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Research Article

Calculation of Misjudgment Probability for Product Inspection Based on Measurement Uncertainty

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It is important to research into the misjudgment probability of product inspection based on measurement uncertainty, which is of great significance to improve the reliability of inspection results. This paper mainly focused on total inspection and sampling inspection methods and regarded the misjudgment probability as the index to provide quantitative misjudgment risk results for both producer and consumer sides. Through the absolute probability and the conditional probability model, the estimation formula of the total inspection misjudgment rate is deduced, respectively, and the calculation methods of qualification determination and misjudgment rate of the full inspection results are studied. According to the total inspection misjudgment rate, the methods of misjudgment rate of sampling inspection and qualification determination of measurement results are researched. The misjudgment rate of measurement results is calculated based on the exhaustive method and the Monte-Carlo simulation. The estimation results show that the misjudgment probabilities calculated by absolute probability models can be used as the basis for the selection of the measurement plan for product inspection. The misjudgment probability calculated by conditional probability models is more directly to reflect the risks for both producer and consumer sides, and it prompts inspectors to make decisions more carefully.

1. Introduction

Measurement uncertainty evaluation is a horizontal activity across all scientific and technological disciplines. It impacts on every area where measurement is employed. As an important parameter to characterize the quality of the measurement results, measurement uncertainty reflects the credibility of the product detection results [1, 2].

Whether the product is determined to be qualified or unqualified, there is a certain risk for product inspection results located near the tolerance limit influenced by the measurement uncertainty. The estimation of the misjudgment rate means to estimate the misjudgment risk that may occur in the inspection based on the measurement uncertainty and the relevant information of the product quality control before the production inspection, so as to determine whether the uncertainty of the selected measurement meets the requirements [3]. It is of great significance to improve the reliability of product inspection results. The ISO14253 standards in the new generation of Geometrical Product Specifications provide the guidance for product inspection based on measurement uncertainty. Part 1 of ISO14253 [4] gives the determination rules for qualified or unqualified product detection results. It is clear that the measurement uncertainty should be taken into account in the determination rules. Part 2 of ISO14253 [5] gives the general guidance on the product measurement uncertainty assessment. Part 3 of ISO14253 [6] gives some consistent guides on the definition of measurement uncertainty in the measurement inspection.

Desimoni and Brunetti [7] studied the US ASME standards, ISO standards, and the European Eurachem/CITAC standards, pointing out that although the qualification

determination in the reasonable consideration of the impact of measurement uncertainty had become an international consensus, it still lacked clear guidance. It was necessary to publish a set of "unified, logical, and widely accepted" guidance rules for product qualification determination, which would greatly facilitate the development of product testing. Pendrill [8, 9] pointed out that the conformity of the product should be determined quantitatively based on measurement uncertainty. The selection of measurement uncertainty should be guided by establishing a cost model which included the test cost and user risk cost. Koshulyan and Malaychuk [10] proposed an approximate formula for calculating the product acceptance limit to balance the risk of product supplier and demanders caused by the measurement uncertainty. Phillips and Krystek [11] summarized the product qualification determination standards, suggesting that the economic loss of the product supplier and the receiver should be fully balanced by the product qualification determination. They also analyzed the misjudgment risk caused by measurement uncertainty for selecting the product receiving area reasonably. Forbes [12] used the Bayesian Decision Method to establish the loss function and quantify the cost of wrong decisions in product inspection, deducing the optimal decision criterion to minimize the economic loss during product inspection. Djapic et al. [13] suggested that the risk management in the product test should be standardized that the uncertainty of the product test should be determined by comprehensively using the statistical and nonstatistical techniques combined with the Bayesian Method. Hinrichs [14] proposed that the measurement method should be carefully selected based on the measurement ability and pointed out that the measurement uncertainty should be made clear at the same time when the measurement result was given, along with a detailed traceability description of the measurement method. It is known from previous researches that misjudgment risk of product inspection caused by measurement uncertainty has been evolved from qualitative analysis [9, 15] to quantitative estimation and controlled reliably based on the result of the misjudgment probability. It will be the future research subjects for product quality engineering.

This paper focuses on the qualification determination method of product inspection based on the measurement uncertainty, quantitatively estimating the misjudgment risk caused by the measurement uncertainty to the total inspection and the sampling inspection. It provides evidences to reasonably optimize the allocation of measurement resources by deducing the estimation formula of the misjudgment probability test and the calculation formula of the misjudgment probability of the detection results, ensuring the reliability of product detection results at the same time.

2. Misjudgment Risk Analysis for Product Inspection

According to ISO14253-1 [4], the tolerance limit of the dimension of product is given in the form of a bilateral tolerance limit $[T_L, T_U]$ among which T_L is lower limit of the tolerance and T_U is the upper limit. For the measurement result x, when $x \in [T_L, T_U]$, the product is determined to be qualified,

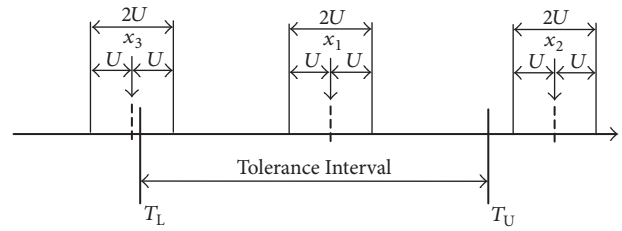

Figure 1: Relationship between product qualification and measurement uncertainty.

and when $x > T_U$ or $x < T_L$, the product is unqualified. The ISO14253 standards [4–6] state that the influence of measurement uncertainty must be considered in the product inspection. The product inspection which is carried out only based on the tolerance limits may result in misjudgment of the dimensions or geometrical tolerances of the workpiece. Figure 1 shows the way product qualification is determined when measurement uncertainty is considered.

In Figure 1, U means the extended uncertainty of the measurement results, and $2U$ is the distribution interval of the measurement results. When $2U$ is located in the tolerance interval (x_1 in Figure 1), it can be directly determined as qualified. When $2U$ is located outside the tolerance interval (x_2 in Figure 1), it can be directly determined as unqualified. When $2U$ is located inside the tolerance interval (x_3 in Figure 1), it cannot be determined as qualified nor qualified. Research on the misjudgment risk of product inspection is to quantify the misjudgment risk that may exist in the results of product qualification determination by calculating the misjudgment probability.

The product qualified intervals can be redivided based on the measurement uncertainty. The qualified intervals are shown in Figure 2. Affected by the measurement uncertainty, the absolute qualified interval of product will be reduced and an uncertainty area can be found near the product tolerance limit. The quantitative analysis of the misjudgment risk caused by the measurement uncertainty is of great significance to improve the reliability of product inspection results.

The role of the misjudgment risk assessment of the product inspection based on the measurement uncertainty is reflected in two aspects.

(1) Before the product inspection, the calculated results based on the misjudgment probability provide a holistic guidance on the product inspection to adjust the allocation of resources in the inspection to improve the efficiency.

(2) After the product inspection, the misjudgment probability of the qualification determination of the single workpiece is calculated to grade its qualification, so as to reasonably approve or reject the workpiece, ensuring the reliability of the workpiece inspection result.

3. Misjudgment Risk Assessment of Total Inspection

There are mainly two types of misjudgment risk caused by the product inspection: producer's risk and consumer's risk.

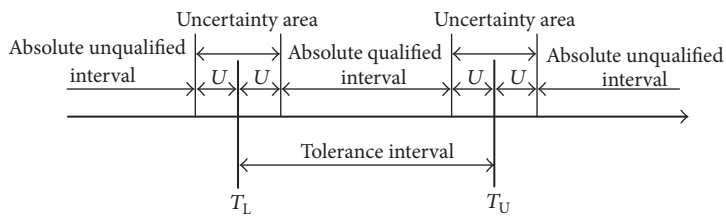

Figure 2: Qualified intervals for product inspection.

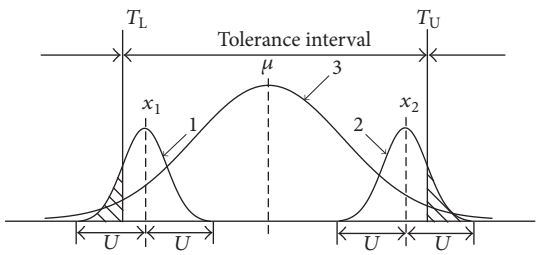

Figure 3: Consumer's risk in total inspection.

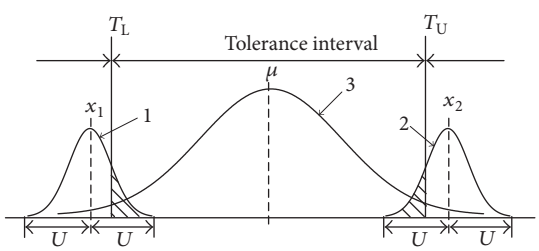

Figure 4: Producer's risk in total inspection.

As shown in Figures 3 and 4, the misjudgment probability is equal to the area of the shaded area in the figure.

For the whole batch of products, when the production process is statistically controlled, the product processing results are generally affected by the random factors and the best estimated result x is a random variable subject to the normal distribution. The probability density function $f(x)$ is as shown in (1). The distribution is shown as Curve 3 in Figures 3 and 4:

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right], \tag{1}$$

where μ is the central value of product quality control and σ is the variation of the product quality characteristic of the batch of products.

For a single workpiece, if the best measured result is x and U is the extended uncertainty, then the true value y is a random variable within the interval $x \pm U$. The distributions are shown as Curve 1 and Curve 2 in Figures 3 and 4. The residual system error and random error of the measuring instrument are regarded as random factors. Based on the central limit theorem, the probability density function $p(y \mid x)$ is expressed in the form of the normal distribution as

$$p(y \mid x) = \frac{1}{\sqrt{2\pi}u_c} \exp\left[-\frac{(y-x)^2}{2u_c^2}\right], \tag{2}$$

where u_c is the combined standard uncertainty.

The modern product quality control theory approximately stipulates that the distribution probability of the normal distribution in the interval $\mu \pm 6\sigma$ is equal to 1. Therefore, x_1 and x_2 in Figure 3 may misjudge the unqualified workpieces as qualified and wrongly approve them, with a misjudgment probability P_{rec}, shown as the shaded area in Figure 3. Also, x_1 and x_2 in Figure 4 may misjudge the qualified workpieces as unqualified and wrongly reject them. The misjudgment probability P_{rej} is shown as the shaded area in Figure 4

$$\begin{aligned}
P_{rec} &= P(T_L \leqslant x < T_L + U)\, P(y < T_L) \\
&\quad + P(T_U - U < x \leqslant T_U)\, P(y > T_U) \\
&= \int_{T_L}^{T_L+U} \left[\int_{x-6u_c}^{T_L} p(y \mid x)\, dy\right] f(x)\, dx \\
&\quad + \int_{T_U-U}^{T_U} \left[\int_{T_U}^{x+6u_c} p(y \mid x)\, dy\right] f(x)\, dx, \\
P_{rej} &= P(T_L - U < x < T_L)\, P(T_L \leqslant y \leqslant T_U) \\
&\quad + P(T_U < x < T_U + U)\, P(T_L \leqslant y \leqslant T_U) \\
&= \int_{T_L-U}^{T_L} \left[\int_{T_L}^{T_U} p(y \mid x)\, dx\right] f(x)\, dx \\
&\quad + \int_{T_U}^{T_U+U} \left[\int_{T_L}^{T_U} p(y \mid x)\, dx\right] f(x)\, dx.
\end{aligned} \tag{3}$$

There is no original function for the integrand in (3). To put it in the language of MATLAB for numerical integration, they can be shown in

$$P_{rec} = \frac{1}{2\pi u_c \sigma} \left\{\int_{T_L}^{T_L+U} \left[\int_{x-6u_c}^{T_L} \exp\left[-\frac{(y-x)^2}{2u_c^2}\right] dy\right] \cdot \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right] dx\right.$$

$$+\int_{T_U-U}^{T_U}\left[\int_{T_U}^{x+6u_c}\exp\left[-\frac{(y-x)^2}{2u_c^2}\right]\mathrm{d}y\right]$$
$$\cdot\exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]\mathrm{d}x\Bigg\},$$
$$P_{\mathrm{rej}}=\frac{1}{2\pi u_c\sigma}\Bigg\{\int_{T_L-U}^{T_L}\left[\int_{T_L}^{T_U}\exp\left[-\frac{(y-x)^2}{2u_c^2}\right]\mathrm{d}y\right]$$
$$\cdot\exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]\mathrm{d}x$$
$$+\int_{T_U}^{T_U+U}\left[\int_{T_L}^{T_U}\exp\left[-\frac{(y-x)^2}{2u_c^2}\right]\mathrm{d}y\right]$$
$$\cdot\exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]\mathrm{d}x\Bigg\}. \tag{4}$$

The misjudgment probability shown in (4) in the form of absolute probability refers to the average probability of misjudgment. The misjudgment probability should be minimized by controlling the measurement uncertainty.

The consumer's risk caused by false approval based on the conditional probability model is

$$P_{\mathrm{Crec}}=\frac{P_{\mathrm{rec}}}{P\left(T_L\leqslant x\leqslant T_U\right)}=\frac{P_{\mathrm{rec}}}{\left(1/\sqrt{2\pi}\sigma\right)\int_{T_L}^{T_U}\exp\left[-(x-\mu)^2/2\sigma^2\right]\mathrm{d}x}. \tag{5}$$

The producer's risk caused by false rejection based on the conditional probability model is

$$P_{\mathrm{Crej}}=\frac{P_{\mathrm{rej}}}{1-P\left(T_L\leqslant x\leqslant T_U\right)}=\frac{P_{\mathrm{rej}}}{1-\left(1/\sqrt{2\pi}\sigma\right)\int_{T_L}^{T_U}\exp\left[-(x-\mu)^2/2\sigma^2\right]\mathrm{d}x}. \tag{6}$$

Equations (5) and (6) show the misjudgment probabilities calculated by the conditional probability models, which indicate the average levels of misjudgment when the qualifications of the workpieces are known, representing the possibility of the qualification misjudgment based on the tolerance limit. During the product inspection, first make sure that P_{rec} and P_{rej} are relatively low, that is, to ensure that measurement uncertainty remains reasonable and reliable. Then calculate P_{Crec} and P_{Crej} to understand the average level of qualified or unqualified misjudgment probability so as to guide the approval or rejection of the products.

4. Misjudgment Risk Assessment of Sampling Inspection

The sampling inspection has the advantages of low cost and high efficiency, which is widely used in the inspection of large quantities of products. The determined object of the sampling inspection is the product batch, and test method can be shown with the three parameters including the number of samples n, the number of products of approved Ac, and the number of products of rejected Re. The information of $(n, \mathrm{Ac}, \mathrm{Re})$ can be obtained by retrieving the ISO standard [16].

Assume that the number of unqualified workpieces in the samples is d. When $d \leqslant \mathrm{Ac}$, the whole batch is accepted as qualified; when $d \geqslant \mathrm{Re}$, the whole batch is rejected as unqualified; when $\mathrm{Ac} < d < \mathrm{Re}$, sampling should be continued until the batch qualification can be determined. The measurement uncertainty will affect the qualification determination of individual workpiece in the samples, resulting in the wrong estimation of the number of unqualified products, thus affecting the qualification determination of the whole batch.

Assuming that the number of misjudged products in the unqualified products is i and that in the qualified products is j, then the actual number of unqualified products in the samples is d_z:

$$d_z = d - i + j. \tag{7}$$

Assume that the probability that the workpiece is judged to be qualified is P_1 and the probability that the workpiece is judged to be unqualified is P_2:

$$P_1 = P\left(T_L \leqslant x \leqslant T_U\right),$$
$$P_2 = P\left(x > T_U\right) + P\left(x < T_L\right). \tag{8}$$

Assume that the probability of the random event that "there are d unqualified products in n samples, and i misjudged workpieces in these unqualified products and j misjudged workpieces in the qualified products" is P_{dij}:

$$P_{dij} = C_n^d P_2^d P_1^{n-d} C_d^i P_{\mathrm{Crej}}^i \left(1-P_{\mathrm{Crej}}\right)^{d-i} \cdot C_{n-d}^j P_{\mathrm{Crec}}^j \left(1-P_{\mathrm{Crec}}\right)^{n-d-j}. \tag{9}$$

In the qualification determination of the sampling test qualification determination, the condition that misjudges the unqualified batch as qualified is

$$d \leqslant \mathrm{Ac},$$
$$d_z \geqslant \mathrm{Re}. \tag{10}$$

The ranges of values d, i, and j can be drawn as

$$0 \leqslant d \leqslant \mathrm{Ac},$$
$$0 \leqslant i \leqslant \min\left(d, n-\mathrm{Re}\right), \tag{11}$$
$$\mathrm{Re} + i - d \leqslant j \leqslant n - d.$$

The probability of an unqualified batch being falsely approved can be calculated through cumulative method:

$$P_{\text{S-rec}} = \sum_{d=0}^{\text{Ac}} \sum_{i=0}^{\min(d,n-\text{Re})} \sum_{j=\text{Re}+i-d}^{n-d} P_{dij}. \tag{12}$$

In the qualification determination of the sampling test, the condition that could lead to the misjudgment of the qualified batch as unqualified is

$$\begin{aligned} d &\geqslant \text{Re}, \\ d_z &\leqslant \text{Ac}. \end{aligned} \tag{13}$$

The ranges of values d, i, and j that can be drawn are

$$\begin{aligned} \text{Re} \leqslant d &\leqslant n, \\ 0 \leqslant j &\leqslant \min(n-d, \text{Ac}), \\ d + j - \text{Ac} \leqslant i &\leqslant d. \end{aligned} \tag{14}$$

The probability of a qualified batch being falsely rejected can be calculated through the cumulative method:

$$P_{\text{S-rej}} = \sum_{d=\text{Re}}^{n} \sum_{j=0}^{\min(n-d,\text{Ac})} \sum_{i=d+j-\text{Ac}}^{d} P_{dij}. \tag{15}$$

Formulas (12) and (15) are misjudgment probabilities in the form of absolute probability models.

Assume that the probability that the workpiece is judged to be qualified is P_3 and the probability that the workpiece is judged to be unqualified is P_4:

$$\begin{aligned} P_3 &= \sum_{d=0}^{\text{Ac}} C_n^d P_2^d P_1^{n-d}, \\ P_4 &= \sum_{d=\text{Re}}^{n} C_n^d P_2^d P_1^{n-d}. \end{aligned} \tag{16}$$

Then the misjudgment probability of the qualified batch in the form of the conditional probability model is shown as (17) and the misjudgment probability of the unqualified batch is shown as (18):

$$P_{\text{S-Crec}} = \frac{P_{\text{S-rec}}}{P_3}, \tag{17}$$

$$P_{\text{S-Crej}} = \frac{P_{\text{S-rej}}}{P_4}. \tag{18}$$

5. Conformity Assessment of Total Inspection Results

The results of the misjudgment probability as described in Section 3 indicate the average misjudgment risk in the measurement results of the total inspection. As each workpiece has a different measuring result in the actual measurement, the corresponding misjudgment probabilities are also different.

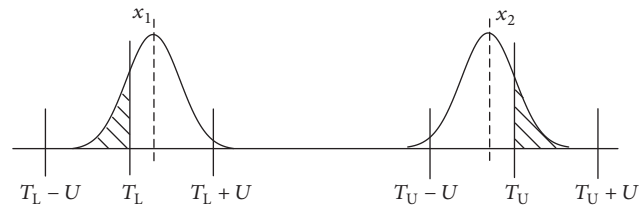

Figure 5: The false acceptance probability when the measurement result is near tolerance.

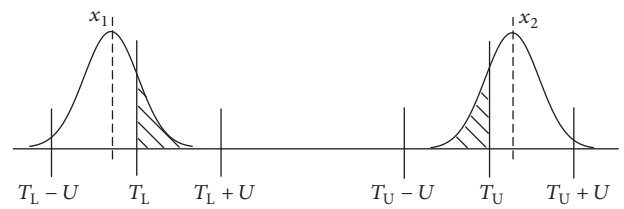

Figure 6: The false rejection probability when the measurement result is near tolerance.

According to ISO standard [4], the misjudgment risk of the estimated value in the absolute qualified area or absolute unqualified area is negligible. In Figure 5, there is a risk of false approval; when $T_L \leqslant x_1 < T_L + U$, the misjudgment probability is shown as the shaded area on the left in Figure 5:

$$P_{\text{Drec}} = \frac{1}{\sqrt{2\pi}u_c} \int_{x-6u_c}^{T_L} \exp\left[-\frac{(y-x)^2}{2u_c^2}\right] \mathrm{d}y. \tag{19}$$

There is a risk of false approval; when $T_U - U < x_2 \leqslant T_U$, the misjudgment probability is shown as the shaded area on the right in Figure 5:

$$P_{\text{Drec}} = 1 - \frac{1}{\sqrt{2\pi}u_c} \int_{x-6u_c}^{T_U} \exp\left[-\frac{(y-x)^2}{2u_c^2}\right] \mathrm{d}y. \tag{20}$$

There is a risk of false rejection; when $T_L - U < x_1 < T_L$, the misjudgment probability is shown as the shaded area on the left in Figure 6:

$$P_{\text{Drej}} = 1 - \frac{1}{\sqrt{2\pi}u_c} \int_{x-6u_c}^{T_L} \exp\left[-\frac{(y-x)^2}{2u_c^2}\right] \mathrm{d}y. \tag{21}$$

There is a risk of false rejection; when $T_U < x_2 \leqslant T_U + U$, the misjudgment probability is shown as the shaded area on the right in Figure 6:

$$P_{\text{Drej}} = \frac{1}{\sqrt{2\pi}u_c} \int_{x-6u_c}^{T_U} \exp\left[-\frac{(y-x)^2}{2u_c^2}\right] \mathrm{d}y. \tag{22}$$

Assume that $F(y) = (1/\sqrt{2\pi}u_c) \int_{x-6u_c}^{y} \exp[-(y-x)^2/2u_c^2]\mathrm{d}y$. The conformity assessment of dimension measurement of products is shown in Table 1.

Table 1: Conformity assessment and misjudgment probability calculation for product inspection.

Interval where x is	Conformity assessment	Misjudgment risk	Misjudgment rate
$0 \leqslant x \leqslant T_L - U$	Unqualified	Negligible	0
$T_L - U < x < T_L$	Unqualified	False rejection	$1 - F(T_L)$
$T_L \leqslant x < T_L + U$	Qualified	False approval	$F(T_L)$
$T_L + U \leqslant x \leqslant T_U - U$	Qualified	Negligible	0
$T_U - U < x \leqslant T_U$	Qualified	False approval	$1 - F(T_U)$
$T_U < x < T_U + U$	Unqualified	False rejection	$F(T_U)$
$x \geqslant T_U + U$	Unqualified	Negligible	0

6. Conformity Assessment of Sampling Inspection Results

After the sampling inspection, the qualification of the whole batch can be determined according to the number of unqualified workpieces in the samples. The calculation results of the misjudgment probability based on the measurement uncertainty can be the supplement of the qualification determination results of the whole batch, qualifying the misjudgment risk of the batch qualification caused by the measurement uncertainty.

In the sampling inspection, the number of unqualified workpieces (d) in the samples is determined according to the sample measurement results and the tolerance limit. After considering the measurement uncertainty, the true number of unqualified workpieces in the samples is d_z. The relationship between d and d_z is shown in (7).

When $d \leqslant \mathrm{Ac}$, there is a risk of false approval and the probability is $P(d_z \geqslant \mathrm{Re})$. The probability of false approval of the sampling inspection results is

$$P_{\text{S-Drec}} = P\left(j - i \geqslant \mathrm{Re} - d\right). \tag{23}$$

When $d \geqslant \mathrm{Re}$, there is a risk of false rejection and the probability is $P(d_z \leqslant \mathrm{Ac})$. The probability of false rejection of the sampling inspection results is

$$P_{\text{S-Drej}} = P\left(i - j \geqslant d - \mathrm{Ac}\right). \tag{24}$$

When the sampling method and the specific measurement results are different, the corresponding expressions of (23) and (24) are different. In the actual test, the misjudgment probability can be calculated by the exhaustive method. The specific process is shown in Figure 7. P_m is the probability that m occurs in the exhaustive situation and M is the number of cases meeting the condition. When the number of the exhaustive cases in the actual test is small, the probability of each case can be calculated by the misjudgment probability of a single workpiece in the total inspection. The whole batch of products can be approved or rejected according to the results of the batch misjudgment probability.

7. Evaluation Methods of Measurement Uncertainty

In the product inspection, the key problem in calculating the misjudgment rate of the product qualification judgment is how to evaluate the synthesis standard uncertainty u_c of the measurement process. The uncertainty of the qualification judgment of the precision measurement products is assessed through the measurement system analysis method, which uses the value characteristics of the measurement data to reflect the influence of each error on the measurement results.

The uncertainty assessment based on the value characteristic index is as follows:

$$u_c = \sqrt{u_E^2 + u_{RT}^2 + u_{RD}^2 + u_S^2 + u_T^2}. \tag{25}$$

The components in (25) are described as follows.

u_E is the uncertainty component caused by linearity and offset, which is expressed by the maximum allowable indication error MPE_E provided by the instrument specification or the verification certificate. According to the uniform distribution, there are $u_E = MPE_E/\sqrt{3}$.

u_{RT} is the uncertainty component caused by repeatability, which needs to be evaluated by experiments. It should be calculated through the results of repeated measurements under the same measurement conditions, such as

$$u_{RT} = \sqrt{\frac{1}{n_1\left(n_1 - 1\right)} \sum_{i=1}^{n_1} \left(x_i - \overline{x}\right)^2}. \tag{26}$$

u_{RD} is the uncertainty component caused by reproducibility, which needs to be evaluated by experiments. It should be calculated according to the measurement results under different conditions:

$$u_{RD} = \sqrt{\frac{1}{\left(n_2 - 1\right)} \sum_{i=1}^{n_2} \left(x_i - \overline{x}\right)^2}. \tag{27}$$

u_S is the uncertainty component caused by stability, which can be obtained by checking the performance of the instrument.

u_T refers to the uncertainty component caused by the impacts of temperature and other factors on the measurement results.

Among them, x_i is the measured value, $\overline{x}$ is the measured mean value, n_1 refers to the number of repeatability assessments, and n_2 refers to the number of measurement results of the reproducibility assessment.

In general, strict quality control is carried out on dimensional machining process, so the distribution parameters and

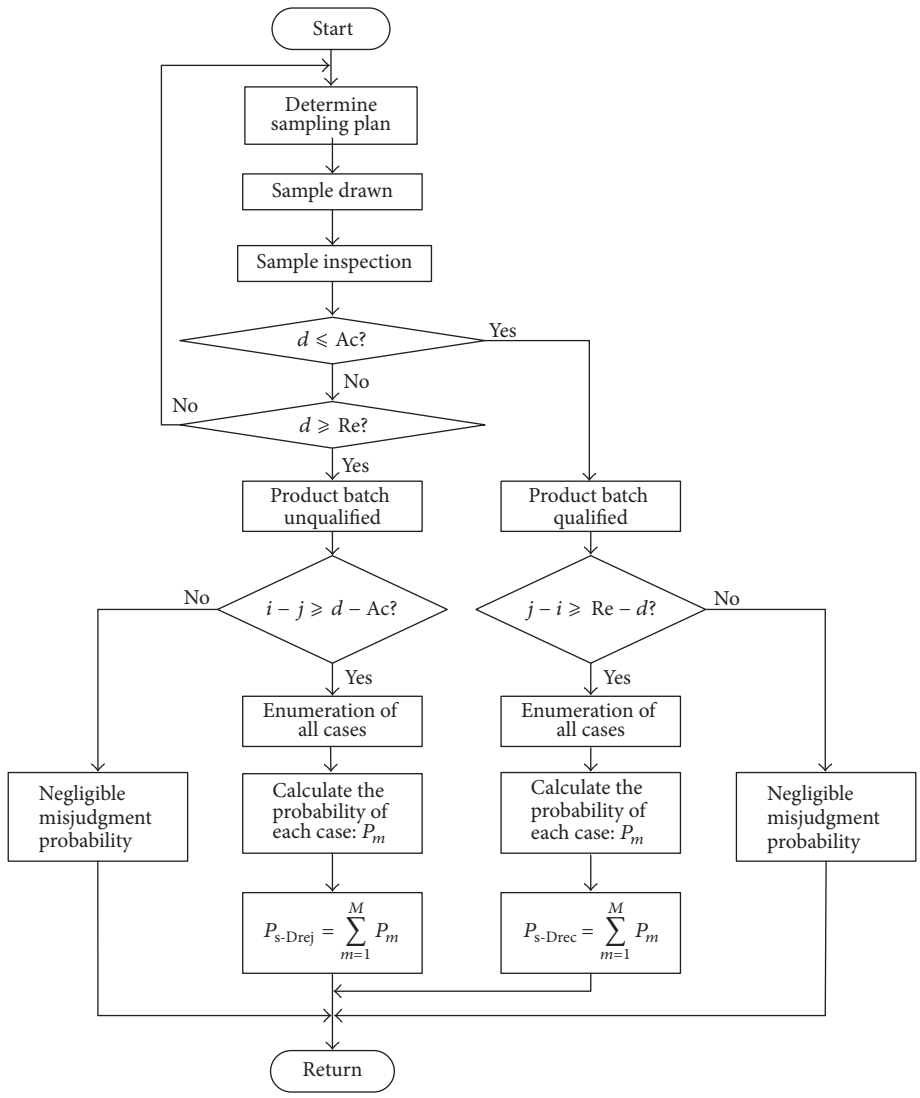

Figure 7: Conformity assessment and misjudgment probability calculation for sampling inspection.

measurement uncertainty of product batch can also be calculated by other Key Performance Indicators (KPIs). The KPIs include unbiased process capability index C_p, biased process capability index C_{pk}, and measurement process performance ratio Q. The calculation formulas of the KPIs are as follows:

$$C_p = \frac{T}{6\sigma},$$
$$C_{pk} = C_p\left(1 - \frac{2\left|M - \mu\right|}{T}\right), \tag{28}$$
$$Q = \frac{2U}{T} \times 100\% = \frac{4u_c}{T} \times 100\%.$$

Among them, M is the central value of the tolerance band. T is the tolerance width of product.

The values of σ and u_c are calculated by

$$\sigma = \frac{T}{6C_p},$$
$$u_c = \frac{QT}{4}. \tag{29}$$

The central value of product quality control is usually deviated from the center of the product tolerance zone for the convenience of processing waste recycling. For instance, the central value of product quality control is generally greater than the central value of product tolerance zone in the machining process of shaft parts, and then

$$\mu = M + \left(1 - \frac{C_{pk}}{C_p}\right) \cdot \frac{T}{2}. \tag{30}$$

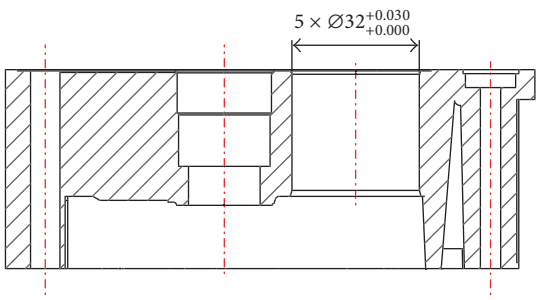

Figure 8: The drawing marking of the measured workpiece.

Table 2: The estimation of misjudgment probabilities for aperture measurement.

	Probability	Estimated results
P_{rec}	Misjudgment probability of false approval	0.34%
P_{rej}	Misjudgment probability of false rejection	0.05%
P_{Crec}	Misjudgment probability of qualified products	0.34%
P_{Crej}	Misjudgment probability of unqualified products	33.35%

In contrast, the central value of product quality control is usually less than the central value of product tolerance zone in the machining process of hole parts, and then

$$\mu = M - \left(1 - \frac{C_{pk}}{C_p}\right) \cdot \frac{T}{2}. \tag{31}$$

8. Results and Discussion

This experiment used the aperture of the workpiece measured by HEAXGON Micro-Hite 3D DCC CMM to demonstrate the validity of the theory mentioned. At present, the uncertainty of CMM measurement task is mainly evaluated according to the relevant standards [17–19] of GPS. The uncertainty of the CMM specific task can also be evaluated according to Section 7 and related research results [20–25].

The drawing and specification of the workpiece measured are shown in Figure 8. The estimation of the aperture measurement uncertainty is $u_c = 0.0022$ mm.

8.1. Examples of Misjudgment Probability Estimation for Total Inspection. According to the manufacturing department, the workpiece processing is statistically controlled. The central value of workpiece quality control is $\mu = 32.0113$ mm, while the batch variation is $\sigma = 0.0038$ mm. When the product tolerance and the batch distribution parameters are determined, the estimated misjudgment probability is only determined by the measurement uncertainty. The relationship between u_c and the estimated misjudgment probability is shown in Figure 9. The misjudgment probability of the measured workpiece when $u_c = 0.0022$ mm is shown in Table 2.

As can be seen from Table 2, the values of P_{rec} and P_{rej} are lower, which indicate that the CMM inspection capability meets the requirement. The value of P_{Crec} is also lower, which indicates that the qualified products can be approved safely. The value of P_{Crej} is higher, which indicates that the manufacturing capacity is strong and the failure probability is low. If there are unqualified products during the inspection, they should be carefully rejected.

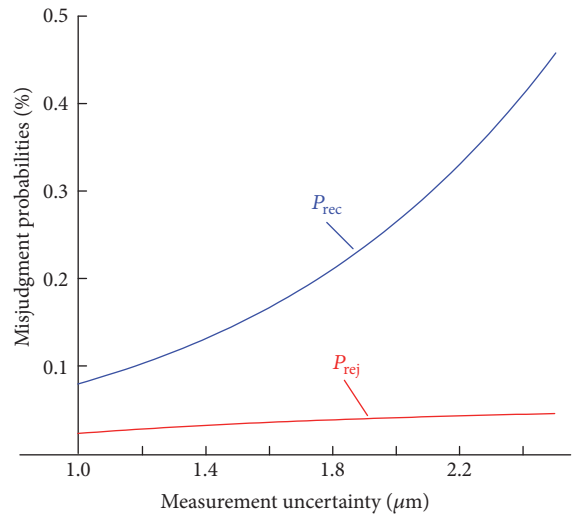

Figure 9: The relationship between the misjudgment probabilities and measurement uncertainty.

8.2. Examples of Misjudgment Probability Estimation for Sampling Inspection. This experiment assumes that the inspected number of each batch is $N = 280$ and the selected inspected level is II [16]. Assume that the workpieces will be provided to customer A and customer B. Customer A requires $AQL = 0.4$ while customer B requires $AQL = 1.0$, where AQL indicates the maximum permissible unqualified probability in a qualified batch.

The sampling plan is determined based on the ISO2859-1. For customer A, $n = 32$, Ac $= 0$, and Re $= 1$; for customer B, $n = 32$, Ac $= 1$, and Re $= 2$. Combined with the results in Section 8.1, the estimated qualified probability and the misjudgment probability of the product batch are shown in Table 3.

It can be seen from Table 3 that the qualified probability P_3 of customer A's product batch is less than that of customer B under the same manufacturing conditions because the value of AQL proposed by customer A is lower. The unqualified probability P_4 of customer A's product batch is larger than that of customer B. It is necessary for the producer to improve the manufacturing capacity of the workpieces when they supply products to customer A.

The false approval probability $P_{S\text{-rec}}$ and false rejection probability $P_{S\text{-rej}}$ of the whole batch of customer A are greater than those of customer B, indicating that the impact of the measurement uncertainty is significantly enlarged under stringent quality requirements. CMM with a higher accuracy should be used when the same batch of products are provided to customer A.

Table 3: The estimation of the qualified probability and misjudgment probabilities for product batch.

	Probability	Calculation results	
		Consumer A	Consumer B
P_3	Qualified probability of the batch	95.77%	99.91%
P_4	Unqualified probability of the batch	4.23%	0.09%
$P_{\text{S-rec}}$	Misjudgment rate of false approval	9.84%	0.79%
$P_{\text{S-rej}}$	Misjudgment rate of false rejection	1.25%	0.04%
$P_{\text{S-Crec}}$	Misjudgment rate of qualified products	10.27%	0.79%
$P_{\text{S-Crej}}$	Misjudgment rate of unqualified products	29.60%	50.85%

Table 4: The calculation formulas of the misjudgment probabilities of total inspection.

Measurement results (mm)	Qualification	Misjudgment risk	Misjudgment rate
$x \leqslant 31.9956$	Unqualified	Negligible	0
$31.9956 < x < 32$	Unqualified	False rejection	$1 - F(32)$
$32 \leqslant x < 32.0044$	Qualified	False approval	$F(32)$
$32.0044 \leqslant x \leqslant 32.0256$	Qualified	Negligible	0
$32.0256 < x \leqslant 32.03$	Qualified	False approval	$1 - F(32.03)$
$32.03 < x < 32.0344$	Unqualified	False rejection	$F(32.03)$
$x \geqslant 32.0344$	Unqualified	Negligible	0

If the misjudgment probability $P_{\text{S-Crej}}$ of the unqualified batch of customer B is larger than that of customer A, it shows that the value of AQL is high. When there are unqualified batch during the inspection, it would be considered whether the test result is affected by the measurement uncertainty, and the products should be carefully rejected.

8.3. Examples of Conformity Assessment for Total Inspection. Measure the workpiece shown in Figure 8 and perform the conformity assessment of total inspection results based on the measurement results. The calculation equation of the misjudgment probability which can be obtained from Table 1 is shown in Table 4.

In Table 4, $F(y)$ is the normal distribution of y which takes the best estimated value x of the measurement as the mean and the measurement uncertainty $u_c = 0.0022$ mm as the standard deviation. The distribution function is

$$F(y) = \frac{1}{\sqrt{2\pi}u_c} \int_{x-6u_c}^{y} \exp\left[-\frac{(y-x)^2}{2u_c^2}\right] \mathrm{d}y. \quad (32)$$

In the experiment, we have measured on five workpieces, and the results of conformity assessment and the misjudgment probabilities based on the measurement results are shown in Table 5.

As can be seen from Table 5, number 2 workpiece and number 5 workpiece can be directly determined as qualified, and the misjudgment risk caused by the measurement uncertainty can be ignored. For number 3 workpiece, if it is judged to be unqualified, the producer will bear 10.16% of the risk. For number 1 workpiece and number 4 workpiece, if they are judged as qualified, the customer will bear 19.39% and 6.68% of the misjudgment risk. If the customer attaches great importance to the quality of the workpieces, the customer should make a decision to reject number 1 workpiece and number 4 workpiece. At this point, in order to reduce the manufacturing cost, the producer can negotiate with the customer on the acceptable misjudgment probability Φ of products. When $\Phi = 10\%$, number 4 workpiece will be approved as a qualified product. It can be seen that the assessment results of the misjudgment probability provide a quantitative basis for the negotiation about the quality of products.

Table 5: The conformity assessment results for single product.

Number	Measurement results (mm)	Qualification	Misjudgment rate
1	32.0019	Qualified	19.39%
2	32.0147	Qualified	0
3	31.9972	Unqualified	10.16%
4	32.0033	Qualified	6.68%
5	32.0116	Qualified	0

8.4. Examples of Conformity Assessment for Sampling Inspection. The sampling inspection is carried out based on the conditions proposed by customer B. The sampling plan is $n = 32$, Ac = 1, and Re = 2. After the one-by-one inspection according to Table 4, there are 31 qualified products and 1 unqualified product in the samples. Among them, 2 of the 31 qualified products may be misjudged and the unqualified product may also be misjudged. The inspection results are shown in Table 6.

When the number of unqualified products in this experiment is $d = 1 \leqslant \text{Ac}$, the batch can be determined as qualified. However, since the maximum number of misjudged items in the qualified products can be $j = 2$, the maximum number of misjudged items in the unqualified products can be

Table 6: The measurement results and misjudgment probabilities of the products with misjudgment risks.

Number	Measurement results (mm)	Qualification	Misjudgment rate
a	32.0037	Qualified	$P_a = 4.63\%$
b	32.0026	Qualified	$P_b = 11.86\%$
c	31.9983	Unqualified	$P_c = 21.98\%$

Table 7: The simulation results of sampling inspection by MCM.

Number of actually unqualified products d_z	Frequency of occurrence
0	184223
1	689971
2	121560
3	4246

$i = 1$, which may meet the conditions of (23), so there is a misjudgment risk to determine the batch as qualified. List all possible situations satisfying (23) by using the exhaustive method:

① a is misjudged, and b and c are correct;

② b is misjudged, and a and c are correct;

③ a and b are misjudged, and c is correct;

④ a, b, and c are all misjudged.

The misjudgment probability to determine the batch as qualified is calculated as

$$\begin{aligned} P_{\text{S-Drec}} &= P\left(j - i \geqslant \text{Re} - d\right) \\ &= P_a\left(1 - P_b\right)\left(1 - P_c\right) + P_b\left(1 - P_a\right)\left(1 - P_c\right) \\ &\quad + P_a P_b\left(1 - P_c\right) + P_a P_b P_c = 12.56\%. \end{aligned} \tag{33}$$

The results of (33) can be verified by using Monte-Carlo simulation. The steps of Monte-Carlo simulation are as follows:

① Generate the uncertainty distribution of the workpieces a, b, and c based on the normal distribution and measured true value is the random numbers within the uncertainty interval.

② Randomly select the simulated true values of the workpieces a, b, and c, and conduct the conformity assessment. Record the number of true unqualified items in the statistical simulation as d_z.

③ Repeat Step ② to conduct the large sample simulation for η times (in this case: $\eta = 1 \times 10^6$).

④ Record the number of true unqualified items in η simulation as d_z.

The simulation results of Monte-Carlo simulation are shown in Table 7.

Based on conformity assessment conditions, when $d_z \leqslant 1$, the batch is qualified; when $d_z \geqslant 2$, the batch is unqualified. If it is determined to be qualified, it is misjudged. According to Table 7, the misjudgment probability of the experiment is

$$\begin{aligned} P_{\text{S-Drec}} &= \frac{121560 + 4246}{184223 + 689971 + 121560 + 4246} \\ &= 12.58\%. \end{aligned} \tag{34}$$

The results of (33) and (34) are basically the same, which verifies the effectiveness of the calculation method of the misjudgment probability proposed in this paper.

Under the measurement uncertainty, there is a 12.56% chance that the qualified batch of the workpiece has been misjudged. The producer and customer can choose to approve or reject the batch of the products flexibly in line with the misjudgment probability they can bear. The measurement uncertainty can be reduced reasonably by using other measurement strategies such as alternative measurement or compensation measurement for the reinspection, preserving the inspection results of samples against the uncertainty area.

9. Conclusions

The present research has carried out the conformity assessment of product inspection and the calculation of misjudgment probability based on measurement uncertainty. It has solved the misjudgment risk assessment of the product inspection based on measurement uncertainty.

(1) The misjudgment rate of the sampling inspection has been estimated based on the misjudgment rate of the full inspection, and the conformity assessment method of the sampling inspection results has been discussed. The effectiveness of the proposed method has been verified by the experimental studies based on the exhaustive method and Monte-Carlo method.

(2) Combined with experimental results, the relevant suggestions for product inspection can be provided. The measurement resources will be allocated reasonably by estimating the misjudgment risk. The reliability of the product inspection results will be ensured by calculating the conformity assessment and the misjudgment rate based on measurement uncertainty.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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