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Research Article

Stabilization of Semi-Markovian Jump Systems with Uncertain Probability Intensities and Its Extension to Quantized Control

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This paper concentrates on the issue of stability analysis and control synthesis for semi-Markovian jump systems (S-MJSs) with uncertain probability intensities. Here, to construct a more applicable transition model for S-MJSs, the probability intensities are taken to be uncertain, and this property is totally reflected in the stabilization condition via a relaxation process established on the basis of time-varying transition rates. Moreover, an extension of the proposed approach is made to tackle the quantized control problem of S-MJSs, where the infinitesimal operator of a stochastic Lyapunov function is clearly discussed with consideration of input quantization errors.

1. Introduction

Over the past few decades, considerable attention has been paid to Markovian jump systems (MJSs) since such systems are suitable for representing a class of dynamic systems subject to random abrupt variations. In addition to the growing interest from their representation ability, MJSs have been widely applied in many practical applications, such as manufacturing systems, aircraft control, target tracking, robotics, networked control systems, solar receiver control, and power systems (see [1–9] and references therein). Following this trend, numerous investigations are underway to deal with the issue of stability analysis and control synthesis for MJSs with complete/incomplete knowledge of transition probabilities in the framework of filter and control design problems: [10–13] with a complete description of transition rates and [14–20] without a complete description. Generally, in MJSs, the sojourn-time is given as a random variable characterized by the continuous exponential probability distribution, which tends to make the transition rates time-invariant due to the memoryless property of the probability distribution. The thing to be noticed here is that the use of constant transition rates plays a limited role in representing a wide range of application systems (see [21–23]). Thus, another interesting topic has recently been studied in semi-Markovian jump systems (S-MJSs) to overcome the limitation of this memoryless property.

As reported in [24–26], the mode transition of S-MJSs is driven by a continuous stochastic process governed by the nonexponential sojourn-time distribution, which leads to the appearance of time-varying transition rates. Thus, it has been well recognized that S-MJSs are more general than MJSs in real situations. Further, with this growing recognition, various problems on S-MJSs have been widely studied for successful utilization of a variety of practical applications (see [21, 22, 25, 27–29] and references therein). Of them, the first attempt to overcome the limits of MJSs was made by [21, 22] for the stability analysis of systems with phase-type (PH) semi-Markovian jump parameters, which was extended to the state estimation and sliding mode control by [29]. Besides, [25] considered the Weibull distribution for the stability analysis of S-MJSs and introduced a sojourn-time partition technique to make the derived stability criterion less conservative. Continuing this, [28] applied the sojourn-time partition technique to the design of $\mathscr{H}_\infty$ state-feedback control for S-MJSs with time-varying delays. After that, another partition technique of dividing the range of transition rates was proposed by [27] to derive the stability and stabilization conditions of S-MJSs with norm-bounded uncertainties. Most recently, [30] designed a reliable mixed passive and $\mathscr{H}_\infty$ filter for semi-Markov jump delayed systems with randomly occurring uncertainties and sensor failures. Also, [26] considered semi-Markovian switching and random measurement while designing a sliding mode control for networked control systems (NCSs). Based on the above observations, it can be found that their key issue mainly lies in finding more applicable transition models for S-MJSs, capable of a broad range of cases. In this light, one needs to explore the impacts of uncertain probability intensities in the study of S-MJSs and then provide a relaxed stability criterion absorbing the property of the resultant time-varying transition rates. However, until now, there have been almost no studies that intensively establish a kind of relaxation process corresponding to the stabilization problem of S-MJSs with uncertain probability intensities.

This paper addresses the issue of stability analysis and control synthesis for S-MJSs with uncertain probability intensities. One of our main contributions is to discover more reliable and scalable transition models for S-MJSs on the basis of their time-varying and boundary properties. To this end, this paper provides a valuable theoretical approach of constructing practical transition models for S-MJSs (1) by taking into account uncertain probability intensities and (2) by reflecting their available bounds in the transition rate description. Further, in a different manner from other works, all constraints on time-varying transition rates are totally incorporated into the stabilization condition via a relaxation process established on the basis of time-varying transition rates. Here, it is worth noticing that our relaxation process is developed in such a way that all possible slack variables can be included therein. In contrast to other works, our relaxation process plays a key role in obtaining a finite and solvable set of linear matrix inequalities (LMIs) from parameterized matrix inequalities (PLMIs) arising from uncertain probability intensities. On the other hand, the quantization module that converts real-valued measurement signals into piecewise constant ones has been commonly used to implement a variety of networked control systems over wired or wireless communications (see [31, 32]). Especially among optical wireless communications, the visible light communication can be applied as a data communication channel to transmit the control input to the S-MJSs under consideration. Thus, as an extension, this paper tackles the quantized control problem of S-MJSs, where the infinitesimal operator of a stochastic Lyapunov function is clearly discussed with consideration on input quantization errors. In addition, this paper proposes a method for reducing the influence of input quantization errors in the control of S-MJSs, which is also one of our main contributions. Finally, simulation examples show the effectiveness of the proposed method.

Notation. The notations $X \geq Y$ and $X - Y$ means that $X - Y$ is positive semidefinite and positive definite, respectively. In symmetric block matrices, $(*)$ is used as an ellipsis for terms induced by symmetry. For any square matrix $\mathcal{Q}$, $\mathbf{He}[\mathcal{Q}] = \mathcal{Q} + \mathcal{Q}^T$. For $\mathbb{N}_s^+ \triangleq \{1, 2, \dots, s\}$,

$$\begin{gathered}
[\mathcal{Q}_i]_{i\in\mathbb{N}_s^+} \triangleq [\mathcal{Q}_1 \mathcal{Q}_2 \cdots \mathcal{Q}_s], \\
[\mathcal{Q}_{ij}]_{i,j\in\mathbb{N}_s^+} \triangleq \begin{bmatrix} \mathcal{Q}_{11} & \cdots & \mathcal{Q}_{1s} \\ \vdots & \ddots & \vdots \\ \mathcal{Q}_{s1} & \cdots & \mathcal{Q}_{ss} \end{bmatrix}, \\
[\mathcal{Q}_i]^{\mathbf{D}}_{i\in\mathbb{N}_s^+} \triangleq \begin{bmatrix} \mathcal{Q}_1 & 0 & \cdots & 0 \\ 0 & \mathcal{Q}_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \mathcal{Q}_s \end{bmatrix}, \\
[\mathcal{Q}_{ij}]^{\mathbf{U}}_{i,j\in\mathbb{N}_s^+} \triangleq \begin{bmatrix} 0 & \mathcal{Q}_{12} & \mathcal{Q}_{13} & \cdots & \mathcal{Q}_{1s} \\ 0 & 0 & \mathcal{Q}_{23} & \cdots & \mathcal{Q}_{2s} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & 0 & \mathcal{Q}_{(s-1)s} \\ 0 & \cdots & \cdots & 0 & 0 \end{bmatrix},
\end{gathered} \tag{1}$$

where $\mathcal{Q}_i$ and $\mathcal{Q}_{ij}$ denote real submatrices with appropriate dimensions or scalar values. The notation $\mathbf{E}[\bullet]$ denotes the mathematical expectation and diag $(\bullet)$ stands for a block-diagonal matrix. The notation $\lambda_{\max}(\bullet)$ denotes the maximum eigenvalue of the argument, and exp $(\bullet)$ indicates the exponential distribution.

2. System Description

Let us consider the following continuous-time semi-Markovian jump linear systems (S-MJSs):

$$\dot{x}(t) = A(\zeta(t))\, x(t) + B(\zeta(t))\, u(t), \tag{2}$$

where $x(t) \in \mathbb{R}^{n_x}$ and $u(t) \in \mathbb{R}^{n_u}$ denote the state and the control input, respectively. Here, $\{\zeta(t), t \geq 0\}$ denotes a continuous-time semi-Markov process that takes values in the finite space $\mathbb{N}_s^+$ and further has the mode transition probabilities:

$$\begin{aligned}
&\mathbf{Pr}(\zeta(t+h) = j \mid \zeta(t) = i) \\
&\quad = \begin{cases} \pi_{ij}(h)\, h + o(h), & \text{if } j \neq i \\ 1 + \pi_{ii}(h)\, h + o(h), & \text{if } j = i, \end{cases}
\end{aligned} \tag{3}$$

where $\lim_{h\to 0}(o(h)/h) = 0$ and $\pi_{ij}(h)$ denotes the transition rate from mode i to mode j at time $t + h$. Further, h indicates the sojourn-time elapsed when the system stays at mode i from the last jump (i.e., h is set to 0 when the system jumps).

In particular, the transition rate matrix $\prod(h) \triangleq [\pi_{ij}(h)]_{i,j\in\mathbb{N}_s^+}$ belongs to the following set:

$$\mathcal{S}_{\Pi}^{(1)} \triangleq \left\{ \left[\pi_{ij}\right]_{i,j\in\mathbb{N}_s^+} \mid 0 = \sum_{j\in\mathbb{N}_s^+} \pi_{ij},\ 0 \le \mu_{ij}\pi_{ij}, \text{ where } \mu_{ij}\big|_{j\neq i} = 1,\ \mu_{ij}\big|_{j=i} = -1,\ \forall i, j \in \mathbb{N}_s^+ \right\}. \quad (4)$$

Before going ahead, for later convenience, we define the system matrix for the ith mode as $(A_i, B_i) \triangleq (A(\zeta_k = i), B(\zeta_k = i))$, and set $\prod_i(h) \triangleq [\pi_{i1}(h) \quad \cdots \quad \pi_{is}(h)]^T = [\pi_{ij}(h)]^T_{j\in\mathbb{N}_s^+}$. Also, to deal with the stability analysis problem in such a stochastic setting, we consider the following definition.

Definition 1. An S-MJS (2) with $u(t) = 0$ is stochastically stable if its solution is such that, for any initial condition x_0 and ζ_0,

$$\lim_{t\to\infty} \mathbf{E}\left[\int_0^t \|x(\tau)\|^2 d\tau \mid x_0, \zeta_0\right] < \infty. \quad (5)$$

3. Stochastic Stability Analysis

First of all, let us consider (2) with $u(t) \equiv 0$:

$$\dot{x}(t) = A(\zeta(t))\, x(t). \quad (6)$$

The following lemma presents the stochastic stability condition for (6) with $\prod(h) \in \mathcal{S}_{\Pi}^{(1)}$.

Lemma 2. *Suppose that there exists $P_i > 0$, for all $i \in \mathbb{N}_s^+$, such that*

$$0 > \mathcal{Q}_i(h), \quad \prod(h) \in \mathcal{S}_{\Pi}^{(1)}, \quad \forall i \in \mathbb{N}_s^+, \quad (7)$$

where $\mathcal{Q}_i(h) \triangleq \mathbf{He}(P_iA_i) + \sum_{j=1}^{s} \pi_{ij}(h)P_j$. Then, S-MJSs (6) with $\prod(h) \in \mathcal{S}_{\Pi}^{(1)}$ are stochastically stable.

Proof. Let us consider a stochastic Lyapunov function candidate of the following form:

$$V(x(t), \zeta(t)) (= V(t)) = x^T(t) P(\zeta(t))\, x(t), \quad (8)$$

where $P(\zeta(t))$ is taken to be positive definite. Then, the infinitesimal generator ∇ of the stochastic process $\{x(t), \zeta(t), t \ge 0\}$ acting on $V(t)$ is given by

$$\begin{aligned}\nabla V(t) &= \lim_{\Delta\to 0} \frac{\mathbf{E}\left[V\left(x(t+\Delta), \zeta(t+\Delta) = j \mid x(t), \zeta(t) = i\right)\right] - V(x(t), r(t))}{\Delta} \\ &= \lim_{\Delta\to 0} \frac{1}{\Delta}\left[\sum_{j=1}^{s} p_{ij}(t,\Delta)\, x^T(t+\Delta) P_j x(t+\Delta) - x^T(t) P_i x(t)\right] = \Psi_1(t) + \Psi_2(t) + \Psi_3(t),\end{aligned} \quad (9)$$

where $p_{ij}(t, \Delta) = \mathbf{Pr}(\zeta(t+\Delta) = j \mid \zeta(t) = i)$,

$$\begin{aligned}\Psi_1(t) &\triangleq \lim_{\Delta\to 0} \frac{1}{\Delta} \sum_{j=1,\, j\neq i}^{s} p_{ij}(t,\Delta)\, x^T(t+\Delta) P_j x(t+\Delta), \\ \Psi_2(t) &\triangleq \lim_{\Delta\to 0} \frac{1}{\Delta} p_{ii}(t,\Delta)\, x^T(t+\Delta) P_i x(t+\Delta), \\ \Psi_3(t) &\triangleq \lim_{\Delta\to 0} \frac{1}{\Delta} \left(-x^T(t) P_i x(t)\right).\end{aligned} \quad (10)$$

Here, the probabilities $p_{ij}(t, \Delta)$ and $p_{ii}(t, \Delta)$ are described as

$$\begin{aligned}p_{ij}(t,\Delta) &= \frac{\mathbf{Pr}\left(\zeta(t+\Delta) = j, \zeta(t) = i\right)}{\mathbf{Pr}(\zeta(t) = i)} \\ &= \frac{q_{ij}\left(G_i(h+\Delta) - G_i(h)\right)}{1 - G_i(h)}, \quad \forall j \neq i,\end{aligned} \quad (11)$$

$$\begin{aligned}p_{ii}(t,\Delta) &= \frac{\mathbf{Pr}\left(\zeta(t+\Delta) = i, \zeta(t) = i\right)}{\mathbf{Pr}(\zeta(t) = i)} \\ &= \frac{1 - G_i(h+\Delta)}{1 - G_i(h)},\end{aligned} \quad (12)$$

where $G_i(h)$ denotes the cumulative distribution function of the sojourn-time h at mode i and q_{ij} stands for the probability intensity such that $\sum_{j=1,\, j\neq i}^{s} q_{ij} = 1$ and $q_{ii} = -1$. Especially in (11), the probability intensity q_{ij} is taken to have uncertainties. Meanwhile, from (6), it follows that $x(t+\Delta) = (A_i\Delta + I)x(t)$, for $\Delta \to 0$, which leads to

$$\begin{aligned}&x^T(t+\Delta) P_j x(t+\Delta) \\ &= x^T(t) \left(A_i\Delta + I\right)^T P_j \left(A_i\Delta + I\right) x(t) \\ &= x^T(t) \left(\Delta^2 \cdot A_i^T P_j A_i + \Delta \cdot \mathbf{He}\left(P_jA_i\right) + P_j\right) x(t).\end{aligned} \quad (13)$$

That is, from (11) and (12), it is clear that

$$\begin{aligned}
&\lim_{\Delta\to 0}\frac{1}{\Delta}p_{ij}(t,\Delta)x^T(t+\Delta)P_jx(t+\Delta)\\
&\quad=\lim_{\Delta\to 0}p_{ij}(t,\Delta)x^T(t)\mathbf{He}(P_jA_i)x(t)\\
&\qquad+\lim_{\Delta\to 0}\frac{1}{\Delta}p_{ij}(t,\Delta)x^T(t)P_jx(t).
\end{aligned}\tag{14}$$

Thus, using (14), $\Psi_1(t)$ and $\Psi_2(t)$ become

$$\begin{aligned}
\Psi_1(t)&=\sum_{j=1,\,j\neq i}^{s}\lim_{\Delta\to 0}p_{ij}(t,\Delta)x^T(t)\mathbf{He}(P_iA_i)x(t)\\
&\quad+\lim_{\Delta\to 0}\frac{1}{\Delta}\sum_{j=1,\,j\neq i}^{s}p_{ij}(t,\Delta)x^T(t)P_jx(t)\\
&=\lim_{\Delta\to 0}\frac{1}{\Delta}\sum_{j=1,\,j\neq i}^{s}p_{ij}(t,\Delta)x^T(t)P_jx(t),\\
\Psi_2(t)&=\lim_{\Delta\to 0}p_{ii}(t,\Delta)x^T(t)\mathbf{He}(P_iA_i)x(t)\\
&\quad+\lim_{\Delta\to 0}\frac{1}{\Delta}p_{ii}(t,\Delta)x^T(t)P_ix(t)\\
&=x^T(t)\mathbf{He}(P_iA_i)x(t)\\
&\quad+\lim_{\Delta\to 0}\frac{1}{\Delta}p_{ii}(t,\Delta)x^T(t)P_ix(t).
\end{aligned}\tag{15}$$

That is, $\nabla V(t)$ becomes

$$\begin{aligned}
\nabla V(t)&=\lim_{\Delta\to 0}\frac{1}{\Delta}\sum_{j=1,\,j\neq 1}^{s}p_{ij}(t,\Delta)x^T(t)P_jx(t)+x^T(t)\\
&\quad\cdot\mathbf{He}(P_iA_i)x(t)\\
&\quad+\lim_{\Delta\to 0}\frac{1}{\Delta}(p_{ii}(t,\Delta)-1)x^T(t)P_ix(t)=x^T(t)\\
&\quad\cdot\Bigg\{\mathbf{He}(P_iA_i)\\
&\quad+\lim_{\Delta\to 0}\frac{1}{\Delta}\Bigg(\sum_{j=1,\,j\neq i}^{s}p_{ij}(t,\Delta)P_j+(p_{ii}(t,\Delta)-1)P_i\Bigg)\Bigg\}\\
&\quad\cdot x(t)=x^T(t)\Bigg\{\mathbf{He}(P_iA_i)\\
&\quad+\lim_{\Delta\to 0}\frac{1}{\Delta}\Bigg(\sum_{j=1,\,j\neq i}^{s}q_{ij}P_j-P_i\Bigg)\frac{G_i(h+\Delta)-G_i(h)}{1-G_i(h)}\Bigg\}\\
&\quad\cdot x(t)=x^T(t)\Bigg\{\mathbf{He}(P_iA_i)\\
&\quad+\Bigg(\sum_{j=1,\,j\neq i}^{s}q_{ij}P_j-P_i\Bigg)\frac{g_i(h)}{1-G_i(h)}\Bigg\}x(t)=x^T(t)\\
&\quad\cdot\Bigg\{\mathbf{He}(P_iA_i)+\Bigg(\sum_{j=1,\,j\neq i}^{s}q_{ij}P_j-P_i\Bigg)\pi_i(h)\Bigg\}x(t),
\end{aligned}\tag{16}$$

where $\pi_i(h)=g_i(h)/1-G_i(h)$ denotes the transition rate of the system jumping from mode i. As a result, by defining $\pi_{ij}(h)=q_{ij}\pi_i(h)$, for $j\neq i$, and $\pi_{ii}(h)=-\sum_{j=1,\,j\neq i}^{s}\pi_{ij}(h)=-\pi_i(h)$, we have

$$\begin{aligned}
\nabla V(t)&=x^T(t)\\
&\quad\cdot\Bigg\{\mathbf{He}(P_iA_i)+\sum_{j=1,\,j\neq i}^{s}\pi_{ij}(h)P_j+\pi_{ii}(h)P_i\Bigg\}x(t)\\
&=x^T(t)\mathcal{Q}_i(h)x(t).
\end{aligned}\tag{17}$$

In what follows, by the generalized Dynkin's formula [33], it is clear that

$$\begin{aligned}
\mathbf{E}[V(t)]-V(0)&=\mathbf{E}\left[\int_0^t\nabla V(\tau)\,d\tau\mid x_0,\zeta_0\right]\\
&\le\max_{i\in\mathbb{N}_s^+,\,0\le h\le t}\left(\lambda_{\max}(\mathcal{Q}_i(h))\right)\\
&\quad\cdot\mathbf{E}\left[\int_0^t\|x(\tau)\|_2^2\,d\tau\mid x_0,\zeta_0\right],
\end{aligned}\tag{18}$$

which results in

$$\begin{aligned}
&-\max_{i\in\mathbb{N}_s^+,\,0\le h\le t}\left(\lambda_{\max}(\mathcal{Q}_i(h))\right)\\
&\quad\cdot\mathbf{E}\left[\int_0^t\|x(\tau)\|_2^2\,d\tau\mid x_0,\zeta_0\right]\le V(0)-\mathbf{E}[V(t)]\\
&\quad\le V(0).
\end{aligned}\tag{19}$$

Thus, from (7), it follows that

$$\begin{aligned}
&\lim_{t\to\infty}\mathbf{E}\left[\int_0^t\|x(\tau)\|_2^2\,d\tau\mid x_0,\zeta_0\right]\\
&\quad\le-\frac{V(0)}{\max_{i\in\mathbb{N}_s^+,\,0\le h}\left(\lambda_{\max}(\mathcal{Q}_i(h))\right)}<\infty.
\end{aligned}\tag{20}$$

Finally, by Definition 1, the proof can be completed. □

In this paper, as a model of probability distribution for the sojourn-time $h\ge 0$, we utilize the Weibull distribution with shape parameter $\beta>0$ and scale parameter $\alpha>0$, since such a distribution has been witnessed as an appropriate choice for representing the stochastic behavior of practical systems. In other words, to represent the probability distribution of

h, its cumulative function $G_i(h)$ and probability distribution function $g_i(h)$ are given as follows: for all i and $j (j \neq i) \in \mathbb{N}_s^+$,

$$G_i(h) = 1 - \exp\left(-\left(\frac{h}{\alpha_i}\right)^{\beta_i}\right),$$
$$g_i(h) = \frac{\beta_i}{\alpha_i^{\beta_i}} h^{\beta_i - 1} \exp\left(-\left(\frac{h}{\alpha_i}\right)^{\beta_i}\right), \tag{21}$$

which leads to

$$\pi_{ij}(h) = q_{ij}\pi_i(h) = q_{ij}\frac{g_i(h)}{1 - G_i(h)} = q_{ij}\frac{\beta_i}{\alpha_i^{\beta_i}} h^{\beta_i - 1}. \tag{22}$$

As a special case, let $\beta_i = 1$. Then, we can represent MJSs from (22); that is, the transition rate $\pi_{ij}(h)$ can be reduced to an h-independent value as follows: $\pi_{ij}(h) = q_{ij}\pi_i(h) = q_{ij}/\alpha_i$. Accordingly, it can be claimed that (22) expresses a more generalized transition model, compared to the case of MJSs.

Remark 3. As shown in (22), the transition rate $\pi_{ij}(h)$ is time-varying and depends on the probability intensity q_{ij}. Thus, to derive a finite number of solvable conditions from (7), there is a need to consider the lower and upper bounds of both $\pi_i(h)$ and q_{ij}, respectively, as follows: $\pi_{i,1} \leq \pi_i(h) \leq \pi_{i,2}$ and $q_{ij,1} \leq q_{ij} \leq q_{ij,2}$. Then, from $\pi_{ij}(h) = q_{ij}\pi_i(h)$, the bounds of $\pi_{ij}(h)$ are decided as follows: $\pi_{ij,1} \leq \pi_{ij}(h) \leq \pi_{ij,2}$, where

$$\pi_{ij,1} = \begin{cases} q_{ij,1} \cdot \pi_{i,1}, & \text{if } j \neq i \\ -\pi_{i,2}, & \text{otherwise,} \end{cases}$$
$$\pi_{ij,2} = \begin{cases} q_{ij,2} \cdot \pi_{i,2}, & \text{if } j \neq i \\ -\pi_{i,1}, & \text{otherwise.} \end{cases} \tag{23}$$

In accordance with Remark 3, an auxiliary constraint can be established as follows: $\prod(h) \in \mathcal{S}_{\Pi}^{(2)}$, where

$$\mathcal{S}_{\Pi}^{(2)} \triangleq \left\{\left[\pi_{ij}\right]_{i,j\in\mathbb{N}_s^+} \mid \pi_{ij,1} \leq \pi_{ij} \leq \pi_{ij,2},\ \forall i, j \in \mathbb{N}_s^+\right\}. \tag{24}$$

The following lemma presents the stochastic stability condition for S-MJSs (6) with $\prod(h) \in \mathcal{S}_{\Pi}^{(1)} \cap \mathcal{S}_{\Pi}^{(2)}$.

Lemma 4. *Suppose that there exists $P_i > 0$, for all $i \in \mathbb{N}_s^+$, such that*

$$0 > \mathcal{Q}_i(h),$$
$$\prod(h) \in \mathcal{S}_{\Pi}^{(1)} \cap \mathcal{S}_{\Pi}^{(2)}, \tag{25}$$
$$\forall i \in \mathbb{N}_s^+,$$

where $\mathcal{Q}_i(h) \triangleq \mathbf{He}(P_iA_i) + \sum_{j=1}^{s}\pi_{ij}(h)P_j$ and $\pi_{ij}(h) \in [\pi_{ij,1}, \pi_{ij,2}]$. Then, S-MJSs (6) with $\mathcal{S}_{\Pi}^{(1)} \cap \mathcal{S}_{\Pi}^{(2)}$ are stochastically stable.

However, it is worth noticing that solving (25) of Lemma 4 is still equivalent to solving an infinite number of LMIs, which is an extremely difficult problem. Thus, it is necessary to find a finite number of solvable LMI-based conditions from (25). To this end, the following theorem provides a relaxed stochastic stability condition for (6) with $\prod(h) \in \mathcal{S}_{\Pi}^{(1)} \cap \mathcal{S}_{\Pi}^{(2)}$.

Theorem 5. *Suppose that there exists matrices $\{G_i, S_{ij}, X_{ij}, Y_{ij}\}_{i,j\in\mathbb{N}_s^+} \in \mathbb{R}^{n_x\times n_x}$ and symmetric matrices $\{P_i > 0\}_{i\in\mathbb{N}_s^+} \in \mathbb{R}^{n_x\times n_x}$ such that*

$$0 > \left[\begin{array}{c|c} (1,1) & (1,2) \\ \hline (*) & (2,2) \end{array}\right], \quad \forall i \in \mathbb{N}_s^+, \tag{26}$$

$$0 \leq \mathbf{He}\left(X_{ij}\right), \quad \forall i, j \in \mathbb{N}_s^+, \tag{27}$$

$$0 \leq \mathbf{He}\left(Y_{ij}\right), \quad \forall i, j \in \mathbb{N}_s^+, \tag{28}$$

where $\mu_{ij}|_{j\neq i} = 1$, $\mu_{ij}|_{j=i} = -1$,

$$(1,1) = \mathbf{He}(P_iA_i) - \sum_{j\in\mathbb{N}_s^+}\mathbf{He}\left(\pi_{ij,1}\pi_{ij,2}X_{ij}\right),$$
$$(1,2) = \left[\frac{1}{2}P_j + G_i + \left(\pi_{ij,1} + \pi_{ij,2}\right)X_{ij} + \mu_{ij}Y_{ij}\right]_{j\in\mathbb{N}_s^+}, \tag{29}$$
$$(2,2) = \left[\mathbf{He}(S_{ia} - X_{ia})\right]^{\mathbf{D}}_{a\in\mathbb{N}_s^+} + \mathbf{He}\left(\left[S_{ia} + S_{ib}\right]^{\mathbf{U}}_{a,b\in\mathbb{N}_s^+}\right).$$

Then, S-MJSs (6) with $\prod(h) \in \mathcal{S}_{\Pi}^{(1)} \cap \mathcal{S}_{\Pi}^{(2)}$ are stochastically stable.

Proof. From the first constraint $\prod(h) \in \mathcal{S}_{\Pi}^{(1)}$, we can obtain, under (28),

$$0 \equiv \mathbf{He}\left(\left(\sum_{j\in\mathbb{N}_s^+}\pi_{ij}(h)\right)\left(G_i + \sum_{j\in\mathbb{N}_s^+}\pi_{ij}(h)S_{ij}\right)\right), \tag{30}$$
$$\forall i \in \mathbb{N}_s^+,$$

$$0 \leq \sum_{j\in\mathbb{N}_s^+}\mu_{ij}\pi_{ij}\mathbf{He}\left(Y_{ij}\right), \quad \forall i \in \mathbb{N}_s^+. \tag{31}$$

Further, under (27), the second constraint $\prod(h) \in \mathcal{S}_{\Pi}^{(2)}$ provides

$$0 \leq -\sum_{j\in\mathbb{N}_s^+}\left(\pi_{ij}(h) - \pi_{ij,1}\right)\left(\pi_{ij}(h) - \pi_{ij,2}\right)\mathbf{He}\left(X_{ij}\right), \tag{32}$$
$$\forall i \in \mathbb{N}_s^+.$$

Here, note that (30)–(32) can be converted, respectively, into

$$0 \equiv \left[\frac{I}{\prod_i(h)\otimes I}\right]^T \left[\begin{array}{c|c} 0 & [G_i]_{i\in\mathbb{N}_s^+} \\ \hline (*) & (2,2)^* \end{array}\right] \left[\frac{I}{\prod_i(h)\otimes I}\right], \tag{33}$$

$$0 \le \left[\frac{I}{\prod_i(h)\otimes I}\right]^T \left[\begin{array}{c|c} 0 & [\mu_{ij}Y_{ij}]_{j\in\mathbb{N}_s^+} \\ \hline (*) & 0 \end{array}\right] \left[\frac{I}{\prod_i(h)\otimes I}\right], \tag{34}$$

$$\begin{aligned} 0 \le{}& \left[\frac{I}{\prod_i(h)\otimes I}\right]^T \\ &\cdot \left[\begin{array}{c|c} (1,1)^* & \left[\left(\pi_{ij,1}+\pi_{ij,2}\right)X_{ij}\right]_{j\in\mathbb{N}_s^+} \\ \hline (*) & [\mathbf{He}(-X_{ia})]^{\mathbf{D}}_{a\in\mathbb{N}_s^+} \end{array}\right] \left[\frac{I}{\prod_i(h)\otimes I}\right], \end{aligned} \tag{35}$$

where $(1,1)^* = -\sum_{j\in\mathbb{N}_s^+}\mathbf{He}(\pi_{ij,1}\pi_{ij,2}X_{ij})$ and $(2,2)^* = [\mathbf{He}(S_{ia})]^{\mathbf{D}}_{a\in\mathbb{N}_s^+} + \mathbf{He}([S_{ia}+S_{ib}]^{\mathbf{U}}_{a,b\in\mathbb{N}_s^+})$. Likewise, according to the form of (33)–(35), we can rewrite $\mathcal{Q}_i(h) < 0$ in Lemma 4 as follows:

$$\begin{aligned} \mathcal{Q}_i(h) ={}& \left[\frac{I}{\prod_i(h)\otimes I}\right]^T \\ &\cdot \left[\begin{array}{c|c} \mathbf{He}(P_iA_i) & [(1/2)P_j]_{j\in\mathbb{N}_s^+} \\ \hline (*) & 0 \end{array}\right] \left[\frac{I}{\prod_i(h)\otimes I}\right] < 0. \end{aligned} \tag{36}$$

As a result, by combining (36) with (33)–(35) through the S-procedure [34], the stochastic stability condition (25) is given by

$$0 > \left[\frac{I}{\prod_i(h)\otimes I}\right]^T \left[\begin{array}{c|c} (1,1) & (1,2) \\ \hline (*) & (2,2) \end{array}\right] \left[\frac{I}{\prod_i(h)\otimes I}\right], \tag{37}$$

where $(1,1)$, $(1,2)$, and $(2,2)$ are given in the body of Theorem 5. Therefore, as (26) implies (37), the proof can be completed. □

4. Control Design

Let us consider the following mode-dependent state-feedback control law:

$$u(t) = F_i x(t), \tag{38}$$

where $F_i \triangleq F(\zeta(t)=i)$. Thereby, the resultant closed-loop system under (2) and (38) is given by

$$\begin{aligned} \dot{x}(t) &= A_i x(t) + B_i u(t) = \left(A_i + B_iF_i\right)x(t) \\ &= \overline{A}_i x(t). \end{aligned} \tag{39}$$

The following theorem provides a relaxed stochastic stabilization condition for S-MJSs (39) with $\prod(h) \in \mathcal{S}_\Pi^{(1)} \cap \mathcal{S}_\Pi^{(2)}$.

Theorem 6. *Suppose that there exist matrices $\{\overline{F}_i\}_{i\in\mathbb{N}_s^+} \in \mathbb{R}^{n_x\times n_x}$ and $\{G_i, S_{ij}, X_{ij}, Y_{ij}, Q_{ij}\}_{i,j\in\mathbb{N}_s^+} \in \mathbb{R}^{n_x\times n_x}$ and symmetric matrices $\{\overline{P}_i > 0\}_{i\in\mathbb{N}_s^+} \in \mathbb{R}^{n_x\times n_x}$ such that*

$$0 > \left[\begin{array}{c|c} (1,1) & (1,2) \\ \hline (*) & (2,2) \end{array}\right], \quad \forall i \in \mathbb{N}_s^+, \tag{40}$$

$$0 \le \mathbf{He}\left(X_{ij}\right), \quad \forall i,j \in \mathbb{N}_s^+, \tag{41}$$

$$0 \le \mathbf{He}\left(Y_{ij}\right), \quad \forall i,j \in \mathbb{N}_s^+, \tag{42}$$

$$0 \le \begin{bmatrix} Q_{ij} & \overline{P}_i \\ (*) & \overline{P}_j \end{bmatrix}, \quad \forall i, j \ne i \in \mathbb{N}_s^+, \tag{43}$$

where $\epsilon_{ij}|_{j\ne i} = 1$, $\epsilon_{ij}|_{j=i} = 0$, $\mu_{ij}|_{j\ne i} = 1$, $\mu_{ij}|_{j=i} = -1$,

$$\begin{aligned} (1,1) &= \mathbf{He}\left(A_i\overline{P}_i + B_i\overline{F}_i\right) - \sum_{j\in\mathbb{N}_s^+}\mathbf{He}\left(\pi_{ij,1}\pi_{ij,2}X_{ij}\right), \\ (1,2) &= \left[\left(\frac{1}{2}\epsilon_{ij}Q_{ij} + \frac{1}{2}\left(1-\epsilon_{ij}\right)\overline{P}_i\right) + G_i \right. \\ &\quad \left. + \left(\pi_{ij,1}+\pi_{ij,2}\right)X_{ij} + \mu_{ij}Y_{ij}\right]_{j\in\mathbb{N}_s^+}, \\ (2,2) &= [\mathbf{He}(S_{ia}-X_{ia})]^{\mathbf{D}}_{a\in\mathbb{N}_s^+} + \mathbf{He}\left([S_{ia}+S_{ib}]^{\mathbf{U}}_{a,b\in\mathbb{N}_s^+}\right). \end{aligned} \tag{44}$$

Then, closed-loop system (39) with $\prod(h) \in \mathcal{S}_\Pi^{(1)} \cap \mathcal{S}_\Pi^{(2)}$ is stochastically stable, where $F_i = \overline{F}_i\overline{P}_i^{-1}$.

Proof. In light of (25), the stabilization condition of (39) is given by

$$0 > \mathbf{He}\left(P_i\overline{A}_i\right) + \sum_{j=1}^{s}\pi_{ij}(h)P_j. \tag{45}$$

Also, pre- and postmultiplying (45) by $\overline{P}_i \triangleq P_i^{-1}$ yields

$$0 > \overline{\mathcal{Q}}_i(h) \triangleq \mathbf{He}\left(A_i\overline{P}_i + B_i\overline{F}_i\right) + \sum_{j=1}^{s}\pi_{ij}(h)\overline{P}_iP_j\overline{P}_i, \tag{46}$$

where $\overline{F}_i \triangleq F_i\overline{P}_i$. Here, by employing Q_{ij} such that (43) holds (i.e., $Q_{ij} \ge \overline{P}_iP_j\overline{P}_i$), we can convert (46) into

$$\begin{aligned} 0 &> \overline{\mathcal{Q}}_i(h) \\ &= \mathbf{He}\left(A_i\overline{P}_i + B_i\overline{F}_i\right) \\ &\quad + \sum_{j=1}^{s}\pi_{ij}(h)\left(\epsilon_{ij}Q_{ij} + \left(1-\epsilon_{ij}\right)\overline{P}_i\right). \end{aligned} \tag{47}$$

Thereupon, $\overline{\mathcal{Q}}_i(h)$ is factorized as follows:

$$\begin{aligned} \overline{\mathcal{Q}}_i(h) ={}& \left[\frac{I}{\prod_i(h)\otimes I}\right]^T \\ &\cdot \left[\begin{array}{c|c} \mathbf{He}\left(A_i\overline{P}_i + B_i\overline{F}_i\right) & (1,2)^* \\ \hline (*) & 0 \end{array}\right] \left[\frac{I}{\prod_i(h)\otimes I}\right], \end{aligned} \tag{48}$$

where $(1,2)^* = [(1/2)\epsilon_{ij}Q_{ij} + (1/2)(1-\epsilon_{ij})\overline{P}_i]_{j\in\mathbb{N}_s^+}$.

Next, as in the proof of Theorem 5, the constraint $\prod(h) \in \mathcal{S}_{\Pi}^{(1)} \cap \mathcal{S}_{\Pi}^{(2)}$ can be represented as

$$\begin{aligned}
0 &\equiv \left[\begin{array}{c} I \\ \hline \prod_i(h) \otimes I \end{array}\right]^T \left[\begin{array}{c|c} 0 & [G_i]_{i\in\mathbb{N}_s^+} \\ \hline (*) & (2,2)^* \end{array}\right] \left[\begin{array}{c} I \\ \hline \prod_i(h) \otimes I \end{array}\right], \\
0 &\le \left[\begin{array}{c} I \\ \hline \prod_i(h) \otimes I \end{array}\right]^T \left[\begin{array}{c|c} 0 & [\mu_{ij}Y_{ij}]_{j\in\mathbb{N}_s^+} \\ \hline (*) & 0 \end{array}\right] \left[\begin{array}{c} I \\ \hline \prod_i(h) \otimes I \end{array}\right], \\
0 &\le \left[\begin{array}{c} I \\ \hline \prod_i(h) \otimes I \end{array}\right]^T \\
&\cdot \left[\begin{array}{c|c} (1,1)^* & \left[\left(\pi_{ij,1}+\pi_{ij,2}\right)X_{ij}\right]_{j\in\mathbb{N}_s^+} \\ \hline (*) & [\mathbf{He}(-X_{ia})]^{\mathbf{D}}_{a\in\mathbb{N}_s^+} \end{array}\right] \left[\begin{array}{c} I \\ \hline \prod_i(h) \otimes I \end{array}\right],
\end{aligned} \tag{49}$$

where $(1,1)^* = -\sum_{j\in\mathbb{N}_s^+}\mathbf{He}(\pi_{ij,1}\pi_{ij,2}X_{ij})$ and $(2,2)^* = [\mathbf{He}(S_{ia})]^{\mathbf{D}}_{a\in\mathbb{N}_s^+} + \mathbf{He}([S_{ia}+S_{ib}]^{\mathbf{U}}_{a,b\in\mathbb{N}_s^+})$. As a result, by the S-procedure, combining (48) with (49) results in

$$0 > \left[\begin{array}{c} I \\ \hline \prod_i(h) \otimes I \end{array}\right]^T \left[\begin{array}{c|c} (1,1) & (1,2) \\ \hline (*) & (2,2) \end{array}\right] \left[\begin{array}{c} I \\ \hline \prod_i(h) \otimes I \end{array}\right], \tag{50}$$

where $(1,1)$, $(1,2)$, and $(2,2)$ are given in the Theorem 6. Therefore, as (40) implies (50), the proof can be completed. □

Hereafter, as a practical extension of the proposed approach, we consider the following input-quantized S-MJSs:

$$\dot{x}(t) = A_i x(t) + B_i \mathbf{q}(u(t)), \tag{51}$$

where $\mathbf{q}(\bullet)$ stands for a uniform quantization operator with the quantization level $\delta > 0$; that is, $\mathbf{q}(u(t)) = \delta \cdot \mathrm{round}(u(t)/\delta)$. Here, note that $\mathbf{q}(u(t)) = u(t) + \varphi(t)$, where the kth element of the quantization error $\varphi(t)$ satisfies

$$\left|\varphi_k(t)\right| \le \frac{\delta}{2}, \quad \forall k \in \mathbb{N}_{n_u}^+. \tag{52}$$

Thus, (51) can be rewritten as

$$\dot{x}(t) = A_i x(t) + B_i\left(u(t) + \varphi(t)\right), \tag{53}$$

where $\varphi(t) = \mathbf{q}(u(t)) - u(t)$ is known. Continuously, as a mode-dependent state-feedback law, we adopt

$$u(t) = F_i x(t) + \nu_i(t). \tag{54}$$

Then, the resultant closed-loop system is described as

$$\dot{x}(t) = \overline{A}_i x(t) + B_i\left(\nu_i(t) + \varphi(t)\right). \tag{55}$$

The following theorem provides a relaxed stochastic stabilization condition for S-MJSs (55) with input quantization error.

Theorem 7. *Let $\nu_{i,k}$ (i.e., the kth element of ν_i) be given as follows:*

$$\begin{aligned}
\nu_{i,k}(t) &= -\delta \cdot \mathrm{sgn}\left(s_{i,k}(t)\right) \\
&\quad \cdot \max\left(0, \mathrm{sgn}\left(s_i^T(t)\varphi(t)\right)\right), \\
&\qquad \forall i \in \mathbb{N}_s^+,\ k \in \mathbb{N}_{n_u}^+,
\end{aligned} \tag{56}$$

where $s_i(t) = B_i^T P_i x(t) \in \mathbb{R}^{n_u}$ and $s_{i,k}(t)$ denotes the kth element of $s_i(t)$. Suppose that there exist matrices $\{\overline{F}_i\}_{i\in\mathbb{N}_s^+} \in \mathbb{R}^{n_x\times n_x}$ and $\{G_i, S_{ij}, X_{ij}, Y_{ij}, Q_{ij}\}_{i,j\in\mathbb{N}_s^+} \in \mathbb{R}^{n_x\times n_x}$ and symmetric matrices $\{\overline{P}_i > 0\}_{i\in\mathbb{N}_s^+} \in \mathbb{R}^{n_x\times n_x}$ such that (40)–(43) hold. Then, the closed-loop system (55) with $\prod(h) \in \mathcal{S}_{\Pi}^{(1)} \cap \mathcal{S}_{\Pi}^{(2)}$ is stochastically stable, where $F_i = \overline{F}_i\overline{P}_i^{-1}$.

Proof. From (55), it follows that $x(t+\Delta) = (\overline{A}_i\Delta + I)x(t) + B_i\Delta(\nu_i(t) + \varphi(t))$, for $\Delta \to 0$, which yields

$$\begin{aligned}
&x^T(t+\Delta)P_j x(t+\Delta) \\
&\quad = \Delta \cdot x^T(t)\,\mathbf{He}\left(P_j\overline{A}_i\right)x(t) + x^T(t)P_j x(t) + \Delta \\
&\qquad \cdot 2x^T(t)P_j B_i\left(\nu_i(t) + \varphi(t)\right) + \Delta^2 \cdot (\bullet).
\end{aligned} \tag{57}$$

Thus, it is given that

$$\begin{aligned}
&\lim_{\Delta\to 0}\frac{1}{\Delta}p_{ij}(t,\Delta)x^T(t+\Delta)P_j x(t+\Delta) = \lim_{\Delta\to 0}p_{ij}(t,\Delta) \\
&\quad \cdot \left(x^T(t)\,\mathbf{He}\left(P_j\overline{A}_i\right)x(t)\right. \\
&\quad \left. + 2x^T(t)P_j B_i\left(\nu_i(t)+\varphi(t)\right)\right) + \lim_{\Delta\to 0}\frac{1}{\Delta}p_{ij}(t,\Delta) \\
&\quad \cdot x^T(t)P_j x(t).
\end{aligned} \tag{58}$$

Then, based on (11) and (12), we have

$$\Psi_1(t) = \sum_{j=1,\,j\ne i}^{s}\lim_{\Delta\to 0}\frac{1}{\Delta}p_{ij}(t,\Delta)x^T(t)P_j x(t), \tag{59}$$

$$\begin{aligned}
&\Psi_2(t) + \Psi_3(t) \\
&\quad = x^T(t)\,\mathbf{He}\left(P_i\overline{A}_i\right)x(t) \\
&\qquad + 2x^T(t)P_i B_i\left(\nu_i(t)+\varphi(t)\right) \\
&\qquad + \lim_{\Delta\to 0}\frac{1}{\Delta}\left(p_{ii}(t,\Delta)-1\right)x^T(t)P_i x(t),
\end{aligned} \tag{60}$$

which leads to

$$\begin{aligned}
\nabla V(t) &= x^T(t)\left(\mathbf{He}\left(P_i\overline{A}_i\right) + \sum_{j=1}^{s}\pi_{ij}(h)P_j\right)x(t) \\
&\quad + 2s_i^T(t)\left(\nu_i(t)+\varphi(t)\right), \\
s_i(t) &= B_i^T P_i x(t) \in \mathbb{R}^{n_u}.
\end{aligned} \tag{61}$$

Hence, for $s_i^T(t)\varphi(t) \le 0$, letting $\nu_i(t) \equiv 0$ implies

$$\nabla V(t) \le x^T(t)\left(\mathbf{He}\left(P_i\overline{A}_i\right) + \sum_{j=1}^{s}\pi_{ij}(h)P_j\right)x(t). \tag{62}$$

Further, for $s_i^T(t)\varphi(t) > 0$, letting $\nu_{i,k}(t) = -\delta \cdot \mathrm{sgn}(s_{i,k}(t))$ yields

$$\begin{aligned} 2s_i^T(t)(\nu_i(t)+\varphi(t)) &= \sum_{k=1}^{n_u} 2s_{i,k}(t)(\nu_{i,k}(t)+\varphi_k(t)) \\ &\leq \sum_{k=1}^{n_u} -2\delta|s_{i,k}(t)| + 2|s_{i,k}(t)| \\ &\quad \cdot |\varphi_k(t)| \leq -\sum_{k=1}^{n_u}\delta|s_{i,k}(t)| < 0. \end{aligned} \tag{63}$$

That is, (56) allows (61) to be reduced to (62) for all $s_i^T(t)\varphi(t)$. In what follows, we need to derive the stabilization condition from (62), which is omitted herein because it is in line with the proof of Theorem 6. □

5. Numerical Examples

Example 1. Consider the following system with three different modes: for $x_0 = (3.0, -5.0)$ and $\zeta_0 = 3$,

$$\begin{aligned} A_1 &= \begin{bmatrix} -0.5 & -0.75 \\ 1.00 & 1.00 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} -2.40 & -0.33 \\ 1.00 & -1.40 \end{bmatrix}, \\ A_3 &= \begin{bmatrix} -0.20 & 0.10 \\ 1.00 & -1.00 \end{bmatrix}, \\ B_1 &= \begin{bmatrix} 5.0 \\ 0.0 \end{bmatrix}, \\ B_2 &= \begin{bmatrix} -2.0 \\ -1.0 \end{bmatrix}, \\ B_3 &= \begin{bmatrix} 1.0 \\ -2.0 \end{bmatrix}. \end{aligned} \tag{64}$$

Here, the Weibull distribution for the sojourn-time is set by $(\alpha_1, \beta_1) = (0.5, 2.0)$, $(\alpha_2, \beta_2) = (1.0, 2.0)$, and $(\alpha_3, \beta_3) = (1.5, 2.0)$ (see Figure 1). Thereby, a reasonable interval $h \in [h_0, h_1]$ can be obtained such that $\int_{h_0}^{h_1} g_i(h)dh \geq 0.99$, from which the lower and upper bounds $(\pi_{i,1}, \pi_{i,2})$ of $\pi_i(h)$ can be also found. As a result, by considering the uncertain probability intensities such that $0.3 \leq q_{ij} \leq 0.7$, for all i and j, the following matrices such that $\pi_{ij}(h) \in [\pi_{ij,1}, \pi_{ij,2}]$ are established from Remark 3:

$$\begin{aligned} [\pi_{ij,1}]_{i,j\in\mathbb{N}_3^+} &= \begin{bmatrix} -8.6880 & 0.0384 & 0.0896 \\ 0.0192 & -4.3420 & 0.0448 \\ 0.0128 & 0.0299 & -2.8951 \end{bmatrix}, \\ [\pi_{ij,2}]_{i,j\in\mathbb{N}_3^+} &= \begin{bmatrix} -0.1280 & 2.6064 & 6.0816 \\ 1.3026 & -0.0640 & 3.0394 \\ 0.8685 & 2.0266 & -0.0427 \end{bmatrix}. \end{aligned} \tag{65}$$

Figure 2(a) shows the mode evolution generated from the stochastic setting. Besides, from Theorem 6, the following control gains are obtained:

$$\begin{aligned} F_1 &= [0.2654 \quad -0.0063], \\ F_2 &= [-0.7178 \quad -0.1854], \\ F_3 &= [-1.9513 \quad 0.6293]. \end{aligned} \tag{66}$$

Figure 2(b) shows the behavior of the state response coupled with the mode transition depicted in Figures 2(a) and 2(c) which presents the Monte Carlo simulation result of the settling time for 5000 different mode transition configurations, where its mean value and standard deviation are 12.2979 and 3.126, respectively. From Figure 2(b), it can be seen that the states converge from the initial condition $(3.0, -5.0)$ to the equilibrium point as time increases. Consequently, Theorem 6 provides an applicable method for deriving a relaxed stabilization condition for S-MJSs with uncertain probability intensities.

Example 2. Consider the following system representing an inverted pendulum: for $x_0 = (1, -1, 0.5)$ and $\zeta_0 = 3$:

$$\begin{aligned} \dot{x}_1(t) &= x_2(t), \\ \dot{x}_2(t) &= \frac{g}{l}\sin x_1(t) + \frac{Nk_m}{ml^2}x_3(t), \\ L_a\dot{x}_3(t) &= k_bNx_2(t) - R(\zeta(t))x_3(t) + \mathbf{q}(u(t)), \end{aligned} \tag{67}$$

where $x_1(t)$ is the angle of the inverted pendulum, $x_2(t)$ is the angular velocity, $x_3(t)$ is the input current, $u(t)$ is the control input voltage, g is the acceleration due to gravity, m and l are the mass and length, respectively, k_b and k_m are the back emf constant and motor torque constant, respectively, L_a and $R(\zeta(t))$ are the inductance and resistance of the DC motor, respectively, and N is the gear ratio. Here, we set $L_a = 1$, $g = 9.8$, $l = 1$, $m = 1$, $N = 10$, $K_m = K_b = 0.1$, and

$$R_i = R(\zeta(t) = i) = \begin{cases} 2, & \text{for } \zeta(t) = 1 \\ 1, & \text{for } \zeta(t) = 2 \\ 0.5, & \text{for } \zeta(t) = 3. \end{cases} \tag{68}$$

Then, the linearized model of (67) is given by

$$\begin{aligned} A_i &= A(\zeta(t) = i) \begin{bmatrix} 0 & 1 & 0 \\ 9.8 & 0 & 1 \\ 0 & 1 & -R_i \end{bmatrix}, \\ B_1 &= B_2 = B_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \end{aligned} \tag{69}$$

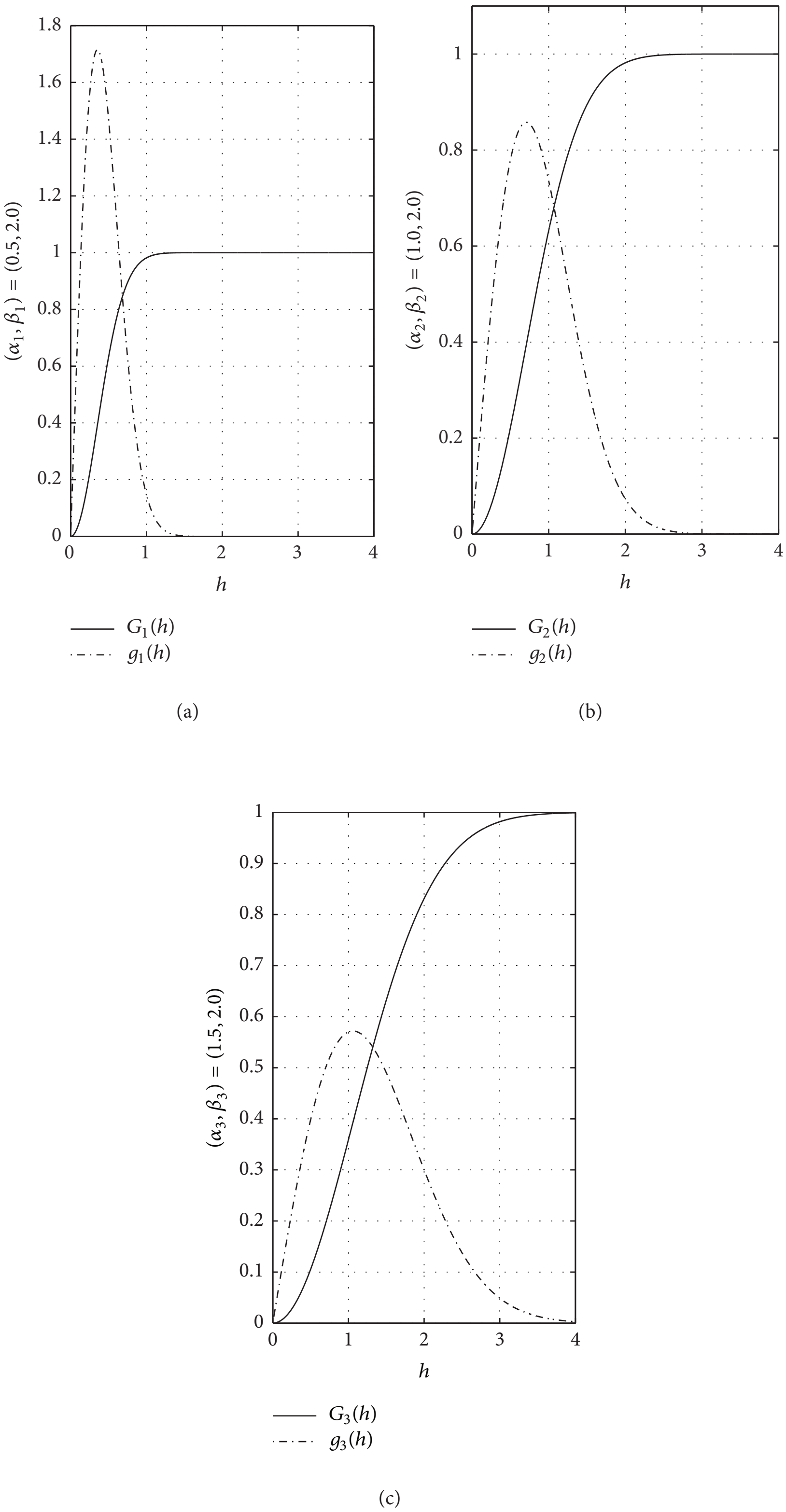

FIGURE 1: Cumulative distribution functions $G_i(h)$ (solid line) and probability distribution functions $g_i(h)$ (dash-dotted line).

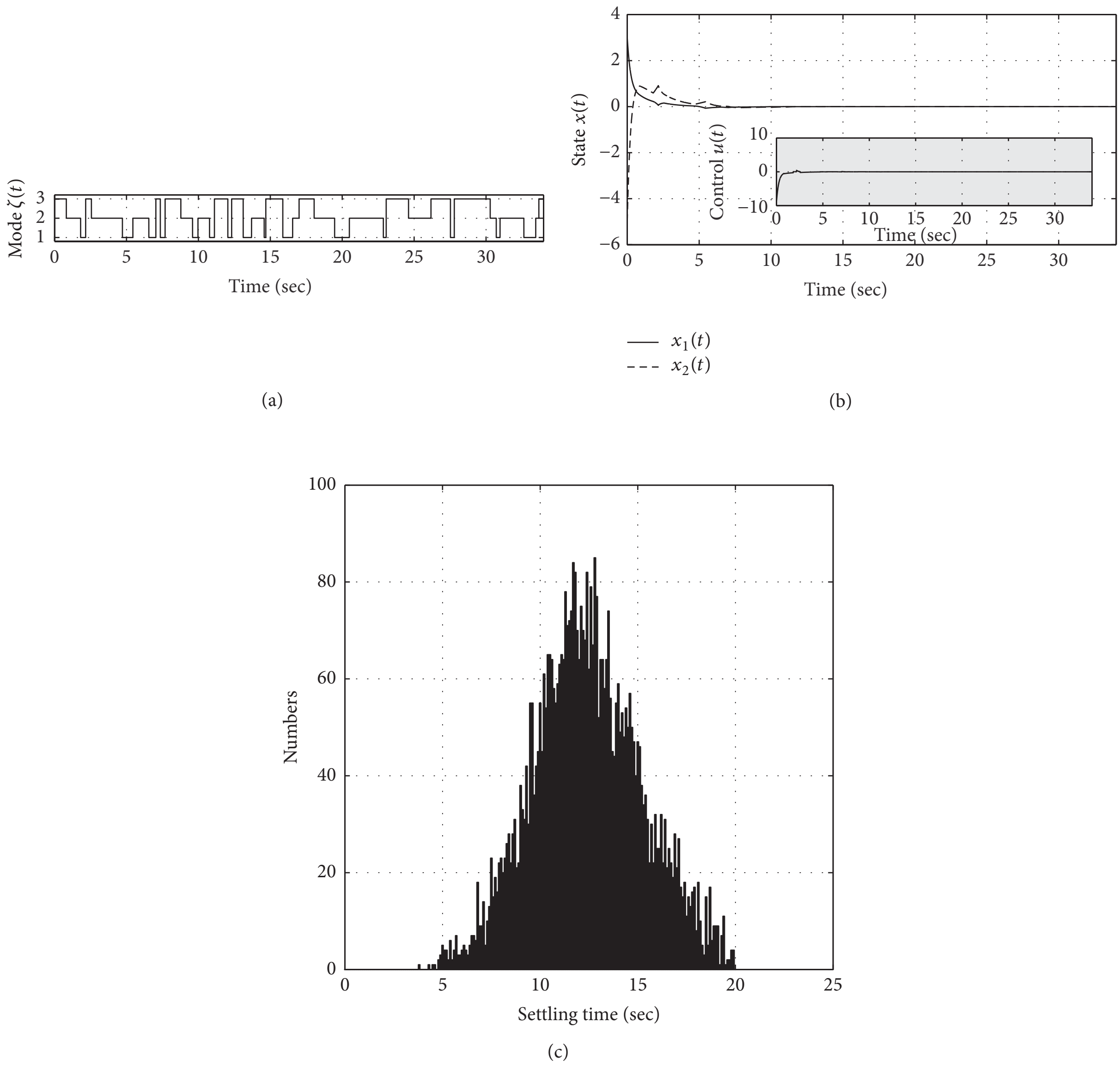

FIGURE 2: (a) Mode evolution, (b) state response and control input, and (c) Monte Carlo simulation results.

Besides, the matrices $[\pi_{ij,1}]_{ij\in\mathbb{N}_3^+}$ and $[\pi_{ij,2}]_{ij\in\mathbb{N}_3^+}$ such that $\pi_{ij}(h) \in [\pi_{ij,1}, \pi_{ij,2}]$ are taken to be the same as in Example 1. Thereupon, Theorem 7 provides the following control gains:

$$\begin{aligned} F_1 &= \begin{bmatrix} -2.7348 & -3.1067 & 2.1744 \end{bmatrix}, \\ F_2 &= \begin{bmatrix} -4.7643 & -3.6490 & 0.5326 \end{bmatrix}, \\ F_3 &= \begin{bmatrix} -243.8434 & -79.7086 & -12.9784 \end{bmatrix}. \end{aligned} \tag{70}$$

Figure 3 shows the behavior of the state response for the mode transition generated according to $(\alpha_1, \beta_1) = (0.5, 2.0)$, $(\alpha_2, \beta_2) = (1.0, 2.0)$, and $(\alpha_3, \beta_3) = (1.5, 2.0)$. Here, the control input $v_i(t)$ is designed in accordance with (56), and the quantization level is assumed to be $\delta = 0.1$. From Figure 3, it can be seen that the states converge from the initial-state condition $(1, -1, 0.5)$ to the origin as time increases. Consequently, Theorem 7 provides a suitable mode-dependent control for the S-MJSs with input quantization errors as well as uncertain probability intensities.

6. Concluding Remarks

The issue of stability analysis and control synthesis for S-MJSs with uncertain probability intensities has been addressed in this paper. Here, the boundary constraints of probability intensities have been totally reflected in the stabilization condition via a relaxation process established on the basis of time-varying transition rates. Furthermore, as an extension, the quantized control problem of S-MJSs has been addressed

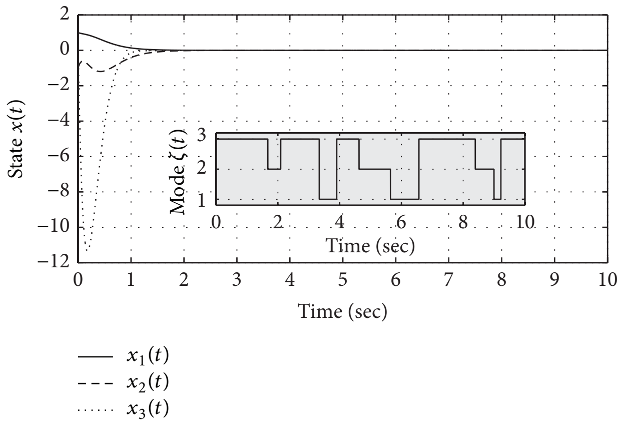

Figure 3: State response and mode evolution.

herein. Through simulation examples, the effectiveness of the proposed method has been shown.

Competing Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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