Hindawi
Mathematical Problems in Engineering
Volume 2017, Article ID 2597071, 13 pages
https://doi.org/10.1155/2017/2597071

Research Article

H_∞ Control for T-S Fuzzy Singularly Perturbed Switched Systems

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Received 20 March 2017; Revised 4 August 2017; Accepted 12 September 2017; Published 15 October 2017

Academic Editor: Asier Ibeas

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This paper is concerned with the design of fuzzy controller with guaranteed H_∞ performance for a class of Takagi-Sugeno (T-S) fuzzy singularly perturbed switched systems. First, by using the average dwell time approach together with the piecewise Lyapunov function technique, a state feedback controller that depends on the singular perturbation parameter ε is developed. This controller is shown to work well for all $\varepsilon \in (0, \varepsilon_0]$. Then, for sufficiently small ε, an ε-independent controller design method is proposed. Furthermore, under the ε-independent controller, the ε-bound estimation problem of the overall switched closed-loop system is solved. Finally, an inverted pendulum system is used to evaluate the feasibility and effectiveness of the obtained results.

1. Introduction

Many practical systems possess multiple time scale characteristics [1–4]. It is well known that control methods for normal systems can not be directly applied to this class of systems, since these methods may cause ill-conditioned numerical problems. To conquer these problems, singular perturbation methods have been widely used in control design of multiple time scale systems (see [5–9] and the references therein).

Singularly perturbed systems (SPSs), whose partial time derivatives involve a small singular perturbation parameter ε, have been widely investigated by many researchers. It is important to obtain the ε-bound such that stability and other performances of SPSs can be ensured. Studies on ε-bound can be mainly divided into two types: one only presents sufficient conditions for the existence of the ε-bound [10, 11], and the other proposes methods to estimate the ε-bound [9, 12].

Singularly perturbed switched systems (SPSSs) consist of a group of SPSs and a certain switching law, which specifies the active SPS at the switching instance. In practical processes, a great number of control systems, whose behavior is simultaneously determined by multiple time scales and switching, can be modeled as SPSSs [13–15]. The control model of the hot strip mill was treated as a SPSS and an H_2 robust controller was designed in [15]. Recently, SPSSs have attained much attention in the literature (see [16–20]). It has been shown in [17–20] that the stability of fast/slow switched subsystems can not guarantee the stability of the original switched systems. In order to guarantee the system stability under an arbitrary switching signal, a common Lyapunov function for individual SPSs or a dwell time scheme has to be considered. In [19], by combining the multiple Lyapunov functions method with the dwell time scheme, sufficient conditions for ensuring exponential stability of time-delay SPSSs with stable fast switched subsystems were derived. These conditions were described by a few ε-dependent algebraic inequalities, which led to a heavy and tedious calculation. The proposed method in [19] was further extended to time-delay SPSSs with impulsive effects [20]. However, few results on the ε-bound estimation are available for SPSSs. The exceptions were given by [21, 22]. A convex optimization based method was presented to make the best estimate of ε-bound for SPSSs whose fast switched subsystems can not depend on the switching signal in [21]. The ε-dependent stabilization and ε-bound problems were addressed by adopting dwell time switching signal and constructing ε-dependent multiple Lyapunov functions for SPSSs in [22].

Fuzzy control has been widely used for engineering practice [23, 24]. The Takagi-Sugeno (T-S) model, which has elegant ability to approximate a certain class of complex

nonlinear functions, was extensively utilized in control system design [25–27]. Based on T-S fuzzy control, many LMI-based fuzzy control design methods have been developed for SPSs. The stabilization problem was addressed for fuzzy SPSs in [28], and some LMI-based control algorithms were proposed. Over past a few years, H_∞ control for fuzzy SPSs has attained a lot of attention. H_∞ control was addressed for fuzzy SPSs in [11, 29–31], and sufficient conditions independent of ε for the existence of H_∞ controller were presented. The resulting conditions in [11, 28–31] are only applicable to stabilizing the system and being with an H_∞ performance for sufficiently small ε. Hence, some researchers have concentrated on the ε-bound design problem for fuzzy SPSs [32–34].

As an important class of hybrid systems, switched fuzzy systems have been a hot spot of present research. Recently, a great number of theoretical results are available for switched fuzzy systems. By using a common or multiple Lyapunov functions method, stability issues for switched fuzzy systems were considered in [35–39]. However, the existing literature on fuzzy SPSSs is rather limited. An LMI-based dynamic state feedback control method was given and the ε-bound problem of system was solved for nonaffine-in-control SPSSs [40]. This method decomposed the switched system into fast and slow subsystems and can not be suitable for nonstandard fuzzy SPSSs. In our previous paper [41], where the stabilization and ε-bound problems were addressed for T-S fuzzy SPSSs without the external disturbance input by constructing the ε-dependent piecewise Lyapunov function. Moreover, to the authors' best knowledge, H_∞ control for fuzzy SPSSs has not been addressed yet.

In this paper, we will investigate the design of fuzzy controller with guaranteed H_∞ performance for a class of T-S fuzzy SPSSs. The problem is composed of stabilization control, H_∞ control, and ε-bound design. First, for a given upper bound ε_0 for ε and a prespecified H_∞ performance bound $\gamma > 0$, an ε-dependent controller is developed, such that, for any $\varepsilon \in (0, \varepsilon_0]$, the switched system is asymptotically stable and the L_2-gain from the disturbance input to the controlled output is less than or equal to γ. This controller is shown to work well for all $\varepsilon \in (0, \varepsilon_0]$. Then, an ε-independent controller design method is proposed in terms of LMIs. Furthermore, under the ε-independent controller, the ε-bound estimation approach is given. Finally, an inverted pendulum system is used to evaluate the feasibility and effectiveness of the obtained results.

The rest of this paper is organized as follows. In Section 2, the problems to be considered are formulated and preliminaries are presented. The main results are given in Section 3. An example is given in Section 4 to illustrate the obtained methods. And Section 5 concludes the paper. The notations used in this paper are standard. The notations T and $\star$ stand for the matrix transpose and the transpose of the off diagonal element of the LMI, respectively. $\lambda_M(Q)$ and $\lambda_m(Q)$ denote the maximal and minimal eigenvalues of a symmetric matrix Q, respectively. $\|\cdot\|$ denotes Euclidean norm for vectors or the spectral norm of matrices. $He\{\cdot\}$ is defined as $He\{\Theta\} = \Theta + \Theta^T$ for a square matrix Θ.

2. Problem Formulation

Consider a T-S fuzzy SPSS, which involves $r_{\sigma(t)}$ rules of the following form.

The lth rule is

$$\begin{aligned}
&\text{Plant Rule } l:\\
&\text{IF } \upsilon_1(t) \text{ is } M^l_{\sigma(t)1},\ \upsilon_2(t) \text{ is } M^l_{\sigma(t)2}, \ldots, \upsilon_\phi(t) \text{ is } M^l_{\sigma(t)\phi}\\
&\text{THEN}\\
&E(\varepsilon)\dot{x}(t) = A_{\sigma(t)l}x(t) + B_{\sigma(t)l}u(t) + E_{\sigma(t)l}w(t)\\
&z(t) = C_{\sigma(t)l}x(t)\\
&\text{for } l = 1, 2, \ldots, r_{\sigma(t)},
\end{aligned} \tag{1}$$

where $E(\varepsilon) = \begin{bmatrix} I_{n_1} & 0 \\ 0 & \varepsilon I_{n_2} \end{bmatrix}$, ε is a small positive scalar which represents the singular perturbation parameter. $\sigma(t)$ is a piecewise constant function with respect to time, referred as to a switching signal, which takes its values in the finite set $S = \{1, 2, \ldots, M\}$, M represents the number of individual subsystems. $M^l_{\sigma(t)m}$ ($l = 1, 2, \ldots, r_{\sigma(t)}$, $m = 1, 2, \ldots, \phi$) are fuzzy sets, $r_{\sigma(t)}$ is the number of fuzzy rules, $\upsilon(t) = [\upsilon_1(t)\ \upsilon_2(t)\ \cdots\ \upsilon_\phi(t)]^T$ is the premise vector that may depend on states in many cases, ϕ is the number of premise variables, $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the control input, $w(t) \in R^r$ is the disturbance input that belongs to $L_2[0, \infty]$, and $z(t) \in R^p$ is the controlled output. For any given T_f, a time sequence of $t_1 < \cdots < t_{k+1}$ ($k \geq 1$) is labeled as the switching instants over the interval $(0, T_f)$. It means that the kth T-S fuzzy subsystem is active as $t \in [t_k, t_{k+1})$. And $A_{\sigma(t)l}$, $B_{\sigma(t)l}$, $E_{\sigma(t)l}$, and $C_{\sigma(t)l}$ are of the following form:

$$\begin{aligned}
A_{il} &= \begin{bmatrix} A_{il11} & A_{il12} \\ A_{il21} & A_{il22} \end{bmatrix},\\
B_{il} &= \begin{bmatrix} B_{il1} \\ B_{il2} \end{bmatrix},\\
E_{il} &= \begin{bmatrix} E_{il1} \\ E_{il2} \end{bmatrix},\\
C_{il} &= [C_{il1}\ \ C_{il2}],\\
&\qquad i \in S.
\end{aligned} \tag{2}$$

Denote

$$\xi_{\sigma(t)l}(\upsilon(t)) = \prod_{k=1}^{\phi} M^l_{\sigma(t)k}(\upsilon_k(t)), \quad l = 1, 2, \ldots, r_{\sigma(t)}, \tag{3}$$

where $M^l_{\sigma(t)k}(\upsilon_k(t))$ is the grade of membership of $\upsilon_k(t)$ in $M^l_{\sigma(t)k}$.

It is assumed in this paper that

$$\xi_{\sigma(t)l}(v(t)) \geq 0,$$
$$\sum_{l=1}^{r_{\sigma(t)}} \xi_{\sigma(t)l}(v(t)) > 0, \tag{4}$$
$$l = 1, 2, \ldots, r_{\sigma(t)}, \ \forall t \geq 0.$$

Let

$$\mu_{\sigma(t)l}(v(t)) = \frac{\xi_{\sigma(t)l}(v(t))}{\sum_{l=1}^{r_{\sigma(t)}} \xi_{\sigma(t)l}(v(t))}, \quad l = 1, 2, \ldots, r_{\sigma(t)}. \tag{5}$$

Then

$$\mu_{\sigma(t)l}(v(t)) \geq 0,$$
$$\sum_{l=1}^{r_{\sigma(t)}} \mu_{\sigma(t)l}(v(t)) = 1, \tag{6}$$
$$l = 1, 2, \ldots, r_{\sigma(t)}, \ \forall t \geq 0.$$

For the convenience of notations, we denote $\mu_{\sigma(t)l} = \mu_{\sigma(t)l}(v(t))$, $l = 1, 2, \ldots, r_{\sigma(t)}$.

Then, the ith T-S fuzzy subsystem can be inferred as

$$\begin{aligned} E(\varepsilon)\dot{x}(t) &= \sum_{l=1}^{r_i} \mu_{il}\left[A_{il}x(t) + B_{il}u(t) + E_{il}w(t)\right] \\ z(t) &= \sum_{l=1}^{r_i} \mu_{il}C_{il}x(t). \end{aligned} \tag{7}$$

Throughout the paper, it is assumed that the singular perturbation parameter ε is available for feedback. By using the concept of parallel distributed compensation (PDC), the state feedback fuzzy controller can be described by the following.

The lth rule is

Controller Rule l:

IF $v_1(t)$ is M_{i1}^l, $v_2(t)$ is $M_{i2}^l, \ldots, v_\phi(t)$ is $M_{i\phi}^l$

THEN (8)

$u(t) = K_{il}(\varepsilon)x(t)$

for $i = 1, 2, \ldots, M$, $l = 1, 2, \ldots, r_i$.

Because the controller rules are the same as the plant rules, the state feedback controller is given as follows:

$$u(t) = \sum_{l=1}^{r_i} \mu_{il}K_{il}(\varepsilon)x(t). \tag{9}$$

Remark 1. In many SPSs, ε is usually a known physical parameter. Based on the fact that ε is available for feedback control, some synthesis problems are considered in [19, 20, 22]. In [22], for a given ε-bound ε_0, under the dwell time switching law, the ε-dependent multiple Lyapunov function method was proposed to ensure exponential stability of the original switched system.

Substituting (9) into (7) yields the closed-loop system

$$\begin{aligned} &E(\varepsilon)\dot{x}(t) \\ &\quad = \sum_{l=1}^{r_i}\sum_{s=1}^{r_i} \mu_{il}\mu_{is}\left[\left(A_{il} + B_{il}K_{is}(\varepsilon)\right)x(t) + E_{il}w(t)\right] \\ &z(t) = \sum_{l=1}^{r_i} \mu_{il}C_{il}x(t). \end{aligned} \tag{10}$$

Upon introducing the indicator function

$$\theta(t) = \left[\theta_1(t), \ldots, \theta_M(t)\right], \tag{11}$$

where $\theta_i(t) = 1$ if the switching system is in mode i and $\theta_i(t) = 0$ if it is in a different mode, and the overall switched closed-loop system can be expressed as follows:

$$\begin{aligned} E(\varepsilon)\dot{x}(t) &= \sum_{l=1}^{r_i}\sum_{s=1}^{r_i}\sum_{i=1}^{M} \mu_{il}\mu_{is}\theta_i(t) \\ &\quad \cdot \left[\left(A_{il} + B_{il}K_{is}(\varepsilon)\right)x(t) + E_{il}w(t)\right] \\ z(t) &= \sum_{l=1}^{r_i}\sum_{i=1}^{M} \mu_{il}\theta_i(t)C_{il}x(t). \end{aligned} \tag{12}$$

We now recall standard notations and preliminaries, which will help formulate our main results.

Definition 2 (see [42]). For $u(t) = 0$, $w(t) = 0$ and the initial condition $x(t_0)$, the equilibrium $x = 0$ of system (7) is said to be asymptotically stable under certain switching signal $\sigma(t)$ if there exist constants $\alpha > 0$, $\delta > 0$ such that the solution of the system satisfies $\|x(t)\| < \alpha e^{-\delta t}\|x(t_0)\|$, $\forall t \geq t_0$.

Definition 3 (see [43]). For any switching signal $\sigma(t)$ and any $t_2 \geq t_1 \geq 0$, let $N_\sigma(t_1, t_2)$ denote the number of discontinuities $\sigma(t)$ in the interval (t_1, t_2). We say that $\sigma(t)$ has the average dwell time property if

$$N_\sigma(t_2, t_1) \leq N_0 + \frac{t_2 - t_1}{\tau_a}, \quad N_0 \geq 0, \ \tau_a > 0, \tag{13}$$

holds, where N_0 and τ_a are called the chatter bound and average dwell time, respectively. As commonly used in the literature, we choose $N_0 = 0$.

Definition 4 (see [30]). Given $\gamma > 0$, a system of the form (7) is said to be with an H_∞-norm less than or equal to γ if

$$\int_0^{T_f} z^T(t)z(t)\,dt \leq \gamma^2 \int_0^{T_f} w^T(t)w(t)\,dt \tag{14}$$

holds for $x(0) = 0$. Where T_f is the terminal time of control and $x(0)$ denotes the initial condition of system (7).

Lemma 5 (see [44]). *Given any constant η and any matrices N, Γ, Y of compatible dimensions, then we have*

$$2x^T N\Gamma Y y \leq \eta x^T N N^T x + \frac{1}{\eta} y^T Y^T Y y, \tag{15}$$

for all $x, y \in R^n$, where Γ is an uncertain matric satisfying $\Gamma^T\Gamma \le I$.

Lemma 6 (see [34]). *For a positive scalar ε_0 and the symmetric matrices S_1 and S_2 of compatible dimensions, if the inequalities*

$$S_1 \ge 0, \qquad S_1 + \varepsilon_0 S_2 > 0 \tag{16}$$

hold, then

$$S_1 + \varepsilon S_2 > 0, \quad \forall \varepsilon \in (0, \varepsilon_0]. \tag{17}$$

Lemma 7 (see [34]). *If there exist matrices Z_i $(i = 1, 2, 3)$ with $Z_i = Z_i^T$ $(i = 1, 2)$ satisfying*

$$Z_1 > 0, \qquad \begin{bmatrix} Z_1 & \varepsilon_0 Z_3^T \\ \varepsilon_0 Z_3 & \varepsilon_0 Z_2 \end{bmatrix} > 0, \tag{18}$$

then

$$E(\varepsilon) Z(\varepsilon) = Z^T(\varepsilon) E(\varepsilon) > 0, \quad \forall \varepsilon \in (0, \varepsilon_0], \tag{19}$$

where $Z(\varepsilon) = \begin{bmatrix} Z_1 & \varepsilon Z_3^T \\ Z_3 & Z_2 \end{bmatrix}$.

The problems under consideration are formulated as follows.

Problem 8. Given an H_∞ performance bound $\gamma > 0$ and an upper bound ε_0 for the singular perturbation parameter ε, under admissible switching signals with ADT property, determine a state feedback controller of form (9), such that, for all $\varepsilon \in (0, \varepsilon_0]$, the overall switched closed-loop system (12) is asymptotically stable and with an H_∞-norm less than or equal to γ.

Problem 9. Given an H_∞ performance bound $\gamma > 0$, determine a state feedback controller of the form (9), such that, under admissible switching signals with ADT property, the overall switched closed-loop system (12) is asymptotically stable and with an H_∞-norm less than or equal to γ for any sufficiently small ε.

Problem 10. Given an H_∞ performance bound $\gamma > 0$ and a controller, determine an ε-bound $\varepsilon_{\max}$, as large as possible, such that, for any $\varepsilon \in (0, \varepsilon_{\max}]$, under admissible switching signals with ADT property, the overall switched closed-loop system (12) is asymptotically stable and with an H_∞-norm less than or equal to γ.

Remark 11. The synthesis problems for T-S fuzzy SPSs have attracted much attention of many researchers. H_∞ control and ε-bound design for T-S fuzzy SPSs with pole placement constraints were considered in [34]. This paper will extend the stability analysis and control methods for normal systems to T-S fuzzy SPSSs. Problem 8 considers the stabilization controller design, ε-bound design, and H_∞ control. Problem 9 aims to design a controller without considering the ε-bound. Problem 10 is used to estimate the ε-bound of the switched system.

3. Controller Design

This section will present a controller design method to solve Problem 8.

Theorem 12. *Give an H_∞ performance bound $\gamma > 0$, an upper bound ε_0, and two constants, $\lambda > 0$ and $\mu \ge 1$, if there exist matrices F_{il} $(l = 1, 2, \ldots, r_i)$, Z_{i1}, Z_{i2}, and Z_{i3} of compatible dimensions with $Z_{ik} = Z_{ik}^T$ $(k = 1, 2)$, such that*

$$Z_{i1} > 0, \tag{20}$$

$$\begin{bmatrix} Z_{i1} & * \\ \varepsilon_0 Z_{i3} & \varepsilon_0 Z_{i2} \end{bmatrix} > 0, \tag{21}$$

$$\begin{bmatrix} He\{A_{il}Z_i(0) + B_{il}F_{il}\} + \lambda Z_i^T(0) E(0) & * & * \\ E_{il}^T & -I_r & * \\ C_{il}Z_i(0) & 0 & -\gamma^2 \end{bmatrix} < 0, \tag{22}$$

$$\begin{bmatrix} He\{A_{il}Z_i(\varepsilon_0) + B_{il}F_{il}\} + \lambda Z_i^T(\varepsilon_0) E(\varepsilon_0) & * & * \\ E_{il}^T & -I_r & * \\ C_{il}Z_i(\varepsilon_0) & 0 & -\gamma^2 \end{bmatrix} < 0, \tag{23}$$

$$\begin{bmatrix} \Upsilon_1 & * & * \\ E_{il}^T + E_{is}^T & -2I_r & * \\ C_{il}Z_i(0) + C_{is}Z_i(0) & 0 & -2\gamma^2 \end{bmatrix} < 0, \quad l < s, \tag{24}$$

$$\begin{bmatrix} \Upsilon_2 & * & * \\ E_{il}^T + E_{is}^T & -2I_r & * \\ C_{il}Z_i(\varepsilon_0) + C_{is}Z_i(\varepsilon_0) & 0 & -2\gamma^2 \end{bmatrix} < 0, \quad l < s, \tag{25}$$

$$Z_{j1} \le \mu Z_{i1}, \quad i \ne j, \tag{26}$$

$$\begin{bmatrix} Z_{j1} & * \\ \varepsilon_0 Z_{j3} & \varepsilon_0 Z_{j2} \end{bmatrix} \le \mu \begin{bmatrix} Z_{i1} & * \\ \varepsilon_0 Z_{i3} & \varepsilon_0 Z_{i2} \end{bmatrix}, \quad i \ne j, \tag{27}$$

where $l, s = 1, 2, \ldots, r_i$, $i, j = 1, 2, \ldots, M$, $\Upsilon_1 = He\{A_{il}Z_i(0) + B_{il}F_{is} + A_{is}Z_i(0) + B_{is}F_{il}\} + \lambda Z_i^T(0)E(0)$, $\Upsilon_2 = He\{A_{il}Z_i(\varepsilon_0) + B_{il}F_{is} + A_{is}Z_i(\varepsilon_0) + B_{is}F_{il}\} + \lambda Z_i^T(\varepsilon_0)E(\varepsilon_0)$, and $Z_i(\varepsilon) = \begin{bmatrix} Z_{i1} & \varepsilon Z_{i3}^T \\ Z_{i3} & Z_{i2} \end{bmatrix}$.

Then, for any $\varepsilon \in (0, \varepsilon_0]$, the overall switched closed-loop system (12) with $K_{il}(\varepsilon) = F_{il}Z_i^{-1}(\varepsilon)$ $(l = 1, 2, \ldots, r_i)$, $Z_i(\varepsilon) = U_{i1} + \varepsilon U_{i2}$ is asymptotically stable and with an H_∞-norm less than or equal to γ under any switching signal with ADT

$$\tau_a \ge \frac{\ln^\mu}{\lambda}. \tag{28}$$

Proof. Based on Lemma 6, LMIs (22) and (23) imply that

$$\begin{bmatrix} He\{A_{il}Z_i(\varepsilon) + B_{il}F_{il}\} + \lambda Z_i^T(\varepsilon) E(\varepsilon) & * & * \\ E_{il}^T & -I_r & * \\ C_{il}Z_i(\varepsilon) & 0 & -\gamma^2 \end{bmatrix} < 0, \quad \forall \varepsilon \in (0, \varepsilon_0]. \tag{29}$$

By the Schur complement, inequality (29) is equivalent to

$$He\left\{A_{il}Z_i(\varepsilon)+B_{il}F_{il}\right\}+\lambda Z_i^T(\varepsilon)E(\varepsilon)+E_{il}E_{il}^T+\frac{1}{\gamma^2}Z_i^T(\varepsilon)C_{il}^TC_{il}Z_i(\varepsilon)<0,\quad \forall\varepsilon\in(0,\varepsilon_0]. \tag{30}$$

Pre- and postmultiplying (30) by $Z_i^{-T}(\varepsilon)$ and its transpose, respectively, we obtain

$$He\left\{P_i^T(\varepsilon)\left(A_{il}+B_{il}K_{il}(\varepsilon)\right)\right\}+\lambda E(\varepsilon)P_i(\varepsilon)+P_i^T(\varepsilon)E_{il}E_{il}^TP_i(\varepsilon)+\frac{1}{\gamma^2}C_{il}^T\times C_{il}<0,\quad \forall\varepsilon\in(0,\varepsilon_0], \tag{31}$$

where $K_{il}(\varepsilon)=F_{il}Z_i^{-1}(\varepsilon)$ and $P_i(\varepsilon)=Z_i^{-1}(\varepsilon)$.

Using Lemma 6 again, it follows from (24) and (25) that

$$\begin{bmatrix}\Upsilon_3 & * & *\\ E_{il}^T+E_{is}^T & -2I_r & *\\ C_{il}Z_i(\varepsilon)+C_{is}Z_i(\varepsilon) & 0 & -2\gamma^2\end{bmatrix}<0,\quad \forall\varepsilon\in(0,\varepsilon_0],\ l<s, \tag{32}$$

where $\Upsilon_3=He\{A_{il}Z_i(\varepsilon)+B_{il}F_{is}+A_{is}Z_i(\varepsilon)+B_{is}F_{il}\}+\lambda Z_i^T(\varepsilon)E(\varepsilon)$.

By the Schur complement, inequality (32) can be replaced by the following inequality:

$$\Upsilon_3+\frac{1}{2}\left(E_{il}+E_{is}\right)\left(E_{il}^T+E_{is}^T\right)+\frac{1}{2\gamma^2}\left(C_{il}Z_i(\varepsilon)+C_{is}Z_i(\varepsilon)\right)^T\cdot\left(C_{il}Z_i(\varepsilon)+C_{is}\times Z_i(\varepsilon)\right)<0,\quad \forall\varepsilon\in(0,\varepsilon_0],\ l<s. \tag{33}$$

By using Lemma 5, we get from inequality (33) that

$$\Upsilon_3+He\left\{E_{il}E_{is}^T+\frac{1}{\gamma^2}Z_i^T(\varepsilon)C_{il}^TC_{is}Z_i(\varepsilon)\right\}<0,\quad \forall\varepsilon\in(0,\varepsilon_0],\ l<s. \tag{34}$$

Pre- and postmultiplying (34) by $Z_i^{-T}(\varepsilon)$ and its transpose, respectively, we have

$$He\left\{P_i^T(\varepsilon)\left(A_{il}+B_{il}K_{is}(\varepsilon)+A_{is}+B_{is}K_{il}(\varepsilon)\right)+P_i^T(\varepsilon)E_{il}E_{is}^TP_i(\varepsilon)+\frac{1}{\gamma^2}\times C_{il}^TC_{is}\right\}+\lambda E(\varepsilon)P_i(\varepsilon)<0,\quad \forall\varepsilon\in(0,\varepsilon_0],\ l<s. \tag{35}$$

By Lemma 7, LMI conditions (20) and (21) guarantee that the inequality

$$E(\varepsilon)Z_i(\varepsilon)=Z_i^T(\varepsilon)E(\varepsilon)>0,\quad \varepsilon\in(0,\varepsilon_0], \tag{36}$$

holds, which implies

$$E(\varepsilon)P_i(\varepsilon)=P_i^T(\varepsilon)E(\varepsilon)>0,\quad \varepsilon\in(0,\varepsilon_0]. \tag{37}$$

Define the piecewise Lyapunov function

$$V(t)=V_{\sigma(t)}(x(t))=x^T(t)E(\varepsilon)P_{\sigma(t)}(\varepsilon)x(t),\quad t\geq 0, \tag{38}$$

where $E(\varepsilon)P_{\sigma(t)}(\varepsilon)$ is switched among $E(\varepsilon)P_i(\varepsilon)$, $i=1,2,\ldots,M$, in accordance with the piecewise constant switching signal $\sigma(t)$.

Computing the derivative of $V_i(x(t))$ with respect to t along the trajectories of system (12), we have

$$\begin{aligned}\dot V_i(x(t))&=2x^T(t)E(\varepsilon)P_i(\varepsilon)\dot x(t)=2x^T(t)P_i^T(\varepsilon)\cdot E(\varepsilon)\dot x(t)=\sum_{l=1}^{r_i}\sum_{i=1}^{M}\mu_{il}^2\theta_i(t)x^T(t)He\left\{P_i^T(\varepsilon)\cdot\left(A_{il}+B_{il}K_{il}(\varepsilon)\right)\right\}x(t)\\&+\sum_{l=1}^{r_i}\sum_{l<s}^{r_i}\sum_{i=1}^{M}\mu_{il}\mu_{is}\theta_i(t)x^T(t)\cdot He\left\{P_i^T(\varepsilon)\left(A_{il}+B_{il}K_{is}(\varepsilon)+A_{is}+B_{is}K_{il}(\varepsilon)\right)\right\}\cdot x(t)+\sum_{l=1}^{r_i}\sum_{i=1}^{M}\mu_{il}\theta_i(t)He\left\{x^T(t)P_i^T(\varepsilon)\times E_{il}w(t)\right\},\\&\qquad\varepsilon\in(0,\varepsilon_0].\end{aligned} \tag{39}$$

Using Lemma 5 again, we obtain

$$2x^T(t)P_i^T(\varepsilon)E_{il}w(t)\leq x^T(t)P_i^T(\varepsilon)E_{il}E_{il}^TP_i(\varepsilon)x(t)+w^T(t)w(t),\quad \varepsilon\in(0,\varepsilon_0]. \tag{40}$$

From equality (39) and inequality (40), it follows that

$$\begin{aligned}\dot V_i(x(t))\leq&\sum_{l=1}^{r_i}\sum_{i=1}^{M}\mu_{il}^2\theta_i(t)x^T(t)\left\{He\left\{P_i^T(\varepsilon)\cdot\left(A_{il}+B_{il}K_{il}(\varepsilon)\right)\right\}+P_i^T(\varepsilon)E_{il}E_{il}^T\times P_i(\varepsilon)\right\}x(t)\\&+\sum_{l=1}^{r_i}\sum_{l<s}^{r_i}\sum_{i=1}^{M}\mu_{il}\mu_{is}\theta_i(t)x^T(t)\left\{He\left\{P_i^T(\varepsilon)\cdot\left(A_{il}+B_{il}\times K_{is}(\varepsilon)+A_{is}+B_{is}K_{il}(\varepsilon)\right)\right\}+P_i^T(\varepsilon)\cdot\left(E_{il}E_{is}^T+E_{is}E_{il}^T\right)P_i(\varepsilon)\right\}x(t)+w^T(t)w(t),\\&\qquad\varepsilon\in(0,\varepsilon_0].\end{aligned} \tag{41}$$

It follows from (31), (35), and (41) that

$$\begin{aligned}\dot{V}_i(x(t)) &\le -\lambda V_i(x(t)) - \sum_{l=1}^{r_i}\sum_{i=1}^{M}\mu_{il}^2\theta_i(t)\frac{1}{\gamma^2}x^T(t)\\ &\quad\cdot C_{il}^T C_{il}x(t) - \sum_{l=1}^{r_i}\sum_{l<s}^{r_i}\sum_{i=1}^{M}\mu_{il}\mu_{is}\theta_i(t)\frac{1}{\gamma^2}x^T(t)\\ &\quad\cdot\left(C_{il}^T C_{is} + C_{is}^T C_{il}\right)x(t) + w^T(t)w(t)\\ &= -\lambda V_i(x(t)) - \frac{1}{\gamma^2}z^T(t)z(t) + w^T(t)w(t),\\ &\qquad t \ge 0.\end{aligned} \tag{42}$$

Furthermore, by Lemma 6, LMIs (26) and (27) imply that

$$E(\varepsilon)Z_j(\varepsilon) \le \mu E(\varepsilon)Z_i(\varepsilon), \quad i \ne j, \ \forall\varepsilon \in (0,\varepsilon_0]. \tag{43}$$

Applying the Schur complement to (43) shows that

$$\begin{bmatrix} -Z_j^{-1}(\varepsilon)E^{-1}(\varepsilon) & * \\ I & -\mu E(\varepsilon)Z_i(\varepsilon)\end{bmatrix} \le 0, \quad i \ne j, \ \forall\varepsilon \in (0,\varepsilon_0]. \tag{44}$$

Pre- and postmultiplying (44) by $\left[\begin{smallmatrix} E(\varepsilon) & 0 \\ 0 & I\end{smallmatrix}\right]^T$ and its transpose, respectively, and taking into account the fact that $P_i(\varepsilon) = Z_i^{-1}(\varepsilon)$, inequality (44) is equivalent to

$$\begin{bmatrix} -E^T(\varepsilon)P_j(\varepsilon) & * \\ E(\varepsilon) & -\mu E(\varepsilon)P_i^{-1}(\varepsilon)\end{bmatrix} \le 0, \quad i \ne j, \ \forall\varepsilon \in (0,\varepsilon_0]. \tag{45}$$

By the Schur complement, it follows from (45) that

$$E(\varepsilon)P_i(\varepsilon) \le \mu E(\varepsilon)P_j(\varepsilon), \quad i \ne j, \ \forall\varepsilon \in (0,\varepsilon_0]. \tag{46}$$

Then, the following properties are obtained for (38):

(1) Each $V_i(x(t)) = x^T(t)E(\varepsilon)P_i(\varepsilon)x(t)$ is continuous and its derivative along the trajectories of the corresponding subsystem satisfies (42).

(2) There exist constant scalars $\kappa_1 > 0$, $\kappa_2 > 0$, such that

$$\kappa_1\|x(t)\|^2 \le V_i(x(t)) \le \kappa_2\|x(t)\|^2, \quad \forall x(t) \in \Omega\left(E(\varepsilon)P_i(\varepsilon)\right), \tag{47}$$

where $\kappa_1 = \inf_{i\in S}\lambda_m(E(\varepsilon)P_i(\varepsilon))$, $\kappa_2 = \sup_{i\in S}\lambda_M(E(\varepsilon)P_i(\varepsilon))$.

(3) There exists a constant scalar $\mu \ge 1$ such that (46) holds.

Thus, $V(t)$ is piecewise monotonically decreasing and its value at switching instants is nonincreasing.

Using the differential inequality (42), we obtain that

$$\begin{aligned}V(T_f) &\le V(t_{k+1})e^{-\lambda(T_f-t_{k+1})} + \int_{t_{k+1}}^{T_f} e^{-\lambda(T_f-\tau)}\Gamma(\tau)\,d\tau\\ &\le \mu V(t_{k+1}^{-1})e^{-\lambda(T_f-t_{k+1})} + \int_{t_{k+1}}^{T_f} e^{-\lambda(T_f-\tau)}\Gamma(\tau)\,d\tau\\ &\le \mu\left[V(t_k)e^{-\lambda(t_{k+1}-t_k)} + \int_{t_k}^{t_{k+1}} e^{-\lambda(t_{k+1}-\tau)}\Gamma(\tau)\,d\tau\right]\\ &\quad\cdot e^{-\lambda(T_f-t_{k+1})} + \int_{t_{k+1}}^{T_f} e^{-\lambda(T_f-\tau)}\Gamma(\tau)\,d\tau = \mu V(t_k)\\ &\quad\cdot e^{-\lambda(T_f-t_k)} + \mu\int_{t_k}^{t_{k+1}} e^{-\lambda(T_f-\tau)}\Gamma(\tau)\,d\tau\\ &\quad+ \int_{t_{k+1}}^{T_f} e^{-\lambda(T_f-\tau)}\Gamma(\tau)\,d\tau,\end{aligned} \tag{48}$$

where $\Gamma(\tau) = w^T(\tau)w(\tau) - (1/\gamma^2)z^T(\tau)z(\tau)$.

It follows from (48) that

$$\begin{aligned}V(T_f) &\le \mu^{k+1}e^{-\lambda T_f}V(0) + \mu^{k+1}\int_0^{t_1} e^{-\lambda(T_f-\tau)}\Gamma(\tau)\,d\tau\\ &\quad+ \mu^k\int_{t_1}^{t_2} e^{-\lambda(T_f-\tau)}\times\Gamma(\tau)\,d\tau + \cdots\\ &\quad+ \mu\int_{t_k}^{t_{k+1}} e^{-\lambda(T_f-\tau)}\Gamma(\tau)\,d\tau + \int_{t_{k+1}}^{T_f} e^{-\lambda(T_f-\tau)}\\ &\quad\times\Gamma(\tau)\,d\tau\\ &= \mu^{N_\sigma(0,T_f)}e^{-\lambda T_f}V(0)\\ &\quad+ \int_0^{T_f}\mu^{N_\sigma(\tau,T_f)}e^{-\lambda(T_f-\tau)}\Gamma(\tau)\,d\tau\\ &= e^{-\lambda T_f + N_\sigma(0,T_f)\ln^\mu}V(0)\\ &\quad+ \int_0^{T_f} e^{-\lambda(T_f-\tau)+N_\sigma(\tau,T_f)\ln\mu}\Gamma(\tau)\,d\tau.\end{aligned} \tag{49}$$

The following proof consists of two parts. First, we will show that the overall switched closed-loop system (12) with $w(t) \equiv 0$ is asymptotically stable under any switching signal with ADT (28). Then, we will verify that system (12) is with an H_∞-norm less than or equal to γ.

Part 1. We consider the following average dwell time scheme: for any $T_f > 0$ and a positive scalar $\lambda^\star$ smaller than λ,

$$N_\sigma(0,T_f) \le \frac{T_f}{\tau_a^*}, \quad \tau_a^* = \frac{\ln^\mu}{\lambda^\star}. \tag{50}$$

It follows from (50) that $N_\sigma(0,T_f)\ln^\mu \le \lambda^\star T_f$. Then, from (49), we obtain

$$V(T_f) \le e^{-(\lambda-\lambda^*)T_f}V(0), \tag{51}$$

where $\lambda - \lambda^* > 0$, and one can see from [45] that

$$\left\|x\left(T_f\right)\right\|^2 \leq \max_{i,j\in[1,M]} \frac{\lambda_M\left(E\left(\varepsilon\right)P_i\left(\varepsilon\right)\right)}{\lambda_m\left(E\left(\varepsilon\right)P_j\left(\varepsilon\right)\right)} e^{-(\lambda-\lambda^*)T_f}\left\|x\left(0\right)\right\|^2, \tag{52}$$

which indicates that system (12) with $w(t) \equiv 0$ is asymptotically stable under any switching signal with ADT (28).

Part 2. Similar to Part 1, for a positive scalar λ^* smaller than λ, the inequality

$$N_\sigma\left(\tau, T_f\right)\ln\mu \leq \lambda^*\left(T_f - \tau\right) \tag{53}$$

holds.

Taking into account the fact that $x(0) = 0$ and $V(T_f) \geq 0$, it follows from (49) that

$$\int_0^{T_f} e^{-\lambda(T_f-\tau)+N_\sigma(\tau,T_f)\ln\mu}\Gamma\left(\tau\right)d\tau \geq 0. \tag{54}$$

It follows from (53) and (54) that

$$\int_0^{T_f} e^{-(\lambda-\lambda^*)(T_f-\tau)}\Gamma\left(\tau\right)d\tau \geq 0, \tag{55}$$

which implies that $\int_0^{T_f} z^T(t)z(t)dt \leq \gamma^2\int_0^{T_f} w^T(t)w(t)dt$. This completes the proof. □

Remark 13. $Z_i(\varepsilon) = \left[\begin{smallmatrix} Z_{i1} & \varepsilon Z_{i3} \\ Z_{i3} & Z_{i2}\end{smallmatrix}\right] = U_{i1} + \varepsilon U_{i2}$, $\varepsilon \in (0, \varepsilon_0]$, where $U_{i1} = \left[\begin{smallmatrix} Z_{i1} & 0 \\ Z_{i3} & Z_{i2}\end{smallmatrix}\right]$, $U_{i2} = \left[\begin{smallmatrix} 0 & Z_{i3} \\ 0 & 0\end{smallmatrix}\right]$. It follows from LMIs (20) and (21) that $Z_{i1} > 0$ and $Z_{i2} > 0$, which imply that the matrices $Z_i(0) = U_{i1}$ are nonsingular. So, the matrices $Z_i(\varepsilon)$ are nonsingular for all $\varepsilon \in (0, \varepsilon_0]$. This nonsingularity can ensure that $K_{il}(\varepsilon) = F_{il}Z_i^{-1}(\varepsilon)$ $(l = 1, 2, \ldots, r_i)$ always work well for all $\varepsilon \in (0, \varepsilon_0]$. For sufficiently small ε, the ε-dependent controller is reduced to an ε-independent one, since $\lim_{\varepsilon\to 0^+} K_{il} = F_{il}U_{i1}^{-1}$ $(l = 1, 2, \ldots, r_i)$.

Remark 14. The multiple Lyapunov functions method has been widely used in control design of switched systems [36–39]. By employing the average dwell time scheme, the problem of extended dissipative state estimation for a class of discrete-time Markov jump neural networks with unreliable links was addressed in [36]. In this paper, the ε-dependent piecewise Lyapunov function will be constructed to solve H_∞ control problem for T-S fuzzy SPSSs.

Remark 15. Theorem 12 is concerned with the situation that ε_0 is known according to prior information. Moreover, the bisectional search algorithm developed in [46] can be used to derive the ε-bound.

Remark 16. ε-bound, which is an essential index of SPSs, has attained much attention. ε-bound was considered in [40, 47]. Both results were derived by constructing a common Lyapunov function, which may lead to conservatism in some cases. In [41], the piecewise Lyapunov function was constructed and ε-bound estimation problem was solved for T-S fuzzy SSPSs without the external disturbance input.

By Theorem 12, under the assumption that the singular perturbation parameter ε is known, sufficient conditions for both stability and H_∞ performance of system (12) are derived. In the following theorem, for sufficiently small and unknown ε, the above sufficient conditions are generalized to design the ε-independent controller.

Theorem 17. *Given an H_∞ performance bound $\gamma > 0$, two constants, $\lambda > 0$ and $\mu \geq 1$, if there exist matrices F_{il}, Z_{i1}, Z_{i2}, and Z_{i3} of compatible dimensions with $Z_{ik} = Z_{ik}^T$ $(k = 1, 2)$, such that*

$$Z_{i1} > 0, \tag{56}$$

$$Z_{i2} > 0, \tag{57}$$

$$\begin{bmatrix} He\left\{A_{il}Z_i\left(0\right) + B_{il}F_{il}\right\} + \lambda Z_i^T\left(0\right)E\left(0\right) & * & * \\ E_{il}^T & -I_r & * \\ C_{il}Z_i\left(0\right) & 0 & -\gamma^2 \end{bmatrix} < 0, \tag{58}$$

$$\begin{bmatrix} \overline{\Upsilon}_1 & * & * \\ E_{il}^T + E_{is}^T & -2I_r & * \\ C_{il}Z_i\left(0\right) + C_{is}Z_i\left(0\right) & 0 & -2\gamma^2 \end{bmatrix} < 0, \quad l < s, \tag{59}$$

$$Z_{j1} \leq \mu Z_{i1}, \quad i \neq j, \tag{60}$$

$$Z_{j2} \leq \mu Z_{i2}, \quad i \neq j, \tag{61}$$

where $l, s = 1, 2, \ldots, r_i$, $i, j = 1, 2, \ldots, M$, $\overline{\Upsilon}_1 = He\{A_{il}Z_i(0) + B_{il}F_{is} + A_{is}Z_i(0) + B_{is}F_{il}\} + \lambda Z_i^T(0)E(0)$, $Z_i(0) = \left[\begin{smallmatrix} Z_{i1} & 0 \\ Z_{i3} & Z_{i2}\end{smallmatrix}\right]$, and $E(0) = \left[\begin{smallmatrix} 1 & * \\ 0 & 0\end{smallmatrix}\right]$.

Then there exists a positive scalar $\varepsilon_{\max}$ such that, for all $\varepsilon \in (0, \varepsilon_{\max}]$, the overall switched closed-loop system (12) with the controller gains of the form $K_{il} = F_{il}Z_i^{-1}(0)$ $(l = 1, 2, \ldots, r_i)$ is asymptotically stable and with an H_∞-norm less than or equal to γ under any switching signal with ADT

$$\tau_a \geq \frac{\ln^\mu}{\lambda}. \tag{62}$$

Proof. For sufficiently small ε, LMI conditions (20)–(27) in Theorem 12 can be reduced to LMI conditions (56)–(61) in Theorem 17. Thus, we omit the proof of Theorem 17 that can be carried out by referring to the standard techniques used in Theorem 12. □

ε-bound is an essential stability index of SPSs. Theorem 17 ensures the existence for ε-bound $\varepsilon_{\max}$. In the following theorem, we will propose a method to estimate the ε-bound of the closed-loop system with the obtained controllers in Theorem 17.

Theorem 18. *Give an H_∞ performance bound $\gamma > 0$, an upper bound $\varepsilon_{\max}$, controller gains K_{il}, and two constants, $\lambda > 0$ and $\mu \geq 1$, if there exist matrices Z_{i1}, Z_{i2}, and Z_{i3} of compatible dimensions with $Z_{ik} = Z_{ik}^T$ $(k = 1, 2)$, such that*

$$Z_{i1} > 0, \tag{63}$$

$$\begin{bmatrix} Z_{i1} & * \\ \varepsilon_{\max} Z_{i3} & \varepsilon_{\max} Z_{i2} \end{bmatrix} > 0, \tag{64}$$

$$\begin{bmatrix} He\left\{\left(A_{il} + B_{il}K_{il}\right) Z_i(0)\right\} + \lambda Z_i^T(0) E(0) & * & * \\ E_{il}^T & -I_r & * \\ C_{il} Z_i(0) & 0 & -\gamma^2 \end{bmatrix} < 0, \tag{65}$$

$$\begin{bmatrix} He\left\{\left(A_{il} + B_{il}K_{il}\right) Z_i(\varepsilon_{\max})\right\} + \lambda Z_i^T(\varepsilon_{\max}) E(\varepsilon_{\max}) & * & * \\ E_{il}^T & -I_r & * \\ C_{il} Z_i(\varepsilon_{\max}) & 0 & -\gamma^2 \end{bmatrix} < 0, \tag{66}$$

$$\begin{bmatrix} \widetilde{Y}_1 & * & * \\ E_{il}^T + E_{is}^T & -2I_r & * \\ C_{il} Z_i(0) + C_{is} Z_i(0) & 0 & -2\gamma^2 \end{bmatrix} < 0, \quad l < s, \tag{67}$$

$$\begin{bmatrix} \widetilde{Y}_2 & * & * \\ E_{il}^T + E_{is}^T & -2I_r & * \\ C_{il} Z_i(\varepsilon_{\max}) + C_{is} Z_i(\varepsilon_{\max}) & 0 & -2\gamma^2 \end{bmatrix} < 0, \quad l < s, \tag{68}$$

$$Z_{j1} \le \mu Z_{i1}, \quad i \ne j, \tag{69}$$

$$\begin{bmatrix} Z_{j1} & * \\ \varepsilon_{\max} Z_{j3} & \varepsilon_{\max} Z_{j2} \end{bmatrix} \le \mu \begin{bmatrix} Z_{i1} & * \\ \varepsilon_{\max} Z_{i3} & \varepsilon_{\max} Z_{i2} \end{bmatrix}, \quad i \ne j, \tag{70}$$

where $l, s = 1, 2, \ldots, r_i$, $i, j = 1, 2, \ldots, M$, $\widetilde{Y}_1 = He\{(A_{il} + B_{il}K_{is} + A_{is} + B_{is}K_{il})Z_i(0)\} + \lambda Z_i^T(0)E(0)$, $\widetilde{Y}_2 = He\{(A_{il} + B_{il}K_{is} + A_{is} + B_{is}K_{il})Z_i(\varepsilon_{\max})\} + \lambda Z_i^T(\varepsilon_{\max})E(\varepsilon_{\max})$, *and* $Z_i(\varepsilon) = \begin{bmatrix} Z_{i1} & \varepsilon Z_{i3}^T \\ Z_{i3} & Z_{i2} \end{bmatrix}$.

Then, for all $\varepsilon \in (0, \varepsilon_{\max}]$, *the overall switched closed-loop system (12) is asymptotically stable and with an* H_∞*-norm less than or equal to* γ *under any switching signal with ADT*

$$\tau_a \ge \frac{\ln^\mu}{\lambda}. \tag{71}$$

4. Example

To illustrate the proposed results, we consider the well-known inverted pendulum system. The equations of motion for the pendulum are given by

$$\begin{aligned} \dot{x}_1(t) &= x_2(t) + 0.1w(t) \\ \dot{x}_2(t) &= \frac{g\sin(x_1(t)) - aml x_2^2(t)\sin(2x_1(t))/2 - a\cos(x_1(t))u(t)}{4l/3 - aml\cos^2(x_1(t))} \\ &\quad + 0.1w(t) \\ z(t) &= 0.1x_1(t) + 0.1x_2(t), \end{aligned} \tag{72}$$

where $x_1(t) = \theta_p(t)$ denotes the angle of the pendulum from the vertical upward, $x_2(t) = \dot{\theta}_p(t)$, g is the gravity acceleration, $a = 1/(m + M)$, m and M are the masses of the pendulum and the cart, respectively, l is the length of the pendulum, u is a horizontal force applied to the cart, and $w(t)$ is the external disturbance variable, which is a piecewise function of time of the form

$$w(t) = \begin{cases} \sin(2\pi t) & 0 \le t < 5 \\ \sin(2\pi t)\, e^{-0.5t} & t \ge 5, \end{cases} \tag{73}$$

$z(t)$ is the controlled output. The parameters for the plant are as follows: $g = 9.8\,\mathrm{m/s^2}$, $m = 2\,\mathrm{Kg}$, $M = 8\,\mathrm{Kg}$, and $l = 0.5\,\mathrm{m}$.

The angle of the pendulum $[-30^\circ \quad 30^\circ]$ is divided into two areas R_1 and R_2, where R_1 is $|x_1| \le 15^\circ$; R_2 is $15^\circ < |x_1| \le 30^\circ$, which results in two fuzzy subsystems [39].

For the individual system, we choose the membership functions of the fuzzy sets as follows.

Mode 1

$$\begin{aligned} M_{11}(x_1(t)) &= 1 - \frac{|x_1(t)|}{15} \\ M_{12}(x_1(t)) &= \frac{|x_1(t)|}{15}. \end{aligned} \tag{74}$$

Mode 2

$$M_{21}(x_1(t)) = \frac{|x_1(t)|}{15} - 1$$
$$M_{22}(x_1(t)) = 2 - \frac{|x_1(t)|}{15}. \tag{75}$$

Then, the dynamics of Mode 1 can be exactly represented by the following T-S fuzzy model under $|x_1(t)| \le 15^\circ$:

$$\begin{aligned}
&\text{Plant Rule 1:}\\
&\text{IF } x_1(t) \text{ is } M_{11}(x_1(t)), \text{ THEN}\\
&\dot{x}(t) = A_{11}x(t) + B_{11}u(t) + E_{11}w(t)\\
&z(t) = C_{11}x(t)\\
&\text{Plant Rule 2:}\\
&\text{IF } x_1(t) \text{ is } M_{12}(x_1(t)), \text{ THEN}\\
&\dot{x}(t) = A_{12}x(t) + B_{12}u(t) + E_{12}w(t),\\
&z(t) = C_{12}x(t),
\end{aligned} \tag{76}$$

where

$$\begin{aligned}
A_{11} &= \begin{bmatrix} 0 & 1 \\ 9.0673 & 0 \end{bmatrix},\\
B_{11} &= \begin{bmatrix} 0 \\ -0.0688 \end{bmatrix},\\
E_{11} &= \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix},\\
C_{11} &= \begin{bmatrix} 0.1 & 0.1 \end{bmatrix},\\
A_{12} &= \begin{bmatrix} 0 & 1 \\ 10.5717 & 0 \end{bmatrix},\\
B_{12} &= \begin{bmatrix} 0 \\ -0.0779 \end{bmatrix},\\
E_{12} &= \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix},\\
C_{12} &= \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}.
\end{aligned} \tag{77}$$

The dynamics of Mode 2 can be exactly represented by the following T-S fuzzy model under $15^\circ < |x_1| \le 30^\circ$:

$$\begin{aligned}
&\text{Plant Rule 1:}\\
&\text{IF } x_1(t) \text{ is } M_{21}(x_1(t)), \text{ THEN}\\
&\dot{x}(t) = A_{21}x(t) + B_{21}u(t) + E_{21}w(t)\\
&z(t) = C_{21}x(t)\\
&\text{Plant Rule 2:}\\
&\text{IF } x_1(t) \text{ is } M_{22}(x_1(t)), \text{ THEN}\\
&\dot{x}(t) = A_{22}x(t) + B_{22}u(t) + E_{22}w(t),\\
&z(t) = C_{22}x(t),
\end{aligned} \tag{78}$$

where

$$\begin{aligned}
A_{21} &= \begin{bmatrix} 0 & 1 \\ 10.5717 & 0 \end{bmatrix},\\
B_{21} &= \begin{bmatrix} 0 \\ -0.0779 \end{bmatrix},\\
E_{21} &= \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix},\\
C_{21} &= \begin{bmatrix} 0.1 & 0.1 \end{bmatrix},\\
A_{22} &= \begin{bmatrix} 0 & 1 \\ 11.1243 & 0 \end{bmatrix},\\
B_{22} &= \begin{bmatrix} 0 \\ -0.0811 \end{bmatrix},\\
E_{22} &= \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix},\\
C_{22} &= \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}.
\end{aligned} \tag{79}$$

Choosing $\varepsilon = 0.1$, the above switched system can be modeled by a SPSS (1) with the following.

Mode 1

$$\begin{aligned}
E(\varepsilon) &= \begin{bmatrix} 1 & * \\ 0 & \varepsilon \end{bmatrix},\\
\overline{A}_{11} &= \begin{bmatrix} 0 & 1 \\ 0.90673 & 0 \end{bmatrix},\\
\overline{B}_{11} &= \begin{bmatrix} 0 \\ -0.00688 \end{bmatrix},\\
\overline{E}_{11} &= \begin{bmatrix} 0.1 \\ 0.01 \end{bmatrix},\\
E(\varepsilon) &= \begin{bmatrix} 1 & * \\ 0 & \varepsilon \end{bmatrix},\\
\overline{A}_{12} &= \begin{bmatrix} 0 & 1 \\ 1.05717 & 0 \end{bmatrix},\\
\overline{B}_{12} &= \begin{bmatrix} 0 \\ -0.00779 \end{bmatrix},\\
\overline{E}_{12} &= \begin{bmatrix} 0.1 \\ 0.01 \end{bmatrix}.
\end{aligned} \tag{80}$$

Mode 2

$$E(\varepsilon) = \begin{bmatrix} 1 & * \\ 0 & \varepsilon \end{bmatrix},$$
$$\overline{A}_{21} = \begin{bmatrix} 0 & 1 \\ 1.05717 & 0 \end{bmatrix},$$
$$\overline{B}_{21} = \begin{bmatrix} 0 \\ -0.00779 \end{bmatrix},$$
$$\overline{E}_{21} = \begin{bmatrix} 0.1 \\ 0.01 \end{bmatrix},$$
$$E(\varepsilon) = \begin{bmatrix} 1 & * \\ 0 & \varepsilon \end{bmatrix},$$
$$\overline{A}_{22} = \begin{bmatrix} 0 & 1 \\ 1.11243 & 0 \end{bmatrix},$$
$$\overline{B}_{22} = \begin{bmatrix} 0 \\ -0.00811 \end{bmatrix},$$
$$\overline{E}_{22} = \begin{bmatrix} 0.1 \\ 0.01 \end{bmatrix}. \tag{81}$$

The fuzzy controller is described as follows.

Mode 1

$$\begin{aligned} &\text{Plant Rule 1:} \\ &\text{IF } x_1(t) \text{ is } M_{11}(x_1(t)), \text{ THEN} \\ &u(t) = K_{11}(\varepsilon)\, x(t) \\ &\text{Plant Rule 2:} \\ &\text{IF } x_1(t) \text{ is } M_{12}(x_1(t)), \text{ THEN} \\ &u(t) = K_{12}(\varepsilon)\, x(t). \end{aligned} \tag{82}$$

Mode 2

$$\begin{aligned} &\text{Plant Rule 1:} \\ &\text{IF } x_2(t) \text{ is } M_{21}(x_1(t)), \text{ THEN} \\ &u(t) = K_{21}(\varepsilon)\, x(t) \\ &\text{Plant Rule 2:} \\ &\text{IF } x_2(t) \text{ is } M_{22}(x_1(t)), \text{ THEN} \\ &u(t) = K_{22}(\varepsilon)\, x(t). \end{aligned} \tag{83}$$

Taking $\varepsilon = 0.1$, $\lambda = 10$, and $\mu = 20$ and solving the LMIs in Theorem 12, we obtain the stabilization controller gains:

$$K_{11} = [7.3418 * 10^3 \quad 5.7720 * 10^2],$$
$$K_{12} = [6.6701 * 10^3 \quad 5.2617 * 10^2],$$
$$K_{21} = [6.4503 * 10^3 \quad 4.8956 * 10^2],$$
$$K_{22} = [6.2518 * 10^3 \quad 4.7497 * 10^2]. \tag{84}$$

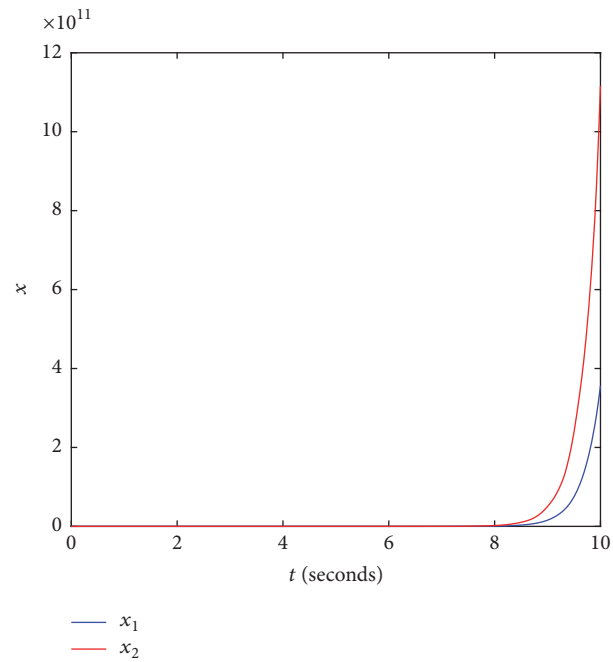

Figure 1: State trajectories without the controller.

Taking $\lambda = 10$ and $\mu = 20$ and solving the LMIs in Theorem 17, we obtain the stabilization controller gains:

$$\overline{K}_{11} = [1.0908 * 10^4 \quad 8.6334 * 10^2],$$
$$\overline{K}_{12} = [1.0237 * 10^4 \quad 8.1009 * 10^2],$$
$$\overline{K}_{21} = [9.8518 * 10^3 \quad 7.7925 * 10^2],$$
$$\overline{K}_{22} = [9.6513 * 10^3 \quad 7.6329 * 10^2]. \tag{85}$$

Under the controller obtained by Theorem 17, the ε-bound of the closed-loop system is $\varepsilon_{\max} = 0.5493$ by using Theorem 18 and the bisectional search algorithm developed in [46].

To illustrate the proposed method, we first consider the simulation of system (10) without the controller and then apply the designed controller to system (10).

Choosing $\varepsilon = 0.1$, $x_1(0) = 0^\circ$, $x_2(0) = 0^\circ$, $\lambda = 10$, $= 20$, $\gamma = 1$, and any switching signal with ADT (71), the simulation result without the controller is shown in Figure 1. It can be seen from Figure 1 that system (10) is not stable. Applying the fuzzy controller obtained by Theorem 12 to the original system, the sate trajectories of the overall switched closed-loop system are shown in Figure 2 and the ratio of the output energy to the disturbance input energy, that is, $\int_0^{T_f} z^T(s)z(s)ds / \int_0^{T_f} w^T(s)w(s)ds$, is depicted in Figure 3. It is easy to find that after 5 seconds the ratio of the output energy to the disturbance input energy is fixed at a constant

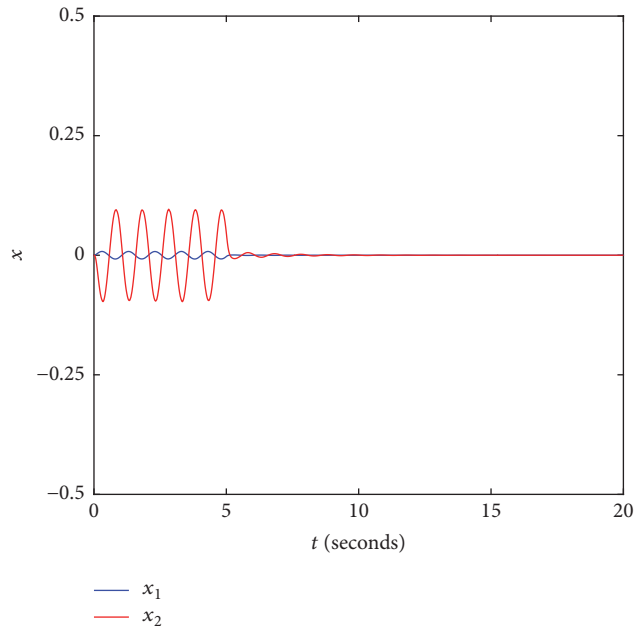

Figure 2: State trajectories.

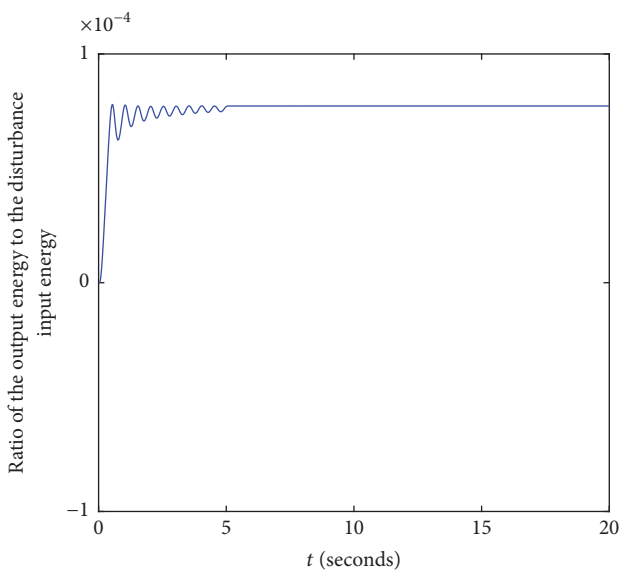

Figure 3: The ratio of the output energy to the disturbance input energy.

value, which is about $7.7265 * 10^{-5}$. So $\gamma = \sqrt{7.7265 * 10^{-5}} = 8.79 * 10^{-3}$, which is less than the prescribed value 1.

5. Conclusion

In this paper, we are concerned with the design of fuzzy controller with guaranteed H_∞ performance for T-S fuzzy SPSSs. An LMI-based method of designing an ε-dependent controller has been proposed. Through this method, the obtained controller can work well for any $\varepsilon \in (0, \varepsilon_0]$. This controller guarantees that, for a given upper bound ε_0 for ε and a prespecified H_∞ performance bound $\gamma > 0$, under admissible switching signals, the switched system is asymptotically stable and with an H_∞-norm less than or equal to γ. Then, for sufficiently small ε, the ε-independent feedback controller has been developed. Furthermore, under this controller, the ε-bound estimation problem of the switched system has been solved. The involved example has shown the feasibility and effectiveness of the obtained results.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This work was supported by the National Natural Science Foundation of China (61374043, 61603392, 61603393, and 61503384), Nature Science Foundation of Jiangsu Province (BK20160275, BK2010275, and BK20150199), the Open Project Foundation of State Key Laboratory of Synthetical Automation for Process Industries under Grant PAL-N201706, Postdoctoral Science Foundation of China (2015M581885), and Ordinary University Graduate Student Scientific Research Innovation Projects of Jiangsu Province (KYLX16−0533).

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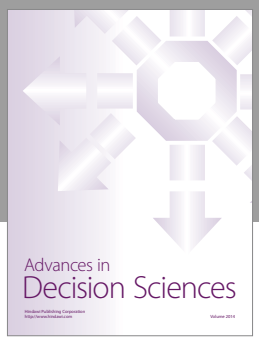

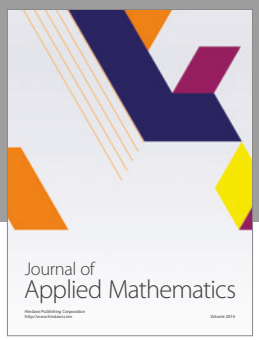

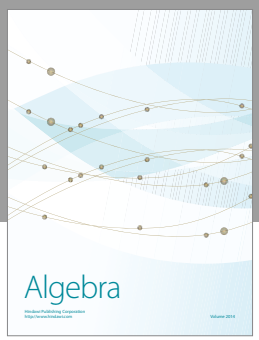

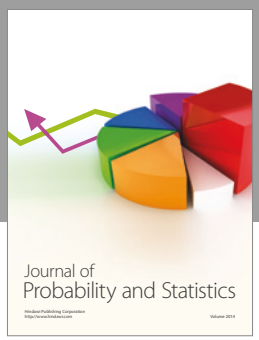

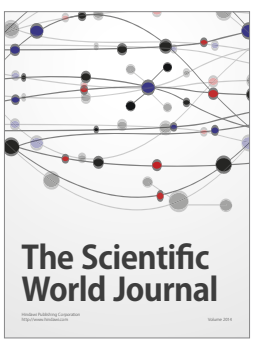

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