Research Article

$H_\infty$ Control for T-S Fuzzy Singularly Perturbed Switched Systems

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This paper is concerned with the design of fuzzy controller with guaranteed $H_\infty$ performance for a class of Takagi-Sugeno (T-S) fuzzy singularly perturbed switched systems. First, by using the average dwell time approach together with the piecewise Lyapunov function technique, a state feedback controller that depends on the singular perturbation parameter $\varepsilon$ is developed. This controller is shown to work well for all $\varepsilon \in (0, \varepsilon_0]$. Then, for sufficiently small $\varepsilon$, an $\varepsilon$-independent controller design method is proposed. Furthermore, under the $\varepsilon$-independent controller, the $\varepsilon$-bound estimation problem of the overall switched closed-loop system is solved. Finally, an inverted pendulum system is used to evaluate the feasibility and effectiveness of the obtained results.

1. Introduction

Many practical systems possess multiple time scale characteristics [1–4]. It is well known that control methods for normal systems can not be directly applied to this class of systems, since these methods may cause ill-conditioned numerical problems. To conquer these problems, singular perturbation methods have been widely used in control design of multiple time scale systems (see [5–9] and the references therein).

Singularly perturbed systems (SPSs), whose partial time derivatives involve a small singular perturbation parameter $\varepsilon$, have been widely investigated by many researchers. It is important to obtain the $\varepsilon$-bound such that stability and other performances of SPSs can be ensured. Studies on $\varepsilon$-bound can be mainly divided into two types: one only presents sufficient conditions for the existence of the $\varepsilon$-bound [10, 11], and the other proposes methods to estimate the $\varepsilon$-bound [9, 12].

Singularly perturbed switched systems (SPSSs) consist of a group of SPSs and a certain switching law, which specifies the active SPS at the switching instance. In practical processes, a great number of control systems, whose behavior is simultaneously determined by multiple time scales and switching, can be modeled as SPSSs [13–15]. The control model of the hot strip mill was treated as a SPS and an $H_2$ robust controller was designed in [15]. Recently, SPSSs have attained much attention in the literature (see [16–20]). It has been shown in [17–20] that the stability of fast/slow switched subsystems can not guarantee the stability of the original switched systems. In order to guarantee the system stability under an arbitrary switching signal, a common Lyapunov function for individual SPSs or a dwell time scheme has to be considered. In [19], by combining the multiple Lyapunov functions method with the dwell time scheme, sufficient conditions for ensuring exponential stability of time-delay SPSSs with stable fast switched subsystems were derived. These conditions were described by a few $\varepsilon$-dependent algebraic inequalities, which led to a heavy and tedious calculation. The proposed method in [19] was further extended to time-delay SPSSs with impulsive effects [20]. However, few results on the $\varepsilon$-bound estimation are available for SPSSs. The exceptions were given by [21, 22]. A convex optimization based method was presented to make the best estimate of $\varepsilon$-bound for SPSSs whose fast switched subsystems can not depend on the switching signal in [21]. The $\varepsilon$-dependent stabilization and $\varepsilon$-bound problems were addressed by adopting dwell time switching signal and constructing $\varepsilon$-dependent multiple Lyapunov functions for SPSSs in [22].

Fuzzy control has been widely used for engineering practice [23, 24]. The Takagi-Sugeno (T-S) model, which has elegant ability to approximate a certain class of complex
nonlinear functions, was extensively utilized in control system design [25–27]. Based on T-S fuzzy control, many LMI-based fuzzy control design methods have been developed for SPSSs. The stabilization problem was addressed for fuzzy SPSSs in [28], and some LMI-based control algorithms were proposed. Over past a few years, $H_{\infty}$ control for fuzzy SPSSs has attained a lot of attention. $H_{\infty}$ control was addressed for fuzzy SPSSs in [11, 29–31], and sufficient conditions independent of $\epsilon$ for the existence of $H_{\infty}$ controller were presented. The resulting conditions in [11, 28–31] are only applicable to stabilizing the system and being with an $H_{\infty}$ performance for sufficiently small $\epsilon$. Hence, some researchers have concentrated on the $\epsilon$-bound design problem for fuzzy SPSSs [32–34].

As an important class of hybrid systems, switched fuzzy systems have been a hot spot of present research. Recently, a great number of theoretical results are available for switched fuzzy systems. By using a common or multiple Lyapunov functions method, stability issues for switched fuzzy systems were considered in [35–39]. However, the existing literature on fuzzy SPSSs is rather limited. An LMI-based dynamic state feedback control method was given and the $\epsilon$-bound problem of system was solved for nonaffine-in-control SPSSs [40]. This method decomposed the switched system into fast and slow subsystems and can not be suitable for nonstandard fuzzy SPSSs. In our previous paper [41], where the stabilization and $\epsilon$-bound problems were addressed for T-S fuzzy SPSSs without the external disturbance input by constructing the $\epsilon$-dependent piecewise Lyapunov function. Moreover, to the authors’ best knowledge, $H_{\infty}$ control for fuzzy SPSSs has not been addressed yet.

In this paper, we will investigate the design of fuzzy controller with guaranteed $H_{\infty}$ performance for a class of T-S fuzzy SPSSs. The problem is composed of stabilization control, $H_{\infty}$ control, and $\epsilon$-bound design. First, for a given upper bound $\varepsilon_0$ for $\varepsilon$ and a prespecified $H_{\infty}$ performance bound $\gamma > 0$, an $\epsilon$-dependent controller is developed, such that, for any $\varepsilon \in (0, \varepsilon_0]$, the switched system is asymptotically stable and the $L_2$-gain from the disturbance input to the controlled output is less than or equal to $\gamma$. This controller is shown to work well for all $\varepsilon \in (0, \varepsilon_0]$. Then, an $\epsilon$-independent controller design method is proposed in terms of LMIs. Furthermore, under the $\epsilon$-independent controller, the $\epsilon$-bound estimation approach is given. Finally, an inverted pendulum system is used to evaluate the feasibility and effectiveness of the obtained results.

The rest of this paper is organized as follows. In Section 2, the problems to be considered are formulated and preliminaries are presented. The main results are given in Section 3. An example is given in Section 4 to illustrate the obtained methods. And Section 5 concludes the paper. The notations used in this paper are standard. The notations $T$ and $*$ stand for the matrix transpose and the transpose of the off diagonal element of the LMI, respectively. $\lambda_M(Q)$ and $\lambda_m(Q)$ denote the maximal and minimal eigenvalues of a symmetric matrix $Q$, respectively. $\| \cdot \|$ denotes Euclidean norm for vectors or the spectral norm of matrices. $H[\cdot]$ is defined as $H[\Theta] = \Theta + \Theta^T$ for a square matrix $\Theta$.

### 2. Problem Formulation

Consider a T-S fuzzy SPSS, which involves $r_{\sigma(t)}$ rules of the following form.

The $l$th rule is

**Plant Rule $l$:**

IF $v_1(t) \in M_{\sigma_l(1)}$, $v_2(t) \in M_{\sigma_l(2)}$, ..., $v_{\phi}(t) \in M_{\sigma_l(\phi)}$ THEN

$$E(\varepsilon)x(t) = A_{\sigma_l(\epsilon)}x(t) + B_{\sigma_l(\epsilon)}u(t) + E_{\sigma_l(\epsilon)}w(t)$$

$$z(t) = C_{\sigma_l(\epsilon)}x(t)$$

for $l = 1, 2, \ldots, r_{\sigma(t)}$

where $E(\varepsilon) = \begin{bmatrix} I_{n_1} & 0 \\ 0 & I_{n_2} \end{bmatrix}$, $\varepsilon$ is a small positive scalar which represents the singular perturbation parameter. $\sigma(t)$ is a piecewise constant function with respect to time, referred as to a switching signal, which takes its values in the finite set $S = \{1, 2, \ldots, M\}$. $M$ represents the number of individual subsystems. $M_{l}^{m}(t)$ ($l = 1, 2, \ldots, r_{\sigma(t)}$, $m = 1, 2, \ldots, \phi$) are fuzzy sets, $r_{\sigma(t)}$ is the number of fuzzy rules, $u(t) = [v_1(t) \ v_2(t) \ \cdots \ v_{\phi}(t)]^T$ is the premise vector that may depend on states in many cases, $\phi$ is the number of premise variables, $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the control input, $w(t) \in R^p$ is the disturbance input that belongs to $L_2(0, \infty)$, and $z(t) \in R^q$ is the controlled output. For any given $T_f$, a time sequence of $t_1 < \cdots < t_{k+1}$ ($k \geq 1$) is labeled as the switching instants over the interval $(0, T_f)$. It means that the $k$th T-S fuzzy subsystem is active as $t \in [t_k, t_{k+1})$. And $A_{\sigma_l(\epsilon)}, B_{\sigma_l(\epsilon)}, E_{\sigma_l(\epsilon)},$ and $C_{\sigma_l(\epsilon)}$ are of the following form:

$$A_{il} = \begin{bmatrix} A_{il1} & A_{il2} \\ A_{il21} & A_{il22} \end{bmatrix},$$

$$B_{il} = \begin{bmatrix} B_{il1} \\ B_{il2} \end{bmatrix},$$

$$E_{il} = \begin{bmatrix} E_{il1} \\ E_{il2} \end{bmatrix},$$

$$C_{il} = \begin{bmatrix} C_{il1} & C_{il2} \end{bmatrix},$$

$i \in S$.

Denote

$$\xi_{\sigma(t)}(v(t)) = \prod_{k=1}^\phi M_{l\sigma(t)(k)}^I(v_k(t)), \ l = 1, 2, \ldots, r_{\sigma(t)}$$

where $M_{\sigma_l(k)}^I(v_k(t))$ is the grade of membership of $v_k(t)$ in $M_{\sigma_l(k)}^I$.
It is assumed in this paper that
\[ \xi_{\sigma(t)}(v(t)) \geq 0, \]
\[ \sum_{l=1}^{r_{\sigma(t)}} \xi_{\sigma(t)}(v(t)) > 0, \quad l = 1, 2, \ldots, r_{\sigma(t)} \geq 0. \]  
(4)

Let
\[ \mu_{\sigma(t)}(v(t)) = \frac{\xi_{\sigma(t)}(v(t))}{\sum_{l=1}^{r_{\sigma(t)}} \xi_{\sigma(t)}(v(t))}, \quad l = 1, 2, \ldots, r_{\sigma(t)}. \]  
(5)

Then
\[ \mu_{\sigma(t)}(v(t)) \geq 0, \]
\[ \sum_{l=1}^{r_{\sigma(t)}} \mu_{\sigma(t)}(v(t)) = 1, \quad l = 1, 2, \ldots, r_{\sigma(t)} \geq 0. \]  
(6)

For the convenience of notations, we denote \( \mu_{\sigma(t)}(v(t)) \). Then, the \( i \)-th T-S fuzzy subsystem can be inferred as

\[ E(e) \dot{x}(t) = \sum_{l=1}^{r_i} \mu_{il} [A_{il} x(t) + B_{il} u(t) + E_{il} w(t)] \]
\[ z(t) = \sum_{l=1}^{r_i} \mu_{il} C_{il} x(t). \]  
(7)

Substituting (9) into (7) yields the closed-loop system

\[ E(e) \dot{x}(t) = \sum_{l=1}^{r_i} \sum_{i=1}^{M} \mu_{il} \theta_i(t) \]
\[ \cdot [(A_{il} + B_{il} K_{il}(e)) x(t) + E_{il} w(t)] \]
\[ z(t) = \sum_{l=1}^{r_i} \mu_{il} C_{il} x(t). \]  
(10)

Upon introducing the indicator function
\[ \theta(t) = [\theta_1(t), \ldots, \theta_M(t)], \]  
(11)

where \( \theta_i(t) = 1 \) if the switching signal is in mode \( i \) and \( \theta_i(t) = 0 \) if it is in a different mode, and the overall switched closed-loop system can be expressed as follows:

\[ E(e) \dot{x}(t) = \sum_{l=1}^{r_i} \sum_{i=1}^{M} \mu_{il} \theta_i(t) \]
\[ \cdot [(A_{il} + B_{il} K_{il}(e)) x(t) + E_{il} w(t)] \]
\[ z(t) = \sum_{l=1}^{r_i} \mu_{il} C_{il} x(t). \]  
(12)

We now recall standard notations and preliminaries, which will help formulate our main results.

**Definition 2** (see [42]). For any \( u(t) = 0, w(t) = 0 \) and the initial condition \( x(t_0) \), the equilibrium \( x = 0 \) of system (7) is said to be asymptotically stable under certain switching signal \( \sigma(t) \) if there exist constants \( \alpha > 0, \delta > 0 \) such that the solution of the system satisfies \( \|x(t)\| < \alpha e^{-\delta t} \|x(t_0)\| \), \( \forall t \geq t_0 \).

**Definition 3** (see [43]). For any switching signal \( \sigma(t) \) and any \( t_2 \geq t_1 \geq 0 \), let \( N_{\sigma}(t_1, t_2) \) denote the number of discontinuities \( \sigma(t) \) in the interval \( (t_1, t_2) \). We say that \( \sigma(t) \) has the average dwell time property if

\[ N_{\sigma}(t_2, t_1) \leq N_0 + \frac{t_2 - t_1}{\tau_a}, \quad N_0 \geq 0, \quad \tau_a > 0, \]
\[ (13) \]

holds, where \( N_0 \) and \( \tau_a \) are called the chatter bound and average dwell time, respectively. As commonly used in the literature, we choose \( N_0 = 0 \).

**Definition 4** (see [30]). Given \( \gamma > 0 \), a system of the form (7) is said to be with an \( H_{\infty} \)-norm less than or equal to \( \gamma \) if

\[ \int_{T_f}^{T_j} \hat{z}(t) z(t) \, dt \leq \gamma^2 \int_{T_0}^{T_f} \hat{w}(t) w(t) \, dt \]
\[ (14) \]

holds for \( x(0) = 0 \). Where \( T_f \) is the terminal time of control and \( x(0) \) denotes the initial condition of system (7).

**Lemma 5** (see [44]). Given any constant \( \eta \) and any matrices \( N, \Gamma, Y \) of compatible dimensions, then we have

\[ 2x^TNY^Ty \leq \eta x^TNN^Tx + \frac{1}{\eta} y^T\Gamma^TYy, \]
\[ (15) \]
for all \( x, y \in \mathbb{R}^n \), where \( T \) is an uncertain matrix satisfying \( T^T T \leq I \).

**Lemma 6** (see [34]). For a positive scalar \( \varepsilon_0 \) and the symmetric matrices \( S_1 \) and \( S_2 \) of compatible dimensions, if the inequalities

\[
S_1 \geq 0,
S_1 + \varepsilon_0 S_2 > 0
\]

hold, then

\[
S_1 + \varepsilon S_2 > 0, \quad \forall \varepsilon \in (0, \varepsilon_0].
\]

**Lemma 7** (see [34]). If there exist matrices \( Z_i \) \((i = 1, 2, 3)\) with \( Z_i = Z_i^T \) \((i = 1, 2)\) satisfying

\[
Z_1 > 0,
\begin{bmatrix}
Z_1 & \varepsilon_0 Z_3^T \\
\varepsilon_0 Z_3 & \varepsilon_0 Z_2
\end{bmatrix} > 0,
\]

then

\[
E(\varepsilon) Z(\varepsilon) = Z^T(\varepsilon) E(\varepsilon) > 0, \quad \forall \varepsilon \in (0, \varepsilon_0],
\]

where \( Z(\varepsilon) = \begin{bmatrix} Z_1 & Z_2^T \end{bmatrix} \).

The problems under consideration are formulated as follows.

**Problem 8.** Given an \( H_\infty \) performance bound \( \gamma > 0 \) and an upper bound \( \varepsilon_0 \) for the singular perturbation parameter \( \varepsilon \), under admissible switching signals with ADT property, determine a state feedback controller of form (9), such that, for all \( \varepsilon \in (0, \varepsilon_0] \), the overall switched closed-loop system (12) is asymptotically stable and with an \( H_\infty \)-norm less than or equal to \( \gamma \).

**Problem 9.** Given an \( H_\infty \) performance bound \( \gamma > 0 \), determine a state feedback controller of the form (9), such that, under admissible switching signals with ADT property, the overall switched closed-loop system (12) is asymptotically stable and with an \( H_\infty \)-norm less than or equal to \( \gamma \) for any sufficiently small \( \varepsilon \).

**Problem 10.** Given an \( H_\infty \) performance bound \( \gamma > 0 \) and a controller, determine an \( \varepsilon \)-bound \( \varepsilon_{\max} \), as large as possible, such that, for any \( \varepsilon \in (0, \varepsilon_{\max}] \), under admissible switching signals with ADT property, the overall switched closed-loop system (12) is asymptotically stable and with an \( H_\infty \)-norm less than or equal to \( \gamma \).

**Remark 11.** The synthesis problems for T-S fuzzy SPSSs have attracted much attention of many researchers. \( H_\infty \) control and \( \varepsilon \)-bound design for T-S fuzzy SPSSs with pole placement constraints were considered in [34]. This paper will extend the stability analysis and control methods for normal systems to T-S fuzzy SPSSs. Problem 8 considers the stabilization controller design, \( \varepsilon \)-bound design, and \( H_\infty \) control. Problem 9 aims to design a controller without considering the \( \varepsilon \)-bound. Problem 10 is used to estimate the \( \varepsilon \)-bound of the switched system.

### 3. Controller Design

This section will present a controller design method to solve Problem 8.

**Theorem 12.** Given an \( H_\infty \) performance bound \( \gamma > 0 \), an upper bound \( \varepsilon_0 \), and two constants, \( \lambda > 0 \) and \( \mu \geq 1 \), if there exist matrices \( F_{i1} \) \((i = 1, 2, \ldots, r)\), \( Z_{i1} \), \( Z_{i2} \), and \( Z_{i3} \) of compatible dimensions with \( Z_{ik} = Z_{ik}^T \) \((k = 1, 2)\), such that

\[
Z_{i1} > 0,
\]

\[
\begin{bmatrix}
Z_{i1} & \varepsilon_0 Z_{i3} \\
\varepsilon_0 Z_{i3} & \varepsilon_0 Z_{i2}
\end{bmatrix} > 0,
\]

\[
\begin{bmatrix}
E_{i1}^T & -I_r & * & * \\
\lambda Z_{i1} & 0 & 0 & 0
\end{bmatrix} < 0,
\]

\[
\begin{bmatrix}
E_{i2}^T & -I_r & * & * \\
\lambda Z_{i2} & 0 & 0 & 0
\end{bmatrix} < 0,
\]

\[
Z_{i1} \leq \mu Z_{i2}, \quad i \neq j.
\]

Then, for any \( \varepsilon \in (0, \varepsilon_0] \), the overall switched closed-loop system (12) with \( K_{\varepsilon}(\varepsilon) = F_{\varepsilon} Z_{i3}^{-T}(\varepsilon) \) \((i = 1, 2, \ldots, r)\), \( Z_\varepsilon(\varepsilon) = U_{\varepsilon} + \varepsilon U_{\varepsilon,2} \) is asymptotically stable and with an \( H_\infty \)-norm less than or equal to \( \gamma \) under any switching signal with ADT

\[
\tau_\varepsilon \geq \ln \frac{\mu}{A_\varepsilon}.
\]

**Proof.** Based on Lemma 6, LMI s (22) and (23) imply that

\[
\begin{bmatrix}
E_{i1}^T & -I_r & * & * \\
\lambda Z_{i1}(\varepsilon) & 0 & 0 & 0
\end{bmatrix} < 0, \quad \forall \varepsilon \in (0, \varepsilon_0].
\]
By the Schur complement, inequality (29) is equivalent to
\[ H e \{ A_{ij} Z_i (e) + B_i F_{ij} (e) + \lambda Z_i^T (e) E (e) + E_i E_i^T \}
+ \frac{1}{\nu^2} Z_i^T (e) C_{ij} C_{ij} Z_i (e) < 0, \quad \forall \varepsilon \in (0, \varepsilon_0]. \quad (30) \]

Pre- and postmultiplying (30) by \( Z_i^{-T} (e) \) and its transpose, respectively, we obtain
\[ H e \left\{ P_i^T (e) \left( A_d + B_i K_d (e) \right) \right\} + \lambda E (e) P_i (e), \]
\[ + P_i^T (e) E_i E_i^T P_i (e) + \frac{1}{\nu^2} C_i^T C_i < 0, \quad (31) \]
\[ \forall \varepsilon \in (0, \varepsilon_0]. \]

where \( K_d (e) = F_d Z_d^{-1} (e) \) and \( P_i (e) = Z_i^{-1} (e) \).

Using Lemma 6 again, it follows from (24) and (25) that
\[
\begin{bmatrix}
Y_3 & * & * \\
E_d^T + E_i^T & -2I_s & * \\
C_{ij} Z_i (e) + C_{is} Z_i (e) & 0 & -2\nu^2
\end{bmatrix} < 0, \quad (32)
\]
\[ \forall \varepsilon \in (0, \varepsilon_0], \quad l < s, \]

where
\[ Y_3 = H e \{ A_d Z_i (e) + B_i F_i + A_{ia} Z_i (e) + B_{ia} F_i \} + \lambda Z_i^T (e) E (e) \]

By the Schur complement, inequality (32) can be replaced by the following inequality:
\[
Y_3 + \frac{1}{2} \left( E_d + E_i \right) \left( E_d^T + E_i^T \right)
+ \frac{1}{2\nu^2} \left( C_d^T Z_i (e) + C_i Z_i (e) \right)^T
\cdot \left( C_d^T Z_i (e) + C_i Z_i (e) \right) < 0, \quad \forall \varepsilon \in (0, \varepsilon_0], \quad l < s. \quad (33)\]

By using Lemma 5, we get from inequality (33) that
\[
Y_3 + H e \left\{ E_d E_i^T + \frac{1}{\nu^2} Z_i^T (e) C_{ij}^T C_{ij} Z_i (e) \right\} < 0, \quad \forall \varepsilon \in (0, \varepsilon_0], \quad l < s. \quad (34)\]

Pre- and postmultiplying (34) by \( Z_i^{-T} (e) \) and its transpose, respectively, we have
\[
H e \left\{ P_i^T (e) \left( A_d + B_i K_d (e) \right) \right\} + \lambda E (e) P_i (e), \]
\[ + P_i^T (e) E_i E_i^T P_i (e) + \frac{1}{\nu^2} C_i^T C_i \right\} + \lambda E (e) P_i (e), \quad \forall \varepsilon \in (0, \varepsilon_0], \quad l < s. \quad (35)\]

By Lemma 7, LMI conditions (20) and (21) guarantee that the inequality
\[ E (e) Z_i (e) = Z_i^T (e) E (e) > 0, \quad \varepsilon \in (0, \varepsilon_0], \quad (36)\]
holds, which implies
\[ E (e) P_i (e) = P_i^T (e) E (e) > 0, \quad \varepsilon \in (0, \varepsilon_0]. \quad (37)\]

Define the piecewise Lyapunov function
\[ V (t) = V_{\sigma(t)} (x (t)) = x^T (t) E (e) P_{\sigma(t)} (e) x (t), \quad (38) \]
\[ t \geq 0, \]
where \( E (e) P_{\sigma(t)} (e) \) is switched among \( E (e) P_i (e), i = 1, 2, \ldots, M \), in accordance with the piecewise constant switching signal \( \sigma(t) \).

Computing the derivative of \( V (x(t)) \) with respect to \( t \) along the trajectories of system (12), we have
\[
\dot{V}_i (x (t)) = 2x^T (t) E (e) P_i (e) x (t) \quad (37)
\[
\cdot (A_d + B_i K_d (e)) \cdot x (t) + \sum_{i=1}^M \sum_{l=1}^M \sum_{d=1}^s \mu_i \mu_d \hat{t}_i (t) x^T (t) \quad (39)
\]
\[
\cdot x (t) + \sum_{i=1}^M \sum_{l=1}^M \sum_{d=1}^s \mu_i \mu_d \hat{t}_i (t) H e \{ x^T (t) P_i^T (e) \times E_i w (t) \},\]
\[ \varepsilon \in (0, \varepsilon_0]. \]

Using Lemma 5 again, we obtain
\[
2x^T (t) P_i^T (e) E_i w (t) \leq x^T (t) P_i^T (e) E_i E_i^T P_i (e) x (t) + w^T (t) w (t), \quad \varepsilon \in (0, \varepsilon_0]. \quad (40)\]

From equality (39) and inequality (40), it follows that
\[
\dot{V}_i (x (t)) \leq \sum_{i=1}^M \sum_{d=1}^s \mu_i \mu_d \hat{t}_i (t) x^T (t) \{ H e \{ P_i^T (e) \}
\cdot (A_d + B_i K_d (e)) \} + P_i^T (e) E_i E_i^T P_i (e) \} x (t)
\[ + \sum_{i=1}^M \sum_{l=1}^M \sum_{d=1}^s \mu_i \mu_d \hat{t}_i (t) x^T (t) \{ H e \{ P_i^T (e) \}
\cdot (A_d + B_i K_d (e)) \} + P_i^T (e) \}
\[ \cdot \left( E_i E_i^T + E_{il} E_{il} \right) P_i (e) \} x (t) + w^T (t) w (t), \]
\[ \varepsilon \in (0, \varepsilon_0]. \quad (41)\]
It follows from (31), (35), and (41) that
\[
\hat{V}_i(x(t)) \leq -\mathcal{L}V_i(x(t)) - \sum_{l=1}^{r_i} \sum_{j=1}^{M} \mu^2 \theta_i^j(t) \frac{1}{y^2} x^T(t)
\]
\[\cdot \left[ C_{j}^T C_{j} x(t) - \sum_{l=1}^{r_i} \sum_{j=1}^{M} \mu^2 \theta_i^j(t) \frac{1}{y^2} x^T(t) \right] \]
\[\cdot \left( C_{j}^T C_{j} + C_{j}^T C_{j} \right) x(t) + w^T(t) w(t) \]
\[= -\mathcal{L}V_i(x(t)) - \frac{1}{y^2} z^T(t) z(t) + w^T(t) w(t), \]
\[t \geq 0.\]

Furthermore, by Lemma 6, LMIs (26) and (27) imply that
\[E(e)Z_i(e) \leq \mu E(e)Z_i(e), \quad i \neq j, \quad \forall e \in (0,e_0]. \quad (43)\]

Applying the Schur complement to (43) shows that
\[
\begin{bmatrix}
-Z_j^{-1}(e)E^{-1}(e) & 0 \\
0 & -\mu E(e)Z_i(e)
\end{bmatrix} \leq 0,
\]
\[i \neq j, \quad \forall e \in (0,e_0]. \quad (44)\]

Pre- and postmultiplying (44) by \[\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^T\] and its transpose, respectively, and taking into account the fact that \(P_i(e) = Z_j^{-1}(e),\) inequality (44) is equivalent to
\[
\begin{bmatrix}
-E^T(e)P_i(e) & 0 \\
0 & -\mu E(e)P_j^{-1}(e)
\end{bmatrix} \leq 0,
\]
\[i \neq j, \quad \forall e \in (0,e_0]. \quad (45)\]

By the Schur complement, it follows from (45) that
\[E(e)P_i(e) \leq \mu E(e)P_j(e), \quad i \neq j, \quad \forall e \in (0,e_0]. \quad (46)\]

Then, the following properties are obtained for (38):
1. Each \(V_i(x(t)) = x^T(t)E(e)P_i(e)x(t)\) is continuous and its derivative along the trajectories of the corresponding subsystem satisfies (42).
2. There exist constant scalars \(\kappa_1 > 0, \kappa_2 > 0,\) such that
\[
\kappa_1 \|x(t)\|^2 \leq V_i(x(t)) \leq \kappa_2 \|x(t)\|^2,
\]
\[\forall x(t) \in \Omega(E(e)P_i(e)), \quad (47)\]

where \(\kappa_1 = \inf_{y \in \lambda_M(E(e)P_i(e))}, \kappa_2 = \sup_{y \in \lambda_M(E(e)P_i(e))} \|x(t)\|^2.\)

3. There exists a constant scalar \(\mu \geq 1\) such that (46) holds.

Thus, \(V(t)\) is piecewise monotonically decreasing and its value at switching instants is nonincreasing.

Using the differential inequality (42), we obtain
\[
V(T_f) \leq V(t_{k+1})e^{-\lambda(T_f-T_{k+1})} + \int_{t_{k+1}}^{T_f} e^{-\lambda(T_f-T)} \Gamma(t) \, dt
\]
\[
= \mu V(t_k) e^{-\lambda(T_f-T_{k+1})} + \int_{t_{k+1}}^{T_f} e^{-\lambda(T_f-T)} \Gamma(t) \, dt
\]
\[\leq \mu \left[ V(t_k) e^{-\lambda(T_f-T_{k+1})} + \int_{t_{k+1}}^{T_f} e^{-\lambda(T_f-T)} \Gamma(t) \, dt \right] + \int_{t_{k+1}}^{T_f} e^{-\lambda(T_f-T)} \Gamma(t) \, dt, \quad (48)\]

where \(\Gamma(t) = w^T(t)w(t) - \frac{1}{y^2} z^T(t)z(t).\)

It follows from (48) that
\[
V(T_f) \leq \mu^{k+1} e^{-\lambda T_f}V(0) + \mu^{k+1} \int_{t_{k+1}}^{t_{k+2}} e^{-\lambda(T_f-T)} \Gamma(t) \, dt
\]
\[+ \mu \int_{t_{k+1}}^{t_{k+2}} e^{-\lambda(T_f-T)} \Gamma(t) \, dt + \cdots
\]
\[+ \mu^{k+1} \int_{t_{k+1}}^{T_f} e^{-\lambda(T_f-T)} \Gamma(t) \, dt + \int_{t_{k+1}}^{T_f} e^{-\lambda(T_f-T)} \times \Gamma(t) \, dt,
\]
\[= \mu^{k+1} e^{-\lambda T_f}V(0) + \int_{t_{k+1}}^{T_f} \mu^{k+1} e^{-\lambda(T_f-T)} \Gamma(t) \, dt
\]
\[+ \int_{t_{k+1}}^{T_f} e^{-\lambda(T_f-T)} \Gamma(t) \, dt,
\]
\[= e^{-\lambda T_f}N_0(0) + \int_{t_{k+1}}^{T_f} e^{-\lambda(T_f-T)} \Gamma(t) \, dt.
\]

The following proof consists of two parts. First, we will show that the overall switched closed-loop system (12) with \(w(t) \equiv 0\) is asymptotically stable under any switching signal with ADT (28). Then, we will verify that system (12) is with an \(H_{\infty}\)-norm less than or equal to \(\gamma.\)

**Part 1.** We consider the following average dwell time scheme: for any \(T_f > 0\) and a positive scalar \(\lambda^* < \lambda,\)
\[
N_0(0,T_f) \leq \frac{T_f}{T_{a}}, \quad T_{a}^* = \frac{\ln \mu}{\lambda^*}, \quad (50)
\]

It follows from (50) that \(N_0(0,T_f)\ln^* \leq \lambda^* T_f.\) Then, from (49), we obtain
\[
V(T_f) \leq e^{-\lambda^* T_f}V(0), \quad (51)
\]
where $\lambda - \lambda^* > 0$, and one can see from [45] that
\[
\left\| x(T_f) \right\|^2 \\
\leq \max_{i\in\{L,M\}} \frac{\lambda_M(E(\epsilon)P_1(\epsilon))}{\lambda_m(E(\epsilon)P_1(\epsilon))} e^{-(\lambda - \lambda^*)T_f} \left\| x(0) \right\|^2,
\]
which indicates that system (12) with $w(t) \equiv 0$ is asymptotically stable under any switching signal with ADT (28).

**Part 2.** Similar to Part 1, for a positive scalar $\lambda^*$ smaller than $\lambda$, the inequality
\[
N_{\alpha}(\tau, T_f) \ln \mu \leq \lambda^* (T_f - \tau)
\]
holds.

Taking into account the fact that $x(0) = 0$ and $V(T_f) \geq 0$, it follows from (49) that
\[
\int_0^{T_f} e^{-(\lambda - \lambda^*)(T_f - \tau)} N_{\alpha}(\tau, T_f) \ln \Gamma(\tau) d\tau \geq 0.
\]
It follows from (53) and (54) that
\[
\int_0^{T_f} e^{-(\lambda - \lambda^*)(T_f - \tau)} \Gamma(\tau) d\tau \geq 0,
\]
which implies that $\int_0^{T_f} z^*(t)z(t)dt \leq \gamma^2 \int_0^{T_f} \omega^*(t)\omega(t)dt$. This completes the proof.

**Remark 13.** $Z_{\alpha}(\epsilon) = \left[ \begin{array}{cc} Z_{\alpha} & \epsilon Z_{\alpha} \\ \epsilon Z_{\alpha} & Z_{\alpha} \end{array} \right] = U_{\alpha} + \epsilon U_{\alpha2}, \epsilon \in (0, \epsilon_0)$, where $U_{\alpha1} = \left[ \begin{array}{cc} Z_{\alpha} & 0 \\ 0 & Z_{\alpha} \end{array} \right]$, $U_{\alpha2} = \left[ \begin{array}{cc} 0 & Z_{\alpha} \\ Z_{\alpha} & 0 \end{array} \right]$. It follows from LMIs (20) and (21) that $Z_{\alpha1} > 0$ and $Z_{\alpha2} > 0$, which imply that the matrices $Z_{\alpha}(0) = U_{\alpha}$ are nonsingular. So, the matrices $Z_{\alpha}(\epsilon)$ are nonsingular for all $\epsilon \in (0, \epsilon_0)$. This nonsingularity can ensure that $K_{\alpha}(\epsilon) = F_{\alpha}Z_{\alpha}^{-1}(\epsilon)$ ($l = 1, 2, \ldots, r_f$) always work well for all $\epsilon \in (0, \epsilon_0)$. For sufficiently small $\epsilon$, the $\epsilon$-dependent controller is reduced to an $\epsilon$-independent one, since $\lim_{\epsilon \to 0} K_{\alpha} = F_{\alpha}U_{\alpha2}^{-1}(\epsilon = 0, 1, 2, \ldots, r_f)$.

**Remark 14.** The multiple Lyapunov functions method has been widely used in control design of switched systems [36–39]. By employing the average dwell time scheme, the problem of extended dissipative state estimation for a class of discrete-time Markov jump networks with unreliable links was addressed in [36]. In this paper, the $\epsilon$-dependent piecewise Lyapunov function will be constructed to solve $H_\infty$ control problem for T-S fuzzy SSPPs.

**Remark 15.** Theorem 12 is concerned with the situation that $\epsilon_0$ is known according to prior information. Moreover, the bisectional search algorithm developed in [46] can be used to derive the $\epsilon$-bound.

**Remark 16.** $\epsilon$-bound, which is an essential index of SSPPs, has attained much attention. $\epsilon$-bound was considered in [40, 47]. Both results were derived by constructing a common Lyapunov function, which may lead to conservatism in some cases. In [41], the piecewise Lyapunov function was constructed and $\epsilon$-bound estimation problem was solved for T-S fuzzy SSPPs without the external disturbance input.

By Theorem 12, under the assumption that the singular perturbation parameter $\epsilon$ is known, sufficient conditions for both stability and $H_\infty$ performance of system (12) are derived. In the following theorem, for sufficiently small and unknown $\epsilon$, the above sufficient conditions are generalized to design the $\epsilon$-independent controller.

**Theorem 17.** Given an $H_\infty$ performance bound $\gamma > 0$, two constants, $\lambda > 0$ and $\mu \geq 1$, if there exist matrices $F_1, Z_{11}, Z_{12},$ and $Z_{13}$ of compatible dimensions with $Z_{1k} = Z_{1k}^T$ ($k = 1, 2$), such that
\[
Z_{11} > 0,
\]
\[
Z_{12} > 0,
\]
\[
\begin{bmatrix}
He \{ A_{ij}Z_{1}(0) + B_{ij}F_1 \} + \lambda Z_{1}^T(0) E(0) & * & *
\\
E_{1i}^T & -I_r & *
\\
C_{il}Z_{1}(0) & 0 & -\gamma^2
\end{bmatrix} < 0,
\]
\[
Z_{2j} \leq \mu Z_{11}, \quad i \neq j
\]
\[
Z_{2i} \leq \mu Z_{12}, \quad i \neq j
\]
where $l, s = 1, 2, \ldots, r_f$, $i, j = 1, 2, \ldots, M$, $\mathbb{Y}_1 = He \{ A_{ij}Z_{1}(0) + B_{ij}F_1 + A_{ij}Z_{1}(0) + B_{ij}F_1 \} + \lambda Z_{1}^T(0)E(0)$, $Z_{1}(0) = \left[ \begin{array}{cc} Z_{11} & 0 \\ 0 & Z_{12} \end{array} \right]$, and $E(0) = \left[ \begin{array}{cc} 1 & \alpha \\ 0 & 0 \end{array} \right]$.

Then there exists a positive scalar $\epsilon_{max}$ such that, for all $\epsilon \in (0, \epsilon_{max})$, the overall switched closed-loop system (12) with the controller gains of the form $K_{\alpha} = F_2Z_{\alpha}^{-1}(\epsilon)$ ($l = 1, 2, \ldots, r_f$) is asymptotically stable and with an $H_\infty$-norm less than or equal to $\gamma$ under any switching signal with ADT
\[
\tau_a \geq \ln^\mu \frac{\ln^\mu}{\lambda}.
\]

**Proof.** For sufficiently small $\epsilon$, LMI conditions (20)-(27) in Theorem 12 can be reduced to LMI conditions (56)-(61) in Theorem 17. Thus, we omit the proof of Theorem 17 that can be carried out by referring to the standard techniques used in Theorem 12.

$\epsilon$-bound is an essential stability index of SSPPs. Theorem 17 ensures the existence for $\epsilon$-bound $\epsilon_{max}$. In the following theorem, we will propose a method to estimate the $\epsilon$-bound of the closed-loop system with the obtained controllers in Theorem 17.

**Theorem 18.** Give an $H_\infty$ performance bound $\gamma > 0$, an upper bound $\epsilon_{max}$ controller gains $K_{\alpha}$, and two constants, $\lambda > 0$ and $\mu \geq 1$, if there exist matrices $Z_{11}, Z_{12},$ and $Z_{13}$ of compatible dimensions with $Z_{1k} = Z_{1k}^T$ ($k = 1, 2$), such that
\( Z_{i1} > 0, \) 
\[
\begin{bmatrix}
Z_{i1} & * \\
\varepsilon_{\text{max}}Z_{i3} & \varepsilon_{\text{max}}Z_{i2}
\end{bmatrix} > 0,
\]
\[
\begin{bmatrix}
E_d^T & -L_r & * & * \\
E_{d_1}^T & -L_r & * & * \\
0 & -2\gamma^2 \\
0 & -2\gamma^2
\end{bmatrix} < 0,
\]
\[
\begin{bmatrix}
-\gamma & * & * \\
E_{d_1}^T + E_{d_2}^T & -2L_r & * & * \\
C_{d_1}Z_i(0) + C_{d_2}Z_i(0) & 0 & -2\gamma^2 \\
C_{d_1}Z_i(\varepsilon_{\text{max}}) + C_{d_2}Z_i(\varepsilon_{\text{max}}) & 0 & -2\gamma^2
\end{bmatrix} < 0,
\]
\[
\begin{bmatrix}
\varepsilon_{\text{max}}Z_{i3} & \varepsilon_{\text{max}}Z_{i2}
\end{bmatrix} \leq \mu \begin{bmatrix}
Z_{i1} & * \\
Z_{i1} & *
\end{bmatrix}, 
\]
\[
t_{a} \geq \frac{\ln \mu}{\lambda},
\]

where \( l, s = 1, 2, \ldots, r_l, i, j = 1, 2, \ldots, M, \) \( \bar{Y}_1 = \text{He}[(A_{i1} + B_{i1}K_{i1})Z_i(0)] + \lambda Z_i^T(0)E(0), \) \( \bar{Y}_2 = \text{He}[(A_{i2} + B_{i2}K_{i2})Z_i(0)] + \lambda Z_i^T(0)E(0) \), and \( Z_i(\varepsilon) = \left[ \begin{array}{c} Z_{i1} \\ \varepsilon_{\text{max}}Z_{i3} \\ \varepsilon_{\text{max}}Z_{i2} \end{array} \right] \).

Then, for all \( \varepsilon \in (0, \varepsilon_{\text{max}}], \) the overall switched closed-loop system (12) is asymptotically stable and with an \( H_{\infty} \) norm less than or equal to \( \gamma \) under any switching signal with ADT

4. Example

To illustrate the proposed results, we consider the well-known inverted pendulum system. The equations of motion for the pendulum are given by

\[
\begin{align*}
\dot{x}_1(t) &= x_1(t) + 0.1\omega(t) \\
\dot{x}_2(t) &= g \sin(x_1(t)) - \frac{amk x_2(t)}{4l} \sin(2x_1(t)) + \frac{2\cos(x_1(t))\mu(t)}{l} + 0.1\omega(t) \\
z(t) &= 0.1x_1(t) + 0.1x_2(t),
\end{align*}
\]

where \( x_1(t) = \theta_p(t) \) denotes the angle of the pendulum from the vertical upward, \( x_2(t) = \dot{\theta}_p(t) \), \( g \) is the gravity acceleration, \( a = 1/(m + M) \), \( m \) and \( M \) are the masses of the pendulum and the cart, respectively, \( l \) is the length of the pendulum, \( \mu \) is a horizontal force applied to the cart, and \( \omega(t) \) is the external disturbance variable, which is a piecewise function of time of the form

\[
\omega(t) = \begin{cases} 
sin(2\pi t) & 0 \leq t < 5 \\
\sin(2\pi t)e^{-0.5t} & t \geq 5,
\end{cases}
\]

\( z(t) \) is the controlled output. The parameters for the plant are as follows: \( g = 9.8 \text{ m/s}^2, \) \( m = 2 \text{ Kg}, \) \( M = 8 \text{ Kg}, \) and \( l = 0.5 \text{ m}. \)

The angle of the pendulum \([-30^\circ \ldots 30^\circ]\) is divided into two areas \( R_1 \) and \( R_2, \) where \( R_1 = [x_1] \leq 15^\circ; R_2 = 15^\circ < |x_1| \leq 30^\circ, \) which results in two fuzzy subsystems [39].

For the individual system, we choose the membership functions of the fuzzy sets as follows.

Mode 1

\[
M_{11}(x_1(t)) = 1 - \frac{|x_1(t)|}{15},
\]

\[
M_{12}(x_1(t)) = \frac{|x_1(t)|}{15}.
\]
Mode 2

\[ M_{21}(x_1(t)) = \frac{|x_1(t)|}{15} - 1 \]
\[ M_{22}(x_1(t)) = 2 - \frac{|x_1(t)|}{15} \]  \hfill (75)

Then, the dynamics of Mode 1 can be exactly represented by the following T-S fuzzy model under \(|x_1(t)| \leq 15^o\):

Plant Rule 1:

IF \( x_1(t) \) is \( M_{11}(x_1(t)) \), THEN
\[
\dot{x}(t) = A_{11}x(t) + B_{11}u(t) + E_{11}w(t)
\]
\[ z(t) = C_{11}x(t) \]  \hfill (76)

Plant Rule 2:

IF \( x_1(t) \) is \( M_{12}(x_1(t)) \), THEN
\[
\dot{x}(t) = A_{12}x(t) + B_{12}u(t) + E_{12}w(t),
\]
\[ z(t) = C_{12}x(t) \]

where
\[
A_{11} = \begin{bmatrix} 0 & 0.90673 \\ 9.0673 & 0 \end{bmatrix},
\]
\[ B_{11} = \begin{bmatrix} 0 \\ -0.0688 \end{bmatrix},
\]
\[ E_{11} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix},
\]
\[ C_{11} = [0.1 \\ 0.1]. \]  \hfill (77)

Choosing \( \varepsilon = 0.1 \), the above switched system can be modeled by a SPSS (1) with the following.

Mode 1

 restrained by
\[ E(\varepsilon) = \begin{bmatrix} 1* \\ 0 \varepsilon \end{bmatrix}, \]
\[ A_{11} = \begin{bmatrix} 0 \\ 10.5717 \end{bmatrix}, \]
\[ B_{11} = \begin{bmatrix} 0 \\ -0.0779 \end{bmatrix},
\]
\[ E_{11} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix},
\]
\[ C_{11} = [0.1 \\ 0.1]. \]  \hfill (80)

The dynamics of Mode 2 can be exactly represented by the following T-S fuzzy model under \( 15^o < |x_1| \leq 30^o \):

Plant Rule 1:

IF \( x_1(t) \) is \( M_{21}(x_1(t)) \), THEN
\[
\dot{x}(t) = A_{21}x(t) + B_{21}u(t) + E_{21}w(t)
\]
\[ z(t) = C_{21}x(t) \]

Plant Rule 2:

IF \( x_1(t) \) is \( M_{22}(x_1(t)) \), THEN
\[
\dot{x}(t) = A_{22}x(t) + B_{22}u(t) + E_{22}w(t),
\]
\[ z(t) = C_{22}x(t) \]  \hfill (78)

where
\[
A_{21} = \begin{bmatrix} 0 \\ 10.5717 \end{bmatrix},
\]
\[ B_{21} = \begin{bmatrix} 0 \\ -0.0779 \end{bmatrix},
\]
\[ E_{21} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix},
\]
\[ C_{21} = [0.1 \\ 0.1]. \]  \hfill (79)

Choosing \( \varepsilon = 0.1 \), the above switched system can be modeled by a SPSS (1) with the following.
\textbf{Mode 2}

\begin{align*}
E(\epsilon) &= \begin{bmatrix} 1 & * \\ 0 & \epsilon \end{bmatrix}, \\
\overline{A}_{21} &= \begin{bmatrix} 0 & 1 \\ 1.05717 & 0 \end{bmatrix}, \\
\overline{B}_{21} &= \begin{bmatrix} 0 \\ -0.00779 \end{bmatrix}, \\
\overline{E}_{21} &= \begin{bmatrix} 0.1 \\ 0.01 \end{bmatrix}, \\
E(\epsilon) &= \begin{bmatrix} 1 & * \\ 0 & \epsilon \end{bmatrix}, \\
\overline{A}_{22} &= \begin{bmatrix} 0 & 1 \\ 1.11243 & 0 \end{bmatrix}, \\
\overline{B}_{22} &= \begin{bmatrix} 0 \\ -0.00811 \end{bmatrix}, \\
\overline{E}_{22} &= \begin{bmatrix} 0.1 \\ 0.01 \end{bmatrix}.
\end{align*}

(81)

The fuzzy controller is described as follows.

\textbf{Mode 1}

Plant Rule 1:

\text{IF } x_1(t) \text{ is } M_{11}(x_1(t)), \text{ THEN } \\
u(t) = K_{11}(\epsilon)x(t)

Plant Rule 2:

\text{IF } x_1(t) \text{ is } M_{12}(x_1(t)), \text{ THEN } \\
u(t) = K_{12}(\epsilon)x(t)

\textbf{Mode 2}

Plant Rule 1:

\text{IF } x_2(t) \text{ is } M_{21}(x_1(t)), \text{ THEN } \\
u(t) = K_{21}(\epsilon)x(t)

Plant Rule 2:

\text{IF } x_2(t) \text{ is } M_{22}(x_1(t)), \text{ THEN } \\
u(t) = K_{22}(\epsilon)x(t)

Taking \( \epsilon = 0.1 \), \( \lambda = 10 \), and \( \mu = 20 \) and solving the LMI s in Theorem 12, we obtain the stabilization controller gains:

\begin{align*}
K_{11} &= \begin{bmatrix} 7.3418 \times 10^3 & 5.7720 \times 10^2 \end{bmatrix}, \\
K_{12} &= \begin{bmatrix} 6.6701 \times 10^3 & 5.2617 \times 10^2 \end{bmatrix}, \\
K_{21} &= \begin{bmatrix} 6.4503 \times 10^3 & 5.2617 \times 10^2 \end{bmatrix}, \\
K_{22} &= \begin{bmatrix} 6.2518 \times 10^3 & 4.7497 \times 10^2 \end{bmatrix}.
\end{align*}

(82)

Taking \( \lambda = 10 \) and \( \mu = 20 \) and solving the LMIs in Theorem 17, we obtain the stabilization controller gains:

\begin{align*}
K_{11} &= \begin{bmatrix} 1.0908 \times 10^4 & 8.1009 \times 10^2 \end{bmatrix}, \\
K_{12} &= \begin{bmatrix} 1.0237 \times 10^4 & 8.1009 \times 10^2 \end{bmatrix}, \\
K_{21} &= \begin{bmatrix} 9.8518 \times 10^3 & 7.7925 \times 10^2 \end{bmatrix}, \\
K_{22} &= \begin{bmatrix} 9.6513 \times 10^3 & 7.6329 \times 10^2 \end{bmatrix}.
\end{align*}

(83)

Under the controller obtained by Theorem 17, the \( \epsilon \) -bound of the closed-loop system is \( \epsilon_{\text{max}} = 0.5493 \) by using Theorem 18 and the bisectional search algorithm developed in [46].

To illustrate the proposed method, we first consider the simulation of system (10) without the controller and then apply the designed controller to system (10). Choosing \( \epsilon = 0.1 \), \( x_1(0) = 0 \), \( x_2(0) = 0 \), \( \lambda = 10 \), \( \gamma = 1 \), and any switching signal with ADT (71), the simulation result without the controller is shown in Figure 1. It can be seen from Figure 1 that system (10) is not stable. Applying the fuzzy controller obtained by Theorem 12 to the original system, the state trajectories of the overall switched closed-loop system are shown in Figure 2 and the ratio of the output energy to the disturbance input energy, that is, \( \int_0^T z^T(s)z(s)ds/\int_0^T w^T(s)w(s)ds \), is depicted in Figure 3. It is easy to find that after 5 seconds the ratio of the output energy to the disturbance input energy is fixed at a constant
value, which is about $7.7265 \times 10^{-5}$. So $\gamma = \sqrt{7.7265 \times 10^{-5}} = 8.79 \times 10^{-3}$, which is less than the prescribed value 1.

### 5. Conclusion

In this paper, we are concerned with the design of fuzzy controller with guaranteed $H_\infty$ performance for T-S fuzzy SPSSs. An LMI-based method of designing an $\epsilon$-dependent controller has been proposed. Through this method, the obtained controller can work well for any $\epsilon \in (0, \epsilon_0]$. This controller guarantees that, for a given upper bound $\epsilon_0$ for $\epsilon$ and a presupposed $H_\infty$ performance bound $\gamma > 0$, under admissible switching signals, the switched system is asymptotically stable and with an $H_\infty$-norm less than or equal to $\gamma$. Then, for sufficiently small $\epsilon$, the $\epsilon$-independent feedback controller has been developed. Furthermore, under this controller, the $\epsilon$-bound estimation problem of the switched system has been solved. The involved example has shown the feasibility and effectiveness of the obtained results.

### Conflicts of Interest

The authors declare that they have no conflicts of interest.

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