Research Article

On the Solution of Local Fractional Differential Equations Using Local Fractional Laplace Variational Iteration Method

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We present iteration formulae of a fractional space-time telegraph equation using the combination of fractional variational iteration method and local fractional Laplace transform.

1. Introduction

Fractional differential equations have number of properties and play master role in different fields of study. These equations and their solutions gained remarkable interest due to its striking role in every science and technology [1–4]. Various fields like diffusion, transport theory, scattering theory, rheology, quantitative biology, and so forth are successfully applying fractional differential equations to define and explain number of phenomena but fractional calculus is not perfectly applicable in the case of fractal functions. In order to deal with fractal problems in various fields, the concept of local fractional derivative was developed. The local fractional calculus was introduced by Yang and further applications of this derivative can be found in the references [5–8]. There are variety of analytical methods for solving them like Adomian decomposition method, generalized differential transform method, homotopy perturbation method, separating variables, local fractional Laplace method, local fractional Sumudu method, variational iteration method, and fractional variational iteration method [5, 6, 9–14].

In this paper, we develop an iteration formula to solve generalized fractional space-time telegraph equation by using the combination of fractional variational iteration method and local fractional Laplace transform.

2. Definitions and Mathematical Preliminaries

Definition 1 (see [7, 8]). The partition of interval $[a, b]$ is denoted as (t_j, t_{j+1}) , $j = 0, \dots, N-1$, $t_0 = a$, and $t_N = b$ with $\Delta t_j = t_{j+1} - t_j$ and $\Delta t = \max\{\Delta t_0, \Delta t_1, \dots\}$. Local fractional integral of $f(x)$ in the interval $[a, b]$ is given by [7, 8]:

$$\begin{aligned} {}_a I_b^{(\alpha)} f(x) &= \frac{1}{\Gamma(1+\alpha)} \int_a^b f(t) (dt)^\alpha \\ &= \frac{1}{\Gamma(1+\alpha)} \lim_{\Delta t \rightarrow 0} \sum_{j=0}^{N-1} f(t_j) (\Delta t_j)_\alpha. \end{aligned} \quad (1)$$

Definition 2. The local fractional derivative of $f(x) \in C_\alpha(a, b)$ of order α at $x = x_0$ is defined [see [7, 8]] as

$$\begin{aligned} f^{(\alpha)}(x_0) &= \frac{d^\alpha}{dx^\alpha} f(x) \Big|_{x=x_0} \\ &= \lim_{x \rightarrow x_0} \frac{\Delta^\alpha (f(x) - f(x_0))}{(x - x_0)^\alpha}, \end{aligned} \quad (2)$$

where $\Delta^\alpha (f(x) - f(x_0)) \cong \Gamma(\alpha + 1)f(x) - f(x_0)$.

Definition 3. The local fractional Laplace transform of $f(x)$ of order α is defined [15] as

$$\bar{L}_\alpha [f(x)] = \frac{1}{\Gamma(1+\alpha)} \int_0^\infty E_\alpha(-s^\alpha x^\alpha) f(x) (dx)^\alpha, \quad (3)$$

$$0 < \alpha \leq 1,$$

where $x \in C$ and $E_\alpha(x)$ is Mittag Leffler function $\sum_{k=0}^\infty (x^k / \Gamma(k+1))$.

And inverse local fractional Laplace transform is defined as

$$\bar{L}_\alpha^{-1} \{f(s)\} = f(x), \quad 0 < \alpha \leq 1. \quad (4)$$

On using the above definition of local fractional Laplace transform, the following result can easily be obtained [15]:

$$\bar{L}_\alpha \{f^{n\alpha}(x)\} = s^{n\alpha} \bar{L}_\alpha \{f(x)\} - \sum_{k=1}^n s^{(k-1)\alpha} f^{(n-k)\alpha}(0), \quad (5)$$

$$\bar{L}_\alpha \{x^{k\alpha}\} = \frac{\Gamma(1+k\alpha)}{s^{(k+1)\alpha}}.$$

3. The Local Fractional Laplace Variational Iteration Method

This section introduces the idea of local fractional Laplace variational method for the following fractional space-time telegraph equation:

$$D_{0+}^{n\alpha} v(x, t) = L_\alpha(v(x, t)) + N_\alpha(v(x, t)) + f(x, t), \quad (6)$$

where $0 < \alpha \leq 1$, $x, t \geq 0$, L_α is a linear operator, N_α is nonlinear operator, and f is a source term.

Taking local fractional Laplace transform of (6), both sides with respect to “ x ” are as follows:

$$s^{n\alpha} \bar{L}_\alpha \{f(x)\} - \sum_{k=1}^n s^{(k-1)\alpha} f^{(n-k)\alpha}(0) = \bar{L}_\alpha [L_\alpha(v(x, t)) + N_\alpha(v(x, t)) + f(x, t)]. \quad (7)$$

For an algebraic equation, the iteration formula can be constructed as

$$x_{n+1} = x_n + \lambda^\alpha f(x_n). \quad (8)$$

The optimality condition for the extreme $\delta x_{n+1} / \delta x_n = 0$, leads to

$$\lambda^\alpha = -\frac{1}{f'(x_n)}, \quad (9)$$

where δ is the classical variational operator.

By the formula of (8), we get the iteration formula for (7) as follows:

$$\bar{L}_\alpha v_{n+1}(s, t) = \bar{L}_\alpha v_n(s, t) + \lambda^\alpha \left\{ s^{n\alpha} \bar{L}_\alpha \{v_n(s, t)\} - \sum_{k=1}^n s^{(k-1)\alpha} v_n^{(n-k)\alpha}(0, t) - \bar{L}_\alpha [L_\alpha(v_n(x, t)) + N_\alpha(v_n(x, t)) + f(x, t)] \right\}. \quad (10)$$

Put $\lambda^\alpha = -s^{-n\alpha}$, the Lagrange multiplier [16] in (10):

$$\bar{L}_\alpha v_{n+1}(s, t) = \left\{ \left[\sum_{k=1}^n s^{(k-1-n)\alpha} v_n^{(n-k)\alpha}(0, t) \right] + s^{-n\alpha} \bar{L}_\alpha [L_\alpha(v_n(x, t)) + N_\alpha(v_n(x, t)) + f(x, t)] \right\}. \quad (11)$$

Taking inverse local fractional Laplace transform into account, we arrived at

$$v_{n+1}(x, t) = \bar{L}_\alpha^{-1} \left\{ \left[\sum_{k=1}^n s^{(k-1-n)\alpha} v_n^{(n-k)\alpha}(0, t) \right] + s^{-n\alpha} \bar{L}_\alpha [L_\alpha(v_n(x, t)) + N_\alpha(v_n(x, t)) + f(x, t)] \right\}. \quad (12)$$

This is the iteration formula for (6).

Example 4. To illustrate the above method, we can consider the following linear equation:

$$\frac{d^\alpha v(x, t)}{dx^\alpha} = av(x, t), \quad (13)$$

$$0 < x \leq T, \quad 0 < \alpha \leq 1, \quad a \in R.$$

We construct the following iteration formula with the help of (12):

$$v_{n+1}(x, t) = v_n(0, t) - \bar{L}_\alpha^{-1} [s^{-\alpha} \bar{L}_\alpha \{av_n(x, t)\}]. \quad (14)$$

Now, applying local fractional Laplace transform to the above equation, find

$$v_{n+1}(x, t) = v_n(0, t) - \bar{L}_\alpha^{-1} [s^{-2\alpha} \{av_n(s, t)\}]. \quad (15)$$

This is the iteration formula for (13).

Let us start from $v_0(x, t) = v(0, t) = v_0$.

Now, by putting the values of “ n ,” we get the iterations; for $v_1(x, t)$, put $n = 0$ in (15), and solving inverse local fractional Laplace transform, we have

$$v_1(x, t) = v_0 - \frac{ax^\alpha v_0}{\Gamma(1+\alpha)}. \quad (16)$$

For $n = 1$, $v_2(x)$ is given by the following iteration:

$$v_2(x, t) = v_1(0, t) - \bar{L}_\alpha^{-1} \left[s^{-2\alpha} \{a v_1(s, t)\} \right]. \quad (17)$$

Using (16) and after easy calculations, we get

$$v_2(x, t) = v_0 - \frac{ax^\alpha v_0}{\Gamma(1+\alpha)} + \frac{(ax^\alpha)^2 v_0}{\Gamma(1+2\alpha)}. \quad (18)$$

Similarly for $n = 2$, $v_2(x)$ is given by the following iteration:

$$\begin{aligned} v_3(x) &= v_0 - \frac{ax^\alpha v_0}{\Gamma(1+\alpha)} + \frac{(ax^\alpha)^2 v_0}{\Gamma(1+2\alpha)} - \frac{(ax^\alpha)^3 v_0}{\Gamma(1+3\alpha)} \\ &\vdots \\ v(x) &= \sum_{n=0}^{\infty} v_n(x) = v_0 \sum_{k=0}^{\infty} \frac{(-a)^k x^{k\alpha}}{\Gamma(1+k\alpha)}. \end{aligned} \quad (19)$$

This is the exact solution of (13).

Example 5. The following space-time fractional homogeneous telegraph equation can also be solved by the above introduced local fractional Laplace variational iteration method:

$$\begin{aligned} \frac{\partial^{2\alpha} v}{\partial x^{2\alpha}} v(x, t) &= \frac{\partial^{2\alpha} v}{\partial t^{2\alpha}}(x, t) + \frac{\partial^\alpha v}{\partial t^\alpha}(x, t) - v(x, t), \\ x, t &\geq 0, \quad 0 < \alpha \leq 1 \end{aligned} \quad (20)$$

with initial conditions

$$\begin{aligned} v(0, t) &= E_\alpha(-t^\alpha), \\ v^\alpha(0, t) &= E_\alpha(-t^\alpha). \end{aligned} \quad (21)$$

We can find the iteration formula for the above with the help of (12) as

$$\begin{aligned} v_{n+1}(x, t) &= \bar{L}_\alpha^{-1} \left[\sum_{k=1}^n s^{(k-1-n)\alpha} v_n^{(n-k)\alpha}(0, t) \right] \\ &+ \bar{L}_\alpha^{-1} \left[s^{-2\alpha} \left\{ \bar{L}_\alpha \left[\frac{\partial^{2\alpha} v_n}{\partial t^{2\alpha}}(x, t) + \frac{\partial^\alpha v_n}{\partial t^\alpha}(x, t) \right. \right. \right. \\ &\left. \left. \left. - v_n(x, t) \right] \right\} \right]. \end{aligned} \quad (22)$$

Consider initial iteration as follows:

$$\begin{aligned} v_0(x, t) &= \bar{L}_\alpha^{-1} \left[\sum_{k=1}^n s^{(k-1-n)\alpha} v^{(n-k)\alpha}(0, t) \right], \\ \bar{L}_\alpha^{-1} \left[s^{-\alpha} v(0, t) + s^{-2\alpha} v^\alpha(0, t) \right] \\ &= E_\alpha(-t^\alpha) + \frac{x^\alpha}{\Gamma(1+\alpha)} E_\alpha(-t^\alpha). \end{aligned} \quad (23)$$

Now, by putting the values of “ n ,” we get the iterations; for $v_1(x, t)$, put $n = 0$ in (22); we have

$$\begin{aligned} v_1(x, t) &= \bar{L}_\alpha^{-1} \left[\sum_{k=1}^n s^{(k-1-n)\alpha} v_0^{(n-k)\alpha}(0, t) \right] \\ &+ \bar{L}_\alpha^{-1} \left[s^{-2\alpha} \left\{ \bar{L}_\alpha \left[\frac{\partial^{2\alpha} v_0}{\partial t^{2\alpha}}(x, t) + \frac{\partial^\alpha v_0}{\partial t^\alpha}(x, t) \right. \right. \right. \\ &\left. \left. \left. - v_0(x, t) \right] \right\} \right]. \end{aligned} \quad (24)$$

Using (23) and applying local fractional Laplace and inverse local fractional Laplace transform, we get

$$\begin{aligned} v_1(x, t) &= E_\alpha(-t^\alpha) + \frac{x^\alpha}{\Gamma(1+\alpha)} E_\alpha(-t^\alpha) \\ &- \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} E_\alpha(-t^\alpha) \\ &- \frac{x^{3\alpha}}{\Gamma(1+3\alpha)} E_\alpha(-t^\alpha). \end{aligned} \quad (25)$$

Similarly, we can find

$$\begin{aligned} v_2(x, t) &= E_\alpha(-t^\alpha) + \frac{x^\alpha}{\Gamma(1+\alpha)} E_\alpha(-t^\alpha) \\ &- \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} E_\alpha(-t^\alpha) \\ &- \frac{x^{3\alpha}}{\Gamma(1+3\alpha)} E_\alpha(-t^\alpha) \\ &+ \frac{x^{4\alpha}}{\Gamma(1+4\alpha)} E_\alpha(-t^\alpha) \\ &+ \frac{x^{5\alpha}}{\Gamma(1+5\alpha)} E_\alpha(-t^\alpha), \\ v_3(x, t) &= E_\alpha(-t^\alpha) + \frac{x^\alpha}{\Gamma(1+\alpha)} E_\alpha(-t^\alpha) \\ &- \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} E_\alpha(-t^\alpha) \\ &- \frac{x^{3\alpha}}{\Gamma(1+3\alpha)} E_\alpha(-t^\alpha) \\ &+ \frac{x^{4\alpha}}{\Gamma(1+4\alpha)} E_\alpha(-t^\alpha) \\ &+ \frac{x^{5\alpha}}{\Gamma(1+5\alpha)} E_\alpha(-t^\alpha) \\ &- \frac{x^{6\alpha}}{\Gamma(1+6\alpha)} E_\alpha(-t^\alpha) \\ &- \frac{x^{7\alpha}}{\Gamma(1+7\alpha)} E_\alpha(-t^\alpha). \end{aligned} \quad (26)$$

Consequently, we obtain

$$\begin{aligned} v(x, t) = E_{\alpha}(-t^{\alpha}) & \left[1 - \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} + \frac{x^{4\alpha}}{\Gamma(1+4\alpha)} \right. \\ & \left. - \frac{x^{6\alpha}}{\Gamma(1+6\alpha)} + \cdots \right] + E_{\alpha}(-t^{\alpha}) \left[\frac{x^{\alpha}}{\Gamma(1+\alpha)} \right. \\ & \left. - \frac{x^{3\alpha}}{\Gamma(1+3\alpha)} + \frac{x^{5\alpha}}{\Gamma(1+5\alpha)} - \frac{x^{7\alpha}}{\Gamma(1+7\alpha)} + \cdots \right], \\ v(x, t) = E_{\alpha}(-t^{\alpha}) \cos_{\alpha}(x^{\alpha}) & + E_{\alpha}(-t^{\alpha}) \sin_{\alpha}(x^{\alpha}). \end{aligned} \quad (27)$$

Remark 6. The result in (27) is the same as the result obtained by Jafari and Jassim [17].

Example 7. The following space-time fractional homogeneous telegraph equation can also be solved by the above introduced local fractional Laplace variational iteration method:

$$\frac{\partial^{2\alpha} v}{\partial x^{2\alpha}} v(x, t) = \frac{\partial^{2\alpha} v}{\partial t^{2\alpha}}(x, t) + 2 \frac{\partial^{\alpha} v}{\partial t^{\alpha}}(x, t) + v(x, t), \quad (28)$$

$$x, t \geq 0, \quad 0 < \alpha \leq 1$$

with initial conditions

$$\begin{aligned} v(0, t) &= 1 - E_{\alpha}(-t^{\alpha}), \\ \frac{\partial^{\alpha} v}{\partial x^{\alpha}} v(0, t) &= 0. \end{aligned} \quad (29)$$

Then, we can find the iteration formula for the above with the help of (12) as

$$\begin{aligned} v_{n+1}(x, t) &= \bar{L}_{\alpha}^{-1} \left[\sum_{k=1}^n s^{(k-1-n)\alpha} v_n^{(n-k)\alpha}(0, t) \right] \\ &+ \bar{L}_{\alpha}^{-1} \left[s^{-2\alpha} \left\{ \bar{L}_{\alpha} \left[\frac{\partial^{2\alpha} v_n}{\partial t^{2\alpha}}(x, t) + 2 \frac{\partial^{\alpha} v_n}{\partial t^{\alpha}}(x, t) \right. \right. \right. \\ &\left. \left. \left. + v_n(x, t) \right\} \right\} \right]. \end{aligned} \quad (30)$$

Consider initial iteration as follows:

$$v_0(x, t) = \bar{L}_{\alpha}^{-1} \left[\sum_{k=1}^n s^{(k-1-n)\alpha} v^{(n-k)\alpha}(0, t) \right], \quad (31)$$

$$\bar{L}_{\alpha}^{-1} \left[s^{-\alpha} v(0, t) + s^{-2\alpha} v^{\alpha}(0, t) \right] = 1 - E_{\alpha}(-t^{\alpha}).$$

Now, by putting the values of “ n ,” we get the iterations; for $v_1(x, t)$, put $n = 0$ in (30); we have

$$\begin{aligned} v_1(x, t) &= \bar{L}_{\alpha}^{-1} \left[\sum_{k=1}^n s^{(k-1-n)\alpha} v_0^{(n-k)\alpha}(0, t) \right] \\ &+ \bar{L}_{\alpha}^{-1} \left[s^{-2\alpha} \left\{ \bar{L}_{\alpha} \left[\frac{\partial^{2\alpha} v_0}{\partial t^{2\alpha}}(x, t) + 2 \frac{\partial^{\alpha} v_0}{\partial t^{\alpha}}(x, t) \right. \right. \right. \\ &\left. \left. \left. + v_0(x, t) \right\} \right\} \right]. \end{aligned} \quad (32)$$

Using (31) and applying local fractional Laplace and inverse local fractional Laplace transform, we get

$$v_1(x, t) = 1 - E_{\alpha}(-t^{\alpha}) + \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}. \quad (33)$$

Similarly, we can find

$$\begin{aligned} v_2(x, t) &= 1 - E_{\alpha}(-t^{\alpha}) + \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} - \frac{x^{4\alpha}}{\Gamma(1+4\alpha)}, \\ v_3(x, t) &= 1 - E_{\alpha}(-t^{\alpha}) + \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} + \frac{x^{4\alpha}}{\Gamma(1+4\alpha)} \\ &+ \frac{x^{6\alpha}}{\Gamma(1+6\alpha)} \\ &\vdots \end{aligned} \quad (34)$$

Consequently, we obtain

$$\begin{aligned} v(x, t) &= [1 - E_{\alpha}(-t^{\alpha})] \left[1 + \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} + \frac{x^{4\alpha}}{\Gamma(1+4\alpha)} \right. \\ &\left. + \frac{x^{6\alpha}}{\Gamma(1+6\alpha)} + \cdots \right], \\ v(x, t) &= \cosh_{\alpha}(x^{\alpha}) - E_{\alpha}(-t^{\alpha}). \end{aligned} \quad (35)$$

Example 8. We consider the following local fractional Laplace equation:

$$\frac{\partial^{2\alpha} v}{\partial x^{2\alpha}} v(x, t) + \frac{\partial^{2\alpha} v}{\partial t^{2\alpha}}(x, t) = 0, \quad x, t \geq 0, \quad 0 < \alpha \leq 1 \quad (36)$$

with the initial condition

$$\begin{aligned} v(0, t) &= -E_{\alpha}(t^{\alpha}), \\ \frac{\partial^{\alpha} v}{\partial x^{\alpha}} v(0, t) &= 0. \end{aligned} \quad (37)$$

Then, we can find the iteration formula for the above with the help of (12) as

$$\begin{aligned} v_{n+1}(x, t) &= \bar{L}_{\alpha}^{-1} \left[\sum_{k=1}^n s^{(k-1-n)\alpha} v_n^{(n-k)\alpha}(0, t) \right] \\ &+ \bar{L}_{\alpha}^{-1} \left[s^{-2\alpha} \left\{ \bar{L}_{\alpha} \left[-\frac{\partial^{2\alpha} v_n}{\partial t^{2\alpha}}(x, t) \right] \right\} \right]. \end{aligned} \quad (38)$$

Consider initial iteration as follows:

$$\begin{aligned} v_0(x, t) &= \bar{L}_{\alpha}^{-1} \left[\sum_{k=1}^n s^{(k-1-n)\alpha} v^{(n-k)\alpha}(0, t) \right], \\ \bar{L}_{\alpha}^{-1} [s^{-\alpha} v(0, t) + s^{-2\alpha} v^{\alpha}(0, t)] &= -E_{\alpha}(t^{\alpha}). \end{aligned} \quad (39)$$

Now, by putting the values of “ n ,” we get the iterations; for $v_1(x, t)$, put $n = 0$ in (38); we have

$$v_1(x, t) = \bar{L}_\alpha^{-1} \left[\sum_{k=1}^n s^{(k-1-n)\alpha} v_0^{(n-k)\alpha}(0, t) \right] + \bar{L}_\alpha^{-1} \left[s^{-2\alpha} \left\{ \bar{L}_\alpha \left[-\frac{\partial^{2\alpha} v_0}{\partial t^{2\alpha}}(x, t) \right] \right\} \right]. \quad (40)$$

Using (39) and applying local fractional Laplace and inverse local fractional Laplace transform, we get

$$v_1(x, t) = -E_\alpha(t^\alpha) + \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} E_\alpha(t^\alpha). \quad (41)$$

Similarly, we can find

$$\begin{aligned} v_2(x, t) &= -E_\alpha(t^\alpha) + \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} E_\alpha(t^\alpha) \\ &\quad - \frac{x^{4\alpha}}{\Gamma(1+4\alpha)} E_\alpha(t^\alpha), \\ v_3(x, t) &= -E_\alpha(t^\alpha) + \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} E_\alpha(t^\alpha) \\ &\quad - \frac{x^{4\alpha}}{\Gamma(1+4\alpha)} E_\alpha(t^\alpha) + \frac{x^{6\alpha}}{\Gamma(1+6\alpha)} E_\alpha(t^\alpha) \\ &\quad \vdots \end{aligned} \quad (42)$$

Consequently, we obtain

$$\begin{aligned} v(x, t) &= [-E_\alpha(t^\alpha)] \left[1 - \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} + \frac{x^{4\alpha}}{\Gamma(1+4\alpha)} \right. \\ &\quad \left. - \frac{x^{6\alpha}}{\Gamma(1+6\alpha)} + \dots \right], \\ v(x, t) &= -E_\alpha(t^\alpha) \cos_\alpha(x^\alpha). \end{aligned} \quad (43)$$

Example 9. We consider the following local fractional Laplace equation:

$$\frac{\partial^{2\alpha} v}{\partial x^{2\alpha}} v(x, t) + \frac{\partial^{2\alpha} v}{\partial t^{2\alpha}}(x, t) = 0 \quad (44)$$

with the initial condition

$$\begin{aligned} v(0, t) &= 0, \\ \frac{\partial^\alpha}{\partial x^\alpha} v(0, t) &= -E_\alpha(t^\alpha). \end{aligned} \quad (45)$$

Then, we can find the iteration formula for the above with the help of (12) as

$$\begin{aligned} v_{n+1}(x, t) &= \bar{L}_\alpha^{-1} \left[\sum_{k=1}^n s^{(k-1-n)\alpha} v_n^{(n-k)\alpha}(0, t) \right] \\ &\quad + \bar{L}_\alpha^{-1} \left[s^{-2\alpha} \left\{ \bar{L}_\alpha \left[-\frac{\partial^{2\alpha} v_n}{\partial t^{2\alpha}}(x, t) \right] \right\} \right]. \end{aligned} \quad (46)$$

Consider initial iteration as follows:

$$\begin{aligned} v_0(x, t) &= \bar{L}_\alpha^{-1} \left[\sum_{k=1}^n s^{(k-1-n)\alpha} v_0^{(n-k)\alpha}(0, t) \right], \\ \bar{L}_\alpha^{-1} \left[s^{-\alpha} v(0, t) + s^{-2\alpha} v^\alpha(0, t) \right] &= -\frac{x^\alpha}{\Gamma(1+\alpha)} E_\alpha(t^\alpha). \end{aligned} \quad (47)$$

Now, by putting the values of “ n ,” we get the iterations; for $v_1(x, t)$, put $n = 0$ in (46); we have

$$\begin{aligned} v_1(x, t) &= \bar{L}_\alpha^{-1} \left[\sum_{k=1}^n s^{(k-1-n)\alpha} v_0^{(n-k)\alpha}(0, t) \right] \\ &\quad + \bar{L}_\alpha^{-1} \left[s^{-2\alpha} \left\{ \bar{L}_\alpha \left[-\frac{\partial^{2\alpha} v_0}{\partial t^{2\alpha}}(x, t) \right] \right\} \right]. \end{aligned} \quad (48)$$

Using (47) and applying local fractional Laplace and inverse local fractional Laplace transform, we get

$$v_1(x, t) = -\frac{x^\alpha}{\Gamma(1+\alpha)} E_\alpha(t^\alpha) + \frac{x^{3\alpha}}{\Gamma(1+3\alpha)} E_\alpha(t^\alpha). \quad (49)$$

Similarly, we can find

$$\begin{aligned} v_2(x, t) &= -\frac{x^\alpha}{\Gamma(1+\alpha)} E_\alpha(t^\alpha) + \frac{x^{3\alpha}}{\Gamma(1+3\alpha)} E_\alpha(t^\alpha) \\ &\quad - \frac{x^{5\alpha}}{\Gamma(1+5\alpha)} E_\alpha(t^\alpha), \\ v_3(x, t) &= -\frac{x^\alpha}{\Gamma(1+\alpha)} E_\alpha(t^\alpha) + \frac{x^{3\alpha}}{\Gamma(1+3\alpha)} E_\alpha(t^\alpha) \\ &\quad - \frac{x^{5\alpha}}{\Gamma(1+5\alpha)} E_\alpha(t^\alpha) + \frac{x^{7\alpha}}{\Gamma(1+7\alpha)} E_\alpha(t^\alpha) \\ &\quad \vdots \end{aligned} \quad (50)$$

Consequently, we obtain

$$\begin{aligned} v(x, t) &= [-E_\alpha(t^\alpha)] \\ &\quad \cdot \left[\frac{x^\alpha}{\Gamma(1+2\alpha)} - \frac{x^{3\alpha}}{\Gamma(1+4\alpha)} + \frac{x^{5\alpha}}{\Gamma(1+6\alpha)} - \dots \right], \\ v(x, t) &= -E_\alpha(t^\alpha) \sin_\alpha(x^\alpha). \end{aligned} \quad (51)$$

4. Conclusion

In this work, we have presented successful demonstration of the local fractional Laplace variational iteration method for solutions of a wide class of problems. Analytical solutions of the telegraph and Laplace equations on Cantor sets involving local fractional derivatives are efficiently developed.

Conflict of Interests

The authors declare no conflict of interests.

Authors' Contribution

Both authors have worked equally in this paper. Both authors have read and approved the final paper.

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