Research Article

Pricing Strategy and Quick Response Adoption System with Strategic Customers

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This study determined the competitive advantage of a quick response (QR) system when a firm faces forward-looking customers with heterogeneous and uncertain valuations for a product, uncertain demand, and two selling periods. We identify two classes of pricing strategies, namely, no-price commitment strategy and price commitment strategy. Interestingly, the unique equilibrium is proven to exist if and only if most customers have high tastes on a product’s value. We also prove that when customers possess beliefs about the markdown in the second period being smaller enough, a firm obtains a high profit with price commitment; otherwise he obtains a high profit without price commitment. Moreover, we distinguish the competitive advantage of a QR system from two strategies. When a firm uses no-price commitment strategy, the value of QR system in the first period decreases and in the second period increases with customer’s strategic behavior. When a firm provides price commitment, the value of QR system in the first period may increase, decrease, or decrease first and then increase with customer’s strategic behavior. And the value of QR in the second period under price commitment strategy decreases or rises first and then decreases with customer’s strategic behavior.

1. Introduction

The value of various products, such as electronic products and fast fashion, decreases in one selling season. Customers who focus on the value of products choose to make a purchase when the product value is high. Others who aim for reduced prices choose to wait for markdowns. Therefore, firms prefer to vary prices over time to increase profit. Pricing policies are generally divided into price commitment and no-price commitment. In the first policy, firms commit to a markdown price before the selling season. In the second policy, customers only become aware of the price in the selling period when they arrive. They are not aware of the markdown price. The current study focuses on the selection of these strategies when a firm sells a new product. The influence of various factors on the strategy choices of a firm is also investigated.

These products utilize a quick response (QR) system, which recently received considerable attention and is widely used in practice. Firms are unaware of exact demands and cannot produce appropriate quantities because of demand uncertainty. Thus, firms attempt to reduce lead times. If they stock out during the selling season, they have a rapid response alternative that can fill an order quickly. However, the issues of whether firms should always use a QR system and the selling period when a firm should use a QR system have not been resolved. We distinguish competitive advantage in the QR system between the two classes of pricing strategies.

In this study, a firm faces forward-looking customers with heterogeneous and uncertain valuations for a product, uncertain product demand, and two selling periods (high-value period and low-value period). A firm can maximize expected profits from sales by continuously changing prices over the course of the season (price commitment or no-price commitment) with or without a QR system in different selling periods. Therefore, a firm has six decision-making options: no-price commitment with a two-period QR system (Strategy \( N2 \)), price commitment with a two-period QR system (Strategy \( C2 \)), no-price commitment with a one-period QR
system (Strategy N1), price commitment with a one-period QR system (Strategy C1), no-price commitment without a QR system (Strategy N0), and price commitment without a QR system (Strategy C0). We formulate two-period models under six different strategies and provide sufficient conditions that guarantee the existence of a unique equilibrium. A substantial number of studies on consumer behavior and multipored pricing assume uniformly distributed valuations; these studies do not establish the existence of equilibrium for general distribution [1]. In the current study, we prove the existence of equilibrium under common conditions. We focus on the effect of strategic behavior of customers on competitive advantage from a QR system. Result indicates that when a firm does not provide price commitment, the competitive advantage obtained from a QR system increases in a low-value period but decreases in a high-value period with the strategic behavior of customers. We found that when a firm provides price commitment, the strategic behavior of customers’ strategic behavior variably influences competitive advantage from a QR system under different postponed production costs in a QR system. We also argue the choice of optimal strategy through a numerical study.

The rest of this study is organized as follows. Section 2 provides a literature review. Section 3 describes the model with strategic consumers. Section 4 establishes the model with a two-period QR system under price commitment. Section 5 provides an analysis on a firm’s strategy selection under different marketing environments. Section 6 concludes the paper.

2. Literature Review

Our work involves several key components, which include pricing policy, quick response system, and strategic customer behavior. We review the relevant literature on these components.

The first stream of literature focuses on pricing policy. Two general pricing policies, namely, price commitment and no-price commitment, are related to this study. The first pricing policy is also known as fixed preannounced pricing. In this policy, a firm announces or commits to a specific price path before the selling season. Early works on this type of policy include those of Coase [2] and Stokey [3]. Yin and Tang [4] develop a model based on a situation when a firm preannounces a price path at the beginning of the selling season and customers arrive in accordance with a Poisson process. Liu and Van Ryzin [5] assume that all customers are risk-averse and present at the beginning of the season. The researchers find that capacity decisions can be more important than price in terms of influencing strategic customer behavior. Whang [6] studies the role of demand uncertainty when a retail firm announces a pair of declining prices for two selling periods. Correa et al. [7] explore preannounced pricing schemes of a firm with strategic customers who are uncertain about the initial inventory level. The second pricing policy is also called dynamic pricing policy. In this policy, customers are not aware of future prices or the price path at the beginning of the selling season. Early works on this topic include Besanko and Winston [8]. Aviv and Pazgal [9] consider two pricing strategies, namely, inventory-contingent discounting strategies and preannounced price paths. Zhang et al. [10] derived optimal dynamic pricing strategy by Pontryagin’s maximum principle and then designed an effective algorithm to obtain the optimal dynamic pricing strategy and replenishment cycle simultaneously. Papanastasiou and Savva [11] investigate how social learning affects strategies under preannounced pricing and responsive pricing strategies. Shum et al. [12] examine the impact of cost reduction under dynamic pricing, price commitment, and price matching. Various studies on dynamic pricing policy (without price commitment) are mainly based on inventory-contingent discounting. By contrast, in the current study, a firm's dynamic pricing path is mainly affected by customers’ decision in the first selling period. Customers are classified according to a firm’s pricing based on the assumption of the present study. Therefore, a firm’s profit in the second period is affected by the preferences of customers who choose to wait.

Another related stream of literature focuses on a QR system. A number of studies [13–15] consider firm decisions under a QR system when customers are nonstrategic. Recent works, such as that of Serel [16], examine a single-period inventory model for multiple products in a QR system with a budget constraint; these studies show that this factor increases order size. Caro and Martínez-de-Albéniz [17] address the effect of QR under competition between two retailers. Choi [18] analyzes the carbon footprint taxation scheme in a QR system; they found that a properly designed carbon footprint taxation scheme can enhance environmental sustainability. Lin and Parlaktürk [19] find that manufacturers who offer a quick response to only one retailer are optimal when demand variability is moderate. Wang et al. [20] find that responsiveness is favorable under demand uncertainty or under conditions of weak product competition. Gong et al. [21] compare a quick response supplier and a regular supplier; they show that supply source diversification or high supplier reliability increases optimal profit and lowers selling price. However, these works did not consider the influence of strategic consumer behavior on a QR system. In practice, some products are suitable for a QR system, whereas others are not. Choi [22] explores the value of QR with the consideration of a bounded rational human manager in the fashion retailing company. A few studies analyze the factors that affect the selection of this strategy, as well as the influence of these factors. In the present study, we investigate the main influencing factor that affects the selection of this strategy.

The extensive literature on strategic purchasing behavior of customers begins with the study of Coase [2], who found that customers will wait for low prices of durable goods in a monopolist market. In recent years, strategic consumer behavior was incorporated into traditional operational models. Existing studies found that a firm’s decision-making can be affected by strategic consumer behavior. For example, Liu and Van Ryzin [5, 23], Lai et al. [24], and Jerath et al. [25] focus on how strategic consumers anticipate future surplus driven by firm inventory decisions. Yin et al. [26] and Su [27]
study firm pricing decisions when consumers anticipate future markdowns. Wu et al. [28] consider a retailer’s markdown pricing and inventory decisions in multiple seasons wherein consumers can learn from reference prices, which may help them decide when to purchase. Liu and Zhang [29] examine a heterogeneous market with two firms; they differentiate products and emphasized the role of product quality and the value of price commitment. Huang and van Mieghem [30] use a newsvendor framework in modeling click tracking practice of strategic website customers. Li et al. [31] employ a structural estimation model to analyze strategic customers; they determine that the presence of strategic customers does not necessarily hurt firm profit. A few studies focus on the effect of strategic consumer behavior on choosing a QR system. The current study explores this aspect. Yang et al. [32] analyze the impact of quick response on supply chain performance for various supply chain structures with strategic customer behavior and found that the value of the QR system would be greater in centralized supply chains systems than in decentralized systems when the extra cost of quick response was relatively low.

To the best of our knowledge, the problem of when to choose the strategy of price commitment or a QR system by considering the influence of strategic consumer behavior was not addressed in previous studies.

### 3. Model Description

#### 3.1. Firm and Consumers

We consider a seller who sells a product in a single selling season. The selling season for a product is divided into two consecutive periods. The first period is denoted as period $H$ and the second period is period $L$. The product is valued by customers at $v_i$ in different periods. We assume $v_H > v_L$ throughout. Thus, the product has a higher value in period $H$ than that in period $L$. A firm has two potential production opportunities, namely, early production and postponed production. Early production is far advanced in a selling season and market size is unknown. We assume $N$ to be a random variable with positive support, distribution function $F(\cdot)$, and density $f(\cdot)$. Postponed production occurs during the selling season and market size is known [33]. Early production incurs unit cost $c$, whereas postponed production incurs a higher unit cost $c_{qr} \geq c$. We assume that production has a sufficiently short lead time during the latter opportunity. Hence, we assume that $c_{qr} \geq c$.

Customers arrive at the beginning of the selling season. Customers have heterogeneous tastes on a product’s value $v_i$, which is denoted by $\theta_i$; this factor is private information of each customer [29, 30]. We assume that $\theta_i$ follows distribution function $G(\cdot)$ and density $g(\cdot)$, which is common knowledge. The prices offered in period $H$ are denoted by $p_H$ (a regular-season price) and $p_L$ (a markdown price) in period $L$. A firm simultaneously determines price $p_H$ and quantity $q$ at the beginning of each period. Customers have opinions about the markdown in the second period; that is, they believe that the price in period $H$ is $r p_H (0 < r < 1)$.

We introduce a discount factor (or a waiting cost) to simultaneously model strategic and nonstrategic customers [31], which is denoted by $\delta$ ($0 \leq \delta \leq 1$). We can interpret $\delta$ as the degree of patience to wait for the markdown or the level of strategic behavior. A high $\delta$ indicates patient or strategic consumers. If $\delta = 0$, customers do not anticipate the opportunity to purchase in period $L$, whereas if $\delta = 1$, they anticipate lack of discounts of future purchases.

#### 3.2. Firm Decision to Use or Not to Use QR System and Price Commitment

Thus, consumers and the firm take part in a game: consumers decide when to buy a product (in period $H$ or in period $L$); the firm decides whether to use a QR system and whether to provide price commitment. In our model, the firm’s decisions include price regular-season price $p_H$, markdown price $p_L$, and quantity $q$. Price $p_H$ can be observed by a customer before the selling season. Price $p_L$ can be observed by the customer before the selling season if the firm provides price commitment. Otherwise, price will be observed in period $L$. Quantity $q$ is private information of a firm and cannot be observed by a customer. Firms may select from six decision-making strategies (as shown in Table 1). The sequence of events in different strategies is depicted in Figures 1(a)–1(f).

#### 3.3. Consumer’s Decision to Wait, Buy, or Not Buy

Customers arrive at the beginning of the selling season. They obtain information, such as regular-season price ($p_H$) or markdown price ($p_L$), when the firm provides price commitment. When a firm does not provide price commitment, consumers believe that the price in period $L$ is $r p_H (0 < r < 1)$. We can interpret

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Price commitment</th>
<th>Period $H$</th>
<th>Period $L$</th>
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<tbody>
<tr>
<td>No-price commitment with two-period QR system (N2)</td>
<td>X</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>Price commitment with two-period QR system (C2)</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>No-price commitment with one-period QR system (N1)</td>
<td>X</td>
<td>√</td>
<td>X</td>
</tr>
<tr>
<td>Price commitment with one-period QR system (C1)</td>
<td>√</td>
<td>√</td>
<td>X</td>
</tr>
<tr>
<td>No-price commitment without QR system (N0)</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Price commitment without QR system (C0)</td>
<td>√</td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

$\sqrt{\cdot}$: choosing the strategy; $X$: not choosing the strategy.
Before the selling season
Consumers arrive and choose to purchase now or wait for period $L$ if their surplus is positive
Consumers who waited to purchase

---

Period $H$
Price: $p_H$

Period $L$
Price: $p_L$

---

Figure 1: Continued.
the demand in period $L$ will be below zero. In period $L$, the customer purchases if and only if $\theta v_L - p_L > 0$; that is, $\theta > p_L/v_L$.

Thus, given market size $N$, we obtain the expected demand in period $H$ as

$$D_H = N\Theta_H,$$

where $\Theta_H = 1 - G_1$.

The expected demand in period $L$ is given by

$$D_L = N\Theta_L,$$

where $\Theta_L = G_1 - G_2$. Denote $\Theta = \Theta_H + \Theta_L = 1 - G_2$.

Therefore, if a firm does not provide price commitment and $r < v_L/v_H$ (we use the symbol $r$ to denote this condition), $G_1 = G_{1r} = G((1 - \delta r)/(v_H - \delta v_L))p_H$ and $G_2 = G_{2r} = G(p_L/v_L)$. If a firm does not provide price commitment and $r \geq v_L/v_H$ (we use symbol $\bar{r}$ to denote this condition), $G_1 = G_{1\bar{r}} = G(p_H - \delta p_L)/(v_H - \delta v_L)$ and $G_2 = G_{2\bar{r}} = G(p_L/v_L)$. If a firm provides price commitment (we use the symbol $C$ to denote this condition), $G_1 = G_{1c} = G((p_H - \delta p_L)/(v_H - \delta v_L))$ and $G_2 = G_{2c} = G(p_L/v_L)$. Hence, $G_2 = G_{2r} = G_{2\bar{r}} = G_{2c} = G(p_L/v_L)$.

Figure 1: (a) Strategy $N2$. (b) Strategy $C2$. (c) Strategy $N1$. (d) Strategy $C1$. (e) Strategy $N0$. (f) Strategy $C0$. 

(1−$r$) as markdown amplitude. High $r$ implies that customers assume a low markdown in the second selling period. The value of $r$ can be influenced by advertising, the popularity of a product, and the type of product ($r$ is low if the product is fast fashion).

Customers decide to purchase products at that time or wait for the next selling period that provides the highest expected surplus. If a firm does not provide price commitment, the surplus to a consumer purchasing in period $H$ is $(\theta v_H - p_H)^\gamma$ and the surplus waiting to be purchased in the period $L$ is $\delta(\theta v_L - r p_H)^\gamma$. Hence, if $(\theta v_H - p_H)^\gamma > \delta(\theta v_L - r p_H)^\gamma$, the customer will purchase in period $H$; otherwise, he or she will wait for the next period. From $(\theta v_H - p_H)^\gamma > \delta(\theta v_L - r p_H)^\gamma$, we obtain $\theta > \max(1 - \delta r)p_H/(v_H - \delta v_L)$, $p_H/v_L$). Therefore, when $r < v_L/v_H$ and $\theta > (1 - \delta r)p_H/(v_H - \delta v_L)$ or when $r \geq v_L/v_H$ and $\theta > p_H/v_H$, the customer will purchase in period $H$. The customer purchases in period $L$ if and only if $\theta v_L - p_L > 0$; that is, $\theta > p_L/v_L$.

If a firm provides price commitment, the surplus to a consumer purchasing in the first period is $(\theta v_H - p_H)^\gamma$, and the surplus waiting to be purchased in the second period is $\delta(\theta v_L - p_H)^\gamma$. Hence, if $(\theta v_H - p_H)^\gamma > \delta(\theta v_L - p_H)^\gamma$, a customer will purchase in period $H$; otherwise, he or she will wait. From $(\theta v_H - p_H)^\gamma > \delta(\theta v_L - p_H)^\gamma$, we obtain $\theta > (p_H - \delta p_L)/(v_H - \delta v_L)$. In this situation, $(p_H - \delta p_L)/(v_H - \delta v_L) \geq p_L/v_L$, that is, $p_L/v_L \leq p_H/v_H$, is always satisfied. Otherwise,
4. Model with Two-Period QR
System under Price Commitment

In the quick response regime, a firm can procure inventory before and after receiving a forecast update prior to the start of the selling season [1, 34]. We define the equilibrium to pricing-inventory-purchasing game without price commitment as follows.

**Definition 1.** Equilibrium with rational expectations and non-zero production to the game between strategic consumers enables the firm to satisfy the following conditions.

(1) The firm sets the price and inventory to maximize expected profit, given that some consumers purchase in period $H$ and other consumers purchase in period $L$.

(2) Consumers believe that a markdown $(1 - r)$ could occur in period $L$ before the selling season and purchases may happen in periods $H$ given selling price $p_H$ and purchase in periods $L$ given selling price $p_L$.

A firm’s expected profit without price commitment under two-period QR strategy and as a function of the initial inventory procurement $(q)$ and price $(p_H, p_L)$ is

$$
\Pi^{N2}(p_H, p_L, q) = E \left[ p_H D_H + p_L D_L - cq - c_q (p_H + D_L - q)^+ \right]
$$

(3) where $L(q)$ is the expected lost sales function (excess demand above $q$):

$$
L(q) = E \left( D_H + D_L - q \right)^+
= \mu \Theta + \int_0^{\mu \Theta} F(x) dx - q.
$$

(4)

Thus, the equilibrium conditions are

(1) $(p_H^{N2*}, p_L^{N2*}, q^{N2*}) = \arg \max_{p_H, p_L} \Pi^{N2}(p_H, p_L, q)$;

(2) $\Theta^*_L = 1 - G_1^*$;

(3) $\Theta^*_H = G_1^* - G_3^*$.

Most studies on consumer behavior and multipored pricing assume uniformly distributed valuations [1]. These works do not establish the existence of equilibrium for general distribution. We attempt to prove the existence of equilibrium with increasing generalized failure rate (IGFR) distributions and obtain the optimal solution under different $r$ (Theorems 2 and 3).

**Theorem 2.** When $r < \frac{v_L}{v_H}$ and $G(\cdot)$ is an IGFR distribution, the firm’s optimal solution is given in the following situations.

(i) When $v_L (1 - \delta r)/(v_H - \delta v_L) > \hat{\psi}$, the firm’s profit function has a local maximum value point. Equilibrium solution $(p_H^{N2*}, p_L^{N2*}, q^{N2*})$ at maximum value point exists and is characterized by (5), (6), and (7).

(ii) When $v_L (1 - \delta r)/(v_H - \delta v_L) < \hat{\psi}$, $v_L (1 - \delta r)/(v_H - \delta v_L) < \psi$, the firm’s profit function has a local maximum value point. Equilibrium solution $(p_H^{N2*}, p_L^{N2*}, q^{N2*})$ at maximum value point exists and is characterized by (5), (6), and (7).

(iii) Otherwise, the firm’s optimal solution is $(p_H^{L2*}, p_L^{L2*}, q^{L2*})$.

$$
\Theta^*_H - \frac{g_2}{v_L} (p_H - p_L) = 0,
\Theta^*_L - \frac{g_2}{v_L} (p_L - M) = 0,
\psi = \min \psi,
$$

(5)

(6)

(7) where

$$
\psi = \frac{1}{g_2^L} \left( 2g_2 + \frac{\Theta^*_L g_2}{g_2} \right) \left( 2g_2^L + g_2^L \frac{\Theta^*_H}{\Theta^*_L} \right),
$$

$$
\hat{\psi} = \min \psi,
$$

(8) $M(\cdot, c_q) = c_q F(\frac{q}{\Theta}) - c = 0$.

Proof. The proof of this theorem is presented in the Appendix.

Situation (iii) in Theorem 2 is an extreme situation if and only if most customers have a low taste on a product’s value. If this situation occurs, a firm’s optimal decision is to sell only in a low-value period, which is inconsistent with actual situations. This condition demonstrates that if the firm wants to use a two-period pricing strategy, consumers should pay sufficient attention to product value, that is, demand distribution must satisfy Situation (i) or (ii). The rest of the paper focuses on Situations (i) and (ii) in Theorem 2; that is, the firm’s optimal solution is $(p_H^{N2*}, p_L^{N2*}, q^{N2*})$.

**Theorem 3.** When $r \geq \frac{v_L}{v_H}$ and $G(\cdot)$ is an IGFR distribution, the firm’s profit function has a maximum value point. The equilibrium solution $(p_H^{N2*}, p_L^{N2*}, q^{N2*})$ at maximum value point exists and is characterized by (9), (10), and (11):

$$
\Theta^*_H - \frac{p_H}{v_H} g_{1H} = 0,
\Theta^*_L - \frac{g_{1L}}{v_L} (p_L - M) = 0,
$$

(9)

(10)

(11) where $M(\cdot, c_q) = c_q F(\frac{q}{\Theta}) - c = 0$.

Proof. The proof of this theorem is presented in the Appendix.
We obtain $F(q/\Theta) = 1 - c/c_{qr}$ from (7) or (11). Therefore, $M(c, c_{qr})$ only depends on the cost of quick response ($c_{qr}$), the cost of early production ($c$), and the distribution function of market size $F^{-1}(1-c/c_{qr}) F(x)dx = ((c_{qr} - c)/c_{qr}) F^{-1}((c_{qr} - c)/c_{qr})$. The QR strategy in period $L$ is not a good choice under such a condition.

**Lemma 4.** When $r < v_L/v_H$, the optimal price in period $L$ ($p^{N2*}_L$) strictly decreases in $r$, and the optimal price in period $H$ ($p^{N2*}_H$) and optimal profit ($\Pi^{N2*}$) strictly increase in $r$.

**Proof.** The proof of this theorem is presented in the Appendix.

Based on Lemma 4, when $r \geq v_L/v_H$, the firm can obtain the maximum profit under Strategy N2. Therefore, firms try to raise the value of $r$ through advertising or other methods to improve profit.

**Special Case.** We obtain $\tilde{\psi} = 4$ when $G(\cdot)$ is a uniform distribution. $v_L(1-\delta^2)/(v_H - \delta v_L) < \tilde{\psi}$ is satisfied. Hence, the equilibrium solution ($p^{N2*}_H$, $p^{N2*}_L$, $q^{N2*}_L$) always exists, and Situations (ii) and (iii) in Theorem 2 will never occur.

When $G(\cdot)$ Is a Beta Distribution ($\Theta \in Be(\alpha, \beta)$). Beta distribution is a family of continuous probability distributions defined on the interval $[0, 1]$ and parameterized by two positive shape parameters denoted by $\alpha$ and $\beta$.

1. When $\alpha = 1, \beta = 1$; the beta distribution turns into a uniform distribution on $[0, 1]$. Thus, equilibrium solution ($p^{N2*}_H$, $p^{N2*}_L$, $q^{N2*}_L$) always exists.

2. When $\alpha > 1, \beta = 1$; that is, the number of customers who have high tastes on a product’s value is always greater than those with low tastes [when $x < y, g(x) > g(y)$]. In this condition, $\tilde{\psi} > 4$. Therefore, equilibrium solution ($p^{N2*}_H$, $p^{N2*}_L$, $q^{N2*}_L$) always exists.

3. When $\beta \neq 1$, that is, the number of customers who have high tastes on a product’s value may be smaller than that with low tastes [when $x < y, g(x) > g(y)$ may happen], in this condition, $\tilde{\psi} > 0$. Therefore, all situations in Theorem 2 may occur.

### 5. Firm’s Strategy Selection under Different Marketing Environments

Two problems are identified in this model based on an earlier discussion. First, does the firm strategy always maximize profit or not? Second, if not, how does the firm change its strategy? These problems are discussed as follows.

#### 5.1. When Should Price Commitment Be Provided?

If a firm commits to a two-period price before the selling season, we obtain the optimal model under Strategy C2. In this model, the firm’s expected profit is

$$\Pi^2_2(p_H, p_L, q) = E \left[ p_H D_{HC} + p_L D_{LC} - c q - c q (D_{HC} + D_{LC} - q)^+ \right]$$

$$= p_H \Theta_{HC} + p_L \Theta_{LC} - c q - c q L(q),$$

where $L(q) = E(D_{HC} + D_{LC} - q)^+ = \mu \Theta + \int_0^q F(x)dx - q$, $\Theta_{HC} = 1 - G_{IC}, \Theta_{LC} = G_{IC} - G_{2C}$, and $\Theta = \Theta_{HC} + \Theta_{LC}$.

**Theorem 5.** If $G(\cdot)$ is an IGFR distribution, a firm’s optimal solution when a price commitment is provided before the selling season under a two-period QR strategy has three situations:

1. When $v_L(1-\delta^2)/(v_H - \delta v_L) < \tilde{\psi}$, the equilibrium solution ($p^{N2*}_H$, $p^{N2*}_L$, $q^{N2*}_L$) at the maximum value point exists and is characterized by (13). When $\psi(\tilde{\psi}) > 4$, the firm’s optimal solution is ($p^{N2*}_H$, $p^{N2*}_L$, $q^{N2*}_L$).

2. When $v_L(1-\delta^2)/(v_H - \delta v_L) > \tilde{\psi}$, $v_L(1-\delta^2) < \psi(\tilde{\psi})$, and $\Pi^2_2(p^{N2*}_H, p^{N2*}_L, q^{N2*}_L)$ is increasing. Based on Lemma 4, when $r < v_L/v_H$, the optimal price in period $L$ is a Beta Distribution. Hence, the equilibrium solution ($p^{N2*}_H$, $p^{N2*}_L$, $q^{N2*}_L$) always exists, and Situations (ii) and (iii) in Theorem 2 will never occur.

3. Otherwise, the firm’s optimal solution is ($\tilde{p}_L, \tilde{q}$).

$$g_{IC} = \frac{v_H - \delta v_L}{\Theta_{LC}}, \frac{\delta (p_H - p_L)}{v_H - \delta v_L} g_{IC} + \Theta_{LC} + \frac{g_2}{v_L} (M - p_L) = 0, \quad (13)$$

$$F \left( \frac{\bar{q}}{\Theta} \right) = 1 - \frac{c}{c_{qr}},$$

where

$$\psi = \frac{1}{g_{IC}} \left( 2 g_2 + \Theta_{LC}/g_2 \right) \left( 2 g_{IC} + g_{IC}' \Theta_{HC}/g_{IC} \right),$$

$$\tilde{\psi} = \min \psi,$$

$$\Pi^C(\tilde{p}_L, \tilde{q}) = \max \left[ \Pi^C(p_L, q) = p_L \tilde{c}_{2q} - c q - c q L(q) \right],$$

$$M(c, c_{qr}) = \frac{c_{qr}}{\mu} + \int_0^{\bar{q}/\Theta} F(x)dx - \frac{\bar{q}}{\Theta} F \left( \frac{\bar{q}}{\Theta} \right).$$

**Proof.** The proof of this theorem is presented in the Appendix.
Proof. The proof of this theorem is presented in the Appendix.

Proposition 7. Decision boundary point $\bar{r}$ increases with customer’s discount factor $\delta$.

5.2. Firm Optimal Decision without QR Strategy. This section analyzes four other pricing and QR strategies (no-price commitment with one-period QR system, price commitment with one-period QR system, no-price commitment without QR system, and price commitment without a QR system).

Under Strategy N1, a firm sets first period price $p_H$ and inventory level $q$ before the selling season. Customers decide whether or not to purchase immediately. In this period, products are replenished with cost $c_q$ if the firm is out of stock. Hence, customers’ demands in first period are satisfied. In the beginning of the second period, a firm observes surplus stock and updates its assumptions over demand in the second period. The firm sets second period price $p_L$ and customers make purchasing decisions.

A firm’s optimal second period pricing policy is defined by

$$p_L = \begin{cases} 
  p_{L1} & (q - D_H)^+ \geq D_{L1} \\
  p_{L2} & (q - D_H)^+ < D_{L1}.
\end{cases}$$

When unsold stocks at the end of the first period are excessive, the firm cannot sell them out at a suitable price. Hence, a customer chooses the second period’s lowest price $p_{L1}$ to maximize $\pi = p_L \theta_L$; that is, $p_{L1}$ satisfies

$$\left( \theta_H - \frac{p_L g_L}{v_L} \right)_{p_L = p_{L1}} = 0.$$ \hspace{1cm} (16)

The lowest price in the second period $p_{L1}$ decreases with price in first period $p_H$.

When unsold stocks at the end of the first period are comparatively few, a firm can sell these out at a suitable price $p_{L2}$ depending on the quality of unsold stocks. Hence, $p_{L2}$ should satisfy equation $q - D_H = D_L$. We then obtain

$$p_{L2} = \frac{v_L g_L (1 - \frac{q}{q^*})}{(1 - \gamma \delta)}.$$ \hspace{1cm} (17)

Based on (16), we obtain

$$\partial p_H / \partial p_{L1} = \left( \frac{v_L - \delta v_L}{1 - \gamma \delta} \right)_{p_L = p_{L1}} > 0.$$ \hspace{1cm} (18)

Therefore, a firm’s profit function under strategy N1 is

$$\Pi_{N1} (p_H, p_L, q) = E \left[ p_H D_H + p_L \min \left( (q - D_H)^+, D_L \right) - c_q - c_q L_H (q) \right].$$

where $L_H (q) = E (D_H - q)^+ = \mu \theta_H + \Theta_H \int_0^{q/\theta_H} F (x) dx - q$. When $r < v_L / v_H$, $G_L = G_L^1$ and $G_L = G_L^2$ when $r \geq v_L / v_H$, $G_L = G_L^1$ when $q$ is sufficiently high.

Therefore, $(p_H, q^*)$ may exist to maximize $\Pi_{N1}$.

When a firm chooses a one-period QR strategy with price commitment (Strategy C1), the profit function is

$$\Pi_C (p_H, p_L, q) = E \left[ p_H D_H + p_L \min \left( (q - D_H)^+, D_L \right) - c_q - c_q L_H (q) \right].$$

Under this strategy, customers who choose to purchase in period H can always buy the product [the first term in (20)], and customers who choose to purchase in period L may not obtain the product. When $D_H > q$, products prepared before the selling season are sold out in the first period. Thus, consumers cannot buy any product in a no-QR strategy in the second period. A consumer who purchases the products in stock are sold out in period H can buy products produced under the QR strategy with product cost $c_q$ [the fourth term in (20)]. When $D_H < q$, that is, $(q - D_H)^+$, surplus products in period H are $q - D_H$ that can be sold in period L and the demand in period L is $D_L$. Therefore, actual sales are the minimum value between $(q - D_H)^+$ and $D_L$ [the second term in (20)].

The firm’s profit function under Strategies C1 and C2 can be rewritten as

$$\Pi_{C2} = \Pi - c_q T,$$ \hspace{1cm} (21)

$$\Pi_{C1} = \Pi - p_L T,$$ \hspace{1cm} (22)

where

$$\Pi = p_H \mu \theta_H + p_L \mu \theta_L - c_q q^* \mu \theta_H + \Theta_H \int_0^{q/\theta_H} F (x) dx - q.$$ \hspace{1cm} (23)

By comparing (21) and (22), we obtain Lemma 8.
Lemma 8. If a firm provides price commitment, \( c_{qr1}^C \in (c_{qr1}^K, c_{qr1}^C) \) exists and the one-period QR strategy is a better choice than the two-period QR strategy when \( c_{qr} > c_{qr1}^C \). Otherwise, the two-period QR strategy is a better choice, where \( p_L^{C2*} (c_{qr1}) = c_{qr1}^C \) and \( p_L^{C1*} (c_{qr2}) = c_{qr2}^C \).

Based on \( p_L^{C2*} (c_{qr}) = c_{qr} \), we obtain the upper bound of \( c_{qr1}^C \), which is denoted as \( \tilde{c}_{qr1}^C \). From \( p_L^{C1*} (c_{qr}) = c_{qr} \), we obtain the lower bound of \( c_{qr1}^C \), which is denoted as \( \hat{c}_{qr1}^C \). From (21) and (22), we have \( \Pi^{C1} (p_{H1}, p_L^{C1*}, q^{C1*}) = \Pi^{C1} (p_{H1}, p_L^{C2*}, q^{C2*}) \) when \( c_{qr} = c_{qr1}^C \). We obtain \( \Pi^{C2} (p_{H2}, p_L^{C2*}, p_L^{C2*}, q^{C2*}) < \Pi^{C1} (p_{H1}, p_L^{C1*}, p_L^{C1*}, q^{C1*}) \) because \( (\partial \Pi^{C1} / \partial p_{H1})|_{p_{H1}, p_L^{C1*}, q^{C1*}, q^{C2*}} = (\partial \Pi^{C2} / \partial p_L^{C2*}) < 0 \). The same procedure may be easily adapted to obtain \( \Pi^{C2} (p_{H1}, p_L^{C2*}, q^{C2*}) > \Pi^{C1} (p_{H1}, p_L^{C1*}, q^{C1*}) \) when \( c_{qr} = c_{qr1} \). If \( c_{qr} > c_{qr1}^C \), \( p_L^{C1*} (c_{qr}) < c_{qr} \), profit with the QR strategy in period L is certainly lower than that without a QR strategy in period L. If \( c_{qr} < c_{qr1}^C \), \( p_L^{C1*} (c_{qr}) > c_{qr} \), profit without the QR strategy in period L is certainly lower than that with the QR strategy in period L. Hence, some \( c_{qr1}^C \in (c_{qr1}^K, c_{qr1}^C) \) exist when \( c_{qr} = c_{qr1}^C \) and \( \Pi^{C2} (c_{qr1}) = \Pi^{C1} (c_{qr1}) \).

Under Strategy N0, products will not be replenished when stocks are out in the two periods. In the beginning of the first period, a firm sets price \( p_H \) and inventory level \( q \). Customers make decisions whether or not to purchase immediately. In the beginning of the second period, a firm observes the surplus stock and updates its assumptions over demand in the second period. A firm sets the second period price \( p_L \) and customers make purchasing decisions.

Similar to the analysis under Strategy N1, a firm’s profit function under strategy N0 is

\[
\Pi^{N0} (p_H, p_L, q) = E \left[ p_H \min (q, D_H) \right] + p_L \min (\{(q - D_H)^+, D_L\} - c_L) - \theta_H \int_0^{\theta_H} f(x) dx + p_L \int_0^{\theta_L} x \theta_L f(x) dx
\]

\[
+ \int_0^{\theta_L} p_L x \theta_L f(x) dx - c_L
\]

where

\[
p_L = \begin{cases} p_{L1} & (q - D_H)^+ \geq D_{L1} \\ p_{L2} & (q - D_H)^+ < D_{L1} \end{cases}
\]

\[
\left( \theta_H - \frac{p_{L1}}{\theta_L} q_{L1} \right)_{p_{L} = p_{L1}} = 0,
\]

\[
p_{L2} = \frac{v_L}{G} \left( 1 - \frac{q}{\chi} \right)
\]

\[
L_H (q) = E (D_H - q)^+ = \mu \theta_H + \theta_H \int_0^{\theta_H} f(x) dx - q.
\]

When \( r < v_L / v_{H1} \), \( G_1 = G_{1*} \) and \( G_2 = G_{2*} \); when \( r \geq v_L / v_{H1} \), \( G_1 = G_{1*} \) and \( G_2 = G_{2*} \).

Similar to the analysis under Strategy C1, a firm’s profit function under strategy C0 is

\[
\Pi^{C0} (p_H, p_L, q) = \Pi - \left( \mu \theta_H + \theta_H \int_0^{\theta_H} f(x) dx - q \right) (p_H - c_{qr}) - p_L T
\]

where

\[
\Pi = p_{H1} \theta_H + p_L \mu \theta_L - c_q
\]

\[
- c_{qr} \left( \mu \theta_H + \theta_H \int_0^{\theta_H} f(x) dx - q \right)
\]

\[
T = \mu \theta_L + \theta_L \int_0^{\theta_L} f(x) dx - \theta_H \int_0^{\theta_H} f(x) dx
\]

We can use the same approach as the proof of Lemma 8. By comparing Strategy C0 with Strategy C1, we can obtain Lemma 9.

Lemma 9. If a firm provides price commitment, \( c_{qr2}^C \in (c_{qr2}^K, c_{qr2}^C) \) exists. When \( c_{qr} > c_{qr2}^C \), the decision not to use a QR system is a better choice than the one-period QR strategy. Otherwise, the one-period QR strategy is a better choice, where \( p_L^{C1*} (c_{qr2}) = c_{qr2}^C \) and \( p_L^{C2*} (c_{qr2}) = c_{qr2}^C \).

Therefore, if the firm does not provide price commitment, \( c_{qr} \) and \( c_{qr2}^N \) exist. When \( c_{qr} < c_{qr2}^N \), \( \Pi^{N2*} > \Pi^{N1*} \); \( \Pi^{N1*} > \Pi^{N0*} \); when \( c_{qr} > c_{qr2}^N \), \( \Pi^{N2*} < \Pi^{N1*} \); \( \Pi^{N1*} < \Pi^{N0*} \).

Under Strategies N1 and N0, when \( \delta \to 1 \) and \( c_{qr} \) is sufficiently high, a firm will prepare sufficient products before the selling season. Hence, loss of stocks in period H almost never occurs; that is, \( L_H (q) \to 0 \) and \( q - \theta_H \int_0^{\theta_H} f(x) dx \to \mu \theta_H \). Therefore, \( \Pi^{N1} (p_H, p_L, q) \to \Pi^{N0} (p_H, p_L, q) \). Under Strategy C1 and Strategy C2, when \( \delta \to 1 \), we obtain \( \theta_L \to 0 \). Only a few customers purchase products in period L. Hence, \( \Pi^{C1} (p_H, p_L, q) \to \Pi^{C2} (p_H, p_L, q) \). We then obtain Lemma 10.

Lemma 10. When \( \delta = 1 \) and \( c_{qr} \) is sufficiently high, \( \Pi^{N1*} \to \Pi^{N0*} \). When \( \delta = 1 \) and \( c_{qr} \) is sufficiently low, \( \Pi^{C1*} \to \Pi^{C2*} \).

Example II. When a customer’s taste for value \( \theta \) follows a uniform distribution on \([0, 1]\) and \( v_H = 30 \), \( v_L = 20 \), \( c = 8 \),
and \( r = 0.6 \). \( N \) is uniformly distributed in \([0, 1000]\). Figure 2 shows that a firm's optimal strategy varies in \( c_{qr} \) and \( \delta \). For example, when \( \delta = 0 \), a firm chooses Strategy N2 if \( c_{qr} < 10.87 \). A firm chooses Strategy N1 if \( 10.87 < c_{qr} < 21.96 \). A firm chooses Strategy N0 if \( c_{qr} > 21.96 \).

When a firm does not provide price commitment, several customers will choose to purchase in period \( L \) if \( \delta \) is high. Hence, profit increase under Strategy N2 increases more with \( \delta \) if \( \Pi^{N2*} > \Pi^{N1*} \) than under Strategy N1 (the value of quick response in the second period). Profit increase under Strategy N1 decreases more with \( \delta \) if \( \Pi^{N1*} > \Pi^{N0*} \) than under Strategy N0 (the value of quick response in the first period). Therefore, we obtain Lemma 12.

**Lemma 12.** If \( \Pi^{N2*} > \Pi^{N1*} \), percentage profit increase \( (\Pi^{N2*} - \Pi^{N1*})/\Pi^{N1*} \) rises with \( \delta \). If \( \Pi^{N1*} > \Pi^{N0*} \), percentage profit increase \( (\Pi^{N1*} - \Pi^{N0*})/\Pi^{N0*} \) decreases with \( \delta \).

**Figure 2:** Optimal profit for various values of \( \delta \) under different \( c_{qr} \).
If \( \tilde{\delta}_2 < 0 \), it always increases with \( \delta \), and if \( \tilde{\delta}_2 > 0 \), it always decreases with \( \delta \). Therefore, we obtain Lemmas 14 and 16.

**Lemma 14.** When \( \Pi^{C2*} > \Pi^{C1*} \), percentage profit increase \( \left( \Pi^{C2*} - \Pi^{C1*} \right)/\Pi^{C1*} \) or the value of quick response in the second period decreases with \( \delta \) if \( \tilde{\delta}_1 < 0 \); the percentage profit increase \( \left( \Pi^{C2*} - \Pi^{C1*} \right)/\Pi^{C1*} \) rises first and decreases with \( \delta \) if \( 0 < \tilde{\delta}_1 < 1 \).

Based on Lemma 14, \( c_{qr1} \) increases with \( \delta \) and \( c_{qr2} \) decreases with \( \delta \).

**Example 15.** Figure 4 shows the percentage profit increase \( \left( \Pi^{C2*} - \Pi^{C1*} \right)/\Pi^{C1*} \) or the value of the quick response in the second period when a firm provides price commitment. If the equilibrium expected profits under Strategy \( C2 \) are higher than those under Strategy \( C0 \), percentage profit increase increases with \( \delta \) when \( c_{qr} \) is sufficiently low (i.e., \( \tilde{\delta}_1 < 0 \)). When \( c_{qr} \) is high (i.e., \( 0 < \tilde{\delta}_1 < 1 \)), the percentage profit increase decreases first and then increases with \( \delta \). Otherwise, when \( c_{qr} \) is sufficiently high (i.e., \( \tilde{\delta}_1 > 1 \)), the percentage profit increase rises with \( \delta \).

**Lemma 16.** When \( \Pi^{C1*} > \Pi^{C0*} \), percentage profit increase \( \left( \Pi^{C1*} - \Pi^{C0*} \right)/\Pi^{C0*} \) or the value of the quick response in the first period increases with \( \delta \) if \( \tilde{\delta}_2 < 0 \); it decreases first and increases with \( \delta \) if \( 0 < \tilde{\delta}_2 < 1 \); it decreases with \( \delta \) if \( \tilde{\delta}_2 > 0 \).

**Example 17.** Figure 5 shows the percentage profit increase \( \left( \Pi^{C1*} - \Pi^{C0*} \right)/\Pi^{C0*} \) or the value of the quick response in the first period when the firm provides a price commitment. If the equilibrium expected profits under Strategy \( C1 \) are higher than those under Strategy \( C0 \), percentage profit increase rises with \( \delta \) when \( c_{qr} \) is sufficiently low (i.e., \( \tilde{\delta}_2 < 0 \)). When \( c_{qr} \) is high (i.e., \( 0 < \tilde{\delta}_2 < 1 \)), the percentage profit increase decreases first and then increases with \( \delta \). Otherwise, when \( c_{qr} \) is sufficiently high (i.e., \( \tilde{\delta}_2 > 1 \)), the percentage profit increase rises with \( \delta \).

**Example 18.** We consider different combinations of \( \delta \) and \( c_{qr} \) while fixing other parameters. We tried different values of other parameters and obtained qualitatively similar results. Figure 6 demonstrates the optimal profit with and without price commitment for various values of \( r \) under different combinations of \( \delta \) and \( c_{qr} \). We can obtain a firm’s optimal strategy in different conditions. For example, when \( \delta = 0.6, c_{qr} = 11 \). A firm’s maximum profit can be obtained under Strategy \( C1 \) or \( C2 \) if \( r < 0.6383 \). If \( r > 0.6383 \), maximum profit can be obtained under Strategy \( N1 \). If a firm does not provide price commitment, Strategy \( N2 \) should be chosen if \( r < 0.614 \) and choose Strategy \( N1 \) if \( r > 0.614 \).

**6. Conclusion**

We explore price commitment and no-price commitment schemes and their equilibrium solution for a firm that faces forward-looking customers who have heterogeneous and uncertain valuations for a product with or without a QR system in two selling periods. We develop different models based on six decision-making options: a firm provides price...
commitment or no-price commitment, and the firm uses or does not use a QR system in two selling periods.

First, unlike many previous studies on consumer behavior and multipoded pricing, we prove that the existence of equilibrium is guaranteed if most customers have a high enough taste on a product’s value; otherwise the firm’s best strategy is to use the one-period selling strategy. Second, when customers possess beliefs about the markdown in the second period being smaller than a certain value, the firm obtains a high profit with price commitment; otherwise he obtains a high profit without price commitment. The certain value increases with customer’s discount factor. Moreover, a stylized analysis of the effects of customers’ strategic behavior on the competitive advantage from a QR system between two classes of pricing strategies is presented. When a firm does not provide price commitment in two selling periods, the competitive advantage from a QR system in a low-value period increases and decreases in a high-value period with customers’ strategic behavior. When a firm provides price commitment, the competitive advantage obtained from a QR system in a low-value period decreases with customers’ strategic behavior if postponed production cost is sufficiently low. Otherwise, it increases first and then decreases with customers’ strategic behavior. The competitive advantage obtained from a QR system in a high-value period increases with customers’ strategic behavior if postponed production cost is sufficiently low and decreases with customers’ strategic behavior if postponed production cost is sufficiently high. Otherwise, it decreases first and then increases with customers’ strategic behavior.

Further, future research can extend our analysis in several directions. First, the firm’s pricing and inventory decisions when the value of product continuously decreases with time can be examined. It would be interesting to investigate an optimal time to mark prices down during a selling season. Second, a firm’s decision, when using a quick response system, should be compared with decisions in a traditional system to gain further insights on the impact of strategic customer behavior.

Appendix

Proof of Theorem 2. When \( r < v_L/v_H \), \( G_1 = G_{1r} = G((p_H - \delta r \eta_H)/(v_H - \delta v_L)) \) and \( G_2 = G_{2r} = G(p_L/v_L) \). The first-order conditions for maximizing \( \Pi^{N2} \) are

\[
\frac{\partial \Pi^{N2}}{\partial p_H} = \mu \left( \Theta \eta_H - \eta_L (1 - \delta r) \frac{(p_H - p_L)}{v_H - \delta v_L} \right), \quad (A.1)
\]
The optimizing condition for the profit of the firm is

\[ \left( -\frac{c_{qr} f \left( \frac{q}{\Theta} \right)}{\Theta} \right) \left( \frac{\partial^2 \Pi^{N2}}{\partial p_H^2} \right) > 0, \quad \left( -\frac{c_{qr} f \left( \frac{q}{\Theta} \right)}{\Theta} \right) \left( \frac{\partial^2 \Pi^{N2}}{\partial p_L^2} \right) < 0. \]  

Therefore, we obtain the Hesse matrices of this problem as

\[ \begin{pmatrix} -\frac{c_{qr} f \left( \frac{q}{\Theta} \right)}{\Theta} & 0 \\ 0 & -\frac{q c_{qr} g_1}{\Theta^2 v_L} f \left( \frac{q}{\Theta} \right) & -\frac{g_1}{\Theta^2 v_L} \mu g_1 v_H - \delta v_L \end{pmatrix} \frac{\partial^2 \Pi^{N2}}{\partial p_H^2} \begin{pmatrix} -\frac{c_{qr} f \left( \frac{q}{\Theta} \right)}{\Theta} \\ 0 & -\frac{q c_{qr} g_1}{\Theta^2 v_L} f \left( \frac{q}{\Theta} \right) & -\frac{g_1}{\Theta^2 v_L} \mu g_1 v_H - \delta v_L \end{pmatrix} \frac{\partial^2 \Pi^{N2}}{\partial p_L^2}. \]  

We obtain the Hesse matrices of this problem as

\[ \begin{pmatrix} -\frac{c_{qr} f \left( \frac{q}{\Theta} \right)}{\Theta} & 0 \\ 0 & -\frac{q c_{qr} g_1}{\Theta^2 v_L} f \left( \frac{q}{\Theta} \right) & -\frac{g_1}{\Theta^2 v_L} \mu g_1 v_H - \delta v_L \end{pmatrix} \frac{\partial^2 \Pi^{N2}}{\partial p_H^2} \begin{pmatrix} -\frac{c_{qr} f \left( \frac{q}{\Theta} \right)}{\Theta} \\ 0 & -\frac{q c_{qr} g_1}{\Theta^2 v_L} f \left( \frac{q}{\Theta} \right) & -\frac{g_1}{\Theta^2 v_L} \mu g_1 v_H - \delta v_L \end{pmatrix} \frac{\partial^2 \Pi^{N2}}{\partial p_L^2}. \]  

Therefore, the optimizing condition for the profit of the firm is

\[ \left( -\frac{c_{qr} f \left( \frac{q}{\Theta} \right)}{\Theta} \right) \left( \frac{\partial^2 \Pi^{N2}}{\partial p_H^2} \right) > 0, \quad \left( -\frac{q c_{qr} g_1}{\Theta^2 v_L} f \left( \frac{q}{\Theta} \right) \right) \left( \frac{\partial^2 \Pi^{N2}}{\partial p_L^2} \right) < 0. \]  

Moreover, we obtain the Hesse matrices of this problem as

\[ \begin{pmatrix} -\frac{c_{qr} f \left( \frac{q}{\Theta} \right)}{\Theta} & 0 \\ 0 & -\frac{q c_{qr} g_1}{\Theta^2 v_L} f \left( \frac{q}{\Theta} \right) & -\frac{g_1}{\Theta^2 v_L} \mu g_1 v_H - \delta v_L \end{pmatrix} \frac{\partial^2 \Pi^{N2}}{\partial p_H^2} \begin{pmatrix} -\frac{c_{qr} f \left( \frac{q}{\Theta} \right)}{\Theta} \\ 0 & -\frac{q c_{qr} g_1}{\Theta^2 v_L} f \left( \frac{q}{\Theta} \right) & -\frac{g_1}{\Theta^2 v_L} \mu g_1 v_H - \delta v_L \end{pmatrix} \frac{\partial^2 \Pi^{N2}}{\partial p_L^2}. \]  

Therefore, the optimizing condition for the profit of the firm is

\[ \left( -\frac{c_{qr} f \left( \frac{q}{\Theta} \right)}{\Theta} \right) \left( \frac{\partial^2 \Pi^{N2}}{\partial p_H^2} \right) > 0, \quad \left( -\frac{q c_{qr} g_1}{\Theta^2 v_L} f \left( \frac{q}{\Theta} \right) \right) \left( \frac{\partial^2 \Pi^{N2}}{\partial p_L^2} \right) < 0. \]  

Moreover, we obtain the Hesse matrices of this problem as

\[ \begin{pmatrix} -\frac{c_{qr} f \left( \frac{q}{\Theta} \right)}{\Theta} & 0 \\ 0 & -\frac{q c_{qr} g_1}{\Theta^2 v_L} f \left( \frac{q}{\Theta} \right) & -\frac{g_1}{\Theta^2 v_L} \mu g_1 v_H - \delta v_L \end{pmatrix} \frac{\partial^2 \Pi^{N2}}{\partial p_H^2} \begin{pmatrix} -\frac{c_{qr} f \left( \frac{q}{\Theta} \right)}{\Theta} \\ 0 & -\frac{q c_{qr} g_1}{\Theta^2 v_L} f \left( \frac{q}{\Theta} \right) & -\frac{g_1}{\Theta^2 v_L} \mu g_1 v_H - \delta v_L \end{pmatrix} \frac{\partial^2 \Pi^{N2}}{\partial p_L^2}. \]
Equation (A.7) is always satisfied. Based on (A.8), we obtain

\[
\frac{v_L (1 - \delta r)}{v_H - \delta v_L} < \psi,
\]  

(A.9)

where \( \psi = (1/g_{1r}^2)(2g_2 + (\Theta_{2r}/g_2)g_{1r}')(2g_1 + g_{1r}'(\Theta_{2r}/g_{1r}')). \)

\( G(\cdot) \) is an IGFR distribution. Thus, \( \psi > 0 \) is satisfied for any value of \( p_H \) and \( p_L \). Denote \( \tilde{\psi} \) as the minimum value of \( \psi \).

Let \( \partial \Pi^{N2}/\partial p_H = 0, \partial \Pi^{N2}/\partial p_L = 0, \) and \( \partial \Pi^{N2}/\partial q = 0 \) ((5), (6), and (7)); we obtain \( p_H^{N2*}, p_L^{N2*}, \) and \( q^{N2*} \). Therefore, the following holds.

1. If the equation \( v_L (1 - \delta r)/(v_H - \delta v_L) < \tilde{\psi} \) is satisfied, the optimal solution is \( (p_H^{N2*}, p_L^{N2*}, q^{N2*}) \).
Moreover, when \( p_H \geq (v_H - \delta v_L)/(1 - r \delta) \), we have \( \partial^2 \Pi^{N2}_L = 0 \), which means that customers do not choose to purchase in period \( H \) for the firm's optimal decisions. Hence, if \((v_H - \delta v_L)/(1 - r \delta), \bar{p}_L, \bar{q}\) is the firm's optimal decision, the optimization problem changes into a one-period decision model as problem (A.10), and the firm's optimal decision is \((\bar{p}_L, \bar{q})\).

We obtain the Hesse matrices of this problem as

\[
\begin{pmatrix}
-\frac{c_q f(q)}{\partial q} & 0 & -q c_q g_2 f(q) \\
0 & \frac{\partial^2 \Pi^{N2}_L}{\partial q^2} & 0 \\
-q c_q g_2 & 0 & \frac{\partial^2 \Pi^{N2}_L}{\partial q^2}
\end{pmatrix}.
\]

Therefore, the optimizing condition for the profit of the firm is

\[
\begin{pmatrix}
-\frac{c_q f(q)}{\partial q} & 0 & -q c_q g_2 f(q) \\
0 & \frac{\partial^2 \Pi^{N2}_L}{\partial q^2} & 0 \\
-q c_q g_2 & 0 & \frac{\partial^2 \Pi^{N2}_L}{\partial q^2}
\end{pmatrix} > 0,
\]

Equations (A.16) are always satisfied. Therefore, the equilibrium solution at the maximum value point exists and is characterized by \( \partial \Pi^{N2}_L / \partial q = 0 \), \( \partial \Pi^{N2}_L / \partial p = 0 \), and \( \Pi^{N2}_L / \partial q = 0 \) [(9), (10), and (11)].

**Proof of Lemma 4.** Let \( A = (1 - \delta r)/(v_H - \delta v_L) \) \( p_H \) and \( B = p_L / v_L \), from (5) and (6). We then obtain

\[
g(A) \quad \left( A - B v_L (1 - \delta r) \right) = 1,
\]

The left side of (A.18) decreases with \( A \), and the right side of (A.18) increases with \( B \). Hence, \( \partial A / \partial B < 0 \); that is, \( A \) decreases with \( B \). Based on (A.17), when \( r \) increases, \( A \) will increase and \( B \) will decrease. Moreover, from \( A = (1 - \delta r)/(v_H - \delta v_L) \) \( p_H \), when \( A \) and \( r \) increase, \( p_H \) rises. From \( B = p_L / v_L \), when \( B \) decreases, \( p_L \) reduces.

Therefore, when \( r \) increases, many customers will purchase in period \( H \) at a high price. The firm will obtain increased profit.

**Proof of Theorem 5.** Theorem 5 can be proved similarly as proof of Theorems 2 and 3 to prove this result. We omit the details.

**Proof of Lemma 6.** When \( r = v_L / v_H \), we have \( \Theta_{1r} = \Theta_{1L} = 1 - G(p_H / v_H) \) and \( \Theta_{1C} = 1 - G((p_H - \delta p_L)/(v_H - \delta v_L)) \). Hence \( \Theta_{1r} \geq \Theta_{1C} \) and \( \Theta_{1r} \leq \Theta_{1C} \) for any \( p_H \) and \( p_L \). Therefore, we obtain \( \Pi^{\star N2} \geq \Pi^{N2} \).

When \( r \to 0 \), \( \Theta_{1r} \to 1 - G(p_L / (v_H - \delta v_L)) \) and \( \Theta_{1C} = 1 - G((p_H - \delta p_L) / (v_H - \delta v_L)) \). Hence \( \Theta_{1r} \leq \Theta_{1C} \) and \( \Theta_{1r} \geq \Theta_{1C} \) for any \( p_H \) and \( p_L \). Therefore, we obtain \( \Pi^{\star N2} \leq \Pi^{N2} \).

Based on Lemma 4, we know that the optimal profit under Strategy N2 (\( \Pi^{N2} \)) is strictly increasing in \( r \). Moreover, the
optimal profit under Strategy $C_2$ ($\Pi^{C_2*}$) is not affected by $r$. As a result, $\bar{r} \in (0, \frac{v_L}{v_H})$ exists, such that $\Pi^{C_2*} < \Pi^{P_2*}$ when $r < \bar{r}$ and $\Pi^{C_2*} > \Pi^{P_2*}$ when $r > \bar{r}$. \hfill $\square$

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors’ Contributions

Junfeng Dong, Li Jiang, and Yan Jiao proposed the idea and conceived the experiments; Junfeng Dong performed the experiments; Li Jiang and Yan Jiao analyzed the data; Junfeng Dong wrote the paper; all the three authors have read and approved the final manuscript.

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References


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