1. Introduction

The computation of the maximum temperature from the grinding conditions in dry surface grinding is essential for predicting thermal damage [1, Section 6.2]. For grinding regimes in which we find a high Peclet number, normally the use of coolant is required to reduce the risk of thermal damage. However, the pressure to reduce production costs and the considerable increase in environmental awareness have led manufacturers to reduce or eliminate cutting fluids in machining operations. In fact, coolant costs are in the range of 10–17% of total manufacturing costs [2]. Therefore, the prediction of the maximum temperature in order to evaluate the risk of thermal damage in dry grinding for a large Peclet number is not negligible. For this purpose, considering a constant heat flux distribution (with a value of $q$ Wm$^{-2}$ in SI units) within the contact width $2\ell$ (m) between wheel and workpiece, the maximum temperature $T_{\text{max}}$ (K) can be approximated for large Peclet numbers ($L \rightarrow \infty$) as

$$T_{\text{max}} \approx \frac{2\sqrt{2}}{\sqrt{\pi}} \frac{q}{k_0} \sqrt{\frac{k}{V_f}},$$

where $V_f$ (ms$^{-1}$) is the constant velocity of the heat source sliding over the workpiece, $k$ is the thermal diffusivity of the workpiece (m$^2$s$^{-1}$), and $k_0$ is its thermal conductivity (Jm$^{-1}$K$^{-1}$s$^{-1}$). This result is first given by Jaeger in [3, Eq. 33], being reported also by Malkin and Guo in [4, Eq. 2]. It is worth noting, on the one hand, that Jaeger omits the derivation of the asymptotic formula (1), being this given in dimensionless form. On the other hand, Malkin and Guo follow Jaeger’s analysis replacing the constant factor $2\sqrt{2/\pi}$ by the approximation 1.595 and stating that (1) provides a good approximation for $L > 5$, although in practice it can be applied down to $L = 1$, which includes most of the actual grinding situations (see also [5]).

Despite the fact that (1) has been used widely to predict thermal damage in surface grinding [1, Section 6.2], it seems to be more realistic to assume a linear heat flux distribution with its maximum at the leading edge. Therefore, considering this linear heat flux profile and assuming a geometrical contact length within the wheel and the workpiece [1, Eq. 3-4]

$$2\ell = \sqrt{aD},$$

where $a$ is the heat source area. For this purpose, we consider the most common heat flux profiles reported in the literature, such as constant, linear, triangular, and parabolic. In the constant case, we provide a refinement of the expression given in the literature, being the latter fitted by using a linear regression. The expressions for the triangular and parabolic cases are novel.
where $D$ is the diameter of the grinding wheel and $a$ is the depth of cut, the maximum temperature $T_{l}^{\text{max}}$ can be approximated for large Peclet numbers as [6, Eq. 3]

$$
T_{l}^{\text{max}} = 1.06 \frac{q^{1/2} a^{1/4} D^{1/4}}{k \nu^{1/2}},
$$

where $q$ is now the average heat flux along the contact zone. It is worth noting that in [6] the constant factor 1.06 is calculated by performing a linear regression, without any ab initio derivation.

Since the most common heat flux profiles reported in the literature are constant [7, 8], linear [8, 9], triangular [10, 11], and parabolic [12] (see Figure 1), this paper is intended, on the one hand, to provide a sound proof for the asymptotic formulas (1) and (3) and, on the other hand, to derive new asymptotic formulas for the triangular and parabolic cases.

This paper is organized as follows. In Section 2, the heat transfer model for surface grinding is presented in order to provide the framework in which the asymptotic formulas of the maximum temperature for large Peclet numbers have to be derived. Section 3 carries out the calculation of these asymptotic expressions considering constant, linear, triangular, and parabolic heat flux profiles. Also, in the constant case, a refinement of (1) is also calculated. Section 4 is devoted to evaluating numerically the accuracy of the asymptotic formulas in order to justify the range in which they are valid. Finally, the conclusions are collected in Section 5.

2. Heat Transfer Model in Surface Grinding

In dry surface grinding, friction due to contact between wheel and workpiece is modelled by an infinite strip heat source of width $2\ell$ that slides over the workpiece surface at the plane $z = 0$ and moves at a constant velocity $\vec{v}_f = -|\vec{v}_f| \vec{i}$ [3]. Assuming a two-dimensional model (see Figure 2), the temperature field of the workpiece $T(x, z)$ with respect to the room temperature in the stationary regime must satisfy the following equation [13, Eqns. 1.6(6) & 1.7(2))]:

$$
k \nabla^2 T(x, z) + \nu_f \partial_z T(x, z) = 0,
$$

where $-\infty < x < \infty$ and $z \geq 0$. Considering a dimensionless heat flux profile $f(x)$ within the contact area between wheel and workpiece, (4) is subjected to the following boundary condition:

$$
k_0 \partial_z T(x, 0) = q f(x) \theta(\ell - x) \theta(x + \ell),
$$

where $\theta(x)$ denotes the Heaviside function and $q$ is the average heat flux entering into the workpiece along the contact width $2\ell$; thus

$$
\frac{1}{2\ell} \int_{-\ell}^{\ell} f(x) dx = 1.
$$

Setting the dimensionless quantities

$$
X = \frac{x}{s},
L = \frac{\ell}{s},
Z = \frac{z}{s},
\mathcal{T} = \frac{\pi k_0 T}{q s},
$$

where $s$ is a characteristic length:

$$
s = \frac{2k}{\nu_f},
$$

the solution of (4) and (5) is expressed as [14]

$$
\mathcal{T}(X, Z) = \int_{X-L}^{X+L} f(s[X - u]) e^{u K_0 \left( \sqrt{u^2 + Z^2} \right)} du, \quad (9)
$$

with $K_0$ being the zeroth-order modified Bessel function of the second kind. On the surface ($Z = 0$), (9) reduces to

$$
\mathcal{T}(X, 0) = \int_{X-L}^{X+L} f(s[X - u]) e^{u K_0 \left( |u| \right)} du. \quad (10)
$$
Assuming that $f(x)$ is an analytic function $\forall x \in [-\ell, \ell]$, then it can be expanded in its Taylor series as

$$f(sX-su) = \sum_{n=0}^{\infty} \frac{f^{(n)}(sX)}{n!} (-su)^n.$$  \hfill (11)

Substituting (11) in (10), we obtain

$$T(X,0) = \sum_{n=0}^{\infty} \frac{s^n}{n!} J_{g_n}(x)|_{x=\pm L},$$  \hfill (12)

where we have defined the following function:

$$J_{g_n}(x) = \int_{0}^{x} u^n e^{\alpha u} K_0(|u|) \, du, \quad n = 0, 1, 2, \ldots.$$  \hfill (13)

According to [14], we can calculate the integral given in (13) as

$$J_{g_n}(x) = \begin{cases} e^{x^{n+1}} \{ K_0 (|x|) \Psi_n (x) + K_1 (|x|) \Phi_n (x) \} + A_n, & x \neq 0, \\ 0, & x = 0, \end{cases}$$  \hfill (14)

where the following polynomials in $1/x$ in terms of hypergeometric functions are defined as

$$\Psi_n (x) = \frac{1}{n+1} \left[ 1 - \frac{n}{2n+1} \binom{1,1-n,-1-n}{\frac{1}{2},1} \right] F_1, \quad \Phi_n (x) = \frac{\text{sgn}(x)}{2n+1} \binom{1,-n,-n}{-n+\frac{1}{2}} F_1,$$  \hfill (15)

as well as the constant

$$A_n = \frac{(-2)^{n+1} (n+1)! (n!)^2}{(2n+2)!}.$$  \hfill (16)

In [14], the solution given in (12) is used for computing the maximum temperature in dry surface grinding for the most common heat flux profiles reported in the literature, provided that this maximum must be found in the stationary regime and on the workpiece surface, within the contact zone. Despite the fact that this computation is very rapid, asymptotic expressions of the maximum temperature for large Peclet number ($L \to \infty$) for any dimensionless heat flux profile $f(x)$ can be calculated. This is quite useful not only because these expressions are extremely rapid to compute, but mainly because they provide a starting point for the development of a model of thermal damage in surface grinding as the one elaborated by Malkin and Guo considering a constant heat flux profile [1, Section 6.2].

3. Large Peclet Number Approximations

3.1. Asymptotic Formula for $J_{g_n}$ within the Grinding Zone.
Notice that, taking into account the definition of the generalized hypergeometric function [15, Eq. 2.1.2]

$$p \int_{\beta_1}^{\beta_2} \left[ \prod_{k=1}^{p} (\alpha_k) z^k \right] \frac{\Gamma(x+k)}{\Gamma(x)} \, dz,$$

and the definition of the Pochhammer symbol

$$(x)_k = \frac{\Gamma(x+k)}{\Gamma(x)},$$

we have the following asymptotic behavior of $\Psi_n(x)$:

$$\lim_{x \to \pm \infty} \Psi_n(x) = \lim_{x \to \infty} \frac{1}{2n+1} \left[ 1 - \frac{n}{2n+1} \sum_{k=0}^{\infty} \left( \frac{1}{2n} \right) \left( \frac{1}{2n+k} \right) \right]$$

$$= \frac{1}{2n+1},$$

and $\Phi_n(x)$

$$\lim_{x \to \pm \infty} \Phi_n(x) = \lim_{x \to \infty} \frac{1}{2n+1} \sum_{k=0}^{\infty} \left( \frac{n}{2n} \right) \left( \frac{n}{2n+k} \right) \left( \frac{1}{2n+k} \right)$$

$$= \frac{1}{2n+1},$$

where in both sums given in (19) and (20) only survives the first term (i.e., $k = 0$), when we perform the limit $x \to \pm \infty$.

Now taking into account the asymptotic behavior of the Macdonald function [16, Eq. 5.16.5],

$$K_0(x) = \sqrt{\frac{\pi}{2x}} e^{-x}, \quad x \to \infty,$$

and the results (19) and (20), we can calculate the asymptotic behavior of the function $J_{g_n}(x)$ given in (14) as

$$\lim_{x \to +\infty} J_{g_n}(x)$$

$$= \lim_{x \to +\infty} e^x x^{n+1} \left[ K_0 (|x|) \Psi_n (x) + K_1 (|x|) \Phi_n (x) \right] + A_n$$

$$= \lim_{x \to +\infty} e^x x^{n+1} \sqrt{2\pi x} + A_n.$$

Similarly

$$\lim_{x \to -\infty} J_{g_n}(x)$$

$$= \lim_{x \to -\infty} e^x x^{n+1} \left[ K_0 (|x|) \Psi_n (x) + K_1 (|x|) \Phi_n (x) \right] + A_n$$

$$= A_n.$$
Notice that within the grinding zone
\[-L < X < L \rightarrow \begin{cases} 
X + L > 0 \rightarrow \lim_{L \to \infty} X + L = +\infty, \\
X - L < 0 \rightarrow \lim_{L \to \infty} X - L = -\infty;
\end{cases} \tag{24}
\]

thus, according to (22)–(24), we conclude that
\[
J_{g_n}(z) \bigg|_{X-L} \approx \sqrt{2\pi} \left( X + L \right)^{n+1/2}, \quad L \to \infty, \quad X \in (-L, L). \tag{25}
\]

3.2. Constant Profile. Notice that the dimensionless function,
\[
f_c(x) = 1, \tag{26}
\]
provides a constant heat flux profile satisfying (6) (see Figure 1). Therefore, substituting (26) in (12), we have
\[
\mathcal{T}_c(X, 0) = J_{g_0}(z) \bigg|_{X-L} \approx \sqrt{2\pi} \left( \sqrt{X + L} - e^{2(x-L)} \right), \quad L \to \infty, \quad X \in (-L, L). \tag{27}
\]

so, according to (25), the asymptotic behavior for large Peclet number within the grinding zone is
\[
\mathcal{T}_c(X, 0) \approx \sqrt{2\pi} (X + L), \quad L \to \infty, \quad X \in (-L, L). \tag{28}
\]

Considering both constant and linear heat flux profiles, the maximum temperature in wet grinding is located on the surface within the grinding zone, in the stationary regime, as it is proved in [17]. Also in [17], wet grinding is modelled assuming a constant heat transfer coefficient \(h\) over all the workpiece surface. Therefore, setting \(h = 0\), we can apply this result of the location of the maximum temperature \(\mathcal{T}_c^{\max}\) also to dry grinding in order to find an estimation of \(\mathcal{T}_c\) for a large Peclet number. Notice that, in (28), the maximum is reached at
\[
X_c^{\max} = L; \tag{29}
\]
since we have a monotonic increasing function within the grinding zone, thus
\[
\mathcal{T}_c^{\max}(L) \approx 2\sqrt{\pi L}, \quad L \to \infty, \tag{30}
\]
and in dimension variables, recalling the definitions (7)-(8), we obtain the result given by Jaeger (1). It is remarkable that Jaeger found this approximation from the integral representation of \(J_g(z)\), that is to say (13), and not from its solution, given in (14).

We can improve the approximation given in (28) taking into account the following asymptotic expansion of the Macdonald function [18, Eq. 10.40.2]:
\[
K_v(z) \approx \sqrt{\frac{\pi}{2z}} e^{-z} \sum_{k=0}^{\infty} \frac{a_k(v)}{x^k}, \quad z \to \infty, \tag{31}
\]
where, according to [18, Eq. 10.17.1],
\[
a_k(v) = \frac{1}{k!18^k} \prod_{l=1}^{k} \left( 4v^2 - (2l - 1)^2 \right), \quad a_0(v) = 1, \tag{32}
\]
so
\[
K_0(z) \approx \sqrt{\frac{\pi}{2z}} e^{-z} \left( 1 - \frac{1}{8z} + \cdots \right), \quad z \to \infty, \tag{33}
\]
\[
K_1(z) \approx \sqrt{\frac{\pi}{2z}} e^{-z} \left( 1 + \frac{3}{8z} + \cdots \right), \quad z \to \infty. \tag{34}
\]

Therefore, the result given in (23) can be refined to the following one:
\[
\lim_{x \to -\infty} J_{g_n}(x) = \lim_{x \to -\infty} \left( -1 \right)^n \sum_{k=0}^{\infty} \frac{a_k(v)}{x^k}, \quad x \to \infty, \tag{34}
\]

and then, taking into account (22), (24), and (34), we arrive at
\[
\mathcal{T}_c(X, 0) = J_{g_0}(z) \bigg|_{X-L} \approx \sqrt{2\pi} \left( \sqrt{X + L} - e^{2(x-L)} \right), \quad L \to \infty, \quad X \in (-L, L). \tag{35}
\]

Despite the fact that we cannot solve exactly the maximum of the function given in (35), let us solve it approximately. First, let us perform the change of variables:
\[
u = L - X, \tag{36}
\]
so that we have to find the maximum of the function:
\[
g(u) = \sqrt{2\pi} \left( \sqrt{2L} - u - e^{-2u} \right). \tag{37}
\]

Second, notice that, according to (29),
\[
u_{\max} = L - X_c^{\max} \approx 0, \tag{38}
\]
so, expanding in Taylor series, we have
\[
\sqrt{2L} - u \approx \sqrt{2L} - \frac{u}{2\sqrt{2L}}, \quad u \to 0, \tag{39}
\]
\[
e^{-2u} \approx 1, \quad u \to 0,
\]
and (37) can be approximated as
\[
g(u) \approx \sqrt{2\pi} \left( \sqrt{2L} - \frac{u}{2\sqrt{2L}} - \frac{1}{4\sqrt{u}} \right), \quad u \to 0. \tag{40}
\]

In order to find out the maximum, let us solve
\[
0 = g'(u) = \sqrt{2\pi} \left( -\frac{1}{2\sqrt{2L}} + \frac{1}{8u^{3/2}} \right); \tag{41}
\]
thus
\[
u_{\max} \approx \frac{L^{1/3}}{2} \to X_c^{\max} \approx L - \frac{L^{1/3}}{2}. \tag{42}
\]

Finally, substituting (42) in (28), we obtain the following refinement of (30):
\[
\mathcal{T}_c^{\max}(L) \approx \sqrt{\pi} \left( 4L - \frac{L^{1/3}}{2} \right), \quad L \to \infty. \tag{43}
\]
3.3. Linear Profile. The following linear dimensionless function,

\[ f_l(x) = 1 - \frac{x}{\ell}, \]  

provides null heat flux at the trailing edge \( f(\ell) = 0 \) and also satisfies (6) (see Figure 1). Therefore, according to (12), we have

\[ T_l(X,0) = \left( 1 - \frac{X}{L} \right) J_0(\ell) + \frac{1}{L} J_1(\ell), \]  

where \( f_\Lambda(\pm\ell) = 0 \) and the profile apex is located in \( x = \ell_{\text{max}} \in [-\ell, \ell] \) (see Figure 1). Also, we have set the dimensionless parameter:

\[ \lambda = \frac{\ell_{\text{max}}}{\ell} \in [-1, 1]. \]  

Notice that when the heat flux occurs in an arbitrary interval, say \( x \in (a, b) \) instead of \( x \in (-\ell, \ell) \), the dimensionless surface temperature is given by

\[ T_A(X,0) = \int_{X-B}^{X-A} f(s) e^{\kappa K_0(|u|)} du, \]  

where we have set the following dimensionless parameters:

\[ A = \frac{a}{s}, \quad B = \frac{b}{s}. \]  

Since (4) is a linear differential equation, the field temperature is given by the superposition of both parts of (51), so according to (12), we have

\[ T_A(X,0) = H_L(X) + H_{-L}(X) + \frac{2\lambda}{1-\lambda^2} H_{-\lambda L}(X), \]  

where we have defined

\[ H_A(X) = 2J_0(X + \Lambda) \left( 1 + \frac{X}{\Lambda} \right), \quad H_{-\lambda}(X) = \frac{2\lambda}{\Lambda} J_0(X + \Lambda). \]  

On the one hand, notice that calculating the limit in (57) when \( \Lambda = L \to \infty \), according to (22) and (24), we can conclude that

\[ H_L(X) \approx \frac{4}{3L} (X + L)^{3/2}, \]  

\( L \to \infty, \) \( X \in (-L, L). \)  

Similarly, taking into account (23) and (24), we arrive at

\[ H_{-L}(X) \approx \frac{2X - L}{L}, \quad L \to \infty, \) \( X \in (-L, L). \)
On the other hand, note that within grinding zone, on the
left-hand side of the apex, we have
\[-L < X < \lambda L \rightarrow X - \lambda L < 0 \rightarrow \lim_{L \to \infty} X - \lambda L = -\infty,\]
(60)
and, on the right-hand side of the apex, we have
\[\lambda L < X < L \rightarrow X - \lambda L > 0 \rightarrow \lim_{L \to \infty} X - \lambda L = +\infty;\]
(61)
thus
\[H_{-\lambda L}(X) \approx \begin{cases} \frac{2 X - \lambda L}{\lambda L}, & X \in (-L, \lambda L), \\ \frac{4 \sqrt{2 \pi}}{3 \lambda L} (X + L)^{3/2}, & X \in (\lambda L, L), \end{cases}\]
(62)
\[L \to \infty.\]

Taking into account (58), (59), and (62) in (56), after some
algebra, we arrive at
\[T_{\triangle}(X,0) \approx \begin{cases} \frac{4 \sqrt{2 \pi}}{3 L (1 + \lambda)} (X + L)^{3/2}, & X \in (-L, \lambda L), \\ \frac{4 \sqrt{2 \pi}}{3 L (1 + \lambda)} \left[ (X + L)^{3/2} - \frac{2 (X - \lambda L)^{3/2}}{1 - \lambda} \right], & X \in (\lambda L, L), \end{cases}\]
(63)
\[L \to \infty.\]

Despite the fact we do not possess any mathematical proof
for the location of the maximum temperature, we have to
assume now that this one must be found in the stationary
regime and on the surface, within the contact zone, as in
the constant and linear cases. In fact, this is a quite natural
assumption, physically speaking. Bearing this in mind, notice
that, according to (63), the maximum cannot lie on the left-
hand side of the apex. Therefore, since the maximum must
have null derivative, let us solve
\[0 = \frac{\partial T_{\triangle}}{\partial X} \approx \frac{\sqrt{2 \pi}}{L (1 + \lambda)} \left[ (X + L)^{1/2} - \frac{2}{1 - \lambda} (X - \lambda L)^{1/2} \right];\]
(64)
thus
\[X_{\triangle}^{\max} \approx \frac{L (1 + \lambda)}{3 - \lambda}, \quad L \to \infty,\]
(65)
and the maximum temperature is then given by
\[T_{\triangle}^{\max}(L) \approx \frac{8}{3} \frac{\sqrt{2 \pi L}}{\sqrt{3 - \lambda}}, \quad L \to \infty.\]
(66)

Notice that if the apex of the triangular profile tends to the
leading edge, that is to say \(\lambda \to -1\), then the profile is linear
and (66) becomes (48).

3.5. Parabolic Profile. The dimensionless parabolic heat flux
profile satisfying (6) is given by
\[f_p(x) = \frac{3}{4} \left( 1 - \frac{x}{\ell} \right)^2,\]
(67)
where the heat flux at the trailing edge fulfills that \(f_p(\ell) = 0\)
and \(f_p'(\ell) = 0\). Therefore, according to (12), we obtain
\[T_p(X,0) = \frac{3}{4\ell^2} \left[ (L - X)^2 G_0(z) + 2 (L - X) G_1(z) + G_2(z) \right]_{z=X-L}^{X+L},\]
(68)

Taking into account (25), we have the following asymptotic
behavior for large Peclet number within the grinding
zone:
\[T_p(X,0) \approx \frac{\sqrt{2 \pi}}{5 \ell^2} \sqrt{X + L} \left( 2 X^2 - 6 L X + 7 L^2 \right),\]
(69)
\[L \to \infty, \quad X \in (-L, L).\]
As in the triangular case, let us consider the fact that the
maximum temperature is located on the surface, within the
grinding zone, in the stationary regime. Therefore, in order
to find out the maximum, let us solve
\[0 = \frac{\partial T_p}{\partial X} \approx \frac{\sqrt{2 \pi}}{2 \ell^2} \frac{\left( 2 X^2 - 2 L X - L^2 \right)}{\sqrt{X + L}},\]
(70)
Since the “+” sign in (71) leads to a value out of the interval \((-L, L)\), we have to choose the “−” sign. Substitution of the latter value in (69) yields

\[
\mathcal{T}^\text{max}_p (L) = \frac{2}{5} \sqrt{6\pi L (3 + \sqrt{3})}, \quad L \to \infty. \tag{72}
\]

It is worth noting that substitution of (71) in (69) taking the “+” sign leads to the same result as (72), but with a “− \sqrt{3}”, so anyway it is clear that (72) is the approximation to the maximum.

4. Numerical Results

In this section we examine the goodness of the asymptotic approximations given in Section 3 for large Peclet number. Figure 3 shows the dimensionless temperature in the stationary regime on the surface \(\mathcal{T}(X,0)\), considering a constant heat flux profile (27), as well as the asymptotic approximations (28) and (35). The graphs are plotted within the grinding zone \(X \in (-L,L)\) and taking \(L = 5\). Note that the maximum lies nearly the trailing edge within the grinding zone. Also, the first and second approximations, (28) and (35), respectively, overlap everywhere except for the zone nearby the maximum, where the second approximation has got a vertical asymptote. Note as well in Figure 3 that the location of the maximum \(X^\text{max}\) can be better estimated by the second approximation \(X^\text{max}_\text{2}\), given in (42), than by the first approximation \(X^\text{max}_\text{1}\) = \(L\), given in (29). However, the maximum temperature is better estimated by using the first approximation (28) at \(X^\text{max}_\text{2}\), that is to say, the approximation given in (43). These considerations justify the procedure followed for the refinement of the maximum temperature approximation performed in Section 3.2.

In Figure 4 is plotted the dimensionless temperature on the surface \(\mathcal{T}(X,0)\) for linear, triangular, and parabolic heat flux profiles, (45), (56), and (68), respectively, as well as its asymptotic approximations, (46), (63), and (69), taking \(L = 10\) for all the graphs and \(\lambda = -0.6\) for the triangular case.

We can appreciate that the asymptotic approximations are quite near to the exact solutions within the contact zone \(X \in (-L,L)\).

In order to evaluate quantitatively the distance between the exact solution and the asymptotic approximation as a function of \(L\), we can use the functional defined in [19]. If \(f(x)\) and \(g(x)\) are two nonnegative functions defined in an interval \(I\), the relative distance between them is defined as

\[
\Delta_I (f,g) = \left\{ \begin{array}{ll}
\int_I \left| f(x) - g(x) \right| dx, & f \neq g, \\
0, & f = g.
\end{array} \right.
\]

It is easy to prove that (see [19])

\[
0 \leq \Delta_I (f,g) \leq 1, \tag{74}
\]

where a value close to 0 means relatively near, but a value close to 1 means relatively infinitely far. Figure 5 shows the computation of (73), taking as functions \(f\) and \(g\) the exact solution \(\mathcal{T}(X,0)\) given in (12) and the asymptotic approximation \(\mathcal{T}_{\text{approx}}(X,0)\) of the surface temperature for the different heat flux profiles considered, within the interval \(I = (-L,L)\). We can see that, for \(L \geq 10\), the asymptotic approximation is relatively quite near to the exact solution, regardless of the heat flux profile considered. Note that second approximation of the constant case (35) is worse than the first approximation (28) because we are considering the distance within the grinding zone \(I = (-L,L)\); and nearby the maximum, the first approximation fits better (see Figure 3).

Nevertheless, the behavior of the maximum is a little bit different from the global behavior of the asymptotic approximation with respect to the exact solution. To see it quantitatively, let us define the relative error as

\[
\delta_{\text{max}} (L) = \frac{\int_{\text{approx}} \mathcal{T}(L) - \int_{\text{exact}} \mathcal{T}(L)}{\mathcal{T}_{\text{exact}} (L)}, \tag{75}
\]

\[
\text{or} \quad \delta_{\text{max}} (L) = 1 - \frac{\mathcal{T}_{\text{approx}} (L)}{\mathcal{T}_{\text{exact}} (L)}. \tag{76}
\]
5. Conclusions

We have considered the heat transfer in dry surface grinding in the stationary regime. Assuming the most common heat flux profiles proposed in the literature (constant, linear, triangular, and parabolic), we have derived very simple asymptotic expressions of the maximum temperature for high Peclet numbers. In the constant case, we have found a refinement of the expression found in the literature, which is quite accurate (an error less than 1% for $L \geq 2$). In the linear case, we have arrived at an expression which is reported in the literature by using a linear regression to fit the multiplicative constant. The expressions for the triangular and parabolic cases seem not to be reported in the literature.

**Conflict of Interests**

The author declares that there is no conflict of interests regarding the publication of this paper.

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