} x^T(\alpha) Q_2(\Theta_\alpha) x(\alpha)\, d\alpha, \\ V_3(t) &= \int_{-d_1}^{0} \int_{t+\alpha}^{t} \dot{x}^T(\beta) R_1(\Theta_\beta) \dot{x}(\beta)\, d\beta\, d\alpha \\ &\quad + \int_{-d_2}^{-d_1} \int_{t+\alpha}^{t} \dot{x}^T(\beta) R_2(\Theta_\beta) \dot{x}(\beta)\, d\beta\, d\alpha, \end{aligned} \tag{13}$$

where $P(\Theta_t)$, $Q_1(\Theta_\alpha)$, $Q_2(\Theta_\alpha)$, $R_1(\Theta_\beta)$, and $R_2(\Theta_\beta)$ are positive definite for all admissible grades. Then, the time derivative of each $V_i(t)$ along the trajectories of (6) is given by

$$\begin{aligned} \dot{V}_1(t) &= \eta^T(t) \left(\mathrm{He}\left(\mathbf{e}_1^T P(\Theta_t) \Phi_t\right) + \mathbf{e}_1^T \dot{P}(\Theta_t) \mathbf{e}_1\right) \eta(t), \\ \dot{V}_2(t) &= \eta^T(t) \left(\mathbf{e}_1^T \left(Q_1(\Theta_t) + Q_2(\Theta_t)\right) \mathbf{e}_1 \right. \\ &\quad \left. - \mathbf{e}_2^T Q_1\left(\Theta_{t-d_1}\right) \mathbf{e}_2 - \mathbf{e}_4^T Q_2\left(\Theta_{t-d_2}\right) \mathbf{e}_4\right) \eta(t), \\ \dot{V}_3(t) &= \eta^T(t) \left(d_1 \Phi_t^T R_1(\Theta_t) \Phi_t + \delta \Phi_t^T R_2(\Theta_t) \Phi_t\right) \\ &\quad \cdot \eta(t) - \int_{t-d_1}^{t} \dot{x}^T(\alpha) R_1(\Theta_\alpha) \dot{x}(\alpha)\, d\alpha \\ &\quad - \int_{t-d_2}^{t-d_1} \dot{x}^T(\alpha) R_2(\Theta_\alpha) \dot{x}(\alpha)\, d\alpha, \end{aligned} \tag{14}$$

which leads to

$$\dot{V}(t) = \eta^T(t) \Pi_1 \eta(t) + \mathcal{O}_1 + \mathcal{O}_2, \tag{15}$$

where

$$\begin{aligned} \Pi_1 &= \mathrm{He}\left(\mathbf{e}_1^T P(\Theta_t) \Phi_t\right) \\ &\quad + \mathbf{e}_1^T \left(\dot{P}(\Theta_t) + Q_1(\Theta_t) + Q_2(\Theta_t)\right) \mathbf{e}_1 \\ &\quad - \mathbf{e}_2^T Q_1\left(\Theta_{t-d_1}\right) \mathbf{e}_2 - \mathbf{e}_4^T Q_2\left(\Theta_{t-d_2}\right) \mathbf{e}_4 \\ &\quad + d_1 \Phi_t^T R_1(\Theta_t) \Phi_t + \delta \Phi_t^T R_2(\Theta_t) \Phi_t, \\ \mathcal{O}_1 &= -\int_{t-d_1}^{t} \dot{x}^T(\alpha) R_1(\Theta_\alpha) \dot{x}(\alpha)\, d\alpha, \quad \delta = d_2 - d_1, \\ \mathcal{O}_2 &= -\int_{t-d_2}^{t-d(t)} \dot{x}^T(\alpha_1) R_2\left(\Theta_{\alpha_1}\right) \dot{x}(\alpha_1)\, d\alpha_1 \\ &\quad - \int_{t-d(t)}^{t-d_1} \dot{x}^T(\alpha_2) R_2\left(\Theta_{\alpha_2}\right) \dot{x}(\alpha_2)\, d\alpha_2. \end{aligned} \tag{16}$$

Remark 4. Indeed, it is hard to directly use Jensen's inequality approach to obtain the upper bounds of $\mathcal{O}_1$ and $\mathcal{O}_2$ because $R_1(\Theta_\alpha)$ and $R_2(\Theta_\alpha)$ are set to be dependent on Θ_α, which motivates the present study.

Lemma 5. *Suppose that there exist matrices $U_0(\Theta_t)$, $U_1(\Theta_t)$, and $U_2(\Theta_t) \in \mathbb{R}^{4n_x \times n_x}$ and symmetric matrices $0 < P(\Theta_t)$, $\dot{P}(\Theta_t)$, $0 < Q_1(\Theta_{t-d_1})$, $0 < Q_2(\Theta_{t-d_2})$, $0 < Q_1(\Theta_t)$, $0 < Q_2(\Theta_t)$, $0 < R_1(\Theta_\alpha)$, $0 < R_2(\Theta_{\alpha_p})$, $0 < R_1(\Theta_t)$, $0 < R_2(\Theta_t) \in \mathbb{R}^{n_x \times n_x}$, $M_0(\Theta_t)$, $M_1(\Theta_t)$, and $M_2(\Theta_t) \in \mathbb{R}^{4n_x \times 4n_x}$ such that*

$$0 > \Pi_1 + \Pi_2 + \Gamma_p, \quad \forall p \in \mathbb{N}_2^+, \tag{17}$$

$$\begin{aligned} & 0 \le \begin{bmatrix} M_0(\Theta_t) & U_0(\Theta_t) \\ (*) & R_1(\Theta_\alpha) \end{bmatrix}, \\ & 0 \le \begin{bmatrix} M_p(\Theta_t) & U_p(\Theta_t) \\ (*) & R_2\left(\Theta_{\alpha_p}\right) \end{bmatrix}, \\ & \qquad \forall p \in \mathbb{N}_2^+, \end{aligned} \tag{18}$$

where

$$\begin{aligned}\Pi_1 &= \mathrm{He}\left(\mathbf{e}_1^T P(\Theta_t)\Phi_t\right) + \mathbf{e}_1^T\left(\dot{P}(\Theta_t) + Q_1(\Theta_t)\right. \\ &\quad \left.+ Q_2(\Theta_t)\right)\mathbf{e}_1 - \mathbf{e}_2^T Q_1\left(\Theta_{t-d_1}\right)\mathbf{e}_2 - \mathbf{e}_4^T Q_2\left(\Theta_{t-d_2}\right)\mathbf{e}_4 \\ &\quad + d_1\Phi_t^T R_1(\Theta_t)\Phi_t + \delta\Phi_t^T R_2(\Theta_t)\Phi_t, \\ \Pi_2 &= \Psi_t^T\Psi_t - \mathbf{e}_5^T\mathbf{e}_5, \\ \Gamma_p &= \mathrm{He}\left(\mathbf{e}_{14}^T U_0(\Theta_t)(\mathbf{e}_1 - \mathbf{e}_2) + \mathbf{e}_{14}^T U_1(\Theta_t)(\mathbf{e}_3 - \mathbf{e}_4)\right. \\ &\quad \left.+ \mathbf{e}_{14}^T U_2(\Theta_t)(\mathbf{e}_2 - \mathbf{e}_3)\right) + d_1\mathbf{e}_{14}^T M_0(\Theta_t)\mathbf{e}_{14} \\ &\quad + \delta\mathbf{e}_{14}^T M_p(\Theta_t)\mathbf{e}_{14}.\end{aligned} \tag{19}$$

Then, (6) is robustly asymptotically stable for $d_1 \le d(t) \le d_2$.

Proof. First of all, by incorporating the following equalities into (15),

$$\begin{aligned}0 &= \overline{x}^T(t) M_0(\Theta_t)\overline{x}(t)\left(d_1 - \int_{t-d_1}^{t} d\alpha\right), \\ 0 &= \overline{x}^T(t) M_1(\Theta_t)\overline{x}(t)\left((d_2 - d(t)) - \int_{t-d_2}^{t-d(t)} d\alpha_1\right), \\ 0 &= \overline{x}^T(t) M_2(\Theta_t)\overline{x}(t)\left((d(t) - d_1) - \int_{t-d(t)}^{t-d_1} d\alpha_2\right), \\ 0 &= 2\overline{x}^T(t) U_0(\Theta_t)\left((\mathbf{e}_1 - \mathbf{e}_2)\eta(t) - \int_{t-d_1}^{t}\dot{x}(\alpha)\,d\alpha\right), \\ 0 &= 2\overline{x}^T(t) U_1(\Theta_t) \\ &\quad \cdot\left((\mathbf{e}_3 - \mathbf{e}_4)\eta(t) - \int_{t-d_2}^{t-d(t)}\dot{x}(\alpha_1)\,d\alpha_1\right), \\ 0 &= 2\overline{x}^T(t) U_2(\Theta_t) \\ &\quad \cdot\left((\mathbf{e}_2 - \mathbf{e}_3)\eta(t) - \int_{t-d(t)}^{t-d_1}\dot{x}(\alpha_2)\,d\alpha_2\right),\end{aligned} \tag{20}$$

we can get

$$\dot{V}(t) = \eta^T(t)\left(\Pi_1 + \overline{\Gamma}_p\right)\eta(t) + \overline{\mathcal{O}}_1 + \overline{\mathcal{O}}_2, \tag{21}$$

where

$$\begin{aligned}\overline{\Gamma}_p &= \mathrm{He}\left(\mathbf{e}_{14}^T U_0(\Theta_t)(\mathbf{e}_1 - \mathbf{e}_2) + \mathbf{e}_{14}^T U_1(\Theta_t)(\mathbf{e}_3 - \mathbf{e}_4)\right. \\ &\quad \left.+ \mathbf{e}_{14}^T U_2(\Theta_t)(\mathbf{e}_2 - \mathbf{e}_3)\right) + d_1\mathbf{e}_{14}^T M_0(\Theta_t)\mathbf{e}_{14} \\ &\quad + \delta\mathbf{e}_{14}^T\left(\sum_{p=1}^{2}\lambda_p(t) M_p(t)\right)\mathbf{e}_{14}, \\ \overline{\mathcal{O}}_1 &= -\int_{t-d_1}^{t}\begin{bmatrix}\overline{x}(t) \\ \dot{x}(\alpha)\end{bmatrix}^T \\ &\quad \cdot\begin{bmatrix}M_0(\Theta_t) & U_0(\Theta_t) \\ (*) & R_1(\Theta_\alpha)\end{bmatrix}\begin{bmatrix}\overline{x}(t) \\ \dot{x}(\alpha)\end{bmatrix} d\alpha, \\ \overline{\mathcal{O}}_2 &= -\int_{t-d_2}^{t-d(t)}\begin{bmatrix}\overline{x}(t) \\ \dot{x}(\alpha_1)\end{bmatrix}^T \\ &\quad \cdot\begin{bmatrix}M_1(\Theta_t) & U_1(\Theta_t) \\ (*) & R_2\left(\Theta_{\alpha_1}\right)\end{bmatrix}\begin{bmatrix}\overline{x}(t) \\ \dot{x}(\alpha_1)\end{bmatrix} d\alpha_1 \\ &\quad - \int_{t-d(t)}^{t-d_1}\begin{bmatrix}\overline{x}(t) \\ \dot{x}(\alpha_2)\end{bmatrix}^T \\ &\quad \cdot\begin{bmatrix}M_2(\Theta_t) & U_2(\Theta_t) \\ (*) & R_2\left(\Theta_{\alpha_2}\right)\end{bmatrix}\begin{bmatrix}\overline{x}(t) \\ \dot{x}(\alpha_2)\end{bmatrix} d\alpha_2\end{aligned} \tag{22}$$

in which $\lambda_1(t) = (d_2 - d(t))/(d_2 - d_1)$ and $\lambda_2(t) = (d(t) - d_1)/(d_2 - d_1)$. Next, the structured feedback uncertainty, given as $0 \le q^T(t)q(t) - p^T(t)p(t)$, can be converted into $0 \le \eta^T(t)(\Psi_t^T\Psi_t - \mathbf{e}_5^T\mathbf{e}_5)\eta(t)$, which yields $\dot{V}(t) \le \eta^T(t)(\Pi_1 + \Pi_2 + \overline{\Gamma}_p)\eta(t) + \overline{\mathcal{O}}_1 + \overline{\mathcal{O}}_2$. That is, the robust stability for (6) is assured by $0 > \eta^T(t)(\Pi_1 + \Pi_2 + \overline{\Gamma}_p)\eta(t) + \overline{\mathcal{O}}_1 + \overline{\mathcal{O}}_2$. Therefore, if (18) holds, then $\overline{\mathcal{O}}_1 + \overline{\mathcal{O}}_2 \le 0$, and hence the robust stability criterion is given by (17) because $\sum_{p=1}^{2}\lambda_p(t)M_p(t) \in \mathrm{Conv}(M_p(\Theta_t))$. □

In the absence of uncertainties, the T-S fuzzy system becomes $\dot{x}(t) = \Phi_t\overline{x}(t)$, where $\Phi_t = A(\Theta_t)\mathbf{e}_1 + A_d(\Theta_t)\mathbf{e}_3$. The following corollary presents the stability criterion for nominal T-S fuzzy systems with time-varying delays.

Corollary 6. *Suppose that there exist matrices $U_0(\Theta_t)$, $U_1(\Theta_t)$, and $U_2(\Theta_t) \in \mathbb{R}^{4n_x\times n_x}$ and symmetric matrices $0 < P(\Theta_t)$, $\dot{P}(\Theta_t)$, $0 < Q_1(\Theta_{t-d_1})$, $0 < Q_2(\Theta_{t-d_2})$, $0 < Q_1(\Theta_t)$, $0 < Q_2(\Theta_t)$, $0 < R_1(\Theta_\alpha)$, $0 < R_2(\Theta_{\alpha_p})$, $0 < R_1(\Theta_t)$, $0 < R_2(\Theta_t) \in \mathbb{R}^{n_x\times n_x}$, $M_0(\Theta_t), M_1(\Theta_t)$, and $M_2(\Theta_t) \in \mathbb{R}^{4n_x\times 4n_x}$ such that*

$$\begin{aligned}& 0 > \Pi_1 + \Gamma_p, \quad \forall p \in \mathbb{N}_2^+, \\ & 0 \le \begin{bmatrix}M_0(\Theta_t) & U_0(\Theta_t) \\ (*) & R_1(\Theta_\alpha)\end{bmatrix}, \\ & 0 \le \begin{bmatrix}M_p(\Theta_t) & U_p(\Theta_t) \\ (*) & R_2\left(\Theta_{\alpha_p}\right)\end{bmatrix}, \\ & \qquad\qquad \forall p \in \mathbb{N}_2^+,\end{aligned} \tag{23}$$

where

$$\begin{aligned}\Pi_1 &= \mathrm{He}\left(\mathbf{e}_1^T P(\Theta_t)\Phi_t\right) + \mathbf{e}_1^T\left(\dot{P}(\Theta_t) + Q_1(\Theta_t)\right. \\ &\quad \left.+ Q_2(\Theta_t)\right)\mathbf{e}_1 - \mathbf{e}_2^T Q_1\left(\Theta_{t-d_1}\right)\mathbf{e}_2 - \mathbf{e}_4^T Q_2\left(\Theta_{t-d_2}\right)\mathbf{e}_4 \\ &\quad + d_1\Phi_t^T R_1(\Theta_t)\Phi_t + \delta\Phi_t^T R_2(\Theta_t)\Phi_t,\end{aligned}$$

$$\begin{aligned}\Gamma_p = {} & \mathrm{He}\left(\mathbf{e}_{14}^T U_0(\Theta_t)(\mathbf{e}_1 - \mathbf{e}_2) + \mathbf{e}_{14}^T U_1(\Theta_t)(\mathbf{e}_3 - \mathbf{e}_4)\right. \\ & \left. + \mathbf{e}_{14}^T U_2(\Theta_t)(\mathbf{e}_2 - \mathbf{e}_3)\right) + d_1 \mathbf{e}_{14}^T M_0(\Theta_t)\mathbf{e}_{14} \\ & + \delta \mathbf{e}_{14}^T M_p(\Theta_t)\mathbf{e}_{14}.\end{aligned} \tag{24}$$

Then, (6) without uncertainties is asymptotically stable for $d_1 \leq d(t) \leq d_2$.

Proof. The proof is omitted since it is analogous to the derivation of Lemma 5. □

4. LMI-Based Stability Criterion

Based on Lemmas 2 and 3, to derive a finite number of solvable LMI conditions from (17), this paper simply sets all the decision variables to be of affine dependence on fuzzy-weighting functions:

$$P(\Theta_t) = \sum_{i=1}^{r}\theta_i P_i,$$

$$\dot{P}(\Theta_t) = \sum_{i=1}^{r}\dot{\theta}_i P_i, \tag{25}$$

$$Q_1(\Theta_t) = \sum_{i=1}^{r}\theta_i Q_{1,i},$$

$$Q_2(\Theta_t) = \sum_{i=1}^{r}\theta_i Q_{2,i},$$

$$Q_1(\Theta_{t-d_1}) = \sum_{q=1}^{r}\theta_q^{d_1} Q_{1,q},$$

$$Q_2(\Theta_{t-d_2}) = \sum_{\phi=1}^{r}\theta_\phi^{d_2} Q_{2,\phi}, \tag{26}$$

$$R_1(\Theta_t) = \sum_{i=1}^{r}\theta_i R_{1,i},$$

$$R_2(\Theta_t) = \sum_{i=1}^{r}\theta_i R_{2,i},$$

$$R_1(\Theta_\alpha) = \sum_{i=1}^{r}\theta_i^{\alpha} R_{1,i}, \tag{27}$$

$$R_2(\Theta_{\alpha_p}) = \sum_{i=1}^{r}\theta_i^{\alpha_p} R_{2,i},$$

$$U_0(\Theta_t) = \sum_{i=1}^{r}\theta_i U_{0,i},$$

$$U_1(\Theta_t) = \sum_{i=1}^{r}\theta_i U_{1,i},$$

$$U_2(\Theta_t) = \sum_{i=1}^{r}\theta_i U_{2,i},$$

$$M_0(\Theta_t) = \sum_{i=1}^{r}\theta_i M_{0,i},$$

$$M_1(\Theta_t) = \sum_{i=1}^{r}\theta_i M_{1,i},$$

$$M_2(\Theta_t) = \sum_{i=1}^{r}\theta_i M_{2,i}. \tag{28}$$

Remark 7. As a way to improve the performance to be considered, we can increase the degree of polynomial dependence on fuzzy-weighting functions, as in [31–33] but this is outside of the intended scope of this paper.

Theorem 8. *Let $\dot{\Theta}_t \in \mathbb{S}_{\dot{\Theta}}$ be satisfied. Suppose that there exist matrices $U_{0,i}, U_{1,i}, U_{2,i} \in \mathbb{R}^{4n_x \times n_x}$ and $S_0, S_i, X_i \in \mathbb{R}^{n_c \times n_c}$ ($n_c = 6n_x + n_p + n_q$), for $i \in \mathbb{N}_r^+$, symmetric matrices $N, 0 < P_i$, $0 < Q_{1,i}$, $0 < Q_{2,i}$, $0 < R_{1,i}$, and $0 < R_{2,i} \in \mathbb{R}^{n_x \times n_x}$, for $i \in \mathbb{N}_r^+$, and $M_{0,i}, M_{1,i}, M_{2,i} \in \mathbb{R}^{4n_x \times 4n_x}$ such that, for all $q, \phi \in \mathbb{N}_r^+$, $p \in \mathbb{N}_2^+$, and $\ell \in \mathbb{L}$,*

$$0 > \left[\begin{array}{c|c} \mathcal{L}_{\ell q\phi,0} & \left[\mathcal{L}_{p,i}\right]_r \\ \hline (*) & \left[\mathcal{L}_{ij}\right]_{r\times r}\end{array}\right], \tag{29}$$

$$\begin{aligned} & 0 < X_q + X_q^T, \\ & 0 \leq \begin{bmatrix} M_{0,\phi} & U_{0,\phi} \\ (*) & R_{1,q}\end{bmatrix}, \\ & 0 \leq \begin{bmatrix} M_{p,\phi} & U_{p,\phi} \\ (*) & R_{2,q}\end{bmatrix},\end{aligned} \tag{30}$$

where $\mathcal{L}_{\ell q\phi,0} = \mathcal{M}_{\ell q\phi,0} + \mathrm{He}(S_0 - \sum_{i=1}^{r}\alpha_i\beta_i X_i)$, $\mathcal{L}_{p,i} = \mathcal{M}_{p,i} + S_i - S_0 + (\alpha_i + \beta_i)X_i$, $\mathcal{L}_{ii} = \mathcal{M}_{ii} + \mathrm{He}(-S_i - X_i)$, and $\mathcal{L}_{ij} = \mathcal{M}_{ij} - S_i - S_j$ in which

$$\mathcal{M}_{\ell q\phi,0} = \mathrm{diag}\left(-I, 0, 0, (4,4)_{\ell q\phi,0}\right), \tag{31}$$

$$\mathcal{M}_{p,i} = \begin{bmatrix} 0 & 0 & 0 & (1,4)_i \\ 0 & -\frac{1}{2}\delta R_{2,i} & 0 & 0 \\ 0 & 0 & -\frac{1}{2}d_1 R_{1,i} & 0 \\ 0 & 0 & 0 & (4,4)_{p,i}\end{bmatrix}, \tag{32}$$

$$\mathcal{M}_{ii} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & (2,4)_{ii} \\ 0 & 0 & 0 & (3,4)_{ii} \\ 0 & (*) & (*) & \mathrm{He}\left((4,4)_{ii}\right)\end{bmatrix},$$

$$\mathscr{M}_{ij} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & (2,4)_{ij} + (2,4)_{ji} \\ 0 & 0 & 0 & (3,4)_{ij} + (3,4)_{ji} \\ 0 & 0 & 0 & (4,4)_{ij} + (4,4)_{ji} \end{bmatrix}, \tag{33}$$

$$\begin{aligned} (4,4)_{\ell q\phi,0} &= \mathbf{e}_1^T \left(\sum_{i=1}^{r} \varrho_{\ell_i,i} (P_i + N) \right) \mathbf{e}_1 - \mathbf{e}_2^T Q_{1,q} \mathbf{e}_2 \\ &\quad - \mathbf{e}_4^T Q_{2,\phi} \mathbf{e}_4 - \mathbf{e}_5^T \mathbf{e}_5, \\ (1,4)_i &= G_i \mathbf{e}_1 + G_{d,i} \mathbf{e}_3, \\ (4,4)_{p,i} &= \mathbf{e}_1^T \left(\frac{1}{2} Q_{1,i} + \frac{1}{2} Q_{2,i} \right) \mathbf{e}_1 + \Gamma_{p,i}, \\ (2,4)_{ij} &= \delta \left(R_{2,j} A_i \mathbf{e}_1 + R_{2,j} A_{d,i} \mathbf{e}_3 + R_{2,j} E_i \mathbf{e}_5 \right), \\ (3,4)_{ij} &= d_1 \left(R_{1,j} A_i \mathbf{e}_1 + R_{1,j} A_{d,i} \mathbf{e}_3 + R_{1,j} E_i \mathbf{e}_5 \right), \\ (4,4)_{ij} &= \mathbf{e}_1^T P_j A_i \mathbf{e}_1 + \mathbf{e}_1^T P_j A_{d,i} \mathbf{e}_3 + \mathbf{e}_1 P_j E_i \mathbf{e}_5, \\ \Gamma_{p,i} &= \mathbf{e}_{14}^T U_{0,i} (\mathbf{e}_1 - \mathbf{e}_2) + \mathbf{e}_{14}^T U_{1,i} (\mathbf{e}_3 - \mathbf{e}_4) \\ &\quad + \mathbf{e}_{14}^T U_{2,i} (\mathbf{e}_2 - \mathbf{e}_3) + \frac{1}{2} d_1 \mathbf{e}_{14}^T M_{0,i} \mathbf{e}_{14} \\ &\quad + \frac{1}{2} \delta \mathbf{e}_{14}^T M_{p,i} \mathbf{e}_{14}. \end{aligned} \tag{34}$$

Then, the system in (6) is robustly asymptotically stable for $d_1 \le d(t) \le d_2$.

Proof. Note that $\dot{\Theta}_t \in \mathbb{S}_{\dot{\Theta}}$. Thus, in view of Lemma 3, applying the Schur complement to (17) is given by

$$0 > \left[\begin{array}{ccc|c} -I & 0 & 0 & \Psi_t \\ 0 & -\delta R_2(\Theta_t) & 0 & \delta R_2(\Theta_t)\Phi_t \\ 0 & 0 & -d_1 R_1(\Theta_t) & d_1 R_1(\Theta_t)\Phi_t \\ \hline (*) & (*) & (*) & \Omega_\ell + \Gamma_p \end{array} \right], \quad \forall p \in \mathbb{N}_2^+,\ \ell \in \mathbb{L}, \tag{35}$$

where $\Omega_\ell = \mathrm{He}(\mathbf{e}_1^T P(\Theta_t)\Phi_t) - \mathbf{e}_2^T Q_1(\Theta_{t-d_1})\mathbf{e}_2 - \mathbf{e}_4^T Q_2(\Theta_{t-d_2})\mathbf{e}_4 - \mathbf{e}_5^T \mathbf{e}_5 + \mathbf{e}_1^T(\sum_{i=1}^{r} \varrho_{\ell_i,i}(P_i + N) + Q_1(\Theta_t) + Q_2(\Theta_t))\mathbf{e}_1$. Further, from (26) and (27), (35) and (18) can be converted into

$$\begin{aligned} 0 &> \sum_{q=1}^{r} \sum_{\phi=1}^{r} \theta_q^{d_1} \theta_\phi^{d_2} \mathscr{M}_{p,\ell q\phi}(\Theta_t) \in \mathbb{R}^{n_c \times n_c}, \quad \forall p \in \mathbb{N}_2^+,\ \ell \in \mathbb{L}, \\ 0 &\le \sum_{\phi=1}^{r} \sum_{q=1}^{r} \theta_\phi \theta_q^{\alpha} \begin{bmatrix} M_{0,\phi} & U_{0,\phi} \\ (*) & R_{1,q} \end{bmatrix}, \\ 0 &\le \sum_{\phi=1}^{r} \sum_{q=1}^{r} \theta_\phi \theta_q^{\alpha_p} \begin{bmatrix} M_{p,\phi} & U_{p,\phi} \\ (*) & R_{2,q} \end{bmatrix}, \end{aligned} \tag{36}$$

where

$$\mathscr{M}_{p,\ell q\phi}(\Theta_t) = \left[\begin{array}{ccc|c} -I & 0 & 0 & \Psi_t \\ 0 & -\delta R_2(\Theta_t) & 0 & \delta R_2(\Theta_t)\Phi_t \\ 0 & 0 & -d_1 R_1(\Theta_t) & d_1 R_1(\Theta_t)\Phi_t \\ \hline (*) & (*) & (*) & \Omega_{\ell q\phi} + \Gamma_p \end{array} \right], \tag{37}$$

$$\begin{aligned} \Omega_{\ell q\phi} &= \mathrm{He}\left(\mathbf{e}_1^T P(\Theta_t)\Phi_t\right) - \mathbf{e}_2^T Q_{1,q}\mathbf{e}_2 - \mathbf{e}_4^T Q_{2,\phi}\mathbf{e}_4 - \mathbf{e}_5^T \mathbf{e}_5 \\ &\quad + \mathbf{e}_1^T \left(\sum_{i=1}^{r} \varrho_{\ell_i,i}(P_i + N) + Q_1(\Theta_t) + Q_2(\Theta_t) \right) \mathbf{e}_1. \end{aligned}$$

As a result, from the convexity of fuzzy-weighting functions, (17) and (18) can be assured by (30),

$$0 > \mathscr{M}_{p,\ell q\phi}(\Theta_t), \quad \forall q,\phi \in \mathbb{N}_r^+,\ p \in \mathbb{N}_2^+,\ \ell \in \mathbb{L}. \tag{38}$$

Further, note that representing (38) in the form of (7) becomes

$$\begin{aligned} 0 &> \mathscr{M}_{\ell q\phi,0} + \sum_{i=1}^{r} \theta_i \mathrm{He}\left(\mathscr{M}_{p,i}\right) + \sum_{i=1}^{r} \theta_i^2 \mathscr{M}_{ii} \\ &\quad + \sum_{i=1}^{r} \left(\sum_{j=i+1}^{r} \theta_i \theta_j \mathscr{M}_{ij} + \sum_{j=1}^{i-1} \theta_i \theta_j \mathscr{M}_{ji}^T \right), \end{aligned} \tag{39}$$

where $\mathscr{M}_{\ell q\phi,0}$, $\mathscr{M}_{p,i}$, $\mathscr{M}_{ii}$, and $\mathscr{M}_{ij}$ are defined in (31)–(33). Therefore, from Lemma 2, we can obtain (29) in the sequel without loss of generality. □

The following corollary presents the LMI-based stability criterion for nominal T-S fuzzy systems with time-varying delays.

Corollary 9. *Let $\dot{\Theta}_t \in \mathbb{S}_{\dot{\Theta}}$ be satisfied. Suppose that there exist matrices $U_{0,i}, U_{1,i}, U_{2,i} \in \mathbb{R}^{4n_x \times n_x}$ and $S_0, S_i, X_i \in \mathbb{R}^{n_c \times n_c}$ ($n_c = 6n_x$), for $i \in \mathbb{N}_r^+$, symmetric matrices $N, 0 < P_i$, $0 < Q_{1,i}$, $0 < Q_{2,i}$, $0 < R_{1,i}$, and $0 < R_{2,i} \in \mathbb{R}^{n_x \times n_x}$, for $i \in \mathbb{N}_r^+$, and $M_{0,i}, M_{1,i}, M_{2,i} \in \mathbb{R}^{4n_x \times 4n_x}$ such that, for all $q, \phi \in \mathbb{N}_r^+$, $p \in \mathbb{N}_2^+$, and $\ell \in \mathbb{L}$,*

$$\begin{aligned} 0 &> \left[\begin{array}{c|c} \mathscr{L}_{\ell q\phi,0} & \left[\mathscr{L}_{p,i}\right]_r \\ \hline (*) & \left[\mathscr{L}_{ij}\right]_{r\times r} \end{array} \right], \\ 0 &< X_q + X_q^T, \\ 0 &\le \begin{bmatrix} M_{0,\phi} & U_{0,\phi} \\ (*) & R_{1,q} \end{bmatrix}, \\ 0 &\le \begin{bmatrix} M_{p,\phi} & U_{p,\phi} \\ (*) & R_{2,q} \end{bmatrix}, \end{aligned} \tag{40}$$

where $\mathscr{L}_{\ell p\phi,0} = \mathscr{M}_{\ell p\phi,0} + \mathrm{He}(S_0 - \sum_{i=1}^{r} \alpha_i\beta_i X_i)$, $\mathscr{L}_{p,i} = \mathscr{M}_{p,i} + S_i - S_0 + (\alpha_i + \beta_i)X_i$, $\mathscr{L}_{ii} = \mathscr{M}_{ii} + \mathrm{He}(-S_i - X_i)$, *and* $\mathscr{L}_{ij} = \mathscr{M}_{ij} - S_i - S_j$ *in which*

$$\begin{aligned}
\mathscr{M}_{\ell p\phi,0} &= \mathrm{diag}\left(0, 0, (3,3)_{\ell p\phi,0}\right),\\
\mathscr{M}_{p,i} &= \begin{bmatrix} -\frac{1}{2}\delta R_{2,i} & 0 & 0 \\ 0 & -\frac{1}{2}d_1 R_{1,i} & 0 \\ 0 & 0 & (3,3)_{p,i}\end{bmatrix},\\
\mathscr{M}_{ii} &= \begin{bmatrix} 0 & 0 & (1,3)_{ii} \\ 0 & 0 & (2,3)_{ii} \\ (*) & (*) & \mathrm{He}\left((3,3)_{ii}\right)\end{bmatrix},\\
\mathscr{M}_{ij} &= \begin{bmatrix} 0 & 0 & (1,3)_{ij} + (1,3)_{ji} \\ 0 & 0 & (2,3)_{ij} + (2,3)_{ji} \\ 0 & 0 & (3,3)_{ij} + (3,3)_{ji}\end{bmatrix},\\
(3,3)_{\ell q\phi,0} &= \mathbf{e}_1^T\left(\sum_{i=1}^{r}\varrho_{\ell,i}\left(P_i + N\right)\right)\mathbf{e}_1 - \mathbf{e}_2^T Q_{1,q}\mathbf{e}_2 \\
&\quad - \mathbf{e}_4^T Q_{2,\phi}\mathbf{e}_4,\\
(3,3)_{p,i} &= \mathbf{e}_1^T\left(\frac{1}{2}Q_{1,i} + \frac{1}{2}Q_{2,i}\right)\mathbf{e}_1 + \Gamma_{p,i},\\
(1,3)_{ij} &= \delta\left(R_{2,j}A_i\mathbf{e}_1 + R_{2,j}A_{d,i}\mathbf{e}_3\right),\\
(2,3)_{ij} &= d_1\left(R_{1,j}A_i\mathbf{e}_1 + R_{1,j}A_{d,i}\mathbf{e}_3\right),\\
(3,3)_{ij} &= \mathbf{e}_1^T P_j A_i \mathbf{e}_1 + \mathbf{e}_1^T P_j A_{d,i}\mathbf{e}_3,\\
\Gamma_{p,i} &= \mathbf{e}_{14}^T U_{0,i}\left(\mathbf{e}_1 - \mathbf{e}_2\right) + \mathbf{e}_{14}^T U_{1,i}\left(\mathbf{e}_3 - \mathbf{e}_4\right) \\
&\quad + \mathbf{e}_{14}^T U_{2,i}\left(\mathbf{e}_2 - \mathbf{e}_3\right) + \frac{1}{2}d_1\mathbf{e}_{14}^T M_{0,i}\mathbf{e}_{14} \\
&\quad + \frac{1}{2}\delta\mathbf{e}_{14}^T M_{p,i}\mathbf{e}_{14}.
\end{aligned} \tag{41}$$

Then, (6) without uncertainties is asymptotically stable for $d_1 \le d(t) \le d_2$.

Proof. The proof is omitted since it is analogous to the derivation of Theorem 8. □

Remark 10. The number of scalar variables involved in Theorem 8 and Corollary 9 is given as follows: $\# = n_c^2(2r+1) + 0.5n_x(n_x+1) + n_x(38.5n_x + 8.5)r$. Table 1 shows the number for each case of (n_x, r). Since the use of slack variables requires more computation cost compared with other methods, there may be the need to balance the tradeoffs between the computational cost and the performance enhancement.

5. Numerical Examples

To verify the effectiveness of our methods, this paper provides two examples that make some comparisons with other results: one is related to the stability analysis for nominal T-S fuzzy systems and the other is related to the robust stability analysis for T-S fuzzy systems with uncertainties.

Example 1. Consider the following T-S fuzzy system, adopted in [25]:

$$\begin{aligned}
A_1 &= \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix},\\
A_2 &= \begin{bmatrix} -1 & 0.5 \\ 0 & -1 \end{bmatrix},\\
A_{d,1} &= \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix},\\
A_{d,2} &= \begin{bmatrix} -1 & 0 \\ 0.1 & -1 \end{bmatrix},
\end{aligned} \tag{42}$$

where $\theta_1 = 1/(1 + \exp(-2x_1(t)))$ and $\theta_2 = 1 - \theta_1$. Table 2 shows the maximum allowable upper bound (MAUB) for each $d_1 \in \{0.0, 0.4, 0.8, 1.0, 1.2\}$, where m denotes the number of delay segments and $(m-1)$ denotes the degree of delay partitioning. From Table 2, we can see that our method (Corollary 9) provides larger MAUBs in comparison with those of [25, 26]. Hence it can be concluded that the stability criterion in Corollary 9, obtained based on the NLKF, is less conservative than other results. In particular, for $d_1 = 1.2$ and $d_2 = 1.531$, Corollary 9 offers the following solutions:

$$\begin{aligned}
P_1 &= 10^{-2}\begin{bmatrix} 1.485 & -0.227 \\ -0.227 & 0.368 \end{bmatrix},\\
P_2 &= 10^{-2}\begin{bmatrix} 1.078 & -0.290 \\ -0.290 & 0.799 \end{bmatrix},\\
Q_{1,1} &= 10^{-3}\begin{bmatrix} 3.315 & 0.253 \\ 0.253 & 0.785 \end{bmatrix},\\
Q_{1,2} &= 10^{-3}\begin{bmatrix} 3.082 & 0.163 \\ 0.163 & 0.930 \end{bmatrix},\\
Q_{2,1} &= 10^{-3}\begin{bmatrix} 5.607 & 0.254 \\ 0.254 & 3.137 \end{bmatrix},\\
Q_{2,2} &= 10^{-3}\begin{bmatrix} 5.708 & -0.001 \\ -0.001 & 3.323 \end{bmatrix},\\
R_{1,1} &= 10^{-3}\begin{bmatrix} 4.661 & -0.847 \\ -0.847 & 1.669 \end{bmatrix},\\
R_{1,2} &= 10^{-3}\begin{bmatrix} 4.647 & -0.813 \\ -0.813 & 1.671 \end{bmatrix},
\end{aligned}$$

Table 1: # involved in Corollary 9 and Theorem 8 ($n_p = 1, n_q = 1$).

(n_x, r)	(2, 2)	(2, 3)	(2, 4)	(3, 2)	(3, 3)	(3, 4)	(4, 2)	(4, 3)	(4, 4)
Corollary 9	1065	1524	1983	2370	3390	4410	4190	5992	7794
Theorem 8	1325	1888	2451	2750	3922	5094	4690	6692	8694

Table 2: Maximum allowable upper bound (MAUB) for each d_1, where m denotes the number of delay segments and $(m-1)$ denotes the degree of delay partitioning.

d_1	0.0	0.4	0.8	1.0	1.2	$(m-1)$
[25]	0.982	1.038	1.158	1.252	1.359	0
[26]	1.221	1.277	1.311	1.358	1.419	1
[26]	1.278	1.303	1.316	1.361	1.425	2
Corollary 9	1.302	1.380	1.413	1.462	1.531	0

Table 3: Maximum allowable upper bound (MAUB) for $d_1 = 0$.

Methods	[27]	[28]	[29]	[30]	Theorem 8
d_2	—	0.443	0.499	1.081	1.132

$$R_{2,1} = 10^{-3}\begin{bmatrix} 7.232 & -1.319 \\ -1.319 & 2.747 \end{bmatrix},$$
$$R_{2,2} = 10^{-3}\begin{bmatrix} 6.538 & -0.704 \\ -0.704 & 2.583 \end{bmatrix}. \tag{43}$$

Example 2. Consider the following T-S fuzzy system:

$$A_1 = \begin{bmatrix} -2 & 1 \\ 0.5 & -1 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix},$$
$$A_{d,1} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix},$$
$$A_{d,2} = \begin{bmatrix} -1.6 & 0 \\ 0 & -1 \end{bmatrix},$$
$$E_1 = \begin{bmatrix} 0.03 & 0 \\ 0 & -0.03 \end{bmatrix},$$
$$E_2 = \begin{bmatrix} 0.03 & 0 \\ 0 & -0.03 \end{bmatrix},$$
$$G_1 = \begin{bmatrix} 1.6 & 0 \\ 0 & 0.05 \end{bmatrix},$$
$$G_2 = \begin{bmatrix} 1.6 & 0 \\ 0 & -0.05 \end{bmatrix},$$
$$G_{d1} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix},$$
$$G_{d2} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix}, \tag{44}$$

where

$$\theta_1 = \left(1 - \frac{1}{1 + \exp\left(-6\left(x_2 - \pi/4\right)\right)}\right) \times \left(\frac{1}{1 + \exp\left(-6\left(x_2 + \pi/4\right)\right)}\right), \tag{45}$$
$$\theta_2 = 1 - \theta_1.$$

The maximum allowable upper bound (MAUB) for each method is tabulated in Table 3. And, from Table 3, we can see that the proposed method (Theorem 8) achieves larger MAUBs than those of other methods [27–30]. Hence, it can be concluded that the robust stability criterion in Theorem 8, established from the NLKF approach and Lemma 2, is less conservative than those of [27–30].

6. Concluding Remarks

This paper proposed an NLKF-based method of deriving a less conservative stability criterion for T-S fuzzy systems with time-varying delays. Of course, the proposed method may increase the burden of numerical computation. However, if the computational complexity is out of the practical problem, then our results can be significantly useful.

Competing Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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