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Research Article

Global Attractor of Thermoelastic Coupled Beam Equations with Structural Damping

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In this paper, we study the existence of a global attractor for a class of n-dimension thermoelastic coupled beam equations with structural damping $u_{tt} + \Delta^2 u + \Delta^2 u_t - [\sigma(\int_\Omega (\nabla u)^2 dx) + \phi(\int_\Omega \nabla u \nabla u_t dx)]\Delta u + f_1(u) + g(u_t) + \nu\Delta\theta = q(x)$, in $\Omega \times R^+$, and $\theta_t - \Delta\theta + f_2(\theta) - \nu\Delta u_t = 0$. Here Ω is a bounded domain of R^N, and $\sigma(\cdot)$ and $\phi(\cdot)$ are both continuous nonnegative nonlinear real functions and q is a static load. The source terms $f_1(u)$ and $f_2(\theta)$ and nonlinear external damping $g(u_t)$ are essentially $|u|^\rho u$, $|\theta|^\varrho \theta$, and $|u_t|^r u_t$ respectively.

1. Introduction

This problem is based on the equation

$$u_{tt} + u_{xxxx} - \left(\alpha + \beta \int_0^L u_x^2 dx\right) u_{xx} = 0, \tag{1}$$

which was proposed by Woinowsky-Krieger [1] as a model for vibrating beam with hinged ends.

Without thermal effects, Ball [2] studied the initial-boundary value problem of more general beam equation

$$u_{tt} + u_{xxxx} - M\left(\int_0^L u_x^2 dx\right) u_{xx} = 0 \tag{2}$$

subjected to homogeneous boundary condition. Ma and Narciso [3] proved the existence of global solutions and the existence of a global attractor for the Kirchhoff-type beam equation

$$\begin{aligned} u_{tt} + \Delta^2 u - M\left(\int_\Omega (\nabla u)^2 dx\right)\Delta u + f(u) + g(u_t) \\ = h(x), \end{aligned} \tag{3}$$

without structural damping, subjected to the conditions

$$u = \frac{\partial u}{\partial \nu} = 0 \quad \text{on } \partial\Omega \times R^+. \tag{4}$$

In fact, the plate equations without thermal effects were studied by several authors; we quote, for instance, [4–8].

In the following we also make some comments about previous works for the long-time dynamics of thermoelastic coupled beam system with thermal effects.

Giorgi et al. [9] studied a class of one-dimensional thermoelastic coupled beam equations

$$\begin{aligned} u_{tt} + \Delta^2 u - \left(\beta + \|\nabla u\|^2_{L^2(0,l)}\right)\Delta u - \Delta u_{tt} + f(u) + \Delta\theta \\ = f, \\ \theta_t - \Delta\theta - \Delta u_t = g \end{aligned} \tag{5}$$

and gave the existence and uniqueness of global weak solution and the existence of global attractor under Dirichlet boundary conditions. Barbosa and Ma [10] studied the long-time behavior for a class of two-dimension thermoelastic coupled beam equation

$$\begin{aligned} u_{tt} + \Delta^2 u - M\left(\int_\Omega |\nabla u|^2 dx\right)\Delta u - \Delta u_{tt} + f(u) + \nu\Delta\theta \\ = h(x), \\ \theta_t - \omega\Delta\theta - (1-\omega)\int_0^t k(t-s)\Delta\theta\, ds - \nu\Delta u_t = 0 \end{aligned} \tag{6}$$

subjected to the conditions

$$u = \Delta u = 0, \qquad \theta = 0. \tag{7}$$

In addition, we also refer the reader to [11–15] and the references therein.

A mathematical problem is the nonlinear n-dimension thermoelastic coupled beam equations with structural damping which arise from the model of the nonlinear vibration beam with Fourier thermal conduction law:

$$\begin{aligned} &u_{tt} + \Delta^2 u + \Delta^2 u_t \\ &\quad - \left[\sigma\left(\int_\Omega (\nabla u)^2\, dx\right) + \phi\left(\int_\Omega \nabla u \nabla u_t dx\right)\right]\Delta u \\ &\quad + f_1(u) + g(u_t) + \nu\Delta\theta = q(x), \quad \text{in } \Omega \times R^+, \end{aligned} \tag{8}$$

$$\theta_t - \Delta\theta + f_2(\theta) - \nu\Delta u_t = 0 \tag{9}$$

with the initial conditions

$$u(x,0) = u^0(x), \quad u_t(x,0) = u^1(x), \quad \theta(x,0) = \theta^0(x) \tag{10}$$

and the boundary conditions

$$u|_{\partial\Omega} = \left.\frac{\partial u}{\partial \nu}\right|_{\partial\Omega} = 0, \qquad \theta|_{\partial\Omega} = 0. \tag{11}$$

To the our best knowledge, the existence of global attractor for thermoelastic coupled beam equations was not considered in the presence of nonlinear structure damping. Here the unknown function $u(x,t)$ is the elevation of the surface of beam; $u^0(x)$ and $u^1(x)$ are the given initial value functions; the subscript t denotes derivative with respect to t and the assumptions on nonlinear functions $\sigma(\cdot)$, $\phi(\cdot)$, $f_1(\cdot)$, $f_2(\cdot)$, $g(\cdot)$, and the external force function $q(x)$ will be specified later.

Our fundamental assumptions on $\sigma(\cdot)$, $\phi(\cdot)$, $f_1(\cdot)$, $f_2(\cdot)$, $g(\cdot)$, and $q(x)$ are given as follows.

Assumption 1. We assume that $\sigma(\cdot) \in C^1(R)$ satisfying

$$\sigma(z)\, z \geq \widetilde{\sigma}(z) \geq 0, \quad \forall z \geq 0, \tag{12}$$

where $\widetilde{\sigma}(z) = \int_0^z \sigma(s)ds$. This condition is promptly satisfied if $\sigma(\cdot)$ is nondecreasing with $\sigma(0) = 0$.

Assumption 2. We also assume that $\phi(\cdot) \in C^1(R)$ satisfying $\phi(0) = 0$ and $\phi(\cdot)$ is nondecreasing and

$$\phi(s)\, s \geq 0, \quad \forall s \in R^+. \tag{13}$$

Assumption 3. The function $f_1(\cdot) : R \to R$ is of class $C^1(R)$ and satisfies $f(0) = 0$, and there exist constants k and $\rho \geq 0$ such that

$$\left|f_1(u) - f_1(v)\right| \leq k_1\left(1 + |u|^\rho + |v|^\rho\right)|u - v|, \quad \forall u, v \in R, \tag{14}$$

$$-a_0 \leq \widehat{f}_1(u) \leq \frac{1}{2} f_1(u)\, u + a_1, \tag{15}$$

where $\widehat{f}_1(z) = \int_0^z f_1(s)ds$.

Assumption 4. The function $f_2(\cdot) : R \to R$ is of class $C^1(R)$ and satisfies $f_2(0) = 0$, and there exist constants k_2, k_3 and $\varrho \geq 0$ such that

$$\left|f_2(\theta) - f_2(\widetilde{\theta})\right| \leq k_2\left(1 + |\theta|^\varrho + \left|\widetilde{\theta}\right|^\varrho\right)\left|\theta - \widetilde{\theta}\right|, \quad \forall \theta, \widetilde{\theta} \in R, \tag{16}$$

$$\left(f_2(\theta) - f_2(\widetilde{\theta})\right)\left(\theta - \widetilde{\theta}\right) \geq k_3\left(\theta - \widetilde{\theta}\right)^{\varrho+2}, \quad \forall \theta, \widetilde{\theta} \in R. \tag{17}$$

Assumption 5. The function $g(\cdot) : R \to R$ is of class $C^1(R)$ and satisfies $g(0) = 0$, and there exist constants k_4, k_5 and $r \geq 0$ such that

$$\left(g(u) - g(v)\right)(u - v) \geq k_4 |u - v|^{r+2}, \quad \forall u, v \in R, \tag{18}$$

$$\left|g(u) - g(v)\right| \leq k_5\left(1 + \left|u^r\right| + |v|^r\right)|u - v|, \quad \forall u, v \in R. \tag{19}$$

Assumption 6. $q(x) \in L^2(\Omega)$.

Under the above assumptions, we prove the existence of global solutions and the existence of a global attractor of extensible beam equation system (8)–(11). And the paper is organized as follows. In Section 2, we introduce some Sobolev spaces. In Section 3, we discuss the existence and uniqueness of global strong solution and weak solution. In Sections 4 and 5, we establish the result of the existence of a global attractor.

2. Basic Spaces

Our analysis is based on the following Sobolev spaces. Let

$$U = \left\{\theta \in H^2(\Omega) \cap L^2(\Omega);\, \theta|_{\partial\Omega} = 0\right\}. \tag{20}$$

Then for regular solutions we consider the phase space

$$H_1 = H^4(\Omega) \cap H_0^2(\Omega) \times H^4(\Omega) \cap H_0^2(\Omega) \times U. \tag{21}$$

In the case of weak solutions we consider the phase space

$$H_0 = H_0^2(\Omega) \times L^2(\Omega) \times L^2(\Omega). \tag{22}$$

In H_0 we adopt the norm defined by

$$\|(u, v, \theta)\|_{H_0}^2 = \|\Delta u\|^2 + \|v\|^2 + \|\theta\|^2. \tag{23}$$

3. The Existence of Global Solutions

Firstly, using the classical Galerkin method, we can establish the existence and uniqueness of regular solution to problem (8)–(11). We state it as follows.

Theorem 7. *Under assumptions (H_1)–(H_6), for any initial data $(u^0, u^1, \theta^0) \in H_1$, then problem (8)–(11) has a unique regular solution (u, θ) with*

$$\begin{aligned} &u \in L^\infty\left(R^+, H^4(\Omega) \cap H_0^2(\Omega)\right), \\ &u_t \in L^\infty\left(R^+, H^4(\Omega) \cap H_0^2(\Omega)\right), \\ &u_{tt} \in L^\infty\left(R^+, L^2(\Omega)\right), \\ &\theta \in L^\infty\left(R^+, U\right), \\ &\theta_t \in L^\infty\left(R^+, L^2(\Omega)\right). \end{aligned} \tag{24}$$

Proof. Let us consider the variational problem associated with (8)–(11): find $(u(t), \theta(t)) \in H^4(\Omega) \cap H_0^2(\Omega) \times U$ such that

$$\begin{aligned} &\int_\Omega u_{tt}\omega\, dx + \int_\Omega \Delta u \Delta\omega\, dx + \int_\Omega \Delta u_t \Delta\omega\, dx \\ &\quad + \sigma\left(\|\nabla u\|^2\right)\int_\Omega \nabla u \nabla\omega\, dx \\ &\quad + \phi\left(\int_\Omega \nabla u_t \nabla\omega\, dx\right)\int_\Omega \nabla u_t \nabla\omega\, dx \\ &\quad + \int_\Omega f_1(u)\,\omega\, dx + \int_\Omega g(u_t)\,\omega\, dx \\ &\quad + \nu\int_\Omega \Delta\theta\omega\, dx = \int_\Omega q(x)\,\omega\, dx, \\ &\int_\Omega \theta_t\varpi\, dx - \int_\Omega \Delta\theta\varpi\, dx + \int_\Omega f_2(\theta)\,\varpi\, dx \\ &\quad - \nu\int_\Omega \Delta u_t\varpi\, dx = 0 \end{aligned} \tag{25}$$

for all $\omega \in H_0^2(\Omega)$ and $\varpi \in U$. This is done with the Galerkin approximation method which is standard. Here we denote the approximate solution by $(u^m(t), \theta^m(t))$. We can get the theorem by proving the existence of approximation solution, the estimate of approximation solution, convergence, and uniqueness. In the following we give the estimates of approximation solution and the proof of uniqueness of solution.

Estimate 1. In the first approximate equation and the second approximate equation of (25), respectively putting $\omega = u_t^m(t)$ and $\varpi = \theta^m(t)$ and making a computation of addition and considering $\widehat{\sigma}(z) = \int_0^z \sigma(s)ds$ and $\widehat{f}_1(z) = \int_0^z f_1(s)ds$, by using Schwarz inequality, and then integrating from 0 to $t < t_m$, we see that

$$\begin{aligned} &\|u_t^m\|^2 + \|\Delta u^m\|^2 + \widehat{\sigma}\left(\|\nabla u^m\|^2\right) + \int_\Omega \widehat{f}_1(u^m)\, dx \\ &\quad + \|\theta^m\|^2 + \int_0^t \|\Delta u_t^m\|^2\, dt + 2\int_0^t \|\nabla\theta^m\|^2\, dt \\ &\quad + 2\int_0^t \phi\left(\int_\Omega \nabla u \nabla u_t dx\right)\int_\Omega \nabla u \nabla u_t dx\, ds \\ &\quad + 2\int_0^t \int_\Omega g(u_t^m)\, u_t^m dx\, ds \\ &\quad + 2\int_0^t \int_\Omega f_2(\theta^m)\,\theta^m dx\, ds \\ &\le \frac{1}{\lambda_1}\int_0^t \|q(x)\|^2 ds + \|u_t^m(0)\|^2 + \|\Delta u^m(0)\|^2 \\ &\quad + \widehat{\sigma}\left(\|\nabla u^m(0)\|^2\right) + \int_\Omega \widehat{f}_1(u^m(0))\, dx \\ &\quad + \|\theta^m(0)\|^2 \end{aligned} \tag{26}$$

independent of m and t, where λ_1 is the same as λ_1 of (47). Taking into account assumptions (13), (17), and (18) of $\phi(\cdot)$, $f_2(\cdot)$, and $g(\cdot)$ and Assumption 6, we see that there exists $M_1 > 0$ depending only on T such that

$$\begin{aligned} &\|u_t^m\|^2 + \|\Delta u^m\|^2 + \widehat{\sigma}\left(\|\nabla u^m\|^2\right) + \int_\Omega \widehat{f}_1(u^m)\, dx \\ &\quad + \|\theta^m\|^2 \le M_1 \end{aligned} \tag{27}$$

for all $t \in [0, T]$ and for all $m \in N$.

Estimate 2. In the first approximate equation and the second approximate equation of (25), respectively, putting $\omega = \Delta^2 u_t^m(t)$ and $\varpi = \Delta\theta^m(t)$ and making a computation of addition by using Schwarz inequality and Young inequality and considering the assumptions of $\sigma(\cdot)$, $\phi(\cdot)$, $f_1(\cdot)$, $g(\cdot)$, $f_2(\cdot)$, and $q(\cdot)$, we see that there exists $M_2 > 0$ depending only on T such that

$$\|\Delta u_t^m\|^2 + \left\|\Delta^2 u^m\right\|^2 + \|\nabla\theta^m\|^2 \le M_2 \tag{28}$$

for all $t \in [0, T]$ and for all $m \in N$.

Estimate 3. In the first approximate equation and the second approximate equation of (25), respectively integrating by parts with $\omega = u_{tt}^m(0)$ and $\varpi = \theta_t^m(0)$ with $t = 0$ and using Schwarz inequality and Young inequality, we see that there exists $M_3, M_4 > 0$ depending only on T such that

$$\begin{aligned} &\|u_{tt}^m(0)\|^2 \le M_3, \\ &\|\theta_t^m(0)\|^2 \le M_4 \end{aligned} \tag{29}$$

for all $t \in [0, T]$ and for all $m \in N$.

Estimate 4. Let us fix $t, \xi > 0$ such that $\xi < T - t$. Respectively taking the difference of the first approximate equation and the second approximate equation of (25) with $t = t + \xi$ and $t = t$ and respectively replacing ω by $u_t^m(t+\xi) - u_t^m(t)$ and ϖ by $\theta^m(t+\xi) - \theta^m(t)$, we can find constants $M_5, M_6 > 0$, depending only on T, such that

$$\begin{gathered}\left\|u_{tt}^m\right\|^2 + \left\|\Delta u_t^m\right\|^2 \le M_5, \\ \left\|\theta_t^m\right\|^2 \le M_6, \\ \forall m \in N,\ \forall t \in [0,T].\end{gathered} \tag{30}$$

Estimate 5. Taking the scalar product in H with $\omega = \Delta\theta^m$ for the second approximate equation of (25), after a computation we can find a constant $M_7 > 0$, depending only on T such that

$$\left\|\Delta\theta^m\right\|^2 \le M_7, \quad \forall m \in N,\ \forall t \in [0,T]. \tag{31}$$

With the estimates 1-2 and 4-5, we can get the necessary compactness in order to pass approximate equation of (25) to the limit. Then it is a matter of routine to conclude the existence of global solutions in $[0,T]$.

Uniqueness. Let (u,θ), $(v,\widetilde{\theta})$ be two solutions of (8)–(11) with the same initial data. Then writing $p = u - v$, $\vartheta = \theta - \widetilde{\theta}$ and taking the difference (25) with $u = u$, $\theta = \theta$ and $u = v$, $\theta = \widetilde{\theta}$ and respectively replacing ω, ϖ by p_t, ϑ and then making a computation of addition, we have

$$\begin{aligned}&\frac{1}{2}\frac{d}{dt}\left[\|p_t\|^2 + \|\Delta p\|^2 + \|\vartheta\|^2\right] + \|\Delta p_t\|^2 + \|\Delta\vartheta\|^2 + J_1 \\ &\quad + J_2 + \int_\Omega \left[f_1(u) - f_1(v)\right] p_t dx \\ &\quad + \int_\Omega \left[g(u_t) - g(v_t)\right] p_t dx \\ &\quad + \int_\Omega \left[f_2(\theta) - f_2\left(\widetilde{\theta}\right)\right] \vartheta_t dx = 0,\end{aligned} \tag{32}$$

where $J_1 = \int_\Omega [\sigma(\int_\Omega (\nabla u)^2 dx)\Delta u - \sigma(\int_\Omega (\nabla v)^2 dx)\Delta v] p_t dx$ and $J_2 = \int_\Omega [\phi(\int_\Omega \nabla u \nabla u_t dx)\Delta u - \phi(\int_\Omega \nabla v \nabla v_t dx)\Delta v] p_t dx$. Using Mean Value Theorem and the Young inequalities combined with the estimates 1-2 and 4-5, we deduce that for some constant $M_8 > 0$,

$$\begin{aligned}&\frac{d}{dt}\left(\|p_t\|^2 + \|\Delta p\|^2 + \|\vartheta\|^2\right) \\ &\quad \le M_8\left(\|p_t\|^2 + \|\Delta p\|^2 + \|\vartheta\|^2\right), \quad \forall t \in (0,T).\end{aligned} \tag{33}$$

Then from Gronwall's Lemma we see that $u = v$, $\theta = \widetilde{\theta}$. The proof of Theorem 7 is completed. □

Theorem 8. *Under the assumptions of Theorem 7, if the initial data $(u^0, u^1, \theta^0) \in H_0$, there exists a unique weak solution of problem (8)–(11) which depends continuously on initial data with respect to the norm of H_0.*

Proof. By using density arguments, we can obtain the existence of a weak solution in H_0.

Let us consider $\{u^0, u^1, \theta^0\} \in \overline{H_1}$. Since H_1 is dense in H_0, then there exists $\{u_\mu^0, u_\mu^1, \theta_\mu^0\} \subset H_1$, such that

$$\begin{aligned}u_\mu^0 &\longrightarrow u^0 \quad \text{in } H_0^2(\Omega); \\ u_\mu^1 &\longrightarrow u^1 \quad \text{in } L^2(\Omega); \\ \theta_\mu^0 &\longrightarrow \theta^0 \quad \text{in } L^2(\Omega).\end{aligned} \tag{34}$$

We observe that for each $\mu \in N$, there exists (u_μ, θ_μ), smooth solution of the initial-boundary value problem (8)–(11) which satisfies

$$\begin{aligned}&u_{tt\mu} + \Delta^2 u_\mu + \Delta^2 u_{t\mu} \\ &\quad - \left[\sigma\left(\int_\Omega \left(\nabla u_\mu\right)^2 dx\right) + \phi\left(\int_\Omega \nabla u_\mu \nabla u_{t\mu} dx\right)\right]\Delta u_\mu \\ &\quad + f_1\left(u_\mu\right) + g\left(u_{t\mu}\right) + \nu\Delta\theta_\mu = q(x), \\ &\theta_{t\mu} - \Delta\theta_\mu + f_2\left(\theta_\mu\right) - \nu\Delta u_{t\mu} = 0.\end{aligned} \tag{35}$$

Respectively multiplying the first equation in (35) by u_μ and multiplying the second equation in (35) by θ_μ and integrating over Ω and taking the sum and then considering the arguments used in the estimate of the existence of solution, we obtain

$$\left\|u_{t\mu}\right\|^2 + \left\|\Delta u_\mu\right\|^2 + \left\|\theta_\mu\right\|^2 \le C_0, \tag{36}$$

where C_0 is a positive constant independent of $\mu \in N$.

Defining $Z_{\mu,\sigma} = u_\mu - u_\sigma$, $\widetilde{Z}_{\mu,\sigma} = \theta_\mu - \widetilde{\theta}_\mu : \mu, \sigma \in N$, following the steps already used in the uniqueness of regular solution for (8)–(11), and considering the convergence given in (34), we deduce that there exists (u,θ) such that

$$\begin{aligned}u_\mu &\longrightarrow u \quad \text{strongly in } C\left([0,T); H_0^2(\Omega)\right), \\ u_{t\mu} &\longrightarrow u_t \quad \text{strongly in } C\left([0,T); L^2(\Omega)\right), \\ \theta_\mu &\longrightarrow \theta \quad \text{strongly in } C\left([0,T); L^2(\Omega)\right).\end{aligned} \tag{37}$$

From the above convergence, we can pass to the limit using standard arguments in order to obtain

$$\begin{aligned}&u_{tt\mu} + \Delta^2 u_\mu + \Delta^2 u_{t\mu} \\ &\quad - \left[\sigma\left(\int_\Omega \left(\nabla u_\mu\right)^2 dx\right) + \phi\left(\int_\Omega \nabla u_\mu \nabla u_{t\mu} dx\right)\right]\Delta u_\mu \\ &\quad + f_1\left(u_\mu\right) + g\left(u_{t\mu}\right) + \nu\Delta\theta_\mu = q(x), \\ &\theta_{t\mu} - \Delta\theta_\mu + f_2\left(\theta_\mu\right) - \nu\Delta u_{t\mu} = 0.\end{aligned} \tag{38}$$

Theorem 8 is proved. □

Remark 9. In both cases

$$\left\|u_t\right\|^2 + \|\Delta u\|^2 + \|u\|_{\rho+2}^{\rho+2} + \|\theta\|^2 \le C, \tag{39}$$

where C is a constant depending on the initial data in different expression.

In addition, in this paper, C denotes different constant in different expression.

Remark 10. Theorem 8 implies that problem (8)–(11) defines a nonlinear C_0-semigroup $S(t)$ on H_0. Indeed, let us set $S(t)(u^0, u^1, \theta^0) = (u(t), u_t(t), \theta(t))$, where u is the unique solution corresponding to initial data $(u^0, u^1, \theta^0) \in H_0$. Moreover, the operator $S(t)$ defined in H_0 meets the usual semigroup properties

$$\begin{aligned} S(t,\tau) &= S(t)\,S(\tau), \quad \forall t,\tau \in R, \\ S(0) &= I. \end{aligned} \tag{40}$$

To prove the main result, we need the following Lemma 11 of Nakao and Lemma 12

Lemma 11 (see [16]). *Let $\varphi(t)$ be a nonnegative continuous function defined on $[0,T)$, $1 < T \le \infty$, which satisfies*

$$\sup_{t\le s\le t+1} \varphi(s)^{1+\eta} \le M_0\left(\varphi(t) - \varphi(t+1)\right) + M_1, \qquad 0 \le t \le T-1, \tag{41}$$

where M_0, M_1, η are positive constants. Then we have

$$\varphi(t) \le \left(M_0^{-1}\eta(t-1)^+ + \left(\sup_{0\le s\le 1}\varphi(s)\right)^{-\eta}\right)^{-1/\eta} + M_1^{1/(\eta+1)}, \quad 0 \le t \le T. \tag{42}$$

Lemma 12 (see [17]). *Assume that for any bounded positive invariant set $B \subset H$ and for any $\varepsilon > 0$, there exists $T = T(\varepsilon, B)$ such that*

$$d\left(S(T)\,x, S(T)\,y\right) \le \varepsilon + \varpi_T(x,y), \quad \forall x,y \in B, \tag{43}$$

where $\varpi_T : H \times H \to R$ satisfies for any sequence $\{z_n\} \subset B$

$$\liminf_{m\to\infty}\liminf_{n\to\infty}\varpi_T(z_n, z_m) = 0. \tag{44}$$

Then $S(t)$ is asymptotically smooth.

4. The Existence of Absorbing Set

The main result of an absorbing set reads as follows.

Theorem 13. *Assume the hypotheses of Theorem 8; then the corresponding semigroup $S(t)$ of problem (8)–(11) has an absorbing set $\mathbb{B}$ in H_0.*

Proof. Now we show that semigroup $S(t)$ has as absorbing set $\mathbb{B}$ in H_0. Firstly, we can calculate the total energy functional

$$E(t) = \frac{1}{2}\left\{\|u_t\|^2 + \|\Delta u\|^2 + \widehat{\sigma}\left(\|\nabla u\|^2\right) + \|\theta\|^2\right\} + \int_\Omega \widehat{f}_1(u)\,dx - \int_\Omega qu(t)\,dx. \tag{45}$$

Let us fix an arbitrary bounded set $B \subset H_0$ and consider the solutions of problem (8)–(11) given by $(u(t), u_t(t), \theta(t)) = S(t)(u^0, u^1, \theta^0)$ with $(u^0, u^1, \theta^0) \in B$. Our analysis is based on the modified energy function

$$\widetilde{E}(t) = E(t) + a_0|\Omega| + \frac{1}{\lambda_1}\|q\|^2, \tag{46}$$

where $\lambda_1 > 0$ is the first eigenvalue of the operator Δ in $H_0^2(\Omega)$; that is, λ_1 satisfies

$$\|u\|^2 \le \frac{1}{\lambda_1}\|\Delta u\|^2, \quad \forall u \in H_0^2(\Omega). \tag{47}$$

□

It is easy to see that $\widetilde{E}(t)$ dominates $\|(u(t), u_t(t), \theta(t))\|_{H_0}^2$ and $\widetilde{E}(t) \ge (1/4)\|\Delta u(t)\|^2$. By multiplying (8) by u and integrating over Ω, we have

$$\begin{aligned} \|\Delta u\|^2 &= -\left[\sigma\left(\int_\Omega |\nabla u|^2\,dx\right) + \phi\left(\int_\Omega \nabla u\nabla u_t\right)dx\right]\|\nabla u\|^2 \\ &\quad - \int_\Omega f_1(u)\,u\,dx + \|u_t\|^2 - \frac{d}{dt}(u_t, u) \\ &\quad - \int_\Omega g(u_t)\,u\,dx + \int_\Omega qu(t)\,dx - \int_\Omega \Delta u\Delta u_t dx \\ &\quad + \upsilon\int_\Omega \nabla\theta\nabla u\,dx. \end{aligned} \tag{48}$$

Inserting (48) into $\widetilde{E}(t)$, we obtain

$$\begin{aligned} \widetilde{E}(t) &= \|u_t\|^2 \\ &\quad + \frac{1}{2}\left[\widehat{\sigma}\left(\int_\Omega |\nabla u|^2\,dx\right) - \sigma\left(\int_\Omega |\nabla u|^2\,dx\right)\|\nabla u\|^2\right] \\ &\quad - \frac{1}{2}\phi\left(\int_\Omega \nabla u\nabla u_t dx\right)\|\nabla u\|^2 + \frac{1}{2}\|\theta\|^2 \\ &\quad - \frac{1}{2}\int_\Omega \Delta u\Delta u_t dx + \int_\Omega\left(\widehat{f}_1(u) - \frac{1}{2}f_1(u)\,u\right)dx \\ &\quad - \frac{1}{2}\frac{d}{dt}\int_\Omega u_t u\,dx - \frac{1}{2}\int_\Omega g(u_t)\,u\,dx \\ &\quad + \frac{\nu}{2}\int_\Omega \nabla\theta\nabla u\,dx + a_0|\Omega| + \frac{1}{\lambda_1}\|q\|^2 \\ &\quad - \frac{1}{2}\int_\Omega qu(t)\,dx. \end{aligned} \tag{49}$$

Considering (12) and (15) and integrating from t_1 to t_2 for (49), we obtain that

$$\begin{aligned} \int_{t_1}^{t_2} \widetilde{E}(t)\,ds &\le \int_{t_1}^{t_2}\|u_t\|^2\,ds - \frac{1}{2}\int_{t_1}^{t_2}\phi\left(\int_\Omega \nabla u\nabla u_t dx\right)\|\nabla u\|^2\,ds \end{aligned}$$

$$
\begin{aligned}
&+\frac{1}{2}\int_{t_1}^{t_2}\|\theta\|^2\,ds-\frac{1}{2}\int_{t_1}^{t_2}\int_\Omega \Delta u\Delta u_t dx\,ds\\
&-\frac{1}{2}\left(\int_\Omega u_t(t_2)\,u(t_2)\,dx-\int_\Omega u_t(t_1)\,u(t_1)\,dx\right)\\
&-\frac{1}{2}\int_{t_1}^{t_2}\int_\Omega g(u_t)\,u\,dx\,ds\\
&+\frac{\nu}{2}\int_{t_1}^{t_2}\int_\Omega \nabla\theta\nabla u\,ds\,dx+(a_0+a_1)\,|\Omega|\\
&+\frac{1}{\lambda_1}\|q\|^2+\frac{1}{2}\int_{t_1}^{t_2}\int_\Omega qu(t)\,dx\,ds,
\end{aligned}
\tag{50}
$$

where $t_1, t_2 \in [t, t+1]$.

Now let us begin to estimate the right hand side of (50) to use the above Lemma 11 of Nakao.

First, by multiplying (8) by u_t and multiplying (9) by θ and integrating over Ω and then taking the sum, we have

$$
\begin{aligned}
&\frac{d}{dt}E(t)+\|\Delta u_t\|^2+\|\nabla\theta\|^2\\
&\quad=-\int_\Omega g(u_t)\,u_t dx\\
&\qquad-\phi\left(\int_\Omega \nabla u\nabla u_t dx\right)\int_\Omega \nabla u\nabla u_t dx\\
&\qquad-\int_\Omega f_2(\theta)\,\theta\,dx.
\end{aligned}
\tag{51}
$$

Then integrating from t to $t+1$, we get

$$
\begin{aligned}
&E(t+1)-E(t)+\int_t^{t+1}\|\Delta u_t\|^2\,ds+\int_t^{t+1}\|\nabla\theta\|^2\,ds\\
&\quad=-\int_t^{t+1}\int_\Omega g(u_t)\,u_t dx\,ds\\
&\qquad-\int_t^{t+1}\phi\left(\int_\Omega \nabla u\nabla u_t dx\right)\int_\Omega \nabla u\nabla u_t dx\,ds\\
&\qquad-\int_t^{t+1}\int_\Omega f_2(\theta)\,\theta\,dx\,ds.
\end{aligned}
\tag{52}
$$

Taking into account assumptions (13), (17), and (18) of $\phi(\cdot)$, $f_2(\cdot)$, and $g(\cdot)$, we have

$$E(t)\geq E(t+1). \tag{53}$$

Then we define an auxiliary function $I^2(t)$ by putting

$$I^2(t)=E(t)-E(t+1)\geq 0. \tag{54}$$

Thus it is obvious that

$$\int_t^{t+1}\|\Delta u_t\|^2\leq I^2(t), \tag{55}$$

$$\int_t^{t+1}\|\nabla\theta\|^2\leq I^2(t). \tag{56}$$

Noting that $r/(r+2)+2/(r+2)=1$ and using twice Holder inequalities and considering assumption (18) of $g(\cdot)$, we have

$$
\begin{aligned}
&\int_t^{t+1}\|u_t\|^2\,ds\\
&\quad\leq|\Omega|^{r/(r+2)}\left(\int_t^{t+1}\int_\Omega |u_t|^{(r+2)}\,dx\,ds\right)^{2/(r+2)}\\
&\quad\leq|\Omega|^{r/(r+2)}\,\frac{1}{k_4}\left(\int_t^{t+1}\int_\Omega g(u_t)\,u_t dx\,ds\right)^{2/(r+2)}\\
&\quad\leq|\Omega|^{r/(r+2)}\,\frac{1}{k_4}I(t)^{4/(r+2)}.
\end{aligned}
\tag{57}
$$

Using the Mean Value Theorem with $\phi(0)=0$ and considering the estimate of (39) and then using Young inequality combined with (55), we have

$$
\begin{aligned}
&\frac{1}{2}\int_{t_1}^{t_2}\phi\left(\int_\Omega \nabla u\nabla u_t dx\right)\|\nabla u\|^2\,ds\\
&\quad=\frac{1}{2}\int_{t_1}^{t_2}\phi'(\xi_1)\int_\Omega \nabla u\nabla u_t dx\,\|\nabla u\|^2\,ds\\
&\quad\leq\frac{1}{2}\int_{t_1}^{t_2}C\,\|\Delta u_t\|\,\|\Delta u\|\,ds\\
&\quad\leq\frac{C}{4\eta}\int_{t_1}^{t_2}\|\Delta u_t\|^2\,ds+\frac{\eta}{4}\int_{t_1}^{t_2}\|\Delta u\|^2\,ds\\
&\quad\leq\frac{C}{4\eta}\int_{t_1}^{t_2}\|\Delta u_t\|^2\,ds+\eta\sup_{t\leq s\leq t+1}\widetilde{E}(s)\\
&\quad\leq\frac{C}{4\eta}I^2(t)+\eta\sup_{t\leq s\leq t+1}\widetilde{E}(s),
\end{aligned}
\tag{58}
$$

where ξ_1 is among 0 and $\int_\Omega \nabla u\nabla u_t dx$.

Since $\|\theta\|^2\leq(1/\lambda_2)\|\nabla\theta\|^2$, $\forall\theta\in H_0^1$, from (56) we obtain

$$\frac{1}{2}\int_{t_1}^{t_2}\|\theta\|^2\,ds\leq\frac{1}{2\lambda_2}I^2(t), \tag{59}$$

where λ_2 is the first eigenvalue of the operator ∇ in $H_0^1(\Omega)$.

Using Young inequality, we get

$$
\begin{aligned}
\frac{1}{2}\int_{t_1}^{t_2}\int_\Omega \Delta u\Delta u_t ds&\leq\frac{1}{4\eta}\int_{t_1}^{t_2}\|\Delta u_t\|^2\,ds\\
&\quad+\frac{\eta}{4}\int_{t_1}^{t_2}\|\Delta u\|^2\,ds\\
&\leq\frac{1}{4\eta}I(t)^2+\eta\sup_{t\leq s\leq t+1}\widetilde{E}(s).
\end{aligned}
\tag{60}
$$

Since (57), in view of the Mean Value Theorem for integral, there exist number $t_1 \in [t, t+1/4]$ and number $t_2 \in [t+3/4, t+1]$ such that

$$\begin{aligned} \|u_t(t_1)\|^2 &\le |\Omega|^{r/(r+2)} \frac{4}{k_4} I(t)^{4/(r+2)}, \\ \|u_t(t_2)\|^2 &\le |\Omega|^{r/(r+2)} \frac{4}{k_4} I(t)^{4/(r+2)}. \end{aligned} \tag{61}$$

Thus from Schwarz inequality combined with (47) and (61), we have

$$\begin{aligned} &\frac{1}{2}\left(\int_\Omega u_t(t_2)\, u(t_2)\, dx - \int_\Omega u_t(t_1)\, u(t_1)\, dx\right) \\ &\quad \le \frac{1}{2\sqrt{\lambda_1}} \left(\|u_t(t_2)\| \|\Delta u(t_2)\| + \|u_t(t_1)\| \|\Delta u(t_1)\|\right) \\ &\quad \le \frac{2}{\sqrt{\lambda_1}\sqrt{k_4}} |\Omega|^{r/2(r+2)} I(t)^{2/(r+2)} \sup_{t\le s\le t+1} \|\Delta u\| \\ &\quad \le \frac{\left(\left(2/\sqrt{\lambda_1}\sqrt{k_4}\right)|\Omega|^{r/2(r+2)}\right)^2}{\eta} I(t)^{4/(r+2)} \\ &\qquad + \eta \sup_{t\le s\le t+1} \widetilde{E}(s). \end{aligned} \tag{62}$$

Considering assumption (19) of $g(\cdot)$ and using Young inequality and Holder inequality with $(r+1)/(r+2) + 1/(r+2) = 1$, then from (47) and (57), we have

$$\begin{aligned} &\frac{1}{2}\int_{t_1}^{t_2}\int_\Omega g(u_t)\, u\, dx\, ds \le \frac{k_5}{2}\int_{t_1}^{t_2}\int_\Omega \left(1+|u_t|^r\right)|u_t|\, u\, dx\, ds \\ &\quad \le \frac{k_5^2}{4\eta\lambda_1}\int_t^{t+1}\int_\Omega u_t(s)^2\, dx\, ds + \frac{\eta\lambda_1}{4}\int_t^{t+1}\int_\Omega u(s)^2\, dx\, ds \\ &\quad + \frac{k_5}{2}\int_{t_1}^{t_2}|u_t|^{r+1} u\, dx\, ds \le \frac{k_5^2}{4\eta\lambda_1}\int_t^{t+1}\int_\Omega u_t(s)^2\, dx\, ds + \frac{\eta\lambda_1}{4} \\ &\quad \cdot \int_t^{t+1}\int_\Omega u(s)^2\, dx\, ds + \frac{k_5}{2} \\ &\quad \cdot \int_t^{t+1}\left(\int_\Omega |u_t|^{r+2} dx\right)^{(r+1)/(r+2)}\left(\int_\Omega u^{r+2} dx\right)^{1/(r+2)} ds \\ &\quad \le \frac{k_5^2}{4\eta\lambda_1}\int_t^{t+1}\int_\Omega u_t(s)^2\, dx\, ds + \frac{\eta\lambda_1}{4}\int_t^{t+1}\int_\Omega u(s)^2\, dx\, ds \\ &\quad + \frac{k_5}{2}\left\{\int_t^{t+1}\left[\left(\int_\Omega u_t^{r+2} dx\right)^{(r+1)/(r+2)}\right]^{(r+2)/(r+1)} ds\right\}^{(r+1)/(r+2)} \\ &\quad \cdot \left\{\int_t^{t+1}\left[\left(\int_\Omega u^{r+2} dx\right)^{1/(r+2)}\right]^{r+2} ds\right\}^{1/(r+2)} = \frac{k_5^2}{4\eta\lambda_1} \\ &\quad \cdot \int_t^{t+1}\int_\Omega u_t(s)^2\, dx\, ds + \frac{\eta\lambda_1}{4}\int_t^{t+1}\int_\Omega u(s)^2\, dx\, ds \\ &\quad + \frac{k_5}{2}\left(\int_t^{t+1}\int_\Omega u_t^{r+2} dx\, ds\right)^{(r+1)/(r+2)} \\ &\quad \cdot \left(\int_t^{t+1}\int_\Omega u^{r+2} dx\, ds\right)^{1/(r+2)} \le \frac{k_5^2}{4\eta\lambda_1}|\Omega|^{r/(r+2)}\frac{1}{k_4} I(t)^{4/(r+2)} \\ &\quad + \eta \sup_{t\le s\le t+1}\widetilde{E}(s) + \frac{k_5\mu}{2k_4} I(t)^{2(r+1)/(r+2)} \sup_{t\le s\le t+1}\|\Delta u\| \\ &\quad \le \frac{k_5^2}{4\eta\lambda_1}|\Omega|^{r/(r+2)}\frac{1}{k_4} I(t)^{4/(r+2)} + \eta \sup_{t\le s\le t+1}\widetilde{E}(s) + \frac{k_5^2\mu^2}{4\eta k_4^2} \\ &\quad \cdot I(t)^{4(r+1)/(r+2)} + \eta \sup_{t\le s\le t+1}\widetilde{E}(s). \end{aligned} \tag{63}$$

Also by Young inequality, we have

$$\begin{aligned} \frac{\nu}{2}\int_{t_1}^{t_2}\int_\Omega \nabla\theta\nabla u\, dx\, ds &\le \frac{\nu}{2}\int_{t_1}^{t_2}\|\nabla\theta\|\,\|\nabla u\|\, ds \\ &\le \int_{t_1}^{t_2}\frac{\nu^2}{4\eta\lambda_3}\|\nabla\theta\|^2\, ds \\ &\quad + \int_{t_1}^{t_2}\frac{\eta\lambda_3}{4}\|\nabla u\|^2\, ds \\ &\le \frac{\nu^2}{4\eta\lambda_3} I^2(t) + \eta \sup_{t\le s\le t+1}\widetilde{E}(s), \end{aligned} \tag{64}$$

where $\lambda_3 > 0$ is the first eigenvalue of the operator ∇ in $H_0^1(\Omega)$; that is, λ_3 satisfies

$$\|\nabla u\|^2 \le \frac{1}{\lambda_3}\|\Delta u\|^2, \quad \forall u \in H_0^1(\Omega). \tag{65}$$

Finally using Young inequality again, we get that

$$\frac{1}{2}\int_{t_1}^{t_2}\int_\Omega qu(t)\, dx\, ds \le \frac{1}{4\eta\lambda_1}\|q\|^2 + \eta \sup_{t\le s\le t+1}\widetilde{E}(s). \tag{66}$$

Inserting (57)–(60) and (62), (63), (64), and (66) into (50), we obtain

$$\begin{aligned} \int_{t_1}^{t_2}\widetilde{E}(s)\, ds &\le \left[\left(|\Omega|^{r/(r+2)}\frac{1}{k_4}\right.\right. \\ &\quad + \frac{\left(\left(2/\sqrt{\lambda_1}\sqrt{k_4}\right)|\Omega|^{r/2(r+2)}\right)^2}{\eta} \\ &\quad \left. + \frac{k_5^2}{4\eta\lambda_1}|\Omega|^{r/(r+2)}\frac{1}{k_4}\right) I(t)^{4/(r+2)} + \left(\frac{C}{4\eta} + \frac{1}{2\lambda_2}\right. \\ &\quad \left.\left. + \frac{1}{4\eta} + \frac{\nu^2}{4\eta\lambda_3}\right) I(t)^2 + \left(\frac{k_5^2\mu^2}{4\eta k_4^2}\right) I(t)^{4(r+1)/(r+2)}\right] \\ &\quad + 6\eta \sup_{t\le s\le t+1}\widetilde{E}(s) + (a_0 + a_1)|\Omega| + \left(\frac{1}{\lambda_1} + \frac{1}{4\eta\lambda_1}\right) \\ &\quad \cdot \|q\|^2. \end{aligned} \tag{67}$$

For the left hand side of (67), we use the Mean Value Theorem; then there exists number $\tau \in [t_1, t_2]$ such that

$$\int_{t_1}^{t_2} \widetilde{E}(s)\,ds \geq \frac{1}{2}\widetilde{E}(t+1) = \frac{1}{2}\left(\widetilde{E}(t) - I(t)^2\right). \tag{68}$$

So we conclude that

$$\widetilde{E}(t) \leq I(t)^2 + 2\int_{t_1}^{t_2} \widetilde{E}(s)\,ds. \tag{69}$$

Inserting (67) into (69), we obtain that

$$\begin{aligned}\widetilde{E}(t) \leq 2\Bigg[\Bigg(&|\Omega|^{r/(r+2)}\frac{1}{k_4} + \frac{\left(\left(2/\sqrt{\lambda_1}\sqrt{k_4}\right)|\Omega|^{r/2(r+2)}\right)^2}{\eta} + \frac{k_5^2}{8\eta\lambda_1}|\Omega|^{r/(r+2)}\frac{1}{k_4}\Bigg) I(t)^{4/(r+2)} + \Bigg(1 + \frac{C}{4\eta} \\ &+ \frac{1}{2\lambda_2} + \frac{1}{4\eta} + \frac{\nu^2}{4\eta\lambda_3}\Bigg) I(t)^2 + \left(\frac{k_5^2\mu^2}{4\eta k_4^2}\right) \\ &\cdot I(t)^{4(r+1)/(r+2)}\Bigg] + 12\eta \sup_{t\leq s\leq t+1}\widetilde{E}(s) + (a_0 + a_1)|\Omega| \\ &+ 2\left(\frac{1}{\lambda_1} + \frac{1}{4\eta\lambda_1}\right)\|q\|^2.\end{aligned} \tag{70}$$

Letting $0 < \eta < 1/12$ and noting that $I(t)^{2r/(r+2)}$ and $I(t)^{4r/(r+2)}$ are bounded with estimate (39), then from (70), we get

$$\begin{aligned}\widetilde{E}(t) \leq \frac{2}{1-12\eta} I(t)^{4/(r+2)}\Bigg[\Bigg(&|\Omega|^{r/(r+2)}\frac{1}{k_4} + \frac{\left(\left(2/\sqrt{\lambda_1}\sqrt{k_4}\right)|\Omega|^{r/2(r+2)}\right)^2}{\eta} \\ &+ \frac{k_5^2}{4\eta\lambda_1}|\Omega|^{r/(r+2)}\frac{1}{k_4}\Bigg) + \Bigg(1 + \frac{C}{4\eta} + \frac{1}{2\lambda_2} + \frac{1}{4\eta} \\ &+ \frac{\nu^2}{4\eta\lambda_3}\Bigg)C + \frac{k_5^2\mu^2}{4\eta k_4^2}C\Bigg] + \frac{1}{1-12\eta}(a_0 + a_1)|\Omega| \\ &+ \frac{2}{1-12\eta}\left(\frac{1}{\lambda_1} + \frac{1}{4\eta\lambda_1}\right)\|q\|^2,\end{aligned} \tag{71}$$

where C is a constant which depends on B.

Set

$$\begin{aligned}C_1 = \frac{2}{1-12\eta}\Bigg[\Bigg(&|\Omega|^{r/(r+2)}\frac{1}{k_4} + \frac{\left(\left(2/\sqrt{\lambda_1}\sqrt{k_4}\right)|\Omega|^{r/2(r+2)}\right)^2}{\eta} \\ &+ \frac{k_5^2}{4\eta\lambda_1}|\Omega|^{r/(r+2)}\frac{1}{k_4}\Bigg) + \Bigg(1 + \frac{C}{4\eta} + \frac{1}{2\lambda_2} + \frac{1}{4\eta} \\ &+ \frac{\nu^2}{4\eta\lambda_3}\Bigg)C + \frac{k_5^2\mu^2}{4\eta k_4^2}C\Bigg];\end{aligned} \tag{72}$$

then (70) can be rewritten as

$$\begin{aligned}\widetilde{E}(t)^{1+r/2} \leq C_1\left(\widetilde{E}(t) - \widetilde{E}(t+1)\right) &+ \Bigg[\frac{2}{1-12\eta}(a_0 + a_1)|\Omega| \\ &+ \frac{2}{1-12\eta}\left(\frac{1}{\lambda_1} + \frac{1}{4\eta\lambda_1}\right)\|q\|^2\Bigg]^{1+r/2}.\end{aligned} \tag{73}$$

Using Nakao's Lemma 11, we conclude that

$$\begin{aligned}\widetilde{E}(t) \leq \left(C_1^{-1}\frac{r}{2}(t-1)^+ + \widetilde{E}(0)^{-r/2}\right)^{-2/r} &+ \frac{1}{1-12\eta}(a_0 + a_1)|\Omega| \\ &+ \frac{2}{1-12\eta}\left(\frac{1}{\lambda_1} + \frac{1}{4\eta\lambda_1}\right)\|q\|^2.\end{aligned} \tag{74}$$

As $r \to \infty$, the first term of the right side of (74) goes to zero; thus, with $\widetilde{E}(t)$, we conclude

$$\begin{aligned}\mathbb{B} = \Big\{(u, v, \theta) \in H_0 \mid \|\Delta u\|^2 + \|v\|^2 + \|\theta\|^2 &\leq \frac{8}{1-12\eta}(a_0 + a_1)|\Omega| \\ &+ \frac{8}{1-12\eta}\left(\frac{1}{\lambda_1} + \frac{1}{4\eta\lambda_1}\right)\|q\|^2\Big\}\end{aligned} \tag{75}$$

is an absorbing set for $S(t)$ in H_0.

5. The Existence of a Global Attractor

The main result of a global attractor reads as follows.

Theorem 14. *Assume the hypotheses of Theorem 8; then the corresponding semigroup $S(t)$ of problem (8)–(11) is asymptotic compactness.*

Proof. We are going to apply Lemmas 11 and 12 to prove the asymptotic smooth. Given initial data (u^0, u^1, θ^0) and

$(v^0, v^1, \widetilde{\theta}^0)$ in a bounded invariant set $B \subset H_0$, let (u,θ), $(v,\widetilde{\theta})$ be the corresponding weak solutions of problem (8)–(11). Then the differences $w = u - v$, $\vartheta = \theta - \widetilde{\theta}$ are the weak solutions of

$$\begin{aligned}
&w_{tt} + \Delta^2 w + \Delta^2 w_t + \nu\Delta\vartheta + \Delta g \\
&\quad = \sigma\left(\|\nabla u\|^2\right)\Delta u - \sigma\left(\|\nabla v\|^2\right)\Delta v + \Delta\phi - \Delta f_1, \\
&\vartheta_t - \Delta\vartheta + \Delta f_2 - \nu\Delta w_t = 0, \\
&w = \frac{\partial w}{\partial \nu} = 0, \\
&\vartheta = 0, \\
&w(0) = u^0 - v^0, \\
&w_t(0) = u^1 - v^1, \\
&\vartheta(0) = \theta^0 - \widetilde{\theta}_0,
\end{aligned} \tag{76}$$

where

$$\begin{aligned}
&\Delta\phi = \phi\left(\int_\Omega \nabla u \nabla u_t dx\right)\Delta u - \phi\left(\int_\Omega \nabla v \nabla v_t dx\right)\Delta v, \\
&\Delta g = g(u_t) - g(v_t), \\
&\Delta f_1 = f_1(u) - f_1(v), \\
&\Delta f_2 = f_2(\theta) - f_2\left(\widetilde{\theta}\right).
\end{aligned} \tag{77}$$

Let us define

$$E_w(t) = \|w_t\|^2 + \|\Delta w\|^2 + \sigma\left(\|\nabla u\|^2\right)\|\nabla w\|^2 + \|\vartheta\|^2. \tag{78}$$

As before, by density, we can assume formally that w is sufficiently regular. Then, multiplying the first equation in (76) by w_t and integrating over Ω and multiplying the second equation in (76) by ϑ and integrating over Ω and then taking the sum, we get

$$\begin{aligned}
&\frac{1}{2}\frac{d}{dt}E_w(t) + \|\Delta w_t\|^2 + \|\nabla\vartheta\|^2 + \int_\Omega \Delta g w_t dx \\
&\quad + \int_\Omega \Delta f_2 \vartheta\, dx \\
&= -\sigma'\left(\|\nabla u\|^2\right)\|\nabla w\|^2 \int_\Omega \Delta u u_t dx \\
&\quad + \Delta\sigma \int_\Omega \Delta v w_t dx + \int_\Omega \Delta\phi w_t dx - \int_\Omega \Delta f_1 w_t dx,
\end{aligned} \tag{79}$$

where

$$\Delta\sigma = \sigma\left(\|\nabla u\|^2\right) - \sigma\left(\|\nabla v\|^2\right). \tag{80}$$

Let us estimate the right hand side of (79).

Considering the continuity of $\sigma'(\cdot)$ and estimate (39)

$$-\sigma'\left(\|\nabla u\|^2\right)\|\nabla w\|^2 \le C\|\nabla w\|^2, \tag{81}$$

Applying the Mean Value Theorem combined with estimate (39), by Young inequality, we get

$$\Delta\sigma \int_\Omega \Delta v w_t dx \le C\|\nabla w\|^{(r+2)/(r+1)} + \frac{k_4}{4}\|w_t\|_{r+2}^{r+2}. \tag{82}$$

Also use the Mean Value Theorem combined with estimate (39) and Young inequality to get

$$\begin{aligned}
&\int_\Omega \Delta\phi w_t dx = \int_\Omega \phi\left(\int_\Omega \nabla u \nabla u_t dx\right)\Delta w w_t dx \\
&\quad - \int_\Omega \left[\phi\left(\int_\Omega \nabla u \nabla u_t dx\right) - \phi\left(\int_\Omega \nabla v \nabla v_t dx\right)\right] \\
&\quad \cdot \Delta v w_t dx = \int_\Omega \phi'(\xi_2)\int_\Omega \nabla u \nabla u_t dx \Delta w w_t dx \\
&\quad - \int_\Omega \phi'(\xi_3)\int_\Omega \nabla w \nabla w_t dx \Delta v w_t dx \le C\|w\|\|\Delta w_t\| \\
&\quad + C\|w\|\|\Delta w_t\|\|w_t\| \le \frac{1}{4}\|\Delta w_t\|^2 + C^2\|w\|^2 \\
&\quad + \frac{1}{4}\|\Delta w_t\|^2 + C^2\|w\|^2\|w_t\|^2 \le \frac{1}{4}\|\Delta w_t\|^2 \\
&\quad + C^2\|\nabla w\|^2 + C^2\|w\|^2\|w_t\|_{r+2}^2 \le \frac{1}{4}\|\Delta w_t\|^2 \\
&\quad + C^2\|\nabla w\|^2 + \frac{C^4}{k_3}\|w\|^{2((r+2)/r)} + \frac{k_3}{4}\|w_t\|_{r+2}^{r+2},
\end{aligned} \tag{83}$$

where ξ_2 is among 0 and $\int_\Omega \nabla u \nabla u_t dx$, and ξ_3 is among $\int_\Omega \nabla u \nabla u_t dx$ and $\int_\Omega \nabla v \nabla v_t dx$.

By the Holder inequality, Minkowski inequality combined with the estimate of (39), and Young inequality, we obtain

$$\int_\Omega \Delta f_1 w_t dx \le C\|\nabla w\|^{(r+2)/(r+1)} + \frac{k_4}{4}\|w_t\|_{r+2}^{r+2}. \tag{84}$$

On the other hand, considering assumptions (17) and (18) of $f_2(\cdot)$ and $g(\cdot)$,

$$\begin{aligned}
&\int_\Omega \Delta f_2 \vartheta\, dx \ge k_3\|\vartheta\|_{\varrho+2}^{\varrho+2}, \\
&\int_\Omega \Delta g w_t dx \ge k_4\|w_t\|_{r+2}^{r+2}.
\end{aligned} \tag{85}$$

Thus by inserting (81)–(85) into (79), we get that

$$\begin{aligned}
&\frac{1}{2}\frac{d}{dt}E_w(t) + \frac{1}{4}\|\Delta w_t\|^2 + \|\nabla\vartheta\|^2 + \frac{k_4}{4}\|w_t\|_{r+2}^{r+2} \\
&\quad + k_3\|\vartheta\|_{\varrho+2}^{\varrho+2} \\
&\le C\left(\|\nabla w\|^2 + \|\nabla w\|^{(r+2)/(r+1)} + \|\nabla w\|^{2((r+2)/r)}\right).
\end{aligned} \tag{86}$$

Then integrating from t to $t+1$ and defining an auxiliary function $F^2(t)$, we get

$$\begin{aligned}&\frac{1}{4}\int_t^{t+1}\|\Delta w_t\|^2\,ds+\int_t^{t+1}\|\nabla\vartheta\|^2\,ds+\frac{k_4}{4}\\&\cdot\int_t^{t+1}\|w_t\|_{r+2}^{r+2}\,ds+k_3\int_t^{t+1}\|\vartheta\|_{\varrho+2}^{\varrho+2}\,ds\le E_w(t)\\&-E_w(t+1)+C\int_t^{t+1}\left(\|\nabla w\|^2+\|\nabla w\|^{(r+2)/(r+1)}\right.\\&\left.+\|\nabla w\|^{2(2(r+2)/r)}\right)ds=F(t)^2.\end{aligned}\tag{87}$$

It is obvious that

$$E_w(t+1)\ge E_t(t),\tag{88}$$

$$\begin{aligned}\frac{1}{4}\int_t^{t+1}\|\Delta w_t\|^2\,ds&\le F(t)^2,\\\int_t^{t+1}\|\nabla\vartheta\|^2\,ds&\le F^2(t),\end{aligned}\tag{89}$$

$$\begin{aligned}\frac{k_4}{4}\int_t^{t+1}\|w_t\|_{r+2}^{r+2}\,ds&\le F(t)^2,\\k_3\int_t^{t+1}\|\vartheta\|_{\varrho+2}^{\varrho+2}\,ds&\le F(t)^2.\end{aligned}\tag{90}$$

Then by multiplying first equation in (76) by w and integrating over Ω again, we obtain that

$$\begin{aligned}&\|\Delta w\|^2+\sigma\left(\|\nabla u\|^2\right)\|\nabla w\|^2=-\frac{d}{dt}\int_\Omega w_t w\,dx+\|w_t\|^2\\&-\int_\Omega\Delta^2 w_t w\,dx+\Delta\sigma\int_\Omega\Delta v w\,dx-\int_\Omega\Delta f_1 w\,dx\\&-\int_\Omega\Delta g w\,dx\\&+\int_\Omega\left[\phi\left(\int_\Omega\nabla u\nabla u_t dx\right)-\phi\left(\int_\Omega\nabla v\nabla v_t dx\right)\right]\\&\cdot\Delta v w\,dx-\phi\left(\int_\Omega\nabla u\nabla u_t dx\right)\|\nabla w\|^2\\&-\nu\int_\Omega\vartheta\Delta w\,dx.\end{aligned}\tag{91}$$

Integrating from t_1 to t_2, we get

$$\begin{aligned}&\int_{t_1}^{t_2}\left(\|\Delta w\|^2+\sigma\left(\|\nabla u\|^2\right)\|\nabla w\|^2\right)ds=\int_\Omega w_t(t_2)\\&\cdot w(t_2)\,dx-\int_\Omega w_t(t_1)\,w(t_1)\,dx\\&+\int_{t_1}^{t_2}\|w_t\|^2\,dt-\int_{t_1}^{t_2}\int_\Omega\Delta w_t\Delta w\,dx\,ds\\&+\int_{t_1}^{t_2}\Delta\sigma\int_\Omega\Delta v w\,dx\,ds-\int_{t_1}^{t_2}\int_\Omega\Delta f_1 w\,dx\,ds\\&-\int_{t_1}^{t_2}\int_\Omega\Delta g w\,dx\,ds\\&+\int_{t_1}^{t_2}\int_\Omega\left[\phi\left(\int_\Omega\nabla u\nabla u_t dx\right)-\phi\left(\int_\Omega\nabla v\nabla v_t dx\right)\right]\\&\cdot\Delta v w\,dx\,ds-\int_{t_1}^{t_2}\phi\left(\int_\Omega\nabla u\nabla u_t dx\right)\|\nabla w\|^2\,ds\\&+\nu\int_{t_1}^{t_2}\int_\Omega\nabla\vartheta\nabla w\,dx\,ds.\end{aligned}\tag{92}$$

Now let us estimate the right hand side of (92). Firstly, from the first inequality of (90), by holder inequality we infer that

$$\begin{aligned}&\int_t^{t+1}\|w_t\|^2\,ds=\int_t^{t+1}\int_\Omega|w_t|^2\,dx\,ds\\&\le|\Omega|^{r/(r+2)}\left(\int_t^{t+1}\int_\Omega|w_t|^{2(r+2)/2}\,dx\,ds\right)^{2/(r+2)}\\&\le CF(t)^{4/(r+2)};\end{aligned}\tag{93}$$

thus there exists $t_1\in[t,t+1/4]$ and $t_2\in[t+3/4,t+1]$ such that

$$\begin{aligned}\|w_t(t_1)\|^2&\le CF^{4/(r+2)}(t),\\\|w_t(t_2)\|^2&\le CF^{4/(r+2)}(t);\end{aligned}\tag{94}$$

then we can deduce that

$$\begin{aligned}&\int_\Omega w_t(t_2)\,w(t_2)\,dx-\int_\Omega w_t(t_1)\,w(t_1)\,dx\\&\le CF(t)^{4/(r+2)}+\frac{1}{8}\sup_{t\le\sigma\le t+1}E_w(\sigma).\end{aligned}\tag{95}$$

Use Schwarz inequality combined with the estimate of (39) and Holder inequality to obtain

$$\begin{aligned}&\int_{t_1}^{t_2}\int_\Omega\Delta w_t\Delta w\,dx\,ds\le C\int_{t_1}^{t_2}\|\Delta w_t\|\,ds\\&\le C\left(\int_{t_1}^{t_2}1\,ds\right)^{1/2}\left(\int_{t_1}^{t_2}\|\Delta w_t\|^2\,ds\right)^{1/2}\le CF(t).\end{aligned}\tag{96}$$

Apply the Mean Value Theorem combined with estimate (39) to get

$$\int_{t_1}^{t_2}\Delta\sigma\int_\Omega\Delta v w\,dx\,ds\le C\int_{t_1}^{t_2}\|\nabla w\|^2\,ds.\tag{97}$$

Assumption (14) of $f(\cdot)$ and the estimate of (39) imply that

$$\int_{t_1}^{t_2}\int_\Omega\Delta f_1 w\,dx\,ds\le C\int_{t_1}^{t_2}\|\nabla w\|^2\,ds.\tag{98}$$

Also from assumption (19) of $g(\cdot)$ and the estimate of (39) combined with (94), we have

$$\int_{t_1}^{t_2}\int_\Omega \Delta g w\,dx\,ds \le C\int_{t_1}^{t_2}\|w_t\|\,\|\Delta w\|\,ds \le CF(t)^{4/(r+2)} + \frac{1}{8}\sup_{t\le\sigma\le t+1} E_w(\sigma). \tag{99}$$

Using the Mean Value Theorem and considering the assumption of $\phi(\cdot)$ and the estimate of (39), we have

$$\begin{aligned}&\int_{t_1}^{t_2}\int_\Omega\left[\phi\left(\int_\Omega \nabla u\nabla u_t dx\right)-\phi\left(\int_\Omega \nabla v\nabla v_t dx\right)\right]\\&\cdot\Delta v w\,dx\,ds = \int_{t_1}^{t_2}\phi'(\xi_4)\int_\Omega \nabla w\nabla w_t dx\\&\cdot\int_\Omega \Delta v w\,dx\,ds \le C\int_{t_1}^{t_2}\|w\|\,\|w_t\|\,ds\\&\le C\int_{t_1}^{t_2}\|\nabla w\|^2 ds + C\int_{t_1}^{t_2}\|w_t\|^2 ds,\end{aligned} \tag{100}$$

$$\int_{t_1}^{t_2}\phi\left(\int_\Omega \nabla u\nabla u_t dx\right)\|\nabla w\|^2 ds \le C\int_{t_1}^{t_2}\|\nabla w\|^2 ds, \tag{101}$$

where ξ_4 is among $\int_\Omega \nabla u\nabla u_t dx$ and $\int_\Omega \nabla v\nabla v_t dx$.

Finally, use Young inequality to get

$$\nu\int_{t_1}^{t_2}\int_\Omega \nabla\vartheta\nabla w\,dx\,ds \le \frac{\nu^2}{2}\int_{t_1}^{t_2}\|\nabla\vartheta\|^2 + \frac{1}{2}\int_{t_1}^{t_2}\|\nabla w\|^2 ds. \tag{102}$$

By inserting (93) and (95)–(102) into (92), we obtain that

$$\begin{aligned}&\int_{t_1}^{t_2}\left[\|\Delta w\|^2+\sigma\left(\|\nabla u\|^2\right)\|\nabla w\|^2\right]ds\\&\le 3C\int_{t_1}^{t_2}\|\nabla w\|^2 ds + 2C\int_{t_1}^{t_2}\|w_t\|^2 ds\\&+C\int_{t_1}^{t_2}\|\nabla\vartheta\|^2 ds + 2CF(t)^{4/(r+2)}\\&+\frac{1}{4}\sup_{t\le\sigma\le t+1}E_w(\sigma)+CF(t).\end{aligned} \tag{103}$$

Considering (89) and (93), from (103), we have

$$\begin{aligned}&\int_{t_1}^{t_2}\left[\|\Delta w\|^2+\sigma\left(\|\nabla u\|^2\right)\|\nabla w\|^2\right]ds\\&\le 3C\int_t^{t+1}\|\nabla w\|\,ds + CF(t)+CF^2(t)\\&+4CF(t)^{4/(r+2)}+\frac{1}{4}\sup_{t\le\sigma\le t+1}E_w(\sigma).\end{aligned} \tag{104}$$

Using Holder inequality with $1/(\varrho+2)+(\varrho+1)/(\varrho+2)=1$,

$$\begin{aligned}\int_{t_1}^{t_2}\|\vartheta\|^2 ds &\le C\int_{t_1}^{t_2}\|\vartheta\|_{\varrho+2}\,ds\\&\le C\left(\int_{t_1}^{t_2}1^{(\varrho+2)/(\varrho+2)}ds\right)^{(\varrho+1)/(\varrho+2)}\int_{t_1}^{t_2}\|\vartheta\|_{\varrho+2}^{\varrho+2}ds\\&\le C\int_{t_1}^{t_2}\|\vartheta\|_{\varrho+2}^{\varrho+2}ds \le CF^2(t).\end{aligned} \tag{105}$$

Then from the definition of $E_w(t)$ and (93), (104), and (105), we obtain that

$$\int_{t_1}^{t_2}E_w(s)\,ds \le 5CF(t)^{4/(r+2)}+2CF^2(t)+CF(t)+\frac{1}{4}\sup_{t\le\sigma\le t+1}E_w(\sigma)+3C\int_t^{t+1}\|\nabla w\|\,ds. \tag{106}$$

For (106), by using Mean Value theorem, there exists $t^*\in[t_1, t_2]$ such that

$$E_w(t^*) \le 10CF(t)^{4/(r+2)}+2CF(t)+4CF^2(t)+\frac{1}{2}\sup_{t\le\sigma\le t+1}E_w(\sigma)+6C\int_t^{t+1}\|\nabla w\|\,ds. \tag{107}$$

From (87), we see that

$$E_w(t) \le E_w(t+1)+F^2(t). \tag{108}$$

Let $E_w = \sup_{t\le\sigma\le t+1}E_w(\sigma)$ with $\tau\in[t,t+1]$; then integrate (86) over $[t,\tau]$ and over $[t^*,t+1]$ to have

$$\begin{aligned}\sup_{t\le\sigma\le t+1}E_w(\sigma) &\le E_w(\tau)\\&\le E_w(t+1)+F^2(t)+C\int_t^{t+1}\left(\|\nabla w\|^2+\|\nabla w\|^{2(r+2)/r}\right)ds\\&\le E_w(t^*)+F^2(t)+C\int_t^{t+1}\left(\|\nabla w\|^2+\|\nabla w\|^{2(r+2)/r}\right)ds.\end{aligned} \tag{109}$$

Inserting (107) into (109), we obtain

$$\begin{aligned}\sup_{t\le\sigma\le t+1}E_w(\sigma) &\le 10CF(t)^{4/(r+2)}+2CF(t)+4CF^2(t)\\&+\frac{1}{2}\sup_{t\le\sigma\le t+1}E_w(\sigma)+6C\int_t^{t+1}\|\nabla w\|\,ds\\&+F^2(t)\\&+C\int_t^{t+1}\left(\|\nabla w\|^2+\|\nabla w\|^{2(r+2)/r}\right)ds.\end{aligned} \tag{110}$$

Therefore from the boundary of $1 + F(t)^{(1-4/(r+2))} + F(t)^{2-4/(r+2)}$, we have

$$\begin{aligned}\sup_{t\le\sigma\le t+1} E_w(\sigma) &\le CF(t)^{4/(r+2)} \\ &\quad + C\int_t^{t+1}\left(\|\nabla w\| + \|\nabla w\|^2 + \|\nabla w\|^{2(r+2)/r}\right)ds.\end{aligned} \tag{111}$$

Therefore

$$\begin{aligned}\sup_{t\le\sigma\le t+1} E_w(\sigma)^{1+r/2} &\le C\left(E_w(t) - E_w(t+1)\right) \\ &\quad + C\sup_{0\le\sigma\le T}\int_\sigma^{\sigma+1}\left(\|\nabla w\| + \|\nabla w\|^2 + \|\nabla w\|^{2(r+2)/r}\right)ds.\end{aligned} \tag{112}$$

From Nakao's Lemma 11, there exists $C_B > 0$ and $C_T > 0$ such that

$$\begin{aligned}E_w(t) &\le C_B\left[(t-1)^+\right]^{-2/r} + C_T\left(\sup_{0\le\sigma\le T}\int_\sigma^{\sigma+1}\left(\|\nabla w\|\right.\right. \\ &\quad \left.\left. + \|\nabla w\|^2 + \|\nabla w\|^{2(r+2)/r}\right)ds\right)^{2/(r+2)}, \\ &\qquad 0 \le t \le T.\end{aligned} \tag{113}$$

From the definition of $E_w(t)$, we have

$$\begin{aligned}\|(w, w_t, \vartheta)\|_{H_0} &\le C_B\left[(t-1)^+\right]^{-2/r} \\ &\quad + C_T\left(\sup_{0\le\sigma\le T}\int_\sigma^{\sigma+1}\left(\|\nabla w\| + \|\nabla w\|^2\right.\right. \\ &\quad \left.\left. + \|\nabla w\|^{2(r+2)/r}\right)ds\right)^{2/(r+2)}.\end{aligned} \tag{114}$$

Given $\varepsilon > 0$, we choose T large such that

$$C_B\left[(t-1)^+\right]^{-2/r} \le \varepsilon, \tag{115}$$

and define $\varpi_T : H_0 \times H_0 \to R$ as

$$\begin{aligned}&\varpi_T\left(\left(u^0, u^1, \theta^0\right), \left(v^0, v^1, \widetilde{\theta}^0\right)\right) \\ &\quad = C_T\left(\sup\int_\sigma^{\sigma+1}\left(\|\nabla w\| + \|\nabla w\|^2\right.\right. \\ &\quad \left.\left. + \|\nabla w\|^{2(r+2)/r}\right)ds\right)^{2/(r+2)}.\end{aligned} \tag{116}$$

Then from (114)–(116), we get

$$\begin{aligned}&\left\|S(T)\left(u^0, u^1, \theta^0\right) - S(T)\left(v^0, v^1, \widetilde{\theta}^0\right)\right\|_{H_0} \\ &\quad \le \varepsilon + \varpi_T\left(\left(u^0, u^1, \theta^0\right), \left(v^0, v^1, \widetilde{\theta}^0\right)\right)\end{aligned} \tag{117}$$

for all $(u^0, u^1, \theta^0), (v^0, v^1, \widetilde{\theta}^0) \in B$.

Let $(u_n^0, u_n^1, \theta_n^0)$ be a given sequence of initial data in B. Then the corresponding sequence (u_n, u_{tn}, θ_n) of solutions of the problem (8)–(11) is uniformly bounded in H_0, because B is bounded and positively invariant. So $\{u_n\}$ is bounded in $C([0,\infty), H_0^2(\Omega)) \cap C^1([0,\infty), L^2(\Omega))$. Since $H_0^2(\Omega) \hookrightarrow H_0^1(\Omega)$ compactly, there exists a subsequence u_{nk} which converges strongly in $C([0,T], H_0^1(\Omega))$. Therefore

$$\begin{aligned}&\lim_{k\to\infty}\lim_{l\to\infty}\int_0^T\left(\|\nabla u_{nk}(s) - \nabla u_{nl}(s)\|\right. \\ &\quad + \|\nabla u_{nk}(s) - \nabla u_{nl}(s)\|^2 \\ &\quad \left. + \|\nabla u_{nk}(s) - \nabla u_{nl}(s)\|^{2(r+2)/r}\right)ds = 0, \\ &\lim_{k\to\infty}\lim_{l\to\infty}\varpi_T\left(\left(u_{nk}^0, u_{nk}^1, \theta_{nk}^0\right), \left(u_{nl}^0, u_{nl}^1, \theta_{nl}^0\right)\right) = 0.\end{aligned} \tag{118}$$

So $S(t)$ is asymptotically smooth in H_0. That is, Lemma 12 holds. Thus Theorem 14 is proved. □

In view of Theorems 13 and 14, we have the following.

Theorem 15. *The corresponding semigroup $S(t)$ of problem (8)–(11) has a compact global attractor in the phase space H_0.*

Competing Interests

The authors declare that they have no competing interests.

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