Research Article

A Distance Model of Intuitionistic Fuzzy Cross Entropy to Solve Preference Problem on Alternatives

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In the field of decision-making, for the multiple attribute decision-making problem with the partially unknown attribute weights, the evaluation information in the form of the intuitionistic fuzzy numbers, and the preference on alternatives, this paper proposes a comprehensive decision model based on the intuitionistic fuzzy cross entropy distance and the grey correlation analysis. The creative model can make up the deficiency that the traditional intuitionistic fuzzy distance measure is easy to cause the confusion of information and can improve the accuracy of distance measure; meanwhile, the grey correlation analysis method, suitable for the small sample and the poor information decision-making, is applied in the evaluation. This paper constructs a mathematical optimization model of maximizing the synthesis grey correlation coefficient between decision-making evaluation values and decision-makers' subjective preference values, calculates the attribute weights with the known partial weight information, and then sorts the alternatives by the grey correlation coefficient values. Taking venture capital firm as an example, through the calculation and the variable disturbance, we can see that the methodology used in this paper has good stability and rationality. This research makes the decision-making process more scientific and further improves the theory of intuitionistic fuzzy multiple attribute decision-making.

1. Introduction

Professor Zadeh [1] pioneered the concept of fuzzy sets and opened up a new area for people to deal with the fuzzy information in 1965. On this basis Professor Atanassov [2, 3] put forward the concepts of intuitionistic fuzzy sets and interval valued intuitionistic fuzzy sets. In the traditional fuzzy set, only the membership degree is considered, and the intuitionistic fuzzy set not only considers the membership degree, but also considers the nonmembership degree and the uncertainty degree. As a result, it is more flexible than the fuzzy set in dealing with the uncertain problems and the fuzzy information. The content of the decision theory with the intuitionistic fuzzy information mainly contains the similarity and the distance measure [4, 5], the determination of the attribute weight [6–8], the application of the intuitionistic fuzzy multiple attribute method [9–12], and the calculation of the score function [13, 14]. However, the intuitionistic fuzzy distance formulas used in these methods still lack sufficient

comparison information which often makes the difference of the results very small so that it is difficult to make decisions.

Besides, in the practice of decision-making, to get more comprehensive and accurate understanding of the alternatives and to achieve the utility maximization better, decision-makers often try to obtain the preference information of the alternatives by all means. The preference information on alternatives usually refers to the tendency and the emotion hidden inside the decision-makers. Rational decision-makers select alternatives in accordance with a predetermined optimization principle. Although their preference information contains subjective elements, the decision criterion is still based on the maximization of the expected utility. Therefore, the decision-making problem with preference information on alternatives still belongs to the multiple attribute decision-making problem under the complete rational perspective.

In recent years, scholars have studied multiple attribute decision-making problems of different data types with preference on alternatives, and such problems have become

a hotspot in research of multiple attribute decision-making problems. Fan et al. [15] and Wang et al. [16], respectively, studied the multiple attribute decision-making problem of the real number with the alternative preference information as the complementary judgment matrix and the reciprocal judgment matrix. Xu and Chen [17, 18] studied the decision-making methods of the interval fuzzy preference relation, the interval utility value, and the interval multiplicative preference relation. In [19, 20], the solving method of the two-tuple linguistic variable multiple attribute decision-making based on the preference information was studied by the method of the aggregation operator and the granularity transformation. The research on the decision-making method with preference on alternatives, utilizing the fuzzy number and the intuitionistic fuzzy number to describe the attribute values, is a hot research field in recent years. Xu [21], using the similarity and the complementary judgment matrix ranking formula, solved the alternative preference problem whose attribute value is noted by triangular fuzzy numbers. Based on this, Liu and He [22] made further improvement and introduced the possibility degree formula for the alternative selection. Wei et al. [23] solved the problem of the intuitionistic fuzzy multiple attribute decision-making with preference on alternatives by establishing the goal programming model based on the minimum deviation and the score function ranking. In [24], a new method of constructing the distance optimization model based on the unification of the partial preference and the global preference of the decision-makers was proposed, which substituted the aggregated global preference into the objective function of the mathematical programming in the form of the intuitionistic fuzzy weighted averaging operator. Although the existing methods of the intuitionistic fuzzy multiple attribute decision-making are improved, the computation is relatively complex, and it is not suitable for the decision-making problem with many alternatives and the attributes.

At the same time, the concept and the theory of the cross entropy in information entropy are applied to the fuzzy multiple attribute decision-making. Shang and Jiang [25] gave the definition of the cross entropy of fuzzy sets in earlier times; then Vlachos and Sergiadis [26] extended it to the field of intuitionistic fuzzy sets. Because the cross entropy may not be calculated when the membership degree, the nonmembership degree, and the uncertainty degree of the intuitionistic fuzzy number take specific parameter values, Zhang and Jiang [27] and Ye [28] made further improvement. Xia and Xu [29] used the cross entropy to determine the weights of experts and the attributes of intuitionistic fuzzy group decision-making. Li [30] improved the interval valued intuitionistic fuzzy entropy by the continuous ordered weighted averaging operator (COWA) and combined it with the TOPSIS decision method to realize the optimum selection of the alternatives. Darvishi et al. [31] combined the intuitionistic fuzzy cross entropy with the principal component analysis method to solve the multiple attribute decision-making problem of a relatively large amount of information. So far, it has been found rarely that the cross entropy theory is applied to the multiple attribute decision-making problems with preference information on alternatives.

From the existing results, the research method of the intuitionistic fuzzy multiple attribute decision-making based on preference on alternatives is not perfect. Firstly, most of the existing decision models based on the deviation are the distance optimization model, while the existing intuitionistic fuzzy number distance measure formula has some defects, which often cannot distinguish the data size or the information confusion; secondly, the research is not deep enough for the unknown or partially unknown attribute weights, which often overlooks the case of the uncertainty of attribute weights. From the perspective of improving the accuracy of decision results, the intuitionistic fuzzy multiple attribute decision-making method based on preference on alternatives still needs further in-depth study.

In view of the above analysis, we propose a comprehensive decision model based on the intuitionistic fuzzy cross entropy distance and the grey correlation analysis to solve the problem of the intuitionistic fuzzy multiple attribute decision-making with preference on alternatives in this paper, which makes up the deficiency of causing the information confusion easily for the traditional intuitionistic fuzzy distance measure, improves the accuracy of the distance measure, solves the attribute weights by combining with the grey correlation analysis suitable for the small sample and the poor information decision-making, setting up the mathematical programming model with the maximum synthesis grey correlation coefficient between the evaluation value and the subjective preference value of the decision-makers, and then sorts the alternatives according to the change of the grey identification coefficients to demonstrate the stability. At last, the validity of the model is proved by the example of a venture capital firm.

2. The Basic Theory of the Intuitionistic Fuzzy Set

In this section, we introduce some basic knowledge and the necessary concepts related to the intuitionistic fuzzy set and the distance measure formula.

2.1. The Intuitionistic Fuzzy Set

Definition 1 (see [2]). Suppose X is a nonempty set and the intuitionistic fuzzy set A on the domain X is defined as follows:

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X \}, \quad (1)$$

where $\mu_A(x)$ and $\nu_A(x)$ denote, respectively, the degrees of membership and nonmembership where element x belonged to A on X : that is, $\mu_A : X \rightarrow [0, 1]$, $x \in X \rightarrow \mu_A(x) \in [0, 1]$, $\nu_A : X \rightarrow [0, 1]$, $x \in X \rightarrow \nu_A(x) \in [0, 1]$, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$; then $\pi_A = 1 - \mu_A(x) - \nu_A(x)$ is called the uncertainty degree where element x belonged to A on X . Obviously, for any $x \in X$, $0 \leq \pi_A(x) \leq 1$. Some basic operations of intuitionistic fuzzy sets are shown in Definition 2.

Definition 2 (see [32]). If $\alpha = (\mu_\alpha, \nu_\alpha)$, $\alpha_1 = (\mu_{\alpha_1}, \nu_{\alpha_1})$, and $\alpha_2 = (\mu_{\alpha_2}, \nu_{\alpha_2})$ are all intuitionistic fuzzy numbers, then

$$(1) \bar{\alpha} = (\nu_\alpha, \mu_\alpha);$$

$$(2) \alpha_1 \wedge \alpha_2 = \{\min(\mu_{\alpha_1}, \mu_{\alpha_2}), \max(\nu_{\alpha_1}, \nu_{\alpha_2})\};$$

$$(3) \alpha_1 \vee \alpha_2 = \{\max(\mu_{\alpha_1}, \mu_{\alpha_2}), \min(\nu_{\alpha_1}, \nu_{\alpha_2})\};$$

$$(4) \alpha_1 \oplus \alpha_2 = (\mu_{\alpha_1} + \mu_{\alpha_2} - \mu_{\alpha_1}\mu_{\alpha_2}, \nu_{\alpha_1}\nu_{\alpha_2});$$

$$(5) \alpha_1 \otimes \alpha_2 = (\mu_{\alpha_1}\mu_{\alpha_2}, \nu_{\alpha_1} + \nu_{\alpha_2} - \nu_{\alpha_1}\nu_{\alpha_2});$$

$$(6) \lambda\alpha = (1 - (1 - \mu_{\alpha})^{\lambda}, \nu_{\alpha}^{\lambda}), \lambda > 0;$$

$$(7) \alpha^{\lambda} = (\mu_{\alpha}^{\lambda}, 1 - (1 - \nu_{\alpha})^{\lambda}), \lambda > 0.$$

2.2. The Distance Measure Formula of Intuitionistic Fuzzy Set. Based on the geometric distance model, Xu [33] proposed a distance measure formula of the intuitionistic fuzzy set. The intuitionistic fuzzy integration operator is defined as follows.

Definition 3. Let d be a mapping: $d : (\Phi(X))^2 \rightarrow [0, 1]$. If there are the intuitionistic fuzzy sets $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle \mid$

$x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x) \rangle \mid x \in X\}$ then the distance measure formula is defined as follows:

$$d_{Xu}(A, B) = \left[\frac{1}{2n} \sum_{j=1}^n \left(|\mu_A(x_j) - \mu_B(x_j)|^{\lambda} + |\nu_A(x_j) - \nu_B(x_j)|^{\lambda} + |\pi_A(x_j) - \pi_B(x_j)|^{\lambda} \right) \right]^{1/\lambda}. \quad (2)$$

When $\lambda = 1$, d_{Xu} is reduced to the Hamming distance:

$$d_H(A, B) = \frac{1}{2n} \sum_{j=1}^n (|\mu_A(x_j) - \mu_B(x_j)| + |\nu_A(x_j) - \nu_B(x_j)| + |\pi_A(x_j) - \pi_B(x_j)|). \quad (3)$$

When $\lambda = 2$, d_{Xu} is reduced to the Euclidean distance:

$$d_E(A, B) = \sqrt{\frac{1}{2n} \sum_{j=1}^n (|\mu_A(x_j) - \mu_B(x_j)|^2 + |\nu_A(x_j) - \nu_B(x_j)|^2 + |\pi_A(x_j) - \pi_B(x_j)|^2)}. \quad (4)$$

3. Intuitionistic Fuzzy Cross Entropy

Definition 4 (see [28]). Assume a domain $X = \{x_1, x_2, \dots, x_n\}$, A and B are two intuitionistic fuzzy sets on X , $A = \{\langle x_i, \mu_A(x_i), \nu_A(x_i) \rangle \mid x_i \in X\}$, $B = \{\langle x_i, \mu_B(x_i), \nu_B(x_i) \rangle \mid x_i \in X\}$, and then the intuitionistic fuzzy cross entropy of A and B is

$$\begin{aligned} CE(A, B) = & \sum_{i=1}^n \left(\frac{1 + \mu_A(x_i) - \nu_A(x_i)}{2} \right. \\ & \cdot \log_2 \frac{1 + \mu_A(x_i) - \nu_A(x_i)}{(1/2)[(1 + \mu_A(x_i) - \nu_A(x_i)) + (1 + \mu_B(x_i) - \nu_B(x_i))]} \\ & + \sum_{i=1}^n \left(\frac{1 - \mu_A(x_i) + \nu_A(x_i)}{2} \right. \\ & \cdot \log_2 \frac{1 - \mu_A(x_i) + \nu_A(x_i)}{(1/2)[(1 - \mu_A(x_i) + \nu_A(x_i)) + (1 - \mu_B(x_i) + \nu_B(x_i))]} \Big). \end{aligned} \quad (5)$$

The intuitionistic fuzzy cross entropy $CE(A, B)$ does not satisfy the symmetry, so let

$$CE^*(A, B) = CE(A, B) + CE(B, A) \quad (6)$$

be the improved form of the intuitionistic fuzzy cross entropy and define it as the intuitionistic fuzzy cross entropy distance.

Theorem 5. The intuitionistic fuzzy cross entropy distance $CE^*(A, B)$ satisfies the following properties:

- (1) $CE^*(A, B) \geq 0$;
- (2) If $A = B$, then $CE^*(A, B) = 0$;

$$(3) CE^*(A, B) = CE^*(\bar{A}, \bar{B});$$

$$(4) CE^*(A, B) = CE^*(B, A).$$

Proof. (1) One has

$$\begin{aligned} CE(A, B) = & \sum_{i=1}^n \left(\frac{1 + \mu_A(x_i) - \nu_A(x_i)}{2} \right. \\ & \cdot \log_2 \frac{1 + \mu_A(x_i) - \nu_A(x_i)}{(1/2)[(1 + \mu_A(x_i) - \nu_A(x_i)) + (1 + \mu_B(x_i) - \nu_B(x_i))]} \\ & + \sum_{i=1}^n \left(\frac{1 - \mu_A(x_i) + \nu_A(x_i)}{2} \right. \\ & \cdot \log_2 \frac{1 - \mu_A(x_i) + \nu_A(x_i)}{(1/2)[(1 - \mu_A(x_i) + \nu_A(x_i)) + (1 - \mu_B(x_i) + \nu_B(x_i))]} \Big), \end{aligned} \quad (7)$$

so

$$\begin{aligned} -CE(A, B) = & - \sum_{i=1}^n \left(\frac{1 + \mu_A(x_i) - \nu_A(x_i)}{2} \right. \\ & \cdot \log_2 \frac{1 + \mu_A(x_i) - \nu_A(x_i)}{(1/2)[(1 + \mu_A(x_i) - \nu_A(x_i)) + (1 + \mu_B(x_i) - \nu_B(x_i))]} \\ & - \sum_{i=1}^n \left(\frac{1 - \mu_A(x_i) + \nu_A(x_i)}{2} \right. \\ & \cdot \log_2 \frac{1 - \mu_A(x_i) + \nu_A(x_i)}{(1/2)[(1 - \mu_A(x_i) + \nu_A(x_i)) + (1 - \mu_B(x_i) + \nu_B(x_i))]} \Big) \\ = & \sum_{i=1}^n \left(\frac{1 + \mu_A(x_i) - \nu_A(x_i)}{2} \right. \end{aligned}$$

$$\begin{aligned}
& \cdot \log_2 \frac{(1/2) [(1 + \mu_A(x_i) - \nu_A(x_i)) + (1 + \mu_B(x_i) - \nu_B(x_i))]}{1 + \mu_A(x_i) - \nu_A(x_i)} \\
& + \sum_{i=1}^n \left(\frac{1 - \mu_A(x_i) + \nu_A(x_i)}{2} \right. \\
& \cdot \log_2 \frac{(1/2) [(1 - \mu_A(x_i) + \nu_A(x_i)) + (1 - \mu_B(x_i) + \nu_B(x_i))]}{1 - \mu_A(x_i) + \nu_A(x_i)} \Bigg). \quad (8)
\end{aligned}$$

According to Jansen's inequality [34], if $f(x)$ is strictly convex, then

$$\begin{aligned}
& f(a_1 x_1 + a_2 x_2 + \cdots + a_n x_n) \\
& \leq a_1 f(x_1) + a_2 f(x_2) + \cdots + a_n f(x_n). \quad (9)
\end{aligned}$$

Because the logarithmic function above is strictly convex, then

$$\begin{aligned}
-\text{CE}(A, B) & \leq \sum_{i=1}^n \log_2 \left(\frac{1 + \mu_A(x_i) - \nu_A(x_i)}{2} \frac{(1/2) [(1 + \mu_A(x_i) - \nu_A(x_i)) + (1 + \mu_B(x_i) - \nu_B(x_i))]}{1 + \mu_A(x_i) - \nu_A(x_i)} \right) \\
& + \sum_{i=1}^n \log_2 \left(\frac{1 - \mu_A(x_i) + \nu_A(x_i)}{2} \frac{(1/2) [(1 - \mu_A(x_i) + \nu_A(x_i)) + (1 - \mu_B(x_i) + \nu_B(x_i))]}{1 - \mu_A(x_i) + \nu_A(x_i)} \right) \quad (10) \\
& \leq \log_2 \frac{(1 + \mu_A(x_i) - \nu_A(x_i)) + (1 + \mu_B(x_i) - \nu_B(x_i)) + (1 - \mu_A(x_i) + \nu_A(x_i)) + (1 - \mu_B(x_i) + \nu_B(x_i))}{4} \\
& = 0,
\end{aligned}$$

so $\text{CE}(A, B) \geq 0$. The same can be proved where $\text{CE}(B, A) \geq 0$, so $\text{CE}^*(A, B) \geq 0$.

(2) When $A = B$, obviously, $\mu_A(x_i) = \mu_B(x_i)$, $\nu_A(x_i) = \nu_B(x_i)$, substitute them into formulas (5) and (6), and $\text{CE}^*(A, B) = 0$.

(3) As can be seen from the complementation of the intuitionistic fuzzy sets, $\bar{A} = \{\langle x_i, \nu_A(x_i), \mu_A(x_i) \rangle\}$, $\bar{B} = \{\langle x_i, \nu_B(x_i), \mu_B(x_i) \rangle\}$, substitute them into formulas (5) and (6), and obviously $\text{CE}^*(A, B) = \text{CE}^*(\bar{A}, \bar{B})$.

(4) Because $\text{CE}^*(A, B)$ satisfies the symmetry, the exchange of intuitionistic fuzzy sets A and B does not affect the entropy result: that is, $\text{CE}^*(A, B) = \text{CE}^*(B, A)$.

As can be seen from property (2), when the two intuitionistic fuzzy sets are exactly equal, the intuitionistic fuzzy cross entropy is the least; therefore, the cross entropy can be used to measure the difference between two intuitionistic fuzzy sets. The intuitionistic fuzzy cross entropy adds the meaning of the information entropy on the basis of the original intuitionistic fuzzy complete information. It can be used to measure the fuzzy degree and the uncertainty degree of the intuitionistic fuzzy sets. The greater the cross entropy of two intuitionistic fuzzy numbers, the further the distance [26]. The following example is proposed to prove that the intuitionistic fuzzy entropy can better reflect the difference between the data than the traditional intuitionistic fuzzy distance measure formula.

For example, there are three intuitionistic fuzzy numbers $\alpha_1 = (0.4, 0.3)$, $\alpha_2 = (0.5, 0.3)$, and $\alpha_3 = (0.5, 0.2)$. With the Hamming distance formula (3), there is $d_H(\alpha_1, \alpha_3) = d_H(\alpha_2, \alpha_3)$. With the Euclidean formula distance formula (4), there is $d_E(\alpha_1, \alpha_3) = d_E(\alpha_2, \alpha_3)$. Obviously, it is difficult to compare the two distances. Compute, with formula (6), $\text{CE}^*(\alpha_1, \alpha_3) = 0.0151$ and $\text{CE}^*(\alpha_2, \alpha_3) = 0.0038$. Comparing with α_1 , α_2 is closer to α_3 , which is consistent with the people's intuition. $\square$

4. Multiple Attribute Decision-Making Method Based on Intuitionistic Fuzzy Cross Entropy

4.1. Problem Description. In this paper, the multiple attribute decision-making problems are assumed to have a certain subjective preference for the decision-makers. Generally the problems can be abstracted as follows: the decision-makers can give the attribute values in the form of the intuitionistic fuzzy numbers (μ_{ij}, ν_{ij}) from the alternatives A_i ($i = 1, 2, \dots, m$) according to the evaluation attributes C_j ($j = 1, 2, \dots, n$), where μ_{ij} denotes the approval degree about A_i under C_j , ν_{ij} denotes the disapproval degree about A_i under C_j , $\pi_{ij} = 1 - \mu_{ij} - \nu_{ij}$ denotes the uncertainty degree, $0 \leq \mu_{ij} \leq 1$, $0 \leq \nu_{ij} \leq 1$, $0 \leq \pi_{ij} \leq 1$, and ω_j ($j = 1, 2, \dots, n$) denotes the attribute weights; meanwhile, $\sum_{j=1}^n \omega_j = 1$. The intuitionistic fuzzy decision matrix R_{mn} is shown in Table 1.

Suppose the decision-makers have some preference on alternatives A_i ($i = 1, 2, \dots, m$) and the preference values are noted by the intuitionistic fuzzy numbers $o_i = (\alpha_i, \beta_i)$ ($i = 1, 2, \dots, m$); then we adopt the optimization model based on the intuitionistic fuzzy cross entropy and the grey correlation analysis method to find the optimum solution among the alternatives with preference.

4.2. Decision Method and Steps. In this paper, the intuitionistic fuzzy multiple attribute decision-making method with preference on alternatives draws lessons from the theory of the intuitionistic fuzzy cross entropy distance and the grey correlation analysis method to solve the optimum alternative selection problem in the case where the weights are partly unknown.

Step 1. Determine the alternatives A_i ($i = 1, 2, \dots, m$), the evaluation attributes C_j ($j = 1, 2, \dots, n$), the subjective evaluation matrix of the decision-makers R_{mn} , and the subjective preference $o_i = (\alpha_i, \beta_i)$.

TABLE 1: Intuitionistic fuzzy decision matrix R_{mn} .

	C_1	C_2	$\dots$	C_n
A_1	(μ_{11}, ν_{11})	(μ_{12}, ν_{12})	$\dots$	(μ_{1n}, ν_{1n})
A_2	(μ_{21}, ν_{21})	(μ_{22}, ν_{22})	$\dots$	(μ_{2n}, ν_{2n})
$\vdots$	$\vdots$	$\vdots$	$\dots$	$\vdots$
A_m	(μ_{m1}, ν_{m1})	(μ_{m2}, ν_{m2})	$\dots$	(μ_{mn}, ν_{mn})

Step 2. Calculate the grey correlation coefficient between the subjective evaluation of each alternative based on the intuitionistic fuzzy cross entropy distance and the subjective preference of the decision-makers. The formula is as follows:

$$\gamma_{ij} = \frac{\min_i \min_j CE_{ij}^* + \xi \max_i \max_j CE_{ij}^*}{CE_{ij}^* + \xi \max_i \max_j CE_{ij}^*}. \quad (11)$$

The grey correlation coefficient here shows the approximation degree of the subjective evaluation to the subjective preference of each A_i under C_j . The greater the grey correlation coefficient γ_{ij} , the higher the approximation degree, and vice versa. In the formula above, CE_{ij}^* is the intuitionistic fuzzy cross entropy distance, and its formula is as follows:

$$\begin{aligned} CE_{ij}^* = & \frac{1 + \mu_{ij} - \nu_{ij}}{2} \\ & \cdot \log_2 \frac{1 + \mu_{ij} - \nu_{ij}}{(1/2) [(1 + \mu_{ij} - \nu_{ij}) + (1 + \alpha_i - \beta_i)]} \\ & + \frac{1 - \mu_{ij} + \nu_{ij}}{2} \\ & \cdot \log_2 \frac{1 - \mu_{ij} + \nu_{ij}}{(1/2) [(1 - \mu_{ij} + \nu_{ij}) + (1 - \alpha_i + \beta_i)]} \\ & + \frac{1 + \alpha_i - \beta_i}{2} \\ & \cdot \log_2 \frac{1 + \alpha_i - \beta_i}{(1/2) [(1 + \alpha_i - \beta_i) + (1 + \mu_{ij} - \nu_{ij})]} \\ & + \frac{1 - \alpha_i + \beta_i}{2} \\ & \cdot \log_2 \frac{1 - \alpha_i + \beta_i}{(1/2) [(1 - \alpha_i + \beta_i) + (1 - \mu_{ij} + \nu_{ij})]}. \end{aligned} \quad (12)$$

In formula (11), ξ is called the distinguishing coefficient: usually $\xi \in (0, 1)$.

Step 3. The actual meaning has been clear by last step; therefore, each attribute weight can be determined by constructing the mathematical programming model with the purpose of maximizing the grey correlation coefficient.

Let the attribute weight be ω_j ($j = 1, 2, \dots, n$); meanwhile, $\sum_{j=1}^n \omega_j = 1$. Calculate the synthesis grey correlation

coefficient of A_i under each attribute using the following formula:

$$\gamma_i = \sum_{j=1}^n \gamma_{ij} \omega_j, \quad (i = 1, 2, \dots, m), \quad (j = 1, 2, \dots, n). \quad (13)$$

If the attribute weight ω_j is known, then sort the alternatives according to the grey correlation coefficient calculated by formula (11). The greater γ_i , the higher the approximation of the subjective evaluation to the subjective preference of alternative A_i , and, therefore, the higher in the rankings. However, in the actual decision-making process, because of the lack of knowledge and the complexity of the objective things, it is often difficult to determine the attribute weights for the decision-makers. In most cases, the attribute weights are partially unknown or even completely unknown. Consequently, how to determine the attribute weights is a key problem in the decision-making. In order to guarantee the feasibility and the validity of the decision-making method, the establishment of the weight should be based on the maximum similarity between the subjective evaluation and the subjective preference value of the alternative A_i . Therefore, the mathematical programming models can be established as

$$\begin{aligned} \max \quad & Z(\omega_j) = \sum_{j=1}^n \gamma_{ij} \omega_j \\ \text{s.t.} \quad & \sum_{j=1}^n \omega_j = 1, \quad \omega_j \in W \\ & \omega_j \geq 0, \\ & j = 1, 2, \dots, n, \\ & i = 1, 2, \dots, m. \end{aligned} \quad (\text{Model 1})$$

Note that s.t. is short for “subject to” (the same as below).

Since there is no preference relationship between the various alternatives, it is fair to compete, so (Model 1) can be transformed into a single objective optimization model as follows:

$$\begin{aligned} \max \quad & Z(\omega_j) = \sum_{i=1}^m \sum_{j=1}^n \gamma_{ij} \omega_j \\ \text{s.t.} \quad & \sum_{j=1}^n \omega_j = 1, \quad \omega_j \in W \\ & \omega_j \geq 0, \\ & j = 1, 2, \dots, n, \\ & i = 1, 2, \dots, m. \end{aligned} \quad (\text{Model 2})$$

The attribute weights can be obtained by using the software MATLAB_R2014a. If the attribute weight part is known, then $\omega_j \in W$ and usually it can be used as a constraint condition in the mathematical programming in 5 cases as follows [35]:

$$(1) \quad \omega_k \geq \omega_l;$$

TABLE 2: The intuitionistic fuzzy decision matrix R_{54} .

	C_1	C_2	C_3	C_4
A_1	(0.4, 0.5)	(0.5, 0.4)	(0.2, 0.7)	(0.3, 0.5)
A_2	(0.7, 0.2)	(0.5, 0.4)	(0.2, 0.5)	(0.1, 0.7)
A_3	(0.5, 0.3)	(0.3, 0.4)	(0.6, 0.2)	(0.4, 0.4)
A_4	(0.6, 0.4)	(0.6, 0.3)	(0.6, 0.3)	(0.3, 0.6)
A_5	(0.5, 0.5)	(0.4, 0.5)	(0.4, 0.4)	(0.5, 0.4)

$$(2) \omega_k - \omega_t \geq c, c > 0;$$

$$(3) \omega_k \geq c\omega_t, c > 0;$$

$$(4) c \leq \omega_i \leq c + \tau, c > 0, \tau > 0;$$

$$(5) \omega_k - \omega_t \geq \omega_l - \omega_e, t \neq l \neq e.$$

Step 4. Substitute the obtained attribute weight ω_j into formula (13) and then calculate the grey correlation coefficient γ_i of the alternative A_i under all the attributes.

Step 5. Sort the alternatives by the synthesis grey correlation coefficient γ_i of each alternative. The greater γ_i , the better the alternative.

Step 6. Make the perturbation analysis according to the variation of the distinguishing coefficient ξ in the formula of the grey correlation coefficient (11) in order to test the stability and the reliability of the method.

5. Example Analysis

In order to prove the accuracy and the validity of the method in this chapter, we use the example in document [24] for calculation and analysis. A venture capital firm intends to make evaluation and selection to 5 enterprises with the investment potential: A_1 : automobile company, A_2 : military manufacturing enterprise, A_3 : TV media company, A_4 : food enterprises, and A_5 : computer software company. The 5 enterprises are evaluated under four conditions including the social and the political factors (C_1), the environmental factors (C_2), the investment risk factors (C_3), and the enterprise growth factors (C_4). The evaluation values are noted in the form of the intuitionistic fuzzy numbers and the obtained intuitionistic fuzzy decision matrix R_{54} is shown in Table 2.

The subjective preference of the decision-makers on alternatives A_i ($i = 1, 2, \dots, 5$) is also noted by the intuitionistic fuzzy numbers: that is, $o_1 = (0.3, 0.5)$, $o_2 = (0.7, 0.2)$, $o_3 = (0.4, 0.3)$, $o_4 = (0.6, 0.2)$, and $o_5 = (0.5, 0.4)$, and the known weights satisfy $0.25 \leq \omega_1 \leq 0.28$, $0.20 \leq \omega_2 \leq 0.25$, $0.22 \leq \omega_3 \leq 0.25$, and $0.25 \leq \omega_4 \leq 0.30$. In order to select the optimal target investment enterprise, the venture capital firm adopts the decision-making method constructed in this paper.

Specific calculation steps are as follows.

Step 1. Determine the alternatives A_i ($i = 1, 2, \dots, 5$), the evaluation attributes C_j ($j = 1, 2, \dots, 4$), the subjective evaluation matrix R_{54} , and the subjective preference $o_i = (\alpha_i, \beta_i)$ ($i = 1, 2, \dots, 5$), as shown in Table 2.

Step 2. Calculate the intuitionistic fuzzy cross entropy distances between the subjective evaluation and the subjective preference of each alternative and form the distance matrix CE^* as follows:

$$CE^* = CE_{54}^* = \begin{bmatrix} 0.0037 & 0.0327 & 0.0372 & 0.0000 \\ 0.0000 & 0.0641 & 0.2402 & 0.4632 \\ 0.0037 & 0.0145 & 0.0348 & 0.0036 \\ 0.0159 & 0.0041 & 0.0041 & 0.1810 \\ 0.0036 & 0.0145 & 0.0036 & 0.0000 \end{bmatrix}. \quad (14)$$

Then calculate the grey correlation coefficient between the subjective evaluation and the subjective preference of each alternative. Suppose $\xi = 0.5$, and the obtained coefficient matrix γ is shown as follows:

$$\gamma = \gamma_{54} = \begin{bmatrix} 0.9843 & 0.8763 & 0.8616 & 1.0000 \\ 1.0000 & 0.7832 & 0.4909 & 0.3333 \\ 0.9843 & 0.9411 & 0.8694 & 0.9847 \\ 0.9358 & 0.9826 & 0.9826 & 0.5613 \\ 0.9847 & 0.9411 & 0.9847 & 1.0000 \end{bmatrix}. \quad (15)$$

Step 3. Construct the mathematical programming model with the goal of maximizing the grey correlation coefficient:

$$\begin{aligned} \max \quad & Z(\omega_j) \\ & = 4.8991\omega_1 + 4.5243\omega_2 + 4.1892\omega_3 \\ & \quad + 3.8793\omega_4 \\ \text{s.t.} \quad & 0.25 \leq \omega_1 \leq 0.28 \\ & 0.20 \leq \omega_2 \leq 0.25 \\ & 0.22 \leq \omega_3 \leq 0.25 \\ & 0.25 \leq \omega_4 \leq 0.30 \\ & \omega_1 + \omega_2 + \omega_3 + \omega_4 = 1 \\ & \omega_j \geq 0, \quad j = 1, 2, 3, 4. \end{aligned} \quad (16)$$

We performed calculation by MATLAB R2014a and the obtained attribute weight values are $\omega_1 = 0.28$, $\omega_2 = 0.25$, $\omega_3 = 0.22$, and $\omega_4 = 0.25$.

Step 4. Substitute the obtained attribute weight ω_j into formula (13) and calculate the synthesis grey correlation coefficient γ_i ($i = 1, 2, \dots, 5$) under all the attributes:

$$\begin{aligned} \gamma_1 &= 0.9342; \\ \gamma_2 &= 0.6671; \\ \gamma_3 &= 0.9483; \\ \gamma_4 &= 0.8642; \\ \gamma_5 &= 0.9776. \end{aligned} \quad (17)$$

TABLE 3: The attribute weight values under different grey resolutions.

	$\xi = 0.05$	$\xi = 0.20$	$\xi = 0.35$	$\xi = 0.50$	$\xi = 0.65$	$\xi = 0.80$	$\xi = 0.95$
ω_1	0.28	0.28	0.28	0.28	0.28	0.28	0.28
ω_2	0.20	0.25	0.25	0.25	0.25	0.25	0.25
ω_3	0.22	0.22	0.22	0.22	0.22	0.22	0.22
ω_4	0.30	0.25	0.25	0.25	0.25	0.25	0.25

TABLE 4: The decision-making results under different grey resolutions.

	$\xi = 0.05$		$\xi = 0.20$		$\xi = 0.35$		$\xi = 0.50$		$\xi = 0.65$		$\xi = 0.80$		$\xi = 0.95$	
	γ_i	Rank	γ_i	Rank	γ_i	Rank	γ_i	Rank	γ_i	Rank	γ_i	Rank	γ_i	Rank
A_1	0.70	3	0.86	3	0.91	3	0.93	3	0.95	3	0.96	3	0.96	3
A_2	0.37	5	0.53	5	0.61	5	0.67	5	0.71	5	0.74	5	0.76	5
A_3	0.71	2	0.89	2	0.93	2	0.95	2	0.96	2	0.97	2	0.97	2
A_4	0.56	4	0.77	4	0.83	4	0.86	4	0.89	4	0.90	4	0.91	4
A_5	0.86	1	0.95	1	0.97	1	0.98	1	0.98	1	0.99	1	0.99	1

Step 5. Sort the alternatives by the synthesis grey correlation coefficient γ_i ($i = 1, 2, \dots, 5$), $\gamma_5 \geq \gamma_3 \geq \gamma_1 \geq \gamma_4 \geq \gamma_2$, so $A_5 > A_3 > A_1 > A_4 > A_2$. γ_3 is the greatest, so A_3 is the optimum: that is, the target the venture capital firm selects to invest is the computer software company.

Step 6. Make the perturbation analysis according to the variation of the distinguishing coefficient ξ in the formula of grey correlation coefficient (5). Let $\xi = 0.05, 0.20, 0.35, 0.50, 0.65, 0.80$, and 0.95 . The attribute weights ω_j ($j = 1, 2, \dots, 4$), the synthesis grey correlation coefficients γ_i ($i = 1, 2, \dots, 5$), and the ranking of the alternatives are shown in Tables 3 and 4.

As can be seen from Table 4, although the grey distinguishing coefficient ξ is assigned 7 dispersed values, the decision result does not vary with the change of the grey distinguishing coefficient, and the ranking result still keeps the same, $A_5 > A_3 > A_1 > A_4 > A_2$, which can demonstrate that the method constructed in this paper is stable and reliable.

However, the result of this paper is difficult compared to that of [24] whose ranking result is $A_5 > A_1 > A_3 > A_4 > A_2$. The investment target the venture capital enterprise selects is still the computer software company, but the ranking of the other alternatives is different. The main reasons lie in the following. (1) The distance measure of the intuitionistic fuzzy multiple attribute decision-making method with preference information on alternatives adopts the intuitionistic fuzzy cross entropy, which is not like the geometric distance such as the Hamming or the Euclidean distance, but a distance of information that can better reflect the difference between the intuitionistic fuzzy sets and not just keep the information. Take the calculation process as an example in this section. The subjective preference on alternative A_3 is $o_3 = (0.4, 0.3)$, but in the initial decision matrix, the subjective evaluation values under attributes C_1 and C_2 are given $(0.5, 0.3)$ and $(0.3, 0.4)$ by the decision-makers. When calculating with the Hamming or the Euclidean distance formula, the two distances between the evaluation value and

the preference value are the same, difficult to distinguish, so the accuracy of the decision result will be affected. However, with the intuitionistic fuzzy cross entropy distance proposed in this section, the distances are 0.0037 and 0.0145, making it easy to distinguish and forming good conditions for improving the accuracy of the decision result. (2) The grey correlation analysis method used in this section is suitable for the small sample and the poor information problem. The specific steps include applying it in the intuitionistic fuzzy multiple attribute decision-making, making full use of the known decision information such as the membership degree, the nonmembership degree, and the uncertainty degree, and then carrying out the perturbation analysis according to the change of the grey distinguishing coefficient. It turns out that combining the grey correlation analysis method with the intuitionistic fuzzy multiple attribute decision-making is suitable for the small sample decision-making; moreover, the effectiveness and the stability of the decision results are both relatively high.

6. Conclusion

In this paper, a comprehensive decision model based on the intuitionistic fuzzy cross entropy distance and the grey correlation analysis is proposed for the multiple attribute decision-making problems with the attribute weights partially unknown, the evaluation information in the form of intuitionistic fuzzy numbers, and the preference information on alternatives. The model introduces the intuitionistic fuzzy cross entropy distance to substitute the traditional geometric distance, calculates the attribute weights by constructing the mathematical optimization model of maximizing the synthesis grey correlation coefficient between the decision-making evaluation values and decision-makers' subjective preference values and making use of the known partial weight information, then sorts the alternatives by the synthesis grey correlation coefficient values, and at last demonstrates the effectiveness of the proposed model through the comparison and the analysis of two calculation examples. In this paper, the model

has strong pertinence, high accuracy, and simple calculation and has further perfected and enriched the intuitionistic fuzzy multiple attribute decision-making theory. In future research, we will focus on the decision model and method constructed in this paper. Comparing to the study of alternatives with the definite preference, this research field will be more complex and more innovative in the multiple attribute decision-making method.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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