Research Article

A Revised Model of the Neutral DEA Model and Its Extension

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The neutral data envelopment analysis (DEA) model is an alternative way to determine the weights in DEA cross-efficiency evaluation, while avoiding the difficulty in making a choice between the aggressive and benevolent formulations. However, the weights determined by the neutral model merely make the efficiency of part output bigger than other sets of weights. The neutral model is not able to make the efficiency of each output of the DMU biggest among the favorable weights. This neutral model is not purely “neutral” and not most favorable to the DMU. We proposed a revised model for the neutral model. Based on the idea that the DMU should choose a set of weights to maximize its own efficiency, this paper proposes a new cross-efficiency model. The weights determined by the two models are neutral, neither aggressive nor benevolent.

1. Introduction

Data envelopment analysis proposed by Charnes et al. [1] is a nonparametric method to identify the production frontier and to measure the efficiency of homogenous decision-making units (DMUs) with multiple inputs and multiple outputs. DEA traditional model (CCR and BCC) measures the relative efficiency of DMU relative to its own subpoint on the production frontier. As long as the DMU is on the production frontier, it is the DEA efficient point and the CCR value is equal to 1. Since there is often more than one DMU on the production frontier, more than one DMU is often evaluated as DEA efficient and cannot be discriminated any further. Lacking of the discrimination power becomes the major drawback of the traditional DEA model.

To solve this problem, some scholars have extended the traditional DEA model and proposed some new approaches to improve the discrimination power of traditional DEA model. One such method is DEA super efficiency model [2] that will lead to the efficiency value of efficient DMUs to be larger than 1, which will further distinguish the efficient DMUs. Another method is DEA common-weight evaluation [3]. Unlike the traditional DEA model in which each DMU uses its own favorable weights to measure the efficiency, the DEA common-weight evaluation adopts the common set of weights to measure the efficiency of all DMUs. There are still other methods, such as the multivariate statistics ranking

technique [4], the benchmark ranking method [5], the DEA multicriteria decision-making method [6], and the context-dependent DEA technique [7].

DEA cross-efficiency evaluation as an extension to DEA is a commonly used method to improve the discriminative power of the traditional DEA model. The cross-efficiency evaluation utilizes self-evaluation and peer-evaluation to evaluate the efficiency of each DMU. The concept of cross-efficiency evaluation was first proposed by Sexton et al. [8] and was later developed by Doyle and Green [9, 10]. In the cross-efficiency evaluation, each DMU determines a set of weights among all the favorable weights based on some ways. That will lead to n sets of weights. N sets of weights will lead to n efficiency values for each DMU. The final average efficiency value for each DMU will be computed by assembling the n efficiency values. The rank of DMUs is determined by the final average efficiency value of DMU. It is believed that the overall efficiency value calculated by cross-efficiency evaluation can guarantee a unique ordering for the DMUs and can be used with few DMUs (e.g., four or five) to produce a unique ranking ordering [10].

Because of the powerful discrimination among DMUs, the cross-efficiency evaluation has been widely applied in efficiency evaluation of nursing homes [8], project ranking and preference voting [11], efficiency evaluation of countries in the Olympic Games (Wu, Liang, and Chen 2009), fixed cost and resource allocation [12], portfolio selection in the Korean

stock market [13], supplier selection under uncertainty [14], and so on. However, the nonunique solution to the traditional DEA model (CCR) limits the application of DEA cross-efficiency. How to solve this problem is the focus of the research of DEA cross-efficiency. Using the secondary goal models is the main method to solve this question. Up to now, there are many secondary goal models proposed by many scholars such as aggressive and benevolent models proposed by Doyle and Green [9], the neutral model by Wang (2010), the weight-balanced model by Wu et al. [15], the maximin model by Lim [16], and the satisfaction degree model by Wu et al. [17]. The aggressive and benevolent models proposed by Doyle and Green [9] are two classical ways among the many models. The aggressive (benevolent) model minimizes (maximizes) the average efficiency of other DMUs. The neutral DEA cross-efficiency model suggested by Ying-Ming Wang (2010) offers the new idea to solve this question. Wang considered that the particular DMU should choose a set of weights which is most favorable to itself not considering whether it is favorable or not to other DMUs. The neutral model maximinimizes the efficiency of each output.

To avoid the difficulty in making a choice between the aggressive and benevolent formulations in DEA cross-efficiency evaluation, Wang and Chin [18] put forward a neutral DEA model for the cross-efficiency evaluation, which maximinimizes the relative efficiency of each output. The neutral DEA model was proposed on the idea that “when a DMU is given an opportunity to decide a set of input and output weights, what the DMU is concerned most about is whether the weights can be as favorable as possible to itself.” However, the weights determined by the neutral DEA model can just make the efficiency of part outputs higher. That leads to the neutral DEA model proposed by Wang and Chin [18] being not very consistent with its modeling idea. To address this issue, we propose a revised model. We further propose a new model based on the idea that according to self-interest principle the DMU will select a set of input and output weights among the favorable weights to maximize its own efficiency.

The rest of the paper is organized as follows. Section 2 briefly introduces the cross-efficiency evaluation and its aggressive and benevolent formulations. Section 3 reviews the neutral DEA mode suggested by Wang and Chin [18] and proposes a revised model. Section 4 proposes a new model based on the self-interest principle. Section 5 makes a comparison among these above models through the numerical examples. Section 6 offers the conclusions.

2. The Cross-Efficiency Evaluation

Suppose there are n DMUs to be evaluated with m inputs and s outputs. Denote by x_{ij} ($i = 1, \dots, m$) and y_{rj} ($r = 1, \dots, s$) the input and output values of DMU _{j} ($j = 1, \dots, n$), based on the input and output ratio, whose efficiencies are defined as

$$\theta_j = \frac{\sum_{r=1}^s u_r y_{rj}}{\sum_{i=1}^m v_i x_{ij}}, \quad j = 1, \dots, n, \quad (1)$$

where v_i ($i = 1, \dots, m$) and u_r ($r = 1, \dots, s$) are, respectively, the input and output weights.

Consider a DMU, say, DMU _{k} , $k \in \{1, \dots, n\}$, whose DEA CCR-efficiency value can be computed by the following CCR model [1]:

$$\begin{aligned} \text{Maximize} \quad & \theta_{kk} = \frac{\sum_{r=1}^s u_{rk} y_{rk}}{\sum_{i=1}^m v_{ik} x_{ik}} \\ \text{Subject to} \quad & \theta_{ik} = \frac{\sum_{r=1}^s u_{rk} y_{rj}}{\sum_{i=1}^m v_{ik} x_{ij}} \leq 1, \quad j = 1, \dots, n \\ & u_{rk} \geq 0, \quad r = 1, \dots, s, \\ & v_{ik} \geq 0, \quad i = 1, \dots, m. \end{aligned} \quad (2)$$

When the value is equal to 1, it means that DMU _{k} is DEA efficient. The input and output weights determined by the model are favorable to DMU _{k} . Because the weights determined by CCR model are favorable to the DMU _{k} , the CCR value has been viewed as the self-evaluation.

By using Charnes and Cooper transformation (Charnes and Cooper, 1962), model (2) can be equivalently transformed into the linear program (LP) below for solution:

$$\begin{aligned} \text{Maximize} \quad & \theta_{kk} = \sum_{r=1}^s u_{rk} y_{rk} \\ \text{Subject to} \quad & \sum_{i=1}^m v_{ik} x_{ik} = 1, \\ & \sum_{r=1}^s u_{rk} y_{rj} - \sum_{i=1}^m v_{ik} x_{ij} \leq 0, \quad j = 1, \dots, n, \\ & u_{rk} \geq 0, \quad r = 1, \dots, s, \\ & v_{ik} \geq 0, \quad i = 1, \dots, m. \end{aligned} \quad (3)$$

Let u_{rk}^* ($r = 1, \dots, s$) and v_{ik}^* ($i = 1, \dots, m$) be the optimal solution to the above model. Then, $\theta_{kk}^* = \sum_{r=1}^s u_{rk}^* y_{rk}$ is referred to as the CCR-efficiency value of DMU _{k} , which is the self-evaluated efficiency of DMU _{k} . As such, $\theta_{jk} = \sum_{r=1}^s u_{rk}^* y_{rj} / \sum_{i=1}^m v_{ik}^* x_{ij}$ is referred to as a cross-efficiency value of DMU _{j} according to the favorable weights of DMU _{k} and reflects the peer-evaluation of DMU _{k} to DMU _{j} ($j = 1, \dots, n; j \neq k$).

Model (3) is solved n times, each time for each DMU. As a result, there will be n sets of input and output weights available for n DMUs and each DMU will have $(n - 1)$ cross-efficiency values that are peer-evaluation and one CCR-efficiency value that is self-evaluation, which form a cross-efficiency matrix, as shown in Table 1, where θ_{kk} ($k = 1, \dots, n$) are the CCR-efficiencies of the n DMUs; that is, $\theta_{kk} = \theta_{kk}^*$. The average efficiency of DMU is measured by the one CCR-efficiency value and $(n - 1)$ cross-efficiency values; namely,

$$\bar{\theta}_k = \frac{1}{n} \left(\theta_{kk}^* + \sum_{j=1, j \neq k}^n \theta_{kj} \right), \quad k = 1, \dots, n. \quad (4)$$

TABLE 1: Cross-efficiency matrix for n DMUs [18].

DMU	Target DMU				Average cross-efficiency
	1	2	...	n	
1	θ_{11}	θ_{12}	...	θ_{1n}	$\frac{1}{n} \sum_{k=1}^n \theta_{1k}$
2	θ_{21}	θ_{22}	...	θ_{2n}	$\frac{1}{n} \sum_{k=1}^n \theta_{2k}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	θ_{n1}	θ_{n2}	...	θ_{nn}	$\frac{1}{n} \sum_{k=1}^n \theta_{nk}$

Based on their average cross-efficiencies, the DMUs can usually be fully ranked.

It is noticed that model (3) may have multiple optimal solutions. This nonuniqueness of input and output weights would damage the use of cross-efficiency evaluation. To solve this problem, one solution suggested by Sexton et al. [8] is to introduce a secondary goal to gain a unique set of the input and output weights among the multiple optimal solutions determined by the CCR-efficiency model (3). The most commonly used secondary goal functions were suggested by Doyle and Green [9] and are shown below:

$$\begin{aligned}
 & \text{Minimize} \quad \sum_{r=1}^s u_{rk} \left(\sum_{j=1, j \neq k}^n y_{rj} \right) \\
 & \text{Subject to} \quad \sum_{i=1}^m v_{ik} \left(\sum_{j=1, j \neq k}^n x_{ij} \right) = 1 \\
 & \quad \sum_{r=1}^s u_{rk} y_{rk} - \theta_{kk}^* \sum_{i=1}^m v_{ik} x_{ik} = 0 \\
 & \quad \sum_{r=1}^s u_{rk} y_{rj} - \sum_{i=1}^m v_{ik} x_{ij} \leq 0, \\
 & \quad \quad \quad j = 1, \dots, n; \quad j \neq k, \\
 & \quad u_{rk} \geq 0, \quad r = 1, \dots, s, \\
 & \quad v_{ik} \geq 0, \quad i = 1, \dots, m, \\
 & \text{Maximize} \quad \sum_{r=1}^s u_{rk} \left(\sum_{j=1, j \neq k}^n y_{rj} \right) \\
 & \text{Subject to} \quad \sum_{i=1}^m v_{ik} \left(\sum_{j=1, j \neq k}^n x_{ij} \right) = 1 \\
 & \quad \sum_{r=1}^s u_{rk} y_{rk} - \theta_{kk}^* \sum_{i=1}^m v_{ik} x_{ik} = 0 \\
 & \quad \sum_{r=1}^s u_{rk} y_{rj} - \sum_{i=1}^m v_{ik} x_{ij} \leq 0, \\
 & \quad \quad \quad j = 1, \dots, n; \quad j \neq k,
 \end{aligned} \tag{5}$$

$$u_{rk} \geq 0, \quad r = 1, \dots, s,$$

$$v_{ik} \geq 0, \quad i = 1, \dots, m.$$

(6)

Model (5) is commonly called the aggressive formulation for cross-efficiency evaluation which aims to minimize the cross-efficiencies of the other DMUs, whereas model (6) is known as the benevolent cross-efficiency evaluation which maximizes the cross-efficiencies of the other DMUs. Since the two models obtain the input and output weights in two different ways, there is obviously no guarantee that they can lead to the same efficiency ranking or conclusion for the n DMUs.

3. The Neutral DEA Model

To avoid the difficulty in making a choice between the aggressive and benevolent formulations, Wang and Chin [18] put forward a neutral DEA model based on the idea that when a DMU is given an opportunity to decide a set of input and output weights, what the DMU is concerned most about is whether the weights can be as favorable as possible to itself. It should not care too much about how to be aggressive or benevolent to the other DMUs. The neutral model is shown below:

$$\begin{aligned}
 & \text{Maximize} \quad \delta = \text{minimum}_{r \in \{1, \dots, s\}} \left\{ \frac{u_{rk} y_{rk}}{\sum_{i=1}^m v_{ik} x_{ik}} \right\} \\
 & \text{subject to} \quad \theta_{kk}^* = \frac{\sum_{r=1}^s u_{rk} y_{rk}}{\sum_{i=1}^m v_{ik} x_{ik}} \\
 & \quad \theta_{jk} = \frac{\sum_{r=1}^s u_{rk} y_{rj}}{\sum_{i=1}^m v_{ik} x_{ij}} \leq 1, \\
 & \quad \quad \quad j = 1, \dots, n, \quad j \neq k, \\
 & \quad u_{rk} \geq 0, \quad r = 1, \dots, s, \\
 & \quad v_{ik} \geq 0, \quad i = 1, \dots, m,
 \end{aligned} \tag{7}$$

where $u_{rk} y_{rk} / \sum_{i=1}^m v_{ik} x_{ik}$ is the efficiency of the r th output of DMU _{k} . Obviously, the goal of the above model has nothing to do with the other DMUs. It determines the input and output weights just from the viewpoint of DMU _{k} itself. The DM has therefore not to make any difficult yet subjective choice between the aggressive and benevolent formulations. Through Charnes and Cooper transformation, model (7) can be converted into the following LP for solution:

$$\begin{aligned}
 & \text{Maximize} \quad \delta \\
 & \text{Subject to} \quad \sum_{i=1}^m v_{ik} x_{ik} = 1, \\
 & \quad \sum_{r=1}^s u_{rk} y_{rk} = \theta_{kk}^* \\
 & \quad \sum_{r=1}^s u_{rk} y_{rj} - \sum_{i=1}^m v_{ik} x_{ij} \leq 0, \\
 & \quad \quad \quad j = 1, \dots, n, \quad j \neq k.
 \end{aligned}$$

$$\begin{aligned}
u_{rk}y_{rk} - \delta &\geq 0, \quad r = 1, \dots, s, \\
v_{ik} &\geq 0, \quad i = 1, \dots, m, \\
\delta &\geq 0,
\end{aligned} \tag{8}$$

where u_{rk} ($r = 1, \dots, s$), v_{ik} ($i = 1, \dots, m$), and δ are decision variables. Like all DEA models for cross-efficiency evaluation, model (8) needs to be solved n times, each time for one different DMU. Accordingly, there will be n sets of input and output weights available for cross-efficiency evaluation.

Since θ_{kk}^* is fixed, the efficiencies of all the outputs determined by this neutral model are not biggest among the favorable weights. For example, if we want to measure the efficiency of the DMUs which consume two inputs to produce two outputs, we supposed that one of the DMUs such as A is the DEA efficiency; that is to say θ_{AA}^* is equal to 1. If there are two sets of weights that are favorable to DMU A , the first set of weights leads to the efficiency of y_1 and y_2 being $1/2$ and $1/2$ that is computed through the formulation $u_{rk}y_{rk} / \sum_{i=1}^m v_{ik}x_{ik}$ according to model (7) and second set of weights leads to the efficiency of y_1 and y_2 being $1/3$ and $2/3$. Because δ of the first set of weights is $1/2$ and δ of the second set of weights is $1/3$, so the DMU A will choose the first set of weights to calculate the cross-efficiency of DMUs but the first set of weights determined by model (7) just ensures that the efficiency of y_1 is bigger ($1/2 > 1/3$, $1/2 < 2/3$). Obviously model (7) just can ensure that the efficiencies of all the outputs are close to each other. Model (7) proposed by Wang and Chin [18] is not fully followed by its modeling idea that when a DMU is given an opportunity to decide a set of input and output weights, what the DMU is concerned most about is whether the weights can be as favorable as possible to itself.

Based on the above analysis, we suggest a revised model. The revised model is shown below:

$$\begin{aligned}
\text{Maximize} \quad & \sum_{r=1}^s u_{rk} \left[\frac{u_{rk}y_{rk}}{\sum_{i=1}^m v_{ik}x_{ik}} \right] \\
\text{Subject to} \quad & \theta_{kk}^* = \frac{\sum_{r=1}^s u_{rk}y_{rk}}{\sum_{i=1}^m v_{ik}x_{ik}} \\
& \theta_{jk} = \frac{\sum_{r=1}^s u_{rk}y_{rj}}{\sum_{i=1}^m v_{ik}x_{ij}} \leq 1, \\
& j = 1, \dots, n, \quad j \neq k \\
& u_{rk} \geq 0, \quad r = 1, \dots, s \\
& v_{ik} \geq 0, \quad i = 1, \dots, m.
\end{aligned} \tag{9}$$

Obviously, model (9) is more consistent with the modeling idea proposed by Wang and Chin.

4. The New DEA Model

According to the weights determined by the second goal to be aggressive or benevolent to the other DMUs, Doyle and Green propose the aggressive and benevolent DEA cross-efficiency

model. However, according to the self-interest principle in economics, the DMU tends to choose a set of input and output weights to maximize its own efficiency among the favorable weights. Based upon this point of view, we construct the following DEA model for cross-efficiency evaluation. The model is shown below:

$$\begin{aligned}
\text{Maximize} \quad & \frac{(\sum_{r=1}^s u_{rk}y_{rk} / \sum_{r=1}^s u_{rk})}{(\sum_{i=1}^m v_{ik}x_{ik} / \sum_{i=1}^m v_{ik})} \\
\text{Subject to} \quad & \theta_{kk}^* = \frac{\sum_{r=1}^s u_{rk}y_{rk}}{\sum_{i=1}^m v_{ik}x_{ik}} \\
& \theta_{jk} = \frac{\sum_{r=1}^s u_{rk}y_{rj}}{\sum_{i=1}^m v_{ik}x_{ij}} \leq 1, \\
& j = 1, \dots, n, \quad j \neq k, \\
& u_{rk} \geq 0, \quad r = 1, \dots, s \\
& v_{ik} \geq 0, \quad i = 1, \dots, m.
\end{aligned} \tag{10}$$

The above model standardized the input and output weights determined by the CCR model. It also means that the DMU chooses a set of weights to maximize its efficiency among the favorable weights where both of the sum of the inputs weights and sum of the outputs weights are all equal to 1. Model (10) recovers the CCR value at the same base that both of the sums of the inputs weights and outputs weights are equal to 1 and then compares the original and absolute values.

5. Numerical Examples

In this section, we provide two numerical examples to illustrate the practical applications of the proposed DEA model in the cross-efficiency evaluation and make a comparison among the mentioned DEA cross-efficiency evaluation models.

Example 1 (efficiency evaluation of seven departments in a university (Wong and Beasley, 1990)). Seven academic departments (DMUs) in a university are evaluated in the case of three inputs and three outputs given below and Table 2 shows their data:

- x_1 : number of academic staff
- x_2 : academic staff salaries in thousands of pounds
- x_3 : support staff salaries in thousands of pounds
- y_1 : number of undergraduate students
- y_2 : number of graduate students
- y_3 : number of research papers

Since the CCR-efficiencies evaluate six of the seven academic departments as efficient and cannot discriminate among them any further, cross-efficiencies are computed. Tables 3 and 4 show the aggressive and benevolent cross-efficiencies results, which are, respectively, obtained through model (5) and model (6). It is very clearly shown that the two

TABLE 2: Data for seven academic departments and their CCR values [18].

Department (DMU)	Inputs			Outputs			CCR-efficiency
	x_1	x_2	x_3	y_1	y_2	y_3	
1	12	400	20	65	35	17	1
2	19	750	70	139	41	40	1
3	42	1500	70	225	68	75	1
4	15	600	100	90	12	17	0.8197
5	45	2000	250	253	145	130	1
6	19	730	50	132	45	45	1
7	41	2350	600	305	159	97	1

TABLE 3: Aggressive cross-efficiency matrix of the seven academic departments in a university [18].

Department (DMU)	Target DMU							Average	Ranking
	1	2	3	4	5	6	7		
1	1.00	0.85	0.93	0.69	1.00	0.93	0.75	0.8788	1
2	0.33	1.00	0.62	1.00	0.70	0.84	0.56	0.7219	4
3	0.56	0.85	1.00	0.74	0.56	1.00	0.42	0.7301	3
4	0.07	0.76	0.28	0.82	0.24	0.44	0.21	0.4018	7
5	0.33	0.66	0.31	0.76	1.00	0.48	0.83	0.6259	5
6	0.51	1.00	0.82	0.95	0.79	1.00	0.61	0.8126	2
7	0.15	0.60	0.16	0.99	0.98	0.28	1.00	0.5966	6

TABLE 4: Benevolent cross-efficiency matrix of the seven academic departments in a university [18].

Department (DMU)	Target DMU							Average	Ranking
	1	2	3	4	5	6	7		
1	1.00	0.92	1.00	0.69	1.00	1.00	1.00	0.9442	3
2	0.98	1.00	0.85	1.00	0.85	0.98	0.98	0.9486	2
3	0.76	0.77	1.00	0.73	0.67	0.77	0.77	0.7827	6
4	0.64	0.70	0.45	0.82	0.41	0.64	0.64	0.6160	7
5	0.94	0.90	0.50	0.77	1.00	0.94	0.94	0.8534	5
6	1.00	1.00	1.00	0.95	0.91	1.00	1.00	0.9801	1
7	1.00	1.00	0.29	1.00	1.00	1.00	1.00	0.8992	4

TABLE 5: Neutral cross-efficiency matrix of the seven academic departments in a university [18].

Department (DMU)	Target DMU							Average	Ranking
	1	2	3	4	5	6	7		
1	1.0000	0.92	1.00	0.69	1.00	1.00	0.94	0.9362	2
2	0.93	1.00	0.62	1.00	0.85	0.95	0.97	0.9026	3
3	0.76	0.77	1.00	0.73	0.67	0.75	0.75	0.7763	6
4	0.61	0.70	0.27	0.82	0.41	0.54	0.60	0.5649	7
5	0.84	0.89	0.34	0.76	1.00	0.99	0.96	0.8272	5
6	0.96	1.00	0.83	0.95	0.91	1.00	1.00	0.9493	1
7	0.82	1.00	0.17	1.00	1.00	1.00	1.00	0.8552	4

TABLE 6: Cross-efficiency matrix of the seven academic departments in a university by model (9).

Department (DMU)	Target DMU							Average	Ranking
	1	2	3	4	5	6	7		
1	1.0000	0.8330	0.9657	0.6878	0.9032	0.8492	1.0000	0.8913	2
2	0.5814	1.0000	0.8567	1.0000	0.8013	1.0000	0.6826	0.8460	3
3	0.5217	0.8331	1.0000	0.7351	0.7287	0.8397	0.5378	0.7423	5
4	0.1898	0.7991	0.4665	0.8197	0.3636	0.7548	0.2514	0.5207	7
5	0.7337	0.6893	0.4846	0.7646	1.0000	0.6911	0.9589	0.7603	4
6	0.6834	0.9981	0.9998	0.9506	0.9256	1.0000	0.7593	0.9024	1
7	0.5695	0.6773	0.2904	0.9986	0.7252	0.6395	1.0000	0.7001	6

TABLE 7: Cross-efficiency matrix of the seven academic departments in a university by the new DEA model (10).

Department (DMU)	Target DMU							Average	Ranking
	1	2	3	4	5	6	7		
1	1.0000	0.8241	0.9685	0.6877	0.7070	0.9103	0.7136	0.8302	3
2	0.6445	1.0000	0.6286	1.0000	0.7890	0.8629	0.9958	0.8458	2
3	0.6889	0.7942	1.0000	0.7352	0.6606	0.9508	0.7641	0.7991	4
4	0.2508	0.7719	0.2022	0.8197	0.4106	0.4632	0.7400	0.5226	7
5	0.6254	0.7663	0.5668	0.7651	1.0000	0.5303	0.9025	0.7366	5
6	0.7914	0.9855	0.9218	0.9509	0.8751	1.0000	0.9915	0.9309	1
7	0.3702	0.8015	0.2105	1.0000	0.8570	0.3210	1.0000	0.6515	6

different formulations result in the different efficiency rankings for the seven academic departments. In particular, the aggressive cross-efficiencies in Table 4 evaluate DMU₁ as the most efficient academic department, whereas DMU₁ is in the third place evaluated by the benevolent cross-efficiencies in Table 5. The two different efficiency rankings to some extent damage the practical application of cross-efficiency evaluation which claims to produce a unique ordering for DMUs.

To avoid the difficulty in making a subjective choice between the two different formulations for cross-efficiency evaluation, Wang and Chin [18] proposed a neutral DEA model (7) to evaluate the seven academic departments. The results are shown in Table 5. Since these efficiencies are neither aggressive nor benevolent, they are more neutral. The average cross-efficiencies in Table 5 clearly show that DMU₁ is in the second place.

We propose a revised model (9) to reevaluate the cross-efficiencies of the seven academic departments. The results are shown in Table 6. The results show that the DMU₆ is the most efficient academic department. Based on the self-interest principle, we suggest model (10). The cross-efficiencies in Table 7 calculated by model (10) also evaluate the DMU₆ as the most efficient academic department. However, the ranks of the DMU₁ and DMU₂ and DMU₃ and DMU₅ are different by the two different formulations.

Using the FPI to measure the false positiveness of DMUs, the results of the FPI value are shown in Table 8, where the FPI was computed by (Baker and Talluri, 1997)

$$FPI_k = \frac{\theta_{kk}^* - ((1/n) \sum_{j=1}^n \theta_{kj})}{((1/n) \sum_{j=1}^n \theta_{kj})} \times 100\%, \quad k = 1, \dots, n. \quad (11)$$

It is seen that none of the six DEA efficient academic departments is highly false positive. However, the non-DEA efficient academic department DMU₄ shows high false positiveness. This means that there is a significant difference between the self-evaluation and peer-evaluation of DMU₄. Usually, a lower value of the FPI is more preferred. The most efficient DMU is undoubtedly the one with the lowest FPI value, as highlighted in Table 8.

Example 2 (efficiency evaluation of 14 international passenger airlines (Tofallis, 1997)). Fourteen major passenger airlines are evaluated in terms of three inputs and two outputs that are defined below:

x_1 : aircraft capacity in ton-kilometres

x_2 : operating cost

x_3 : nonflight assets such as reservation system, facilities, and current assets

y_1 : passenger kilometres

y_2 : nonpassenger revenue

Table 9 shows the input and output data of the 14 passenger airlines in the year 1990 along with their CCR-efficiencies, which evaluate seven out of the 14 passenger airlines as DEA efficient and cannot distinguish them any further.

Tables 10 and 11 show, respectively, the aggressive and benevolent cross-efficiencies of the 14 passenger airlines. It is found that the two different formulations for cross-efficiency evaluation lead to two different efficiency rankings again.

TABLE 8: False positiveness of the seven academic departments in a university.

Department (DMU)	FPI (%) for cross-efficiency evaluation				
	Aggressive	Benevolent	Neutral	Revised	New
1	13.79	5.91	6.81	12.2	20.45
2	38.52	5.41	10.79	18.2	18.23
3	36.96	27.76	28.81	34.72	25.14
4	104.01	33.07	45.11	57.42	56.85
5	59.76	17.18	20.88	31.53	35.76
6	23.06	2.03	5.34	10.82	7.42
7	67.62	11.21	16.94	42.84	53.49

TABLE 9: Data for 14 passenger airlines and CCR values [18].

Airline (DMU)	Inputs			Outputs		CCR-efficiency
	x_1	x_2	x_3	y_1	y_2	
1	5273	3239	2003	26,677	697	0.8684
2	5895	4225	4557	3081	539	0.3379
3	24,099	9560	6267	124,055	1266	0.9475
4	13,565	7499	3213	64,734	1563	0.9581
5	5183	1880	783	23,604	513	1.0000
6	19,080	8032	3272	95,011	572	0.9766
7	4603	3457	2360	22,112	969	1.0000
8	12,097	6779	6474	52,363	2001	0.8588
9	6587	3341	3581	26,504	1297	0.9477
10	5654	1878	1916	19,277	972	1.0000
11	12,599	8098	3310	41,925	3398	1.0000
12	5728	2481	2254	27,754	982	1.0000
13	4715	1792	2485	31,332	543	1.0000
14	22,793	9874	4145	122,528	1404	1.0000

According to the aggressive cross- efficiencies in Table 10, the DMU₅ is the most efficient passenger airline. However, the benevolent cross-efficiencies in Table 11 evaluate DMU₁₁ as the most efficient passenger airline. The inconsistent results in efficiency ranking obviously cause confusion for the DM. This issue causes the choice difficulty for the DM.

In comparison with the aggressive and benevolent cross-efficiencies in Tables 10 and 11, the neutral cross-efficiencies are presented in Table 12, which are obtained by the neutral DEA model (8). Neutral cross-efficiencies in Table 12 evaluate DMU₁₁ as the most efficient airline.

The cross-efficiencies in Table 13 computed by the revised model (9) show that the DMU₁₁ is the most efficient airline. Table 14 obtained by solving the new DEA cross-efficiency model (10) evaluates the DMU₁₃ as the most efficient airline.

Table 15 shows the values of FPI of the 14 passenger airlines, from which it is seen that none of the seven DEA efficient passenger airlines is highly false positive, but a non-DEA efficient passenger airline DMU₂ reveals the highest false positiveness value. This implies that the self-evaluation of DMU₂ is significantly different from its peer-evaluation. The most efficient passenger airline is the DMU with the lowest FPI value, as highlighted in Table 15.

6. Conclusions

Cross-efficiency evaluation is an important and popular tool to evaluate and rank the efficiency of the DMUs. The classical DEA cross-efficiency evaluation formulations are the aggressive and benevolent models proposed by the Doyle and Green. However, the two models often cause different ranking of the DMUs. This issue causes a choice difficulty for the DM between the two different formulations. To solve this issue, Wang and Chin [18] propose a neutral model. Through the deep analysis of this neutral model, we suggest a revised model. Based on the self-interest principle, we further propose a new model. The weights are determined by the new model purely from its own point of view without the need to be aggressive or benevolent to the other DMUs. The model is more logical and more in line with code of the conduct of rational DM.

Because the models proposed in this paper are the nonlinear programming models, it is a little difficult to compute relative to linear programming models. This paper determines the importance level of the outputs by the different weights of output computed by the neutral model. There may be some other feasible ways such as the prices of the outputs.

TABLE 10: Aggressive cross-efficiency matrix of 14 passenger airlines [18].

Airline (DMU)	Target DMU														Average cross-efficiency	Rank
	1	2	3	4	5	6	7	8	9	10	11	12	13	14		
1	0.8684	0.4501	0.6225	0.8684	0.4418	0.4726	0.7679	0.7881	0.7031	0.4158	0.3390	0.7043	0.4711	0.4726	0.5990	12
2	0.1719	0.3379	0.0472	0.1719	0.0224	0.0247	0.2770	0.2724	0.2808	0.2465	0.1152	0.2789	0.0417	0.0247	0.1652	14
3	0.8826	0.1942	0.9475	0.8826	0.6567	0.6898	0.6468	0.6833	0.6225	0.2559	0.1968	0.6261	0.7422	0.6898	0.6226	11
4	0.9581	0.4259	0.7034	0.9581	0.6683	0.6973	0.7629	0.7850	0.6991	0.4027	0.4739	0.7016	0.4937	0.6973	0.6734	7
5	0.9653	0.3658	1.0000	0.9653	1.0000	1.0000	0.7011	0.7359	0.7778	0.5272	0.6382	0.7820	0.7181	1.0000	0.7983	1
6	0.8818	0.1108	0.9563	0.8818	0.9632	0.9766	0.5745	0.6084	0.5099	0.1376	0.1703	0.5141	0.6766	0.9766	0.6385	9
7	0.9211	0.7781	0.4773	0.9211	0.3108	0.3382	1.0000	1.0000	0.8395	0.5416	0.4000	0.8383	0.3658	0.3382	0.6478	8
8	0.7813	0.6114	0.5162	0.7813	0.2683	0.2924	0.8415	0.8588	0.8208	0.5703	0.3011	0.8194	0.4418	0.2924	0.5855	13
9	0.7855	0.7278	0.5076	0.7855	0.2455	0.2677	0.8881	0.9072	0.9477	0.7501	0.3528	0.9452	0.4537	0.2677	0.6309	10
10	0.7821	0.6354	0.6520	0.7821	0.3338	0.3564	0.7650	0.7944	1.0000	1.0000	0.4942	1.0000	0.5871	0.3564	0.6813	6
11	1.0000	1.0000	0.4287	1.0000	0.4202	0.4418	1.0000	1.0000	1.0000	0.8107	1.0000	1.0000	0.2961	0.4418	0.7742	2
12	0.9462	0.6336	0.7500	0.9462	0.4085	0.4395	0.9082	0.9395	0.9998	0.7647	0.4244	1.0000	0.6398	0.4395	0.7314	5
13	1.0000	0.4257	1.0000	1.0000	0.4183	0.4555	0.9511	1.0000	1.0000	0.5855	0.2129	1.0000	1.0000	0.4555	0.7503	3
14	1.0000	0.2277	1.0000	1.0000	0.9806	1.0000	0.6919	0.7275	0.6478	0.2747	0.3300	0.6521	0.7097	1.0000	0.7316	4

TABLE II: Benevolent cross-efficiency matrix of 14 passenger airlines [18].

Airline (DMU)	Target DMU														Average cross-efficiency	Rank
	1	2	3	4	5	6	7	8	9	10	11	12	13	14		
1	0.8684	0.4501	0.6225	0.8684	0.8492	0.4726	0.8108	0.7881	0.7031	0.7512	0.8684	0.7713	0.8684	0.8684	0.7543	12
2	0.1719	0.3379	0.0472	0.1719	0.1735	0.0247	0.2479	0.2724	0.2808	0.2058	0.1719	0.2025	0.1719	0.1719	0.1894	14
3	0.8826	0.1942	0.9475	0.8826	0.8844	0.6898	0.7232	0.6833	0.6225	0.7846	0.8826	0.8072	0.8826	0.8826	0.7678	9
4	0.9581	0.4259	0.7034	0.9581	0.9413	0.6973	0.8228	0.7850	0.6991	0.8113	0.9581	0.8341	0.9581	0.9581	0.8222	6
5	0.9653	0.3658	1.0000	0.9653	1.0000	1.0000	0.7704	0.7359	0.7778	1.0000	0.9653	1.0000	0.9653	0.9653	0.8912	3
6	0.8818	0.1108	0.9563	0.8818	0.8780	0.9766	0.6615	0.6084	0.5099	0.7176	0.8818	0.7478	0.8818	0.8818	0.7554	11
7	0.9211	0.7781	0.4773	0.9211	0.8795	0.3382	1.0000	1.0000	0.8395	0.7808	0.9211	0.8012	0.9211	0.9211	0.8214	7
8	0.7813	0.6114	0.5162	0.7813	0.7703	0.2924	0.8458	0.8588	0.8208	0.7532	0.7813	0.7631	0.7813	0.7813	0.7242	13
9	0.7855	0.7278	0.5076	0.7855	0.7889	0.2677	0.8782	0.9072	0.9477	0.8375	0.7855	0.8369	0.7855	0.7855	0.7590	10
10	0.7821	0.6354	0.6520	0.7821	0.8250	0.3564	0.7780	0.7944	1.0000	1.0000	0.7821	0.9719	0.7821	0.7821	0.7803	8
11	1.0000	1.0000	0.4287	1.0000	1.0000	0.4418	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9193	1
12	0.9462	0.6336	0.7500	0.9462	0.9602	0.4395	0.9362	0.9395	0.9998	1.0000	0.9462	1.0000	0.9462	0.9462	0.8850	4
13	1.0000	0.4257	1.0000	1.0000	1.0000	0.4555	1.0000	1.0000	1.0000	0.9843	1.0000	1.0000	1.0000	1.0000	0.9190	2
14	1.0000	0.2277	1.0000	1.0000	1.0000	1.0000	0.7795	0.7275	0.6478	0.8569	1.0000	0.8838	1.0000	1.0000	0.8659	5

TABLE 12: Neutral cross-efficiency matrix of 14 passenger airlines [18].

Airline (DMU)	Target DMU														Average cross-efficiency	Rank
	1	2	3	4	5	6	7	8	9	10	11	12	13	14		
1	0.8684	0.4501	0.6225	0.8684	0.4907	0.4726	0.7744	0.7881	0.7031	0.6603	0.8639	0.7533	0.7031	0.8492	0.7049	11
2	0.1719	0.3379	0.0472	0.1719	0.1027	0.0247	0.2716	0.2724	0.2808	0.1641	0.1723	0.2054	0.2808	0.1735	0.1912	14
3	0.8826	0.1942	0.9475	0.8826	0.4856	0.6898	0.6606	0.6833	0.6225	0.7894	0.8830	0.7871	0.6225	0.8844	0.7154	10
4	0.9581	0.4259	0.7034	0.9581	0.7093	0.6973	0.7704	0.7850	0.6991	0.7109	0.9542	0.8137	0.6991	0.9413	0.7733	7
5	0.9653	0.3658	1.0000	0.9653	1.0000	1.0000	0.7091	0.7359	0.7778	1.0000	0.9730	1.0000	0.7778	1.0000	0.8764	2
6	0.8818	0.1108	0.9563	0.8818	0.6162	0.9766	0.5895	0.6084	0.5099	0.7125	0.8809	0.7209	0.5099	0.8780	0.7024	12
7	0.9211	0.7781	0.4773	0.9211	0.4738	0.3382	1.0000	1.0000	0.8395	0.6340	0.9110	0.7829	0.8395	0.8795	0.7711	8
8	0.7813	0.6114	0.5162	0.7813	0.3741	0.2924	0.8434	0.8588	0.8208	0.6645	0.7787	0.7542	0.8208	0.7703	0.6906	13
9	0.7855	0.7278	0.5076	0.7855	0.4036	0.2677	0.8865	0.9072	0.9477	0.7504	0.7863	0.8374	0.9477	0.7889	0.7378	9
10	0.7821	0.6354	0.6520	0.7821	0.5587	0.3564	0.7632	0.7944	1.0000	1.0000	0.7918	0.9969	1.0000	0.8250	0.7813	6
11	1.0000	1.0000	0.4287	1.0000	1.0000	0.4418	0.9901	1.0000	1.0000	0.7970	1.0000	1.0000	1.0000	1.0000	0.9041	1
12	0.9462	0.6336	0.7500	0.9462	0.5418	0.4395	0.9115	0.9395	0.9998	0.9399	0.9494	1.0000	0.9998	0.9602	0.8541	4
13	1.0000	0.4257	1.0000	1.0000	0.3800	0.4555	0.9652	1.0000	1.0000	1.0000	1.0000	0.9860	1.0000	1.0000	0.8723	3
14	1.0000	0.2277	1.0000	1.0000	0.7507	1.0000	0.7058	0.7275	0.6478	0.8286	1.0000	0.8598	0.6478	1.0000	0.8140	5

TABLE 13: Cross-efficiency matrix of 14 passenger airlines by model (9).

Airline (DMU)	Target DMU														Average cross-efficiency	Rank
	1	2	3	4	5	6	7	8	9	10	11	12	13	14		
1	0.8684	0.4501	0.6225	0.8684	0.7705	0.4726	0.7962	0.7881	0.7031	0.7473	0.8307	0.7513	0.8322	0.8534	0.7396	11
2	0.1719	0.3379	0.0472	0.1719	0.1988	0.0247	0.2630	0.2724	0.2808	0.2059	0.1805	0.2075	0.1818	0.1719	0.1940	14
3	0.8826	0.1942	0.9475	0.8826	0.8064	0.6898	0.6972	0.6833	0.6225	0.7796	0.8674	0.7810	0.8628	0.8820	0.7556	10
4	0.9581	0.4259	0.7034	0.9581	0.8375	0.6973	0.7985	0.7850	0.6991	0.8063	0.9154	0.8089	0.9145	0.9464	0.8039	6
5	0.9653	0.3658	1.0000	0.9653	1.0000	1.0000	0.7476	0.7359	0.7778	0.9956	1.0000	0.9906	0.9879	0.9882	0.8943	3
6	0.8818	0.1108	0.9563	0.8818	0.7528	0.9766	0.6273	0.6084	0.5099	0.71156	0.8485	0.7124	0.8431	0.8792	0.7357	12
7	0.9211	0.7781	0.4773	0.9211	0.79858	0.3382	1.0000	1.0000	0.8395	0.7775	0.8599	0.7843	0.8674	0.8896	0.8038	7
8	0.7813	0.6114	0.5162	0.7813	0.7552	0.2924	0.8532	0.8588	0.8208	0.7510	0.7686	0.7562	0.7717	0.7701	0.7206	13
9	0.7855	0.7278	0.5076	0.7855	0.8250	0.2677	0.8954	0.9072	0.9477	0.8361	0.7999	0.8408	0.8012	0.7836	0.7651	9
10	0.7821	0.6354	0.6520	0.7821	0.9564	0.3564	0.78670	0.7944	1.0000	1.0000	0.8558	0.9971	0.8481	0.8077	0.8039	8
11	1.0000	1.0000	0.4287	1.0000	1.0000	0.4418	1.0000	1.0000	1.0000	0.9970	1.0000	0.9990	1.0000	1.0000	0.9190	1
12	0.9462	0.6336	0.7500	0.9462	0.9894	0.4395	0.9370	0.9395	0.9998	0.9974	0.9691	1.0000	0.9664	0.9519	0.8904	4
13	1.0000	0.4257	1.0000	1.0000	0.9850	0.4555	1.0000	1.0000	1.0000	0.9809	1.0000	0.9869	1.0000	0.9940	0.9163	2
14	1.0000	0.2277	1.0000	1.0000	0.8880	1.0000	0.7459	0.7275	0.6478	0.8507	0.9734	0.8512	0.9674	1.0000	0.8485	5

TABLE 14: Cross-efficiency matrix of 14 passenger airlines by the new DEA model (10).

Airline (DMU)	Target DMU														Average cross-efficiency	Rank
	1	2	3	4	5	6	7	8	9	10	11	12	13	14		
1	0.8684	0.4501	0.6225	0.8684	0.5914	0.4726	0.7707	0.7881	0.7031	0.6231	0.6892	0.7392	0.6820	0.7414	0.6864	12
2	0.1719	0.3379	0.0472	0.1719	0.0813	0.0247	0.2756	0.2724	0.2808	0.1893	0.1606	0.2245	0.0825	0.0814	0.1716	14
3	0.8826	0.1942	0.9475	0.8826	0.7563	0.6898	0.6520	0.6833	0.6225	0.6755	0.6784	0.7374	0.7973	0.8851	0.7203	9
4	0.9581	0.4259	0.7034	0.9581	0.7371	0.6973	0.7663	0.7850	0.6991	0.6543	0.8022	0.7785	0.7126	0.8440	0.7516	6
5	0.9653	0.3658	1.0000	0.9653	1.0000	1.0000	0.7053	0.7359	0.7778	0.8835	0.8927	0.9258	0.7414	0.9161	0.8482	2
6	0.8818	0.1108	0.9563	0.8818	0.8500	0.9766	0.5798	0.6084	0.5099	0.5807	0.6884	0.6537	0.7710	0.9260	0.7125	10
7	0.9211	0.7781	0.4773	0.9211	0.5061	0.3382	1.0000	1.0000	0.8395	0.6404	0.7235	0.7995	0.6584	0.6759	0.7342	8
8	0.7813	0.6114	0.5162	0.7813	0.4637	0.2924	0.8429	0.8588	0.8208	0.6668	0.6136	0.7731	0.6269	0.6081	0.6612	13
9	0.7855	0.7278	0.5076	0.7855	0.4706	0.2677	0.8890	0.9072	0.9477	0.7732	0.6492	0.8675	0.6022	0.5814	0.6973	11
10	0.7821	0.6354	0.6520	0.7821	0.6102	0.3564	0.7668	0.7944	1.0000	1.0000	0.7373	1.0000	0.5660	0.6085	0.7351	7
11	1.0000	1.0000	0.4287	1.0000	0.6707	0.4418	0.9990	1.0000	1.0000	0.8257	1.0000	1.0000	0.5189	0.6476	0.8237	3
12	0.9462	0.6336	0.7500	0.9462	0.6643	0.4395	0.9111	0.9395	0.9998	0.9193	0.7953	1.0000	0.7498	0.7772	0.8194	4
13	1.0000	0.4257	1.0000	1.0000	0.6654	0.4555	0.9566	1.0000	1.0000	0.9372	0.7171	1.0000	1.0000	0.9166	0.8624	1
14	1.0000	0.2277	1.0000	1.0000	0.9288	1.0000	0.6973	0.7275	0.6478	0.7045	0.8146	0.79241	0.8334	1.0000	0.8124	5

TABLE 15: False positiveness of the 14 passenger airlines.

Airline (DMU)	FPI (%) for cross-efficiency evaluation				
	Aggressive	Benevolent	Neutral	Revised	New
1	44.98	15.12	26.10	17.41	26.52
2	104.52	78.39	111.58	74.18	96.91
3	104.52	23.40	40.38	25.40	31.54
4	42.28	16.53	26.30	19.18	27.47
5	25.26	12.21	17.49	11.82	17.90
6	52.96	29.28	53.16	32.74	37.07
7	54.36	21.74	35.55	24.41	36.20
8	46.67	18.59	29.76	19.18	29.89
9	50.23	24.86	35.18	23.87	35.91
10	46.77	28.16	32.27	24.39	36.04
11	29.16	8.78	14.54	8.81	21.40
12	36.72	13.00	20.40	12.31	22.04
13	33.28	8.82	20.90	9.13	15.96
14	36.69	15.48	29.70	17.86	23.09

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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