Research Article

Testing and Micromechanical Modelling of Rockfill Materials Considering the Effect of Stress Path

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We have extended the micromechanics-based analytical (M-A) model to make it capable of simulating Nuozhadu rockfill material (NRFM) under different stress paths. Two types of drained triaxial tests on NRFM were conducted, namely, the stress paths of constant stress ratio (CSR) and the complex stress paths with transitional features. The model was improved by considering the interparticle parameter variation with the unloading-reloading cycles and the effect of the stress transition path. The evolution of local dilatancy at interparticle planes due to an externally applied load is also discussed. Compared with Duncan-Chang’s E-u and E-B models, the improved model could not only better describe the deformation properties of NRFM under the stress path loading, but also present the volumetric strain changing from dilatancy to contractancy with increasing transitional confining pressures. All simulations have demonstrated that the proposed M-A model is capable of modelling the mechanical behaviour of NRFM in the dam.

1. Introduction

Rockfill materials (RFMs) are widely used in the constructions of rockfill dams because of their high strength and low cost [1–7]. Normally, the large-scale conventional triaxial compression (LCTC) test (i.e., the confining pressure is constant) is used to study the mechanical properties of RFMs [8–14]. However, during such tests, the loading path is not consistent with that in actual engineering applications. Observation data of dams and the results of finite element calculation analyses have shown that the stress path of RFMs can be approximated as the stress ratio remaining constant during dam construction, and it will become a transitional stress path during reservoir filling (Figure 1) [15]. Moreover, the mechanical properties of RFMs are stress-path dependent [16]. In other words, the stress-strain relationships of the RFMs vary greatly under different loading paths. Therefore, it is necessary to consider the influence of the stress path in the constitutive modelling of RFM.

Many LCTC tests have been conducted for examining the behaviour of RFMs. Marsal [17, 18] conducted a series of super LCTC tests (the sample was 1130 mm in height and 180 mm in diameter). He found that the peak state friction angle of the RFMs decreases with increasing confining pressures. Indraratna and Salim [19] investigated the stress-dilatancy phenomenon of RFMs during the LCTC test. Then, they proposed a relative constitutive equation to improve the stress-dilatancy relationship by incorporating particle breakage. Araei et al. [20] conducted the LCTC tests of RFMs under cyclic loading conditions and studied the effect of loading frequency on the sample. Xiao et al. [21, 22] performed several LCTC tests of RFMs with different initial void ratios and investigated the effects of density and pressure on the strength and deformation of RFMs. However, the confining pressure of the LCTC test was constant, and under the stress path, the changing confining pressure had a significant impact on the mechanical behaviour (such as strength, stress-strain relationship, and dilatancy) of the RFMs. Although many
In recent years, increasingly more researchers have begun to model granular materials from the particle length scale. The discrete element method is a commonly used approach for studying the effect of particle properties on macroparticle assemblies. Liu et al. [28] investigated breakage of the rockfill particles in a dam model using Particle Flow Code (PFC) software. Discrete element modelling of sand under one-dimensional compression and creep conditions was studied by McDowell and De Bono [29] using the PFC software; however, the calculation speed of PFC is slow; it is difficult to analyse the actual engineering. The M-A model not only appropriately considers the microstructure of soils but also is easier to be applied in engineering practice. Based on this model, Chang and Hicher [30] simulated the conventional triaxial test of Hostun sand under drained and undrained conditions. The simulation results showed that the model could well describe the stress-strain behaviour of the sand during the test. Chang et al. [31, 32] introduced a new formulation that accounts for the stress reversal on a contact plane and the density state-dependent dilatancy. Subsequently, the M-A model was extended to be capable of simulating sand under cyclic loading. Previous studies of the M-A model primarily focused on the mechanical properties of sand. The models for RFMs have not been investigated as intensively as the models for sand. Li proposed the M-A model for RFMs based on LCTC tests, and the simulation results indicated that the model could well describe the deformation properties of RFMs during the test [34]. However, modelling of RFMs under the stress path has not been conducted.

In this paper, the M-A model is extended to make it capable of simulating the triaxial tests of NRFM under the stress path. The model is improved by considering the parameter variations under unloading-reloading conditions and the stress transition path. The local dilatancy relations at interparticle planes due to applied load are also discussed. The features of the M-A model are described in comparison with Duncan-Chang’s models. Finally, the overall applicability of the proposed model is evaluated through comparisons of the prediction and test results.

2. Two Types of Large Triaxial Tests

RFMs were taken from the site of Nuozaodu rockfill dam, which is the highest rockfill dam in China (ranked the third highest in the world). Specific details of the study site are presented in Table 1. These materials are used as the main rockfill of the core-dam body. Table 2 presents the basic properties of the NRFM, including particle shape, specific density, initial void ratio, and compressive strength. As shown in Figure 2, the particle size distribution of the prototype NRFM is reduced using the parallel gradation technique [35], with a maximum particle size of 60 mm. The uniformity coefficient Cu and the curvature coefficient Cc of the NRFM are 8.8 and 1.47, respectively.
### Table 1: Project and site for NRFM.

<table>
<thead>
<tr>
<th>Project detail</th>
<th>Depiction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Project name</td>
<td>Nuozhadu Hydropower Station</td>
</tr>
<tr>
<td>Dam type</td>
<td>Rockfill dam with clay core</td>
</tr>
<tr>
<td>Site</td>
<td>Puer City, Yunnan Province, China</td>
</tr>
<tr>
<td>Type of NRFM</td>
<td>Blasted rock</td>
</tr>
</tbody>
</table>

### Table 2: Physical and mechanical properties of rockfill.

<table>
<thead>
<tr>
<th>Material</th>
<th>Nuozhadu rockfill</th>
</tr>
</thead>
<tbody>
<tr>
<td>Particle shape</td>
<td>Angular</td>
</tr>
<tr>
<td>Specific density $\rho_i$ (g/cm$^3$)</td>
<td>2.63</td>
</tr>
<tr>
<td>Initial void ratio $\varepsilon_0$</td>
<td>0.32</td>
</tr>
<tr>
<td>Compressive strength (Mpa)</td>
<td>118.6</td>
</tr>
</tbody>
</table>

The test equipment is a large triaxial apparatus. The specimen is 302 mm in diameter and 655 mm in height. The maximum confining pressure of the device is 2500 kPa, the maximum axial force is 1000 kN (compression), and the maximum axial displacement is 200 mm. The specimen was compacted and then saturated with CO$_2$ and air-free water ($B$-value was above 0.96 after saturation). The specimen was consolidated at an initial confining pressure of 40 kPa. Then, it was loaded at the set value of stress ratio (as shown in Figures 3 and 4) under the drained condition, which was consistent with the practical working conditions of NRFM in the dam. Because the pore water pressure was zero, all the stresses in this paper refer to the effective stresses.

Ten samples were prepared for the tests. According to the loading method, the samples could be divided into two types. For the first type of test (Figure 3), the sample was loaded under the paths of a constant stress ratio (CSR). When the confining pressure reached 1600 kPa, a cycle of unloading and reloading was applied under the same CSR. After the cyclic loading, the loading was continued until the confining pressure $\sigma_3$ reached 2500 kPa. Because there were six different stress ratios in the tests, six samples were required.

For the second type of test (Figure 4), the stress ratio was maintained constant ($q/p = dq/dp = 1.00$) during the initial stage. Then, at each confining pressure, $\sigma_3 = 300, 800, 1400$, and 2000 kPa, the confining pressure was held constant and the axial load was increased ($dq/dp = 3.00$) until the samples reached failure. Four samples were required according to the four transitional points.

### 3. Constitutive Model

In the proposed constitutive model, the rockfill sample is considered to be a collection of grains. The deformation of the sample is generated by moving contacted grains in all orientations (Figure 5). Thus, the stress-strain relationship of the NRFM can be obtained from the average of the mobilized deformation of all contact planes between the rockfill grains. The forces and movements at all contact planes are used to obtain the macroscopic stress and strain tensors (Figure 7).

The macroscopic stiffness tensor is obtained by integrating the stiffness properties at interparticle contacts. A statically constrained microstructure is assumed for the micro-macro links, which means that the forces on each contact plane are assumed to be equal to the resolved components of the macroscopic stress tensor [30].

#### 3.1. Local Coordinates and Definitions.

As shown in Figure 5, an auxiliary local coordinate is established on each contact plane. The orientation of a contact plane between two grains is defined by the vector perpendicular to the plane. For the $a$ contact plane, the local forces $F^a_n$ and the local movements $\delta^a_n$ can be denoted as follows: $F^a_n = \{F^a_n, F^a_s, F^a_t\}$ and $\delta^a_n = \{\delta^a_n, \delta^a_s, \delta^a_t\}$, where the subscripts $n$, $s$, and $t$ represent three orthogonal unit vectors that form the local coordinate system.
Deviatoric stress $q$

$q_f = M pf + B$

$\Delta q/\Delta p = 3.0$

$\Delta q/\Delta p = 1.0$

Mean principal stress $p$

Figure 4: Stress paths of constant stress ratio with a transition.

The vector $n$ is inward normal to the contact plane. Vectors $s$ and $t$ are on the contact plane.

3.2. Force-Displacement Relations of the Contacted Grains

3.2.1. Elastic Part. Suppose that the relationship of the local forces $F^\alpha_i$ and the local movements $\delta^\alpha_i$ is nonlinear elasticity. The contact stiffness of the plane includes normal stiffness $K_n^\alpha$ and shear stiffness $K_s^\alpha$ and $K_t^\alpha$. The elastic stiffness is given by

$$
\begin{align*}
\delta F_n^\alpha &= K_n^\alpha d\Delta_n^a,
\delta F_s^\alpha &= K_s^\alpha d\Delta_s^a,
\delta F_t^\alpha &= K_t^\alpha d\Delta_t^a.
\end{align*}
$$

The tensor forms of (1) are as follows:

$$
\begin{align*}
\delta F_I^\alpha &= K_I^\alpha d\Delta_I^a, \quad (I, J = n, s, t).
\end{align*}
$$

It is clear that $K_I^\alpha$ is a diagonal matrix, which is as follows:

$$
K_I^\alpha = \begin{bmatrix}
K_n^\alpha & 0 & 0 \\
0 & K_s^\alpha & 0 \\
0 & 0 & K_t^\alpha
\end{bmatrix}.
$$

In the global coordinate system, (2) can be expressed as

$$
\delta f_I^\alpha = k_I^\alpha \delta \sigma^a.
$$

Then

$$
\begin{align*}
k^\alpha_{ij} &= B^\alpha_{ij} K^\alpha_{ii} B^\alpha_{jj}, \\
B^\alpha &= \begin{bmatrix}
-cos(y) & -sin(y) & cos(\beta) & -sin(y) & sin(\beta) \\
-sin(y) & cos(y) & cos(\beta) & cos(y) & sin(\beta) \\
0 & -sin(\beta) & cos(\beta)
\end{bmatrix},
\end{align*}
$$

where, $\delta f_I^\alpha$ and $\delta \sigma^a$ are the increments of the local forces and the local movements in the global coordinate system. $k^\alpha_{ij}$ is the stiffness matrix in the global coordinate system. $B^\alpha$ is the coordinate transformation matrix. $\beta$ and $\gamma$ are the angles between the vectors and coordinate system (Figure 5).

The value of the elastic stiffness can be estimated from Hertz-Mindlin’s formulation [36]

$$
\begin{align*}
K_n^\alpha &= K_{n0}\left(\frac{F_n^\alpha}{l}\right)^n, \\
K_s^\alpha &= K_{s0}\left(\frac{F_s^\alpha}{l}\right)^n, \\
K_t^\alpha &= K_{t0}\left(\frac{F_t^\alpha}{l}\right)^n,
\end{align*}
$$

where $F_n^\alpha$ is the contact force in the normal direction. $K_{n0}$, $K_{s0}$, and $n$ are material constants. $l$ is the branch length between two grains. For two spherical grains, $l$ is the same as the grain size $d$. According to Hertz-Mindlin’s theory [36], $n = 1/3$.

3.2.2. Plastic Part. Stress dilatancy is a well-known phenomenon in RFMs [37–39]. During triaxial tests, the plastic sliding of the particles made the contact plane move, and then shear dilation occurs [31, 32]. Assuming that that plastic work for a contact plane due to both normal and shear movements is equal to the energy loss due to friction at the contact, the dilatancy effect can be described by

$$
\begin{align*}
F_n^\alpha d\sigma_n^a + T^\alpha dS^a = F_n^\alpha \tan \phi dS^a.
\end{align*}
$$

Then

$$
\begin{align*}
\frac{dS_n^a}{dS^a} &= \tan \phi - \frac{T^\alpha}{F_n^\alpha},
\end{align*}
$$

where $dS_n^a$ is the increment of the normal plastic movement. $\phi$ is the dilatancy angle. $\tan \phi$ represents the obliquity when the plastic normal movement is zero, which is related to the phase transformation line of the rockfill assembly.
force $T^\alpha$ and the increase of the share plastic movement $dS^{ap}$ can be given by

$$T^\alpha = \sqrt{(F^\alpha_n)^2 + (F^\alpha_s)^2},$$
$$dS^{ap} = \sqrt{(dS^{ap}_n)^2 + (dS^{ap}_s)^2}. \quad (9)$$

The yield function is assumed to be of Mohr-Coulomb type

$$F(F^\alpha_n, \kappa) = T^\alpha - F^\alpha_n \kappa (S^{ap}) = 0, \quad (10)$$
where $\kappa$ is the hardening function, which is defined by a hyperbolic curve (Figure 6) as follows:

$$\kappa = K_{p0} \tan \phi_u \cdot S^{ap} - F^\alpha_n \tan \phi_u + K_{p0} S^{ap}, \quad (11)$$
where $\phi_u$ is the interparticle friction angle and $K_{p0}$ is the initial slope of the hyperbolic curve, which can be expressed as follows:

$$K_{p0} = kn \left( 0.0125 + \frac{10}{F_n} \right). \quad (12)$$

The plastic flow in the direction normal to the contact plane is governed by the stress-dilatancy equation in (8). Thus the flow rule is nonassociated. In the global coordinate system the incremental plastic movements $d\delta^{ap}$ can be expressed by

$$\begin{bmatrix} d\delta^{ap}_n \\ d\delta^{ap}_s \\ d\delta^{ap}_t \end{bmatrix} = dS^{ap} \begin{bmatrix} \tan \phi - \frac{T^\alpha}{F^\alpha_n} \\ \frac{F^\alpha_s}{F^\alpha_n} \\ \frac{F^\alpha_t}{F^\alpha_n} \end{bmatrix} B^\alpha. \quad (13)$$

3.3. Macro-Micro Relationship. The stress-strain relationship for the sample can be determined by integrating the behaviour of NRFM interparticle contacts in all orientations.

During the integration process, a relationship is required to link the macro and micro variables. Liao et al. [40] proposed that the stress-strain relationship for a granular system could be derived based on the hypothesis that the mean field of displacement is the best fit of actual particle displacements. This relationship has been introduced in the M-A model by Chang et al. and could well describe the stress-strain behaviour of the sand [30–33]. As a kind of granular materials, this hypothesis is also applied to the NRFM. The relation between the macrostrain and interparticle displacement is as follows:

$$d\varepsilon_{ij} = A^{-1}_{ik} \sum_{\alpha=1}^{N} d\delta^\alpha_{f_{ik}}, \quad (14)$$
$$A_{ik} = \sum_{\alpha=1}^{N} b_i^\alpha b_k^\alpha, \quad (15)$$
where $d\delta^\alpha_{f_{ik}}$ is the incremental relative displacement between two contacted particles. $l_k^\alpha$ is the branch vector joining the centres of two contacted particles. $N$ is the total number of contact orientations, according to Oda’s research [41], which can be expressed as

$$N = \frac{12}{\pi d^3 (1 + e) e}. \quad (15)$$

Using this hypothesis, the mean force on the contact plane of a given orientation is

$$d f^\alpha_f = d\sigma_{f_{ik}} A_{ik}^{-1} b_k^\alpha V. \quad (16)$$
The stress increment can be obtained by the contact forces and branch vectors for all contacts as follows [42, 43]:

\[
d\sigma_{ij} = \frac{1}{V} \sum_{\alpha=0}^{N} d\sigma_{ij}^{\alpha}.
\]  

(17)

When the contact number \( N \) is sufficiently large in an isotropic packing, the summation of flexibility tensor in (14) can be written in integral form [44], given by

\[
de_{ij} = A_{ik}^{-1} N \int_{0}^{2\pi} \int_{0}^{\pi} d\delta_{ij}^{\alpha}(y, \beta) \xi_k(y, \beta) \xi(y, \beta) \cdot \sin(y) dy d\beta,
\]  

(18)

\[
A_{ik} = N \int_{0}^{2\pi} \int_{0}^{\pi} l_{ik}^\gamma(y, \beta) l_{ik}^\beta(y, \beta) \xi(y, \beta) \cdot \sin(y) dy d\beta,
\]  

(19)

where \( \xi \) is a distribution density function, and in this paper, its value is 1/4\(\pi \), which means that the sample is isotropic.

4. Calibrations of Model Parameters

The model parameters are divided into two parts: the global parameters, which can be chosen from the test, and the interparticle parameters, which cannot be obtained from the test directly. Here, a particle swarm optimization (PSO) algorithm (Figure 8) was used to determine the interparticle parameters.

PSO is a population-based heuristic search technique. Its original concept came from the movements of birds or particles [45]. This algorithm possesses a fast convergence rate and high estimate precision of sensor bias. Therefore, it is widely used in geotechnical engineering. In this algorithm (Figure 8), each potential solution is called a “particle.” The solution is searched by a particle swarm flying in the hyperspace. Based on the optimization problem, the objective function of the optimization problem is evaluated for each particles position in the search space. The fitness values of each position are calculated. The best-known positions of each particle are denoted \( p_{best} \), and the global best position of the entire particle swarm is denoted \( g_{best} \). For each iteration, the velocity and particle location are expressed by

\[
V_{sid}^{k+1} = \omega V_{sid}^{k} + c_1 r_1 \left( p_{id}^{k} - X_{sid}^{k} \right) + c_2 r_2 \left( g_{best} - X_{sid}^{k} \right),
\]

\[
X_{sid}^{k+1} = X_{sid}^{k} + V_{sid}^{k+1},
\]  

(20)

where \( V_{sid}^{k} \) and \( X_{sid}^{k} \) are the velocity and the position of the \( i \)th particle in the previous iteration, respectively. \( r_1 \) and \( r_2 \) are random numbers ranging from 0 to 1. \( c_1 \) and \( c_2 \) are acceleration coefficients, which are the positive parameters; here, they are equal to 2. \( p_{id}^{k} \) and \( g_{best} \) are \( p_{best} \) and \( g_{best} \), respectively.

In this paper, the fitness value of the objective function, which determines the POS performance, is represented by the mean squared error, and it is expressed as

\[
E_r = \frac{1}{n} \sum_{i=1}^{n} \left( x - x_i \right)^2,
\]  

(21)

where \( x \) is the calculated value, which represents the axial strain and volume strain, respectively; \( x_i \) is the test data; and \( n \) is the total number of test data. As the number of search steps increases, \( E_r \) will gradually become smaller and satisfy the end conditions.

Table 3 shows the search range of the parameters. According to the initial stress increments and the constitutive model, the optimizer calculates the strain increments. Then, the error \( E_r \) can be calculated through comparison with test data. The candidate solution is improved throughout the iteration process. When the solution reaches the best fitness, the model parameters are obtained. Considerable efforts were devoted to determining the interparticle parameters of the model under the stress paths of different CSRs. In particular, under unloading-reloading conditions and the stress transition paths, the interparticle parameters might be different; specific instructions are as follows.

Under unloading-reloading conditions, the stress-strain curve is nearly a straight line during the test (Figure 14). Therefore, we assumed that only linear elastic deformation of the particle occurred during this stage (plastic deformation could be neglected). From the analysis of the fitting parameter
Table 3: Parameter-search range of the M-A model.

<table>
<thead>
<tr>
<th>Material</th>
<th>$K_{n0}$ (N/m)</th>
<th>$\Phi_u$ (°)</th>
<th>$K_{n1}$ (N/m)</th>
<th>$\Phi$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nuozhadu rockfill</td>
<td>$2.0 \times 10^3$–$3.0 \times 10^4$</td>
<td>39–40</td>
<td>$1.5 \times 10^2$–$2.5 \times 10^7$</td>
<td>35.6–36.6</td>
</tr>
</tbody>
</table>

$K_{n1}$, a significant phenomenon was found: parameter $K_{n1}$ varies regularly with the mean principal stress $P_1$ at the unloading point. After plotting $\lg(K_{n1}/P_a) - \lg(P_1/P_a)$ in Figure 9, a good linear relationship is observed. Therefore, the value of the normal stiffness for two elastic spheres could be expressed as follows:

$$K_{n1} = k_n P_a \left( \frac{P_1}{P_a} \right)^w,$$

where $K_{n1}$ is the intercept of a straight line and $w$ is the slope. The normal stiffness $K_{n1}$ is similar to the loading and unloading modulus of the conventional triaxial test [26], because the two types of tests have similarly shaped deformation curves (see Figure 13(a)).

Under the stress transition path, as the transition confining pressures decrease, the volumetric strain changes from continuous contractancy to final regular dilatancy (as shown in Figure 15). The employed local dilatancy equation (shown as (8)) could well describe the volume changes of sand [31, 32] under compression and extension loading conditions. However, for the NRFM, the large rockfill grains are easily broken as the pressure increases. The abrasion of the rock might reduce the friction coefficient of the contacted particles [46–48]. From the analysis of the fitting parameter $\varphi$, we found that the dilatancy angle $\varphi$ decreased as the transitional confining pressure $\sigma_{3t}$ increased. As shown in Figure 10, the dilatancy angle $\varphi$ and the transitional confining pressure $\sigma_{3t}$ exhibit a good linear relationship under the logarithmic coordinates. Here, the dilatancy angle of the contacted particles $\varphi$ can be determined as follows:

$$\varphi = \varphi_0 - \Delta\varphi \lg \left( \frac{\sigma_{3t}}{P_n} \right),$$

where $\varphi_0$ and $\Delta\varphi$ are parameters whose values are equal to the intercept and slope of the line, respectively.

5. The Evolution of Local Dilatancy at Interparticle Planes

The stress-dilatancy behaviour of the NRFM calculated using the M-A model is based on the mobilization of contacted particles. Therefore, it is necessary to study the local dilatancy behaviour on individual contact planes between two particles. In the large triaxial test, the applied loads are axisymmetric about the $x$-axis. Therefore, the orientation of the contact plane can be represented by an inclined angle $\gamma$, which can be measured by the branch vector and the $x$-axis, as shown in Figure II(a). Eight contact planes were selected for the investigation: $\gamma = 15^\circ, 25^\circ, 35^\circ, 45^\circ, 55^\circ, 65^\circ, 75^\circ$, and $85^\circ$ (shown in Figure II(a)).

In order to study the local dilatancy behaviour of the particles, the triaxial test of the type two with the transition confining pressure $\sigma_{3t} = 300$ kPa (see Figure 4) was chosen to be calculated. According to the employed local dilatancy equation (see (8)), the local dilatancy rate $dS_{ap}/dS_{ap}$ was defined here. When the value of the local dilatancy rate is negative, the contact plane moves upward and shear-induced dilatancy occurs; conversely, when the value is positive, the contact plane moves downward and the contraction occurs (Figures II(g) and II(h)).

The stress-dilatancy relations for the eight selected contact orientations are plotted in Figures II(b)–II(i), respectively. During the CSR stage with $q/p = 1$, as the load increased, the local dilatancy rate also increased. All the values are positive, which indicate that the contact planes...
Figure II: Continued.
are contracted. The $65^\circ$ contact plane reaches the highest mobilized ratio (close to $\pi/4 + \Phi/2 = 64.6^\circ$). When stress transition occurred, a clear turning point was observed, as shown in Figures 11(b)–11(i). As the axial force increased, the local dilatancy rate decreased. The contact planes are contracted/dilated at different degree for the given applied stress ratio. The $65^\circ$ and $75^\circ$ contact planes initially exhibited contraction. Then, they mobilized to higher stress levels and reached the dilation region (Figures 11(g) and 11(h)). The other planes only exhibited contraction.

Figure 12 shows the relationship of the local dilatancy rate $dS_{\alpha p}^q / dS_{\alpha p}^p$ and the global stress ratio $q/p$. The contact planes are contracted/dilated to different degrees for a given applied stress ratio. Note that the local dilatancy relations, with the planes greater than $55^\circ$, are no longer linear when plotted against the global stress ratio. Similar nonlinear relationships were also found during the compression and extension test of Hostun sand simulated by Chang et al. [31, 32].

6. Comparison and Verification of Constitutive Models

6.1. Comparison of Predicted Results with Three Models. Duncan-Chang’s E-u and E-B models are widely used for calculating the deformation of RFMs. Here, we verify the validity of the M-A models by comparing the predicted results with those of Duncan-Chang’s models. The parameters of the models could be obtained from the conventional triaxial tests ($\sigma_3 = \text{constant}$) of the NRFM. Detailed information is available in the literature [49]. The values of the parameters are listed in Table 5. Because there are too many samples to be tested, two samples were chosen to be simulated. One was loaded under a principal stress ratio of $\sigma_1/\sigma_3 = 4.0$. The other sample had a transition confining pressure of $\sigma_3 = 800$ kPa (Figure 13). The comparisons of the predicted results are as follows.

As shown in Figures 13(a) and 13(c), the shapes of the $q \sim \varepsilon_1$ curves simulated using the E-u and E-B models are the same. The reason for this result is that, under the same stress conditions, the values of the tangent modulus are equal for the two models. The shapes of the $q \sim \varepsilon_1$ curves are similar to that of the test result. However, the calculated values are clearly lower than the test results. The reason might be that the parameters are chosen from the conventional triaxial test; thus, the model cannot reflect the effect of the stress path on the deformation of the samples. The parameters of the M-A model were selected from the stress path test; thus, the simulation results and the experimental results are well fitted.

As shown in Figures 13(b) and 13(d), the $\varepsilon_3 \sim \varepsilon_1$ curves simulated using the E-u and E-B models are both significantly different from the test results. One reason for this result is that the models cannot reflect the effect of the stress path. Another reason is that they cannot describe the dilatancy behaviours of the NRFM due to the limit of elasticity theory. The dilatancy equation (see (8)) of the M-A model could well describe the dilatancy behaviours of the NRFM. Therefore, better simulation results can be obtained.

6.2. M-A Model Predictions. Under CSR loading, the parameters of the extended M-A model were calibrated by using the test data of $\sigma_1/\sigma_3 = 1.5, 2.5, \text{ and } 4.0$. Equation (22)
was chosen to determine the normal stiffness $K_n^a$ under the unloading-reloading conditions. The dilatancy angle $\varphi$ under the stress transition path could be obtained from (23). The diameter of the particle was chosen as the characteristic particle size $D_{50} = 18$ mm suggested by Chang and Yin [31]. All the parameters are listed in Tables 2 and 4. For several tests, without being used for the parameter determination, they were calculated; the calculations could be considered as genuine model predictions. For the unextended M-A model, the details information could be seen in the literature [30].

The comparisons between model predictions and type one test are shown in Figure 14 in terms of the stress-strain-volume relationship on NRFM. The unextended M-A model overestimates the deviatoric stress especially under unloading-reloading conditions (Figure 14(a)), while the extended model proposed in this paper could well predict the

![Figure 13: Comparisons between three types of model simulations and test results on NRFM: type one test, (a) and (c): type two test, (b) and (d).](image-url)

![Graphs showing comparisons between model simulations and test results on NRFM.](image-url)

<table>
<thead>
<tr>
<th>Simulation</th>
<th>Test data</th>
</tr>
</thead>
<tbody>
<tr>
<td>M-A model</td>
<td>$\sigma_1/\sigma_3 = 4.0$</td>
</tr>
<tr>
<td>E-u model</td>
<td></td>
</tr>
<tr>
<td>E-B model</td>
<td></td>
</tr>
</tbody>
</table>

![Graph showing deviatoric stress q (kPa) vs Axial strain $\varepsilon_1$ (%).](image-url)

![Graph showing Volumetric strain $\varepsilon_{13}$ (%) vs Axial strain $\varepsilon_1$ (%).](image-url)

![Graph showing Deviatoric stress q (kPa) vs Volumetric strain $\varepsilon_{13}$ (%).](image-url)

![Graph showing Deviatoric stress q (kPa) vs Volumetric strain $\varepsilon_{13}$ (%).](image-url)

**Table 4: Parameters of the M-A model.**

\[
\begin{array}{cccc}
K_n (N/m) & n & K_c/K_m & \Phi_c (\degree) \\
\end{array}
\]

(a)

\[
\begin{array}{cccc}
2.5 \times 10^3 & 0.33 & 0.5 & 39.17 \\
\end{array}
\]

(b)

\[
\begin{array}{cccc}
K_m (N/m) & w & \varphi_0 (\degree) & \Delta\varphi (\degree) \\
3.98 \times 10^4 & 0.52 & 36.91 & -0.99 \\
\end{array}
\]
stress-strain-volume behaviour of NRFM under CSR loading (Figure 14(b)). Under CSR loading, all the axial strains are less than 15%. This result indicates that no failure occurred throughout the entire range of tests. Under the unloading-reloading condition, the elastic contact of the particles (proposed in the extended model) could well reflect the linear relationship of the stress-strain curves of the sample. The calculated volumetric-strain curves appear to be linear relationships and match the test data well (Figure 14(b)). Because little changes in the volume occurred during this stage, the calculated volume changes are neglected.

Figure 15 shows the comparisons between the model predictions and type two test. The extended model considering the effect of stress path can well capture the stress-strain

<table>
<thead>
<tr>
<th>Material</th>
<th>Φ (°)</th>
<th>ΔΦ (°)</th>
<th>R_f</th>
<th>K</th>
<th>N</th>
<th>D</th>
<th>G</th>
<th>F</th>
<th>K_p</th>
<th>m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nuozhadu rockfill</td>
<td>54.37°</td>
<td>10.47°</td>
<td>0.72</td>
<td>1491</td>
<td>0.24</td>
<td>7.65</td>
<td>0.41</td>
<td>0.2</td>
<td>683</td>
<td>0.10</td>
</tr>
</tbody>
</table>
relationships of NRFM (Figure 15(b)), while the model without considering the effect of stress path cannot well represent the stress-strain relationship (overestimating the dilation of the NRFM). Under the CSR path, the change in the behaviour of the sample is similar to that of the type one test at $\sigma_1/\sigma_3 = 2.5$. However, at the transition confining pressure, the stress path moves towards the failure line and the axial strain rapidly increases. The volume of the sample changes from contraction to dilatancy as the transition confining pressure decreases. Because the load path of the test is similar to the stress variation of the actual RFM in the dam, this model could well describe the constitutive relationship of the NRFM under the paths of a constant stress ratio during dam construction and the transitional stress paths upon reservoir filling.

7. Conclusions and Discussion

A micromechanical model considering the effect of stress path was proposed for NRFM. The main conclusions and discussion are summarized below.

The stress path of RFMs during dam construction can be approximated as the CSR load, and it will become a transitional feature upon reservoir filling. Corresponding to these engineering conditions, two types of triaxial drained tests were conducted to simulate the actual stress variations of the NRFM. In the first type of test, no failure occurred, and the NRFM exhibited contraction during the entire loading period. However, in the second type of test, the samples reached the failure point. Dilatancy became increasingly more serious as the transition confining pressure decreased.
At the particle length scale, the movements of the local contacted planes can well reflect the deformation behaviour of the NRFM sample. The calculation speed of the model is fast, for the mean force and the movements of the contact plane were used to obtain the macro stress and strains. However, the model cannot reflect the deformation characteristics of specific particles.

The M-A model has been extended. For the NRFM under unloading-reloading conditions, the new stiffness of the particles can well predict the linear relationship of the stress-strain curves; for the tests under the stress transition path, the local dilatancy angle can well describe the volume of the sample changing from contractancy to dilatancy. Compared with Duncan-Chang’s models, the M-A model could better describe the deformation behaviour of the NRFM under the stress path. Model predictions for the drained triaxial tests have demonstrated that the present micromechanical approach is capable of modelling the deformation behaviours of NRFM under actual stress paths in the dam.

Competing Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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