Research Article

An Evaluation Method for Sortie Generation Capacity of Carrier Aircrafts with Principal Component Reduction and Catastrophe Progression Method

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This study proposes a new comprehensive evaluation method with principal component reduction and catastrophe progression method, considering the complexity, hierarchy, contradiction, and relevance of the factors in sortie generation of carrier aircrafts. First of all, the index system of sortie generation capacity is ascertained, which contains correlative indexes. The principal component reduction method is applied to transform the correlative indexes into independent indexes. This method eliminates the effect of correlativity among indexes. The principal components are determined with their contributions. Then the reduced principal components are evaluated in catastrophe progression method. This method is easy to realize without weights, which is more objective. In fact, catastrophe progression method is a multidimensional fuzzy membership function, which is suitable for the incompatible multiobjective evaluation. Thus, a two-level evaluation method for sortie generation capacity of carrier aircrafts is realized with principal component reduction and catastrophe progression method. Finally, the Surge operation of aircraft carrier “Nimitz” is taken as an example to evaluate the sortie generation capacity. The results of the proposed method are compared with those of Analytic Hierarchy Process, which verify the usefulness and reliability of the proposed method.

1. Introduction

Aircraft carrier is the important part in the modern naval warfare. The research on the warfare capacity of aircraft carrier has become a hot issue with the increasing attention of the security in the territorial sea [1–3]. The comparison of sortie generation capacity of aircraft carrier in different operational schemes is helpful to determine the final plan [4–7]. Therefore, the evaluation for sortie generation capacity of aircraft carrier has important theoretical significance and application value.

The sortie generation capacity of aircraft carrier is mostly evaluated by Analytic Hierarchical Process (AHP) at present. The evaluation results are obtained by subjective scores of experts. Reference [8] studied the application of AHP in the measurement process. Reference [9] evaluated the original purchase process with AHP. Reference [10] developed an evaluation tool for the information sharing of supply chain using AHP. Reference [11] discussed the application of AHP in the process of risk assessment. An improved AHP in [12] was used in the priority scheduling problems. Reference [13] researched the application of AHP in business management. Reference [14] proposed the combination of fuzzy theory and AHP and discussed the consistency problem of the evaluation method. Reference [15] pointed out the shortcomings and improvements of AHP. Reference [16] studied the evaluation process of comprehensive method of fuzzy AHP. Reference [17] solved mining selection problem based on AHP and fuzzy mathematics. However, these evaluation methods are one-sided and subjective, which ignore the correlation and contradiction of indexes. AHP is difficult to evaluate the multivariate evaluation objects objectively [18–26].

A new evaluation method of principal component reduction (PCR) and catastrophe progression method (CPM) is proposed to evaluate the sortie generation capacity of carrier aircrafts in this study. The proposed method can avoid the subjectivity and complexity in the traditional evaluation method. The main contents are as follows. Firstly, the hierarchy structure of index system for sortie generation capacity is
determined. Secondly, the related indexes are transformed to independent principal components by PCR. Then, independent principal components are evaluated by CPM. Finally, the usefulness and reliability of the new method are verified by comparing with the traditional evaluation method.

2. Index System for Sortie Generation Capacity of Carrier Aircrafts

The index system for sortie generation capacity of carrier aircrafts is established with related research results. A three-level index system with complexity, hierarchy, contradiction, and relevance is established by recursive hierarchy method. The index system for sortie generation capacity of carrier aircrafts is shown in Figure 1.

These indexes are defined as follows.

(1) Emergency sortie generation rate (ESGR): it is the maximum number of ready aircrafts taking off in a few minutes.

(2) Surge sortie generation rate (SSGR): it is the average number of aircrafts per day in the Surge operation (4 days).

(3) Last sortie generation rate (LSGR): it is the average number of aircrafts per day in the continuous operation (30 days).

(4) Performing tasks proportion (PTP): it is the time proportion that the aircrafts can carry out one task at least under a certain flight plan and logistics condition.

(5) Missing tasks proportion waiting for parts (MTPWP): it is the proportion of aircrafts missing the tasks due to waiting for parts.

(6) Missing tasks proportion waiting for repair (MTPWR): it is the proportion of aircrafts missing the tasks due to waiting for repair.

(7) Scheduled completion proportion (SCP): it is the proportion of completed number in the planned number of aircrafts.

(8) Pilot utilization rate (PUR): it is the average utilization rate of the pilots per day.

(9) Plan implementation probability per aircraft (PIPA): it is the plan implementation probability per aircraft under the certain constraints in a given period of time.

(10) Sortie generation rate per aircraft (SGRA): it is the sortie generation rate per aircraft under the certain constraints.

(11) Preparation time for next sortie (PTNS): it is the preparation time for next sortie under the condition of a certain resource allocation.

(12) Ejection interval (EI): it is the average time for ejecting a single aircraft per catapult.

(13) Take-off outage proportion (TOOP): it is the proportion of canceled number in the ready number of aircrafts.

(14) Recovery interval (RI): it is the average time for recovering a single aircraft.

(15) Overshoot proportion (OP): it is the proportion of number of aircrafts failed to recover in the number of aircrafts ready to recover.

3. Principal Component Reduction Method

3.1. Principal Component Reduction Principle. There are correlations between various indexes for sortie generation capacity of carrier aircrafts, which will bring repetitive information. The independent indexes can be obtained from related indexes using principal component reduction method. This method can minimize the information loss after reduction.

Principal component reduction uses dimension reduction techniques to obtain less comprehensive variables instead of the original variables. These comprehensive variables cover the most information of the original variables. Then the objective phenomenon is evaluated by calculating the score of comprehensive principal component.

3.2. Steps of Principal Component Reduction. Steps of principal component reduction are as shown in Figure 2.
Specific steps are as follows.

Step 1 (parameters standardization). Each index is nondimensionalized due to the different dimensions of indexes. The numerical transformation can eliminate the dimensional effect of indexes. Z-Score method is applied to transform the original matrix $X = [x_{ij}]_{n \times m}$ to standardized matrix $Z = [z_{ij}]_{n \times m}$, where $n$ is the number of scenarios and $m$ is the number of indexes:

$$z_{ij} = \frac{x_{ij} - \overline{x}_j}{s_j}, \quad (1)$$

where $\overline{x}_j$ is the mean of $j$th index and $s_j$ is the standard deviation of $j$th index.

Step 2 (correlation coefficient matrix $R$).

$$r_{jk} = \frac{1}{n-1} \sum_{i=1}^{n} z_{ij} z_{ik}, \quad (2)$$

where $R = [r_{jk}]_{m \times m}$, $r_{ii} = 1$, and $r_{jk} = r_{kj}$.

Step 3 (characteristic roots of $R$). The characteristic roots of $R$ can be calculated by

$$|\lambda_g I_m - R| = 0, \quad (3)$$

where $\lambda_g$ $(g = 1, 2, \ldots, m)$ is the characteristic root, which is the variance of principal component. It denotes the effect of each principal component on the evaluated object.

Step 4 (feature vectors of $R$). The feature vectors of $R$ can be obtained from

$$[\lambda_g I_m - R] L_g = 0, \quad (4)$$

where $L$ is a real-valued vector of $m$ dimensions. $L_g$ is the feature vector of $\lambda_g$ and the coefficient of $z_j$ in the new coordinate system.

Step 5 (contribution of $R$). $\alpha_g$ is the information amount of each component in the total information amount, which is the contribution

$$\alpha_g = \frac{\lambda_g}{\sum_{g=1}^{m} \lambda_g}. \quad (5)$$

Step 6 (number of principal components $K$). If the number of original variables is more, the first $K$ principal components are analyzed instead of the original variables and the other variables are ignored. The proportion of the $K$ principal components in the original variable information is $\alpha(K)$:

$$\alpha(K) = \left( \frac{\sum_{g=1}^{K} \lambda_g}{\sum_{g=1}^{m} \lambda_g} \right)^{-1}. \quad (6)$$

Thus, the number of principal components is determined with a balance between $K$ and $\alpha(K)$. On the one hand, the smaller $K$ is better. On the other hand, the larger $\alpha(K)$ is better. It will keep enough information with few components in this way. In this study, $\alpha(K) \geq 95\%$.

Finally, $m$ related indexes can be transformed to $K$ independent principal components $f_{i1} - f_{iK}$:

$$f_{ig} = \sum_{j=1}^{m} L_{ij} z_{ij}. \quad (7)$$

The index system after reduction is shown in Figure 3.
4. Catastrophe Progression Evaluation Method

4.1. Description of Catastrophe Progression Method. The index system for sortie generation capacity of carrier aircrafts after reduction is applied to evaluate. The contradiction decomposition of evaluated objects is the first step of catastrophe progression method. Then catastrophe fuzzy membership function is the combination of catastrophe theory and fuzzy mathematics. This method considers the relative importance of evaluation indexes instead of index weight, which reduces the subjectivity and simplifies calculation.

In the process of formulating combat scenario, a variety of scenarios are designed due to the influence of various factors. The scenarios are evaluated comprehensively in the process of selecting the optimal scenario. The evaluation process is conducted from the indexes in lower levels to the indexes in upper levels according to catastrophe progression method. Finally, a catastrophe progression between 0 and 1 can be obtained. If the catastrophe progression is bigger, the scenario is better.

4.2. Steps of Catastrophe Progression Method. The steps of catastrophe progression method are as shown in Figure 4.

Step 1 (type of catastrophe system). The type of catastrophe system is determined by the number of subindexes, which is shown in Table 1.

In Table 1, $f(x)$ is the potential function of $x$, $a$, $b$, $c$, and $d$ are subindexes, which are sorted from high importance to low importance.

Step 2 (unitary formula). The critical points of potential function $f(x)$ gather to a balance surface based on catastrophe theory, which can be obtained from the first-order derivative of $f(x)$:

$$f'(x) = 0.$$  (8)

The singular points of potential function $f(x)$ can be obtained by the second-order derivative:

$$f''(x) = 0.\quad (9)$$

The unitary formula can be derived from bifurcation set, which will transform different states of subindex to the same state.

(1) Bifurcation set equations of sharp point system are

$$a = -6x^2,\quad b = 8x^3.$$  (10)

Then the normalization formula can be derived from

$$x_a = a^{1/2},\quad x_b = b^{1/3},$$  (11)

where $x_a$ is the value of $x$ corresponding $a$, $x_b$ is the value of $x$ corresponding $b$.

(2) Bifurcation set equations of dovetail system are

$$a = -6x^2,\quad b = 8x^3,$$  (12)

$$c = -3x^4.$$
Then the normalization formula can be derived from
\[ x_a = a^{1/2}, \]
\[ x_b = b^{1/3}, \]
\[ x_c = c^{1/4}, \]
\[ x_d = d^{1/5}. \] (13)

(3) Bifurcation set equations of butterfly system are
\[ a = -10x^2, \]
\[ b = 20x^3, \]
\[ c = -15x^4, \]
\[ d = 4x^5. \] (14)

Then the normalization formula can be derived from
\[ x_a = a^{1/2}, \]
\[ x_b = b^{1/3}, \]
\[ x_c = c^{1/4}, \]
\[ x_d = d^{1/5}. \] (15)

Normalization formula is a multidimensional fuzzy membership function.

**Step 3** (comprehensive evaluation with normalization formula). The ideal strategy is obtained from (16), when the fuzzy targets are \( A_1, A_2, \ldots, A_m \) in the same scenario
\[ C = A_1 \cap A_2 \cap \cdots \cap A_m. \] (16)

The membership function is
\[ \mu(x) = \mu_{A_1}(x) \land \mu_{A_2}(x) \land \cdots \land \mu_{A_m}(x), \] (17)
where \( \mu_{A_i}(x) \) is the membership function of \( A_i \). If the indexes are complementary, the membership function is the average value of \( \mu_{A_i}(x) \).

### 5. Evaluation for Sortie Generation Capacity of Carrier Aircrafts

#### 5.1. Evaluation Samples

The object of evaluation is the Surge operation of “Nimitz” carrier in 1997. Ten scenarios are selected randomly in order to ensure the scientific nature, which are shown in Tables 2-5. In Tables 2-5, \( X_{11} \) is emergency sortie generation rate, \( X_{12} \) is surge sortie generation rate, \( X_{13} \) is last sortie generation rate, \( X_{21} \) is performing tasks proportion, \( X_{22} \) is missing tasks proportion waiting for parts, \( X_{23} \) is missing tasks proportion waiting for repair, \( X_{31} \) is scheduled completion proportion, \( X_{32} \) is pilot utilization rate, \( X_{33} \) is plan implementation probability per aircraft, \( X_{34} \) is sortie generation rate per aircraft, \( X_{41} \) is preparation time for next sortie, \( X_{42} \) is ejection interval, \( X_{43} \) is take-off outage proportion, \( X_{44} \) is recovery interval, and \( X_{45} \) is overshoot proportion. The data in Tables 2-5 are the original data.

### Table 2: Index \( X_1 \) of sortie generation rate capacity.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>( X_{11} ) (sortie)</th>
<th>( X_{12} ) (sortie/day)</th>
<th>( X_{13} ) (sortie/day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30</td>
<td>250</td>
<td>200</td>
</tr>
<tr>
<td>2</td>
<td>31</td>
<td>240</td>
<td>180</td>
</tr>
<tr>
<td>3</td>
<td>29</td>
<td>235</td>
<td>210</td>
</tr>
<tr>
<td>4</td>
<td>33</td>
<td>260</td>
<td>220</td>
</tr>
<tr>
<td>5</td>
<td>32</td>
<td>210</td>
<td>170</td>
</tr>
<tr>
<td>6</td>
<td>29</td>
<td>245</td>
<td>194</td>
</tr>
<tr>
<td>7</td>
<td>27</td>
<td>267</td>
<td>230</td>
</tr>
<tr>
<td>8</td>
<td>32</td>
<td>211</td>
<td>183</td>
</tr>
<tr>
<td>9</td>
<td>25</td>
<td>261</td>
<td>201</td>
</tr>
<tr>
<td>10</td>
<td>32</td>
<td>232</td>
<td>196</td>
</tr>
</tbody>
</table>

### Table 3: Index \( X_2 \) of aircraft availability capacity.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>( X_{21} ) (%)</th>
<th>( X_{22} ) (%)</th>
<th>( X_{23} ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>80</td>
<td>11</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>85</td>
<td>20</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>90</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>75</td>
<td>11</td>
<td>14</td>
</tr>
<tr>
<td>5</td>
<td>82</td>
<td>10</td>
<td>18</td>
</tr>
<tr>
<td>6</td>
<td>91</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>78</td>
<td>23</td>
<td>9</td>
</tr>
<tr>
<td>8</td>
<td>84</td>
<td>9</td>
<td>7</td>
</tr>
<tr>
<td>9</td>
<td>85</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>10</td>
<td>72</td>
<td>2</td>
<td>16</td>
</tr>
</tbody>
</table>

### Table 4: Index \( X_3 \) of tasks completion capacity.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>( X_{31} ) (%)</th>
<th>( X_{32} ) (sortie/day)</th>
<th>( X_{33} ) (%)</th>
<th>( X_{34} ) (sortie/day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>85</td>
<td>2.5</td>
<td>90</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>74</td>
<td>2.2</td>
<td>80</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>81</td>
<td>2.0</td>
<td>84</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>90</td>
<td>1.8</td>
<td>75</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>61</td>
<td>1.5</td>
<td>68</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>86</td>
<td>1.9</td>
<td>86</td>
<td>8</td>
</tr>
<tr>
<td>7</td>
<td>78</td>
<td>2.1</td>
<td>88</td>
<td>5</td>
</tr>
<tr>
<td>8</td>
<td>65</td>
<td>2.3</td>
<td>94</td>
<td>6</td>
</tr>
<tr>
<td>9</td>
<td>79</td>
<td>2.4</td>
<td>81</td>
<td>7</td>
</tr>
<tr>
<td>10</td>
<td>83</td>
<td>1.7</td>
<td>82</td>
<td>5</td>
</tr>
</tbody>
</table>

### 5.2. Indexes Reduction

Take the reduction process of index \( X_4 \) as an example.

**Step 1** (standardization). The Z-Score method is used to standardize indexes, and the results are as shown in Table 6. In Table 6, \( Z_{41}, Z_{42}, Z_{43}, \) and \( Z_{44} \) are standardize indexes.
Step 2 (correlation coefficient matrix). One has

\[
R = \begin{bmatrix}
1 & 0.0159 & 0.7398 & -0.2916 & -0.3731 \\
0.0159 & 1 & 0.1600 & 0.5919 & 0.0279 \\
0.7398 & 0.1600 & 1 & -0.2186 & -0.3401 \\
-0.2916 & 0.5919 & -0.2186 & 1 & 0.2588 \\
-0.3731 & 0.0279 & -0.3401 & 0.2588 & 1
\end{bmatrix}
\]

Step 3 (characteristic roots). One has

\[
\lambda_1 \lambda_2 \lambda_3 \lambda_4 \lambda_5 = [2.1781 1.5206 0.7310 0.2381 0.3322].
\]

Step 4 (feature vectors). One has

\[
L_1 L_2 L_3 L_4 L_5 = \begin{bmatrix}
-0.5778 & -0.2117 & -0.2907 & -0.6141 & -0.3996 \\
0.1322 & -0.7342 & 0.0912 & -0.2976 & 0.5888 \\
-0.5450 & -0.3264 & -0.3113 & 0.6972 & 0.1161 \\
0.4056 & -0.5542 & 0.0927 & 0.2136 & -0.6886 \\
0.4326 & 0.0500 & -0.8954 & -0.0512 & 0.0780
\end{bmatrix}
\]

Step 5 (contribution). One has

\[
[\alpha_1 \alpha_2 \alpha_3 \alpha_4 \alpha_5] = \begin{bmatrix} 0.4356 & 0.3041 & 0.1462 & 0.0476 & 0.0664 \end{bmatrix}.
\]

Step 6 (number of principal components). Let \(\alpha(K) \geq 95\%\); then sort \(\alpha\) from big to small:

\[
[\alpha_1 \alpha_2 \alpha_3 \alpha_4 \alpha_5] = \begin{bmatrix} 0.4356 & 0.3041 & 0.1462 & 0.0664 & 0.0476 \end{bmatrix}.
\]

When \(K = 4\), \(\alpha(K) = 95.24\% \geq 95\%\).

Step 7 (principal components). The principal components of index \(X_4\) are \(f_{14}, f_{24}, f_{34}, \text{ and } f_{44}\), which are determined by characteristic roots, feature vectors, and the number of principal components:

\[
f_{41} = -0.3915X_{41} + 0.1072X_{42} - 0.6375X_{43} + 0.8312X_{44} + 0.7505X_{45},
\]

\[
f_{42} = -0.1434X_{41} - 0.5954X_{42} - 0.3818X_{43} - 1.1359X_{44} + 0.0868X_{45},
\]

\[
f_{43} = -0.1970X_{41} + 0.0740X_{42} - 0.3641X_{43} + 0.1900X_{44} - 1.5535X_{45},
\]

\[
f_{44} = -0.2708X_{41} + 0.4774X_{42} + 0.1357X_{43} - 1.4112X_{44} + 0.1354X_{45}.
\]

Similarly, the principal components of index \(X_1, X_2, \text{ and } X_3\) can be derived by repeating Steps 1–7:

\[
f_{11} = -1.2109X_{11} + 0.7943X_{12} + 0.3977X_{13},
\]

\[
f_{12} = 2.0214X_{11} + 0.2061X_{12} + 0.3569X_{13},
\]

\[
f_{13} = 0.5796X_{11} + 0.9406X_{12} - 0.4140X_{13},
\]

\[
f_{21} = 0.5329X_{21} - 0.4290X_{22} - 0.6711X_{23},
\]

\[
f_{22} = -0.0995X_{21} - 1.8995X_{22} + 0.2937X_{23},
\]

\[
f_{23} = -0.5352X_{21} - 0.3935X_{22} - 0.6795X_{23},
\]

\[
f_{31} = 0.5573X_{31} + 0.7273X_{32} + 0.5758X_{33} + 0.3004X_{34},
\]

\[
f_{32} = 1.4342X_{31} - 0.3351X_{32} - 0.3749X_{33} + 0.3459X_{34},
\]

\[
f_{33} = 1.1895X_{31} - 0.0913X_{32} + 0.3033X_{33} - 0.5228X_{34},
\]

\[
f_{34} = 0.2282X_{31} + 0.8156X_{32} - 0.6306X_{33} - 0.1828X_{34}.
\]
The comprehensive scoring model can be obtained from (23) and (24) and the contributions:

\[ Y_1 = 0.4180X_{11} + 0.6767X_{12} + 0.3430X_{13}, \]
\[ Y_2 = 0.2630X_{21} - 0.9130X_{22} - 0.3558X_{23}, \]
\[ Y_3 = 0.8846X_{31} + 0.2985X_{32} + 0.1979X_{33} + 0.1235X_{34}, \]
\[ Y_4 = -0.2740X_{41} - 0.0964X_{42} - 0.4599X_{43} - 0.0518X_{44} - 0.1420X_{45}. \]  

The weights of indexes are shown in Figure 5 according to (25). In Figure 5, the horizontal axis is the evaluated index and the vertical axis is the weight of index.

Figure 5 shows that the most important subindexes are the pilot utilization rate and scheduled completion proportion, the weights of which are greater than weights of other indexes.

5.3. Catastrophe Progression Evaluation. The index system after reduction is shown in Figure 6.

The steps of catastrophe progression evaluation are as follows.

Step 1 (normalization). Take the principal components \( f_{11}, f_{12}, \) and \( f_{13} \) of index \( X_1 \) as the example. The results are shown in Table 7. In Table 7, \( f_{11}, f_{12}, \) and \( f_{13} \) are normalization of principal components.

Step 2 (calculate evaluation value). The evaluation values of catastrophe progression for indexes \( X_1, X_2, X_3, \) and \( X_4 \) are calculated. The number of subindexes for indexes \( X_1 \) and \( X_2 \) is three; then the type of catastrophe is dovetail type. The number of subindexes for indexes \( X_3 \) and \( X_4 \) is four; then the type of catastrophe is butterfly type. Thus the evaluation values are shown in Table 8. In Table 8, \( X_1 \) is sortie generation rate capacity, \( X_2 \) is availability capacity, \( X_3 \) is tasks completion capacity, and \( X_4 \) is support, ejection, and recovery capacity. The data in Table 8 are the evaluation values of the above four indexes.

Step 3 (calculate evaluation value of \( X \)). The number of subindexes for index \( X \) is four; then the type of catastrophe is butterfly type. The evaluation results of 10 scenarios are shown in Table 9. \( X \) is sortie generation capacity. The data in Table 9 are the evaluation values of sortie generation capacity.

5.4. Analysis of Evaluation Results. The evaluation results of the proposed method are compared with that of AHP, in order to verify the usefulness of the proposed method. The comparison is shown in Figure 7 and the deviations of evaluation results are shown in Figure 8. In Figure 7, the horizontal axis is the evaluated scenario and the vertical axis is the evaluation value. In Figure 8, the horizontal axis is the evaluated scenario and the vertical axis is the deviation of proposed method and AHP.

Figure 7 shows that the evaluation results of two methods are similar. Figure 8 shows the deviations of evaluation results, and the maximum absolute value of deviation is less than 0.05, which verifies usefulness and reliability of the principal component reduction and catastrophe progression evaluation method. The proposed method can evaluate scenarios more objectively.
### Table 9: Evaluation values of index $X$.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X$</td>
<td>0.9663</td>
<td>0.9554</td>
<td>0.7237</td>
<td>0.9719</td>
<td>0.6233</td>
<td>0.9268</td>
<td>0.9239</td>
<td>0.8832</td>
<td>0.9453</td>
<td>0.9380</td>
</tr>
</tbody>
</table>

#### Figure 6: Index system after reduction.

The principal component reduction and catastrophe progression evaluation method can analyze the importance of indexes for sortie generation capacity and sort the selected scenarios objectively and reliably.

### 6. Conclusions

This study proposes a new comprehensive evaluation method based on principal component reduction and catastrophe progression method. First of all, the index system of sortie generation capacity is ascertained in Figure 1 and Tables 2–5,
which contains correlative indexes. The principal component reduction method is applied to transform the correlative indexes into independent indexes in Figures 2, 5, and 6 and Table 6. This method eliminates the effect of correlativity among indexes. The principal components are determined with their contributions. Then the reduced principal components are evaluated in catastrophe progression method in Figures 3 and 4 and Table 1. This method is easy to realize without weights, which is more objective. In fact, catastrophe progression method is a multidimensional fuzzy membership function, which is suitable for the incompatible multijobjective evaluation. Thus, a two-level evaluation method for sortie generation capacity of carrier aircrafts is realized with principal component reduction and catastrophe progression method. The principal component reduction and catastrophe progression evaluation method can analyze the importance of indexes and sort the selected scenarios objectively and reliably in Figures 7–9 and Tables 7–9. At the same time, the proposed method is suitable for other evaluated objects.

### Nomenclature

- \( X_1 \): Sortie generation rate capacity
- \( X_2 \): Availability capacity
- \( X_3 \): Tasks completion capacity
- \( X_4 \): Support, ejection, and recovery capacity
- \( X_{11} \): Emergency sortie generation rate
- \( X_{12} \): Surge sortie generation rate
- \( X_3 \): Last sortie generation rate
- \( X_{21} \): Performing tasks proportion
- \( X_{22} \): Missing tasks proportion waiting for parts
- \( X_{33} \): Missing tasks proportion waiting for repair
- \( X_{31} \): Scheduled completion proportion
- \( X_{32} \): Pilot utilization rate
- \( X_{33} \): Plan implementation probability per aircraft
- \( X_{44} \): Recovery interval
- \( X_{45} \): Overshoot proportion
- \( X_{44} \): Recovery interval
- \( X \): Original input matrix
- \( Z \): Standardized matrix
- \( n \): Number of scenarios
- \( m \): Number of indexes
- \( \bar{X}_j \): Mean of jth index
- \( s_j \): Standard deviation of jth index
- \( R \): Correlation coefficient matrix
- \( \lambda_g \): Characteristic root
- \( L \): Real-valued vector
- \( L_g \): Feature vector
- \( \alpha_g \): Information amount of each component in the total information amount
- \( K \): Number of principal components
- \( \alpha(K) \): Proportion of the K principal components in the original variable information
- \( f_{iy} \): Independent principal components
- \( f(x) \): Potential function
- \( a, b, c, d \): Subindex
- \( A_1, A_2, \ldots, A_m \): Fuzzy targets
- \( \mu_A(x) \): Membership function.

### Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this study.

### References


