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Research Article

Constrained Adaptive Neural Control of Nonlinear Strict-Feedback Systems with Input Dead-Zone

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This paper focuses on a single neural network tracking control for a class of nonlinear strict-feedback systems with input dead-zone and time-varying output constraint via prescribed performance method. To release the limit condition on previous performance function that the initial tracking error needs to be known, a new modified performance function is first constructed. Further, to reduce the computational burden of traditional neural back-stepping control approaches which require all the virtual controllers to be necessarily carried out in each step, the nonlinear items are transmitted to the last step such that only one neural network is required in this design. By regarding the input-coefficients of the dead-zone slopes as a system uncertainty and introducing a new concise system transformation technique, a composite adaptive neural state-feedback control approach is developed. The most prominent feature of this scheme is that it not only owes low-computational property but also releases the previous limitations on performance function and is capable of guaranteeing the output confined within the new form of prescribed bound. Moreover, the closed-loop stability is proved using Lyapunov function. Comparative simulation is induced to verify the effectiveness.

1. Introduction

In recent years, actuated by practical requirements and theoretical developments, numerous adaptive back-stepping control schemes have been proposed for uncertain nonlinear systems in lower-triangular form including strict-feedback and pure-feedback systems [1]. Specially, fuzzy logic systems (FLS) or neural networks (NNs) based control schemes have attracted great concern due to their inherent approximation capabilities as well as relaxing linearly in parameters assumption [2, 3]. The benefit of applying FLS or NNs is that the problem of spending much effort on system modeling can be elegantly overcome. Although there has been significant progress in aforementioned literatures, the problem of complexity (POC) is the main drawback of traditional adaptive back-stepping schemes caused by two reasons [4, 5]: the first one is the reduplicative derivations of virtual controllers, and the other one is that there exist numerous NNs/FLS. In the existing literature, several so-called dynamic surface control [6], sliding mode differentiator (or tracking differentiator) [7], and command filter techniques [8] have been incorporated into adaptive back-stepping control design to avoid the repeated derivations problem. Meanwhile, those techniques have been extensively employed to multifarious practical systems such as flexible-joint robot system [9], flight control [10], and static Var compensator [11]. By using those techniques, the repeated differentiations of virtual control laws can be avoided. However, numerous NNs employed to construct the virtual and actual control laws are still necessary for high-order systems. To thoroughly refrain from this problem, single neural (SN) approximation-based adaptive back-stepping control has been proposed for strict-feedback nonlinear systems where the control gains in each subsystem are equal to one [4]. After that, Pan and Sun have extended SN control structure to a general class of uncertain strict-feedback [5] and pure-feedback nonlinear systems [12], respectively. Meanwhile, some researchers have also utilized the single neural control technique (SNC) to discrete-time nonlinear systems [13, 14]. However, the works described in [4, 5, 12–14] only focus on the stabilization of interested systems, and the output-constraint problems are rarely considered. Meanwhile, it must be pointed out that violation of the output constraint may lower the system performance or even lead to systems instability [15, 16].

To manage the problem of output constraint, a number of attempts regarding model predictive control [17], barrier Lyapunov function (BLF) [18–24], and prescribed performance control (PPC) [25–28] techniques have been investigated from both academia and industries. Using the property of BLF, a constrained back-stepping control approach is proposed for nonlinear strict-feedback systems to ensure that the static constraint is not transgressed [20]. More specifically, some constrained adaptive neural/fuzzy back-stepping schemes are investigated for nonlinear systems subject to external disturbance and unknown functions [19, 22, 29]. Apart from the technique about BLF, Bechiloulis et al. have also proposed an alternative approach named PPC to conquer the problem of output constraint [25].

The dead-zone characteristic on behalf of the most important input nonlinearities widely exists in actuators and sensors among numerous industrial processes, which also severely deteriorate the system performance [30–34]. Thus, many researchers have devoted themselves to improving the performance of the control systems in the presence of dead-zones. Some researches apply an inverse dead-zone model to minimize the influence of the dead-zone; see reference [30]. On the other hand, by exploring the bounds of the dead-zone slopes and regarding the coefficients of the dead-zone as a system uncertainty, some robust adaptive neural/fuzzy control schemes are proposed of nonlinear systems with nonsymmetric dead-zone inputs [31, 35–37]. Na [38] and Chen [39] both proposed an adaptive neural prescribed performance control for nonlinear system with dead-zone input. Unfortunately, aforementioned PPC approaches still suffer from the POC and use numerous NNs, which result in a complex control structure. Moreover, there also exists restriction that the initial tracking errors must be known in advance. However, actually, the initial tracking error is hard to obtain in the presence of uncertainties and additional disturbances. Thus, how to concurrently tackle such restriction and aforementioned problems needs further investigations.

To simultaneously solve the aforementioned problems, a systematic design procedure is developed to derive a composite single neural network control scheme of nonlinear strict-feedback systems subject to dead-zone input, system uncertainties, and time-varying output constraint. In the controller design, to release the limit condition on previous performance function that the initial tracking error needs to be known, a modified performance function is first constructed. Meanwhile, depending on a new system transformation technique, the prescribed performance of the constrained system can be equivalent to ensure the stability of the transformed system. Regarding the coefficients of the dead-zone as a system uncertainty, an adaptive NN approach is developed via single NN control structure to stabilize the transformed system. Finally, stability analysis is conducted by using Lyapunov theory. The main contributions of this paper are summarized as follows:

(1) A new modified prescribed performance function is constructed to stipulate the predefined region of the tracking error. Thus, the previous restriction on initial tracking error is removed.

(2) In construct to previous output-constraint neural/fuzzy back-stepping control approach [15, 25, 38, 40, 41], the proposed control scheme does not need numerous NNs/FLS to construct virtual and practical control law in each step, only one neural network including one adaptive laws is required to approximate the lumped unknown function, thus deriving a low-computational control scheme.

(3) By regarding time-varying input-coefficients of input dead-zone as a system uncertainty and using a new system transformation technique in the control design, an integrated single NN adaptive controller, which is capable of arbitrarily prescribing the system performance and dealing with dead-zone input nonlinearity, simultaneously, is first presented for a class of nonlinear strict-feedback systems.

2. Problem Formulation and Preliminaries

2.1. Problem Formulation. Consider uncertain nonlinear strict-feedback systems in the following form:

$$\begin{aligned} \dot{x}_1 &= f_1(x_1) + g_1(x_1)x_2 \\ \dot{x}_i &= f_i(\overline{x}_i) + g_i(\overline{x}_i)x_{i+1}, \quad 2 \le i \le n-1 \\ \dot{x}_n &= f_n(\overline{x}_n) + g_n(\overline{x}_n)u \\ u &= D(v) \\ y &= x_1, \end{aligned} \tag{1}$$

where $\overline{x}_i = [x_1, x_2, \ldots, x_i]^T \in R^i$, $i = 2, \ldots, n$, $u \in R$, and $y \in R$ are state variables, system input, and output, respectively. $f_i(\overline{x}_i)$ are the unknown smooth functions; $g_i(\overline{x}_i)$ denote unknown smooth virtual control gain functions. v is the output from the controller. According to [30], u is a nonsymmetric dead-zone input nonlinearity which is defined as follows:

$$u = D(v) = \begin{cases} m_l(v + b_l), & v \le -b_l \\ 0, & -b_l < v < b_r \\ m_r(v - b_r), & v \ge b_r, \end{cases} \tag{2}$$

where b_l and b_r denote breakpoints of the input nonlinearity.

To facilitate control design, we need the following assumptions.

Assumption 1. The desired trajectory $y_d(t)$ and its derivatives $y_d^{(1)}, y_d^{(2)}, \ldots, y_d^{(n)}$ are bounded and known.

Assumption 2. There exist constants $g_{i0}, g_{i1} \in R^+$ such that $0 < g_{i0} \le |g_i(\overline{x}_i)| \le g_{i1}$ with $i = 1, 2, \ldots, n$. Without loss of generality, it is assumed that $g_i(\overline{x}_i) > 0$.

Assumption 3 (see [36]). The dead-zone parameters m_l, m_r, b_r, and b_l, are unknown bounded positive constants.

The dead-zone nonlinearity (2) can be rewritten as a slowly time-varying function:

$$u = D(v) = Hv + d_v, \tag{3}$$

where

$$H = \begin{cases} m_l, & v \le 0 \\ m_r, & v \ge 0, \end{cases}$$
$$d_v = \begin{cases} m_l b_l, & v \le -b_l \\ -Hv, & b_l < v < b_r \\ -m_r b_r, & v \ge b_r. \end{cases} \tag{4}$$

Here, we can easily conclude that $|d_v| \le d^*$, $d^* = \max\{m_l b_l, m_r b_r\}$, and there exist positive constants $\overline{H}$ and $\underline{H}$ such that $\underline{H} \le H \le \overline{H}$.

Control Objective. The control goal is to design a controller such that (1) the output y tracks the desired trajectory y_d well; (2) meanwhile, the output y can be always confined within the predefined time-varying constraint. That is to say, the output y is needed to satisfy

$$\underline{y}(t) < y(t) < \overline{y}(t), \tag{5}$$

where $\underline{y}(t)$ and $\overline{y}(t)$ are the bounds of the output.

Assumption 4 (see [15]). The output bounds $\underline{y}(t)$ and $\overline{y}(t)$ are smooth functions and their derivatives from 1 to nth are all available.

2.2. Some Preliminaries

Lemma 5 (see [42]). *Assume that $y_d(t)$ and its $n-1$ derivatives are bounded. Consider system*

$$\begin{aligned} \lambda\dot{\varsigma}_1 &= \varsigma_2 \\ \lambda\dot{\varsigma}_2 &= \varsigma_3 \\ &\vdots \\ \lambda\dot{\varsigma}_n &= -d_1\varsigma_n - d_2\varsigma_{n-1} - \cdots - d_{n-1}\varsigma_2 - \varsigma_1 + y_d(t), \end{aligned} \tag{6}$$

where λ is a constant and $d_1, \ldots, d_{n-1}$ are selected such that $s^n + d_1 s^{n-1} + \cdots + d_{n-1}s + 1$ is Hurwitz. Thus, there exist positive constants θ and t^* such that $\forall t > t^*$ one has

$$\left|Y_d - \widehat{Y}_d\right| \le \lambda\theta, \tag{7}$$

where $Y_d = [y_d, y_d^{(1)}, \ldots, y_d^{(n-1)}]$ and $\widehat{Y}_d = [y_d, \varsigma_2/\lambda, \ldots, \varsigma_n/\lambda^{n-1}]$ are the estimation of Y_d.

Neural Network. Radial basis function (RBF) NN is often employed as an effective tool to approximate nonlinear functions. The following NN is applied to approximate $f(Z) : R^q \to R$

$$\widehat{f}(Z) = \widehat{W}^T \Phi(Z), \tag{8}$$

where $Z \in \Omega_Z \subset R^q$ is the input vector; $\widehat{W} = [w_1, \ldots, w_l]^T \in R^l$ represents weight vector; $l > 1$ denotes the node number; and $\Phi(Z) = [\Phi_1(Z), \ldots, \Phi_l(Z)]^T$ has the form

$$\Phi_i(Z) = \exp\left[\frac{-(Z-\mu_i)^T(Z-\mu_i)}{2\eta_i^2}\right], \quad i = 1, \ldots, l, \tag{9}$$

where $\mu_i = [\mu_{i1}, \ldots, \mu_{iq}]^T$ and η_i denote the center and width of the Gaussian function.

RBF network (8) has the ability to approximate any continuous function to arbitrary accuracy over the set $\Omega_Z \subset R^q$

$$f(Z) = W^{*T}\Phi(Z) + \varepsilon, \quad \forall Z \in \Omega_Z, \tag{10}$$

where W^* is ideal constant weights and ε is the approximation error.

Assumption 6. There are ideal constant weights such that $|\varepsilon| \le \varepsilon^*$ for all $Z \in \Omega_Z$.

Lemma 7 (see [43]). *The Gaussian function described in (9) with $\widehat{Z} = Z - r\psi$ being the input vector, where ψ is a bounded vector and r is a positive, is given as*

$$\Phi_i\left(\widehat{Z}\right) = \exp\left[\frac{-\left(\widehat{Z}-\mu_i\right)^T\left(\widehat{Z}-\mu_i\right)}{2\eta_i^2}\right], \quad i = 1, \ldots, l \tag{11}$$

$$\Phi\left(\widehat{Z}\right) = \Phi(Z) + r\Phi_t, \tag{12}$$

where Φ_t is a bounded vector.

Remark 8. Assumption 1 imposes a controllability condition for system (1) and can be found in most existing adaptive back-stepping neural network control approaches.

3. Controller Design and Stability Analysis

3.1. System Transformation. Inspired by [15], the tracking error is defined as

$$z_1 = y(t) - y_d(t). \tag{13}$$

By subtracting y_d from both sides of (5), we have

$$\underline{y}(t) - y_d(t) < z_1(t) < \overline{y}(t) - y_d(t). \tag{14}$$

For clarity, we define $\overline{\varphi}_1(t) = \overline{y}(t) - y_d(t)$ and $\underline{\varphi}_1(t) = \underline{y}(t) - y_d(t)$, where $\overline{\varphi}_1(t) > 0$ and $\underline{\varphi}_1(t) < 0$.

Then, (14) becomes

$$\underline{\varphi}_1(t) < z_1(t) < \overline{\varphi}_1(t). \tag{15}$$

Therefore, the output constraints given by (5) are now transformed into tracking error constraints shown as (15). To establish the relationship between $z_1(t)$ and $\overline{\varphi}_1(t)$, $\underline{\varphi}_1(t)$, we first define $\varphi_1(t) = (z_1(t) - \underline{\varphi}_1(t))/(\overline{\varphi}_1(t) - \underline{\varphi}_1(t))$. Then, the following novel transformed error is established as

$$s_1(t) = \ln\left(\frac{\varphi_1(t)}{1-\varphi_1(t)}\right), \tag{16}$$

where $s_1(t)$ is an error variable.

The inverse transformation of (16) with respect to $\varphi_1(t)$ is shown as

$$\varphi_1(t) = \frac{e^{s_1(t)}}{1+e^{s_1(t)}}. \tag{17}$$

Therefore, if $s_1(t)$ is bounded, we have

$$\begin{gathered} 0 < \varphi_1(t) < 1, \\ 0 < \frac{z_1(t) - \underline{\varphi}_1(t)}{\overline{\varphi}_1(t) - \underline{\varphi}_1(t)} < 1, \\ \underline{\varphi}_1(t) < z_1(t) < \overline{\varphi}_1(t). \end{gathered} \tag{18}$$

Adding y_d to every side of (18), condition (5) can be easily got. Therefore, the goal is presently converted to designing a controller to ensure s_1 to be bounded.

The derivative of (16) is shown as

$$\dot{s}_1 = \frac{\left[f_1(x_1) + g_1(x_1)x_2 - \dot{y}_d + \left(\underline{\varphi}_1(t)\dot{\overline{\varphi}}_1(t) - \dot{\underline{\varphi}}_1(t)\overline{\varphi}_1(t) - z_1(t)\left(\dot{\overline{\varphi}}_1(t) - \dot{\underline{\varphi}}_1(t)\right)\right)/\left(\overline{\varphi}_1(t) - \underline{\varphi}_1(t)\right)\right]}{\varphi_1(t)\left(1-\varphi_1(t)\right)\left(\overline{\varphi}_1(t) - \underline{\varphi}_1(t)\right)}. \tag{19}$$

Therefore, (19) can be rewritten as

$$\dot{s}_1 = P + \overline{R}\left[f_1(x_1) + g_1(x_1)x_2\right], \tag{20}$$

where

$$\begin{aligned} \overline{R} &= \frac{1}{\varphi_1(t)\left(1-\varphi_1(t)\right)\left(\overline{\varphi}_1(t) - \underline{\varphi}_1(t)\right)}, \\ P &= \overline{R}\left[-\dot{y}_d \right. \\ &\quad \left. + \frac{\underline{\varphi}_1(t)\dot{\overline{\varphi}}_1(t) - \dot{\underline{\varphi}}_1(t)\overline{\varphi}_1(t) - z_1(t)\left(\dot{\overline{\varphi}}_1(t) - \dot{\underline{\varphi}}_1(t)\right)}{\left(\overline{\varphi}_1(t) - \underline{\varphi}_1(t)\right)}\right]. \end{aligned} \tag{21}$$

Apparently, we have the transformed systems shown as

$$\begin{aligned} \dot{s}_1 &= P + \overline{R}\left[f_1(x_1) + g_1(x_1)x_2\right] \\ \dot{x}_n &= f_n(\overline{x}_n) + g_n(\overline{x}_n)u, \quad n \geq 2. \end{aligned} \tag{22}$$

Remark 9. It should be noted that $z_1, M_1^{-1}, \dot{\overline{\varphi}}_1, \varphi_1, \dot{\varphi}_1, \overline{R}$, and P are definitely known and can be used in the controller design.

Remark 10. From (20) and (18), we know that $\overline{R} = 1/[\varphi_1(1-\varphi_1)(\overline{\varphi}_1 - \underline{\varphi}_1)]$, $\underline{\varphi}_1 < z_1 < \overline{\varphi}_1$. The expression of $\overline{R}$ can be rewritten as

$$\overline{R} = \frac{\left(\overline{\varphi}_1 - \underline{\varphi}_1\right)}{\left(z_1 - \underline{\varphi}_1\right)\left(\overline{\varphi}_1 - z_1\right)}. \tag{23}$$

Define a new function $f_{z_1} = 1/(z_1 - \underline{\varphi}_1)(\overline{\varphi}_1 - z_1)$ with $\underline{\varphi}_1 < z_1 < \overline{\varphi}_1$; the derivative of f_{z_1} with respect to z_1 can be described as

$$\frac{\partial f_{z_1}}{\partial z_1} = \frac{\left(2z_1 - \overline{\varphi}_1 - \underline{\varphi}_1\right)}{\left[\left(z_1 - \underline{\varphi}_1\right)\left(\overline{\varphi}_1 - z_1\right)\right]^2}. \tag{24}$$

It is easy for us to obtain that function f_{z_1} will get minimum in the set $\underline{\varphi}_1 < z_1 < \overline{\varphi}_1$ as long as $z_1 = (\overline{\varphi}_1 + \underline{\varphi}_1)/2$. By considering Assumption 4, therefore, we can conclude that $\overline{R} \geq 2/(\overline{\varphi}_1 - \underline{\varphi}_1)$, which also means that there exists a time instant t' such that $\overline{R} \geq \overline{R}_{\min}$ with $\overline{R}_{\min} = 2/(\overline{\varphi}_1(t') - \underline{\varphi}_1(t'))$.

3.2. Constrained Functions Design. To study the constrained character of the tracking error $z_1(t)$, the following positive functions are chosen as

$$\begin{aligned} \rho(t) &= \coth\left(l_0 t + \mu_0\right) - 1 + \rho_\infty \\ \sigma_1(t) &= \left(\sigma_{10} - \sigma_{1\infty}\right)e^{-l_1 t} + \sigma_{1\infty} \\ \sigma_2(t) &= \left(\sigma_{20} - \sigma_{2\infty}\right)e^{-l_2 t} + \sigma_{2\infty}, \end{aligned} \tag{25}$$

where l_0, μ_0, ρ_∞, σ_{10}, $\sigma_{1\infty}$, l_1, σ_{20}, $\sigma_{2\infty}$, and l_2 are positive design constants and $\sigma_{10} > \sigma_{1\infty}$, $\sigma_{20} > \sigma_{2\infty}$; $\coth(\cdot)$ denotes the hyperbolic cotangent function. Then, the control objective can be achieved if the following conditions hold:

$$-\rho(t)\sigma_1(t) < z_1(t) < \rho(t)\sigma_2(t), \tag{26}$$

where $\underline{\varphi}_1(t) = -\rho(t)\sigma_1(t)$, $\overline{\varphi}_1(t) = \rho(t)\sigma_2(t)$.

Remark 11. From the definition of $\rho(t)$, apparently, $\rho(t)$ satisfies $\rho(0) = ((e^{\mu_0} + e^{-\mu_0})/(e^{\mu_0} - e^{-\mu_0})) - 1 + \rho_\infty > \rho_\infty$ and $\lim_{\mu_0\to 0}\rho(0) \to \infty$ and $\lim_{t\to\infty}\rho(t) = \rho_\infty$. Thus, the initial and ultimate tracking accuracy of (26) are confined within the bounds $-\infty < \lim_{t\to 0} z_1(t) < +\infty$ and $-\rho_\infty\sigma_{1\infty} < \lim_{t\to\infty} z_1(t) < \rho_\infty\sigma_{2\infty}$, respectively. It is worth mentioning that traditional prescribed performance function [25, 44, 45] requires the initial tracking error $z_1(0)$ to be precisely known in advance and needs the initial value of performance function satisfying strict condition such as $-\rho_0\sigma_{10} < z_1(0) < \rho_0\sigma_{20}$. However, in practical, the precise initial tracking error is uneasy to obtain in real systems with consideration of the uncertainty. Fortunately, the proposed prescribed performance can naturally release this limit condition.

Remark 12. In contrast to previous prescribed performance functions with constant parameters σ_1 and σ_2, where the designed $\sigma_1(t)$ and $\sigma_2(t)$ are time-varying functions. It implies that the proposed constrained function owes more degrees of freedom to adjust the prescribed performance bounds of the tracking error. This is quite beneficial for the controller design.

3.3. Controller Design. Before the controller design, we define $\kappa_i^0 := 1$, $\kappa_i^j := k_i k_{i-1}\cdots k_{i-j-1}$, $j \le i$, and $\kappa_i^j = k_i\kappa_{i-1}^{j-1}$ for $i > 1$, where $k_m \in R^+$ with $m = 1, 2, \ldots, i$ are design control gains, i is a positive integer, and j is a nonnegative integer. Moreover, let $Y_{di}(t) := [y_{d1}(t), \ldots, y_{di}(t)] \in R^i$, $\overline{Y}_i(t) := [\overline{y}_1(t), \ldots, \overline{y}_i(t)] \in R^i$, $\underline{Y}_i(t) := [\underline{y}_1(t), \ldots, \underline{y}_i(t)] \in R^i$, $Y_{d(n+1)}(t) := [Y_{dn}(t), y_d^{(n)}(t)]^T \in R^{n+1}$, $\overline{Y}_{n+1}(t) := [\overline{Y}_n(t), \overline{y}^{(n)}(t)]^T \in R^{n+1}$, and $\underline{Y}_{n+1}(t) := [\underline{Y}_n(t), \underline{y}^{(n)}(t)]^T \in R^{n+1}$, where $y_{d1}(t) = y_d(t)$, $y_{di}(t) = y_d^{(i-1)}(t)$, $\overline{y}_1(t) = \overline{y}(t)$, $\overline{y}_i(t) = \overline{y}^{(i-1)}(t)$, $\underline{y}_1(t) = \underline{y}(t)$, and $\underline{y}_i(t) = \underline{y}^{(i-1)}(t)$ with $i = 2, \ldots, n$. Here, $y_{di}(t)$, $\overline{y}_i(t)$, and $\underline{y}_i(t)$ denote the ith derivative of $y_d(t)$, $\overline{y}_1(t)$, and $\underline{y}_1(t)$, respectively. Meanwhile, we define the virtual tracking errors $z_i = x_i - \alpha_i$ with $i = 2, \ldots, n$, where α_i are virtual control inputs which will be defined later. Let $X_{ei} = [\overline{x}_i^T, Y_{d(i+1)}^T, \overline{Y}_{(i+1)}(t), \underline{Y}_{(i+1)}(t)]^T$ with $i = 1, 2, \ldots, n$. X_{ei} denotes the neural input vector which will be used in the following derivation.

Step 1. Consider the transformed system (22), $\dot{s}_1 = P + \overline{R}[f_1(x_1) + g_1(x_1)x_2]$; the first virtual control input is chosen as

$$\alpha_2 = -k_1 s_1 - h_1(X_{e1}), \tag{27}$$

where $h_1(X_{e1}) = [P + \overline{R}f_1(x_1)]/(\overline{R}g_1(x_1))$.

Applying $x_2 = z_2 + \alpha_2$ and (27), we have

$$\dot{s}_1 = g_1(x_1)\overline{R}[-k_1 s_1 + z_2]. \tag{28}$$

Applying (27) to z_2, one obtains

$$\begin{aligned} z_2 &= x_2 + k_1 s_1 + h_1(X_{e1}) \\ &= k_1 s_1 + \sum_{j=2}^{2}\kappa_1^{2-j}(x_j - y_{dj}) + h_1^*(X_{e1}), \end{aligned} \tag{29}$$

where $h_1^*(X_{e1}) = h_1(X_{e1}) + y_{d2}$.

Step 2. The derivative of z_2 is shown as follows:

$$\dot{z}_2 = f_2(\overline{x}_2) + g_2(\overline{x}_2)x_3 - \dot{\alpha}_2. \tag{30}$$

From (27), (28), and (13), one get $\alpha_2 = \alpha_2(X_{e1})$; thus we have

$$\begin{aligned} \dot{\alpha}_2 = &\left(\frac{\partial\alpha_2}{\partial x_1^T}\right)\dot{x}_1 + \left(\frac{\partial\alpha_2}{\partial Y_{d2}^T}\right)\dot{Y}_{d2} + \left(\frac{\partial\alpha_2}{\partial\overline{Y}_3^T}\right)\overline{Y}_3 \\ &+ \left(\frac{\partial\alpha_2}{\partial\underline{Y}_3^T}\right)\underline{Y}_3 \end{aligned} \tag{31}$$

Defining $\alpha_2^*(X_{e2}) = \dot{\alpha}_2$ and using (31), (30) can be rewritten as

$$\dot{z}_2 = f_2(\overline{x}_2) + g_2(\overline{x}_2)x_3 - \alpha_2^*(X_{e2}). \tag{32}$$

To remove the term $\overline{R}g_1(x_1)z_2$ in (28), a related term $\overline{R}g_1(x_1)s_1$ is added and subtracted in equality (32), then we have

$$\begin{aligned} \dot{z}_2 = &f_2(\overline{x}_2) + \overline{R}g_1(x_1)s_1 + g_2(\overline{x}_2)x_3 - \alpha_2^*(X_{e2}) \\ &- \overline{R}g_1(x_1)s_1. \end{aligned} \tag{33}$$

Consider the virtual control input

$$\alpha_3 = -k_2 z_2 - h_2(X_{e2}), \tag{34}$$

where $h_2(X_{e2}) = (f_2(\overline{x}_2) + \overline{R}g_1(x_1)s_1 - \alpha_2^*(X_{e2}))/g_2(\overline{x}_2)$.

Applying $x_3 = z_3 + \alpha_3$ and (34) to (33) yields

$$\dot{z}_2 = g_2(\overline{x}_2)(-k_2 z_2 + z_3) - \overline{R}g_1(x_1)s_1. \tag{35}$$

Utilizing (34) and (29), z_3 can be changed to

$$\begin{aligned} z_3 &= x_3 + k_2 z_2 + h_2(X_{e2}) \\ &= x_3 - y_{d3} + \kappa_2^2 s_1 \\ &\quad + \sum_{j=2}^{2}\kappa_2^{3-j}(x_j - y_{dj}) + k_2 h_1^*(X_{e1}) + h_2^*(X_{e2}) \\ &= \kappa_2^2 s_1 + \sum_{j=2}^{3}\kappa_2^{3-j}(x_j - y_{dj}) + \sum_{j=1}^{2}\kappa_2^{2-j}h_j^*(X_{ej}), \end{aligned} \tag{36}$$

where $h_2^*(X_{e2}) = h_2(X_{e2}) + y_{d3}$.

Step i ($3 \le i \le n-1$). The derivative of z_i is as follows:

$$\dot{z}_i = f_i(\overline{x}_i) + g_i(\overline{x}_i)x_{i+1} - \dot{\alpha}_i, \tag{37}$$

where $\dot{\alpha}_i = (\partial\alpha_i/\partial\overline{x}_i^T)\dot{\overline{x}}_i + (\partial\alpha_i/\partial Y_{i+1}^T)\dot{Y}_{i+1} + (\partial\alpha_i/\partial\overline{Y}_{i+1}^T)\dot{\overline{Y}}_{i+1} + (\partial\alpha_i/\partial\underline{Y}_{i+1}^T)\dot{\underline{Y}}_{i+1}$.

Define $\dot{\alpha}_i = \alpha_i^*(X_{ei})$, and (37) can be rewritten as

$$\dot{z}_i = f_i(\overline{x}_i) + g_i(\overline{x}_i)x_{i+1} - \alpha_i^*(X_{ei}). \tag{38}$$

As done previously, we add and subtract $g_{i-1}(\overline{x}_{i-1})z_i$ in equality (38) to remove this interconnected term $g_{i-1}(\overline{x}_{i-1})z_i$, then (38) can be changed to

$$\begin{aligned}\dot{z}_i = {} & f_i(\overline{x}_i) + g_{i-1}(\overline{x}_{i-1})z_i + g_i(\overline{x}_i)x_{i+1} - \alpha_i^*(X_{ei}) \\ & - g_{i-1}(\overline{x}_{i-1})z_i.\end{aligned} \tag{39}$$

The virtual control is designed as

$$\alpha_{i+1} = -k_i z_1 - h_i(X_{ei}), \tag{40}$$

where $h_i(X_{ei}) = (f_i(\overline{x}_i) + g_{i-1}(\overline{x}_{i-1})z_i - \alpha_i^*(X_{ei}))/g_i(\overline{x}_i)$.

Using $x_i = z_i + \alpha_i$ and (40), (39) can be written as

$$\dot{z}_i = g_i(\overline{x}_i)(-k_i z_i + z_{i+1}) - g_{i-1}(\overline{x}_{i-1})z_{i-1}. \tag{41}$$

Consider (41) and (36), we can obtain that

$$z_i = \kappa_{i-1}^{i-1}s_1 + \sum_{j=2}^{i}\kappa_{i-1}^{i-j}(x_j - y_{dj}) + \sum_{j=1}^{i-1}\kappa_{i-1}^{i-1-j}h_j^*(X_{ej}), \tag{42}$$

where $h_j^*(X_{ej}) = h_i(X_{ei}) + y_{d(i+1)}$.

Step n. The derivative of z_n is shown as

$$\dot{z}_n = f_n(\overline{x}_n) + g_n(\overline{x}_n)u - \dot{\alpha}_n, \tag{43}$$

where $\dot{\alpha}_n$ can be written as

$$\begin{aligned}\dot{\alpha}_n = {} & \left(\frac{\partial\alpha_n}{\partial\overline{x}_{n-1}^T}\right)\dot{\overline{x}}_{n-1} + \left(\frac{\partial\alpha_n}{\partial Y_{n+1}^T}\right)\dot{Y}_{n+1} \\ & + \left(\frac{\partial\alpha_n}{\partial\overline{Y}_{n+1}^T}\right)\dot{\overline{Y}}_{n+1} + \left(\frac{\partial\alpha_n}{\partial\underline{Y}_{n+1}^T}\right)\dot{\underline{Y}}_{n+1}.\end{aligned} \tag{44}$$

Define $\alpha_n^*(X_{en}) = \dot{\alpha}_n$, then, we can get

$$\dot{z}_n = f_n(\overline{x}_n) + g_n(\overline{x}_n)u - \alpha_n^*(X_{en}). \tag{45}$$

As done previously, we add and subtract $g_{n-1}(\overline{x}_{n-1})z_{n-1}$ in equality (45) to remove this interconnected term and consider the dead-zone input (3), then (45) can be changed to

$$\begin{aligned}\dot{z}_n = {} & f_n(\overline{x}_n) + g_{n-1}(\overline{x}_{n-1})z_{n-1} - \alpha_n^*(X_{en}) \\ & + g_n(\overline{x}_n)Hv + g_n(\overline{x}_n)d_v - g_{n-1}(\overline{x}_{n-1})z_{n-1}.\end{aligned} \tag{46}$$

The desired control law v^* is chosen as

$$v^* = -k_n z_n - h_n(X_{en}), \tag{47}$$

where $h_n(X_{en}) = (f_n(\overline{x}_n) + g_{n-1}(\overline{x}_{n-1})z_{n-1} - \alpha_n^*(X_{en}))/(g_n(\overline{x}_n)H)$.

Applying (42) and (40), z_n can be rewritten as

$$z_n = \kappa_{n-1}^{n-1}s_1 + \sum_{j=2}^{n}\kappa_{n-1}^{n-j}(x_j - y_{dj}) + \sum_{j=1}^{n-1}\kappa_{n-1}^{n-1-j}h_j^*(X_{ej}). \tag{48}$$

It must be noted that v^* is unreliable due to the unknown functions f_i, g_i, and H with $i = 1, 2, \ldots, n$. The expression of z_n in (48) is substituted into (47), then we have

$$v^* = -\kappa_n^n s_1 - \sum_{j=2}^{n}\kappa_n^{n+1-j}(x_j - y_{dj}) - F(X_{en}), \tag{49}$$

where $F(X_{en}) = \sum_{j=1}^{n}\kappa_n^{n-j}h_j^*(X_{ej})$ and $h_n^*(X_{en}) = h_n(X_{en})$.

Hence, we use NN to approximate the lumped unknown uncertainty $F(X_{en})$.

$$\widehat{F}(X_{en}, \widehat{W}) = \widehat{W}\Phi(X_{en}), \tag{50}$$

where $\Phi(\cdot)$ satisfies $\|\Phi(\cdot)\| \le \overline{\Phi}$; $\overline{\Phi}$ is a certain positive constant.

Therefore, the actual control law and adaptive law are shown as

$$v = -\kappa_n^n s_1 - \sum_{j=2}^{n}\kappa_n^{n+1-j}(x_j - y_{dj}) - \widehat{W}\Phi(X_{en}) \tag{51}$$

$$\dot{\widehat{W}} = \gamma\left(s_1\Phi(X_{en}) - \sigma|s_1|\widehat{W}\right), \tag{52}$$

where γ and σ are positive design constants.

Consider (47), (49), and (51); (46) can be converted to

$$\begin{aligned}\dot{z}_n = {} & g_n(\overline{x}_n) \\ & \cdot H\left(\frac{f_n(\overline{x}_n) + g_{n-1}(\overline{x}_{n-1})z_{n-1} - \alpha_n^*(X_{en})}{g_n(\overline{x}_n)H} + v\right) \\ & + g_n(\overline{x}_n)d_v - g_{n-1}(\overline{x}_{n-1})z_{n-1} = g_n(\overline{x}_n) \\ & \cdot H\left(-k_n z_n + \widetilde{W}\Phi(X_{en}) + \varepsilon\right) + g_n(\overline{x}_n)d_v \\ & - g_{n-1}(\overline{x}_{n-1})z_{n-1},\end{aligned} \tag{53}$$

where $\widetilde{W} = W^* - \widehat{W}$.

3.4. Stability Analysis

Lemma 13. *For updated law (52), there exists a compact set*

$$\Omega_W = \left\{\widehat{W} \mid \left\|\widehat{W}\right\| \le \frac{\overline{\Phi}}{\sigma}\right\}, \tag{54}$$

where $\|\Phi(X_{en})\| \le \overline{\Phi}$, *such that* $\widehat{W}(t) \in \Omega_W$, $\forall t \ge 0$ *in case that* $\widehat{W}(0) \in \Omega_W$.

Proof. Consider the following Lyapunov candidate $V_W = (1/2\gamma)\widetilde{W}^T\widetilde{W}$. The time derivative of function V_W along with (52) is derived as

$$\begin{aligned}\dot{V}_W = {} & \frac{1}{\gamma}\widetilde{W}^T\dot{\widehat{W}} = \widetilde{W}^T s_1\Phi(X_{en}) - \sigma|s_1|\left\|\widehat{W}\right\|^2 \\ & \le -|s_1|\left\|\widehat{W}\right\|\left(\sigma\left\|\widehat{W}\right\| - \overline{\Phi}\right).\end{aligned} \tag{55}$$

It follows that $\dot{V}_W < 0$ as long as $\|\widehat{W}\| > \overline{\Phi}/\sigma$. Therefore, $\widehat{W}(t) \in \Omega_W$ if $\widehat{W}(0) \in \Omega_W \forall t \geq 0$. □

Theorem 14. *Consider system (1) satisfying Assumptions 1 to 4, controller (51), and the updated law (52). Then, we know that the output time-varying output constraint can be achieved and the signals s_1, z_i $(i = 2, \ldots, n)$ in the closed-loop are bounded.*

Proof. Consider the following Lyapunov candidate:

$$V = \frac{1}{2}s_1^2 + \sum_{i=2}^{n}\frac{1}{2}z_i^2. \tag{56}$$

By using (28), (35), (41), and (53), the derivative of (56) is shown

$$\begin{aligned}\dot{V} &= s_1\dot{s}_1 + \sum_{i=2}^{n} z_i\dot{z}_i \\ &= -k_1 g_1 \overline{R} s_1^2 - \sum_{i=2}^{n-1} k_i g_i z_i^2 - k_n g_n H z_n^2 + z_n g_n H \widetilde{W}\Phi \\ &\quad + z_n g_n H\varepsilon + z_n g_n d_v.\end{aligned} \tag{57}$$

Since $\widehat{W}_1$ is bounded as presented in Lemma 13, it follows that $\|\widetilde{W}_1\| \leq \varepsilon_W$, where ε_W denote the upper bound of the approximation error. We note that $\|\Phi(X_{en})\| \leq \overline{\Phi}$. Consider the following facts, we have

$$\begin{aligned} z_n g_n H\widetilde{W}\Phi &\leq g_{n1}\overline{H}\left(\frac{z_n^2}{4} + \varepsilon_W^2\overline{\Phi}^2\right), \\ z_n g_n H\varepsilon &\leq \overline{H} g_{n1}\left(\frac{z_n^2}{4} + \varepsilon^{*2}\right), \\ z_n g_n d_v &\leq g_{n1}\left(\frac{\overline{H} z_n^2}{2} + \frac{d^{*2}}{2\overline{H}}\right).\end{aligned} \tag{58}$$

we have

$$\begin{aligned}\dot{V} &\leq -k_1\overline{R}_{\min} g_{10} s_1^2 - \sum_{i=2}^{n-1} k_i g_{i0} z_i^2 \\ &\quad - \left(k_n g_{n0}\underline{H} - g_{n1}\overline{H}\right) z_n^2 + \varepsilon_W^2 \overline{H} g_{n1}\overline{\Phi}^2 + \overline{H} g_{n1}\varepsilon^{*2} \\ &\quad + \frac{g_{n1} d^{*2}}{2\overline{H}}.\end{aligned} \tag{59}$$

By choosing k_i and σ such that $k_i > 0$, $i = 1, 2, \ldots, n-1$, $k_n > g_{n1}\overline{H}/g_{n0}\underline{H}$, and $\sigma > 0$ and defining $C_1 = \min\{2\overline{R}_{\min} g_{10}, 2k_i, 2(k_n g_{n0}\underline{H} - g_{n1}\overline{H})\}$, $i = 2, \ldots, n$, $C_2 = \varepsilon_W^2\overline{H} g_{n1}\overline{\Phi}^2 + \overline{H} g_{n1}\varepsilon^{*2} + g_{n1} d^{*2}/2\overline{H}$. The derivative of V can be reformed as

$$\begin{aligned}\dot{V} &\leq -k_1\overline{R}_{\min} g_{10} s_1^2 - \sum_{i=2}^{n-1} k_i g_{i0} z_i^2 \\ &\quad - \left(k_n g_{n0}\underline{H} - g_{n1}\overline{H}\right) g_{n0} z_n^2 + C_2 \leq -C_1 V + C_2.\end{aligned} \tag{60}$$

If $s_1 \notin \Omega_{s_1} = \{s_1 \mid |s_1| \leq \sqrt{C_2/k_1\overline{R}_{\min} g_{10}}\}$, $z_i \notin \Omega_{z_i} = \{z_i \mid |z_i| \leq \sqrt{C_2/k_i g_{i0}}\}$, $i = 2, \ldots, n-1$, and $z_n \notin \Omega_{z_n} = \{z_n \mid |z_n| \leq \sqrt{C_2/(k_n g_{n0}\underline{H} - g_{n1}\overline{H})}\}$, $\dot{V}$ will become negative. Thus, we can conclude that the transformed error s_1, tracking errors z_i, and $\widetilde{W}$ are uniformly ultimately boundedness. Then, the boundedness of s_1 can ensure the time-varying output constraints. □

3.5. Further Design. It must be noted that the control design described in (51) needs the high-order derivatives of the reference signal y_d. In practice, high-order derivatives are hard to obtain in the implantation of the controller. Motivated by [46, 47], we can employ a high-gain observer (HGO) or high-order sliding mode observer (HOSM) to estimate $y^{(i)}$, $i = 2, \ldots, n$. As indicated in [46], the prominent features of those observers lie in the fact that they can achieve finite-time observer error convergence and thus can be utilized in almost any feedback with separation principle being trivially fulfilled. If we use HGO to estimate the high-order derivatives of y_d, the actual control law (51) and updated law (52) can be modified as

$$\begin{aligned} v &= -\kappa_n^n s_1 - \sum_{j=2}^{n}\kappa_n^{n+1-j}\left(x_j - \widehat{y}_{dj}\right) - \widehat{W}\Phi\left(\widehat{X}_{en}\right) \\ \dot{\widehat{W}} &= \gamma\left(s_1\Phi\left(\widehat{X}_{en}\right) - \sigma\left|s_1\right|\widehat{W}\right),\end{aligned} \tag{61}$$

where $\widehat{y}_{dj}$ and $\widehat{X}_{en}$ are the estimation of y_{dj} and X_{en}, respectively.

By using HGO to estimate the high-order derivative of y_d, the actual controller and updated law have been changed to (61). The stability of the closed-loop system can be demonstrated the same as Section 3.4. From Lemma 7, it has $\|\Phi(\widehat{X}_{en})\| \leq \overline{\overline{\Phi}}$ with $\overline{\overline{\Phi}} = \|\Phi(X_{en})\| + r\|\Phi_t\|$. Employing the same procedure of Lemma 13, we can easily prove that there also exists a compact set

$$\Omega_W = \left\{\widehat{W} \mid \left\|\widehat{W}\right\| \leq \frac{\overline{\overline{\Phi}}}{\sigma}\right\}, \tag{62}$$

where $\|\Phi(\widehat{X}_{en})\| \leq \overline{\overline{\Phi}}$, such that $\widehat{W}(t) \in \Omega_W$, $\forall t \geq 0$ in case that $\widehat{W}(0) \in \Omega_W$.

Using the property of Gaussian radial basis function (12), (53) can be rewritten as

$$\begin{aligned}\dot{z}_n &= g_n\left(\overline{x}_n\right) \\ &\quad \cdot H\left(-k_n z_n + W\Phi\left(X_{en}\right) + \varepsilon - \widehat{W}\Phi\left(\widehat{X}_{en}\right) + M_1\right) \\ &\quad + g_n\left(\overline{x}_n\right) d_v - g_{n-1}\left(\overline{x}_{n-1}\right) z_{n-1} = g_n\left(\overline{x}_n\right) \\ &\quad \cdot H\left(-k_n z_n + \widetilde{W}\Phi\left(\widehat{X}_{en}\right) + \varepsilon - rW\Phi_t + M_1\right) \\ &\quad + g_n\left(\overline{x}_n\right) d_v - g_{n-1}\left(\overline{x}_{n-1}\right) z_{n-1},\end{aligned} \tag{63}$$

where $M_1 = \sum_{j=2}^{n}\kappa_n^{n+1-j}(y_{dj} - \widehat{y}_{dj})$.

Substituting (63) into the derivative of Lyapunov function, we have

$$\begin{aligned}\dot{V} &= s_1\dot{s}_1 + \sum_{i=2}^{n} z_i\dot{z}_i \\ &= -k_1 g_1 \overline{R} s_1^2 - \sum_{i=2}^{n-1} k_i g_i z_i^2 - k_n g_n H z_n^2 + z_n g_n H M \\ &\quad + z_n g_n d_v,\end{aligned} \tag{64}$$

where $M = \varepsilon - rW\Phi_t + M_1 + \widetilde{W}\Phi(\widehat{X}_{en})$.

Since $\widetilde{W}_1$ is bounded, it follows that $\|\widetilde{W}_1\| \le \varepsilon_W$ where ε_W denote the upper bound of the approximation error. We note that $\|\Phi(\widehat{X}_{en})\| \le \overline{\overline{\Phi}}$. Using Lemma 5, we can also get that there exist a constant $\overline{M}_1$ such that $|M_1| \le \overline{M}_1$. Until now, it can be easily concluded that there is a constant $\overline{M}$ with $\overline{M} = \|\varepsilon\| + \|rW\Phi_t\| + \overline{M}_1 + \|\widetilde{W}\Phi(\widehat{X}_{en})\|$ such that $|M| \le \overline{M}$.

Consider the following fact:

$$z_n g_n H M \le \frac{g_{n1}\overline{H} z_n^2}{2} + \frac{g_{n1}\overline{H}\,\overline{M}^2}{2}. \tag{65}$$

we have

$$\begin{aligned}\dot{V} &\le -k_1 \overline{R}_{\min} g_{10} s_1^2 - \sum_{i=2}^{n-1} k_i g_{i0} z_i^2 \\ &\quad - \left(k_n g_{n0}\underline{H} - g_{n1}\overline{H}\right) z_n^2 + \frac{\overline{H} g_{n1}\overline{M}^2}{2} + \frac{g_{n1} d^{*2}}{2\overline{H}}.\end{aligned} \tag{66}$$

The same analysis with Theorem 14, the signals s_1, and z_i $(i = 2, \ldots, n)$ in the closed-loop are bounded.

Remark 15. Compared with the previously proposed output-constraint [40] or prescribed performance control [41, 44, 45] adaptive back-stepping approach, the proposed structure is extremely simple and the computational burden is really low since there is only actual controller required to be implemented and there exists single neural network to approximate the lump uncertainty. It also be noted that the issue of explosion of the complexity inherent in the traditional back-stepping approach is completely removed without employing DSC, command filter, or differentiator technique.

Remark 16. In contrast to the results [4, 5, 12] using cascade low-pass filter to approximate the high-order derivatives of the reference without considering the influence of estimated error in the closed-loop stability analysis, the HGO is applied to approach the derivatives of the reference and the estimated error effect is also taken into account in the control design and stability analysis.

4. Simulation Studies

In this section, the effectiveness of the proposed method is illustrated by one-link manipulator with a brush DC (BDC) motor. One-link manipulator actuated by a BDC motor can be expressed as [48]

$$\begin{aligned}D\ddot{q} + B\dot{q} + N\sin(q) &= I \\ M\dot{I} &= -HI - K_m\dot{q} + V,\end{aligned} \tag{67}$$

where q, $\dot{q}$, and $\ddot{q}$ represent the link angular position, velocity, and acceleration, respectively. I denotes the motor current. V represents the input control voltage. The parameter values with appropriate units are given by $D = 1$, $B = 1$, $M = 0.05$, $H = 0.5$, $N = 10$, and $K_m = 10$.

Setting $x_1 = q$, $x_2 = \dot{q}$, and $x_3 = I$, (67) can be written as

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= f_2(x_1, x_2) + g_2 x_3 \\ \dot{x}_3 &= f_3(x_1, x_2, x_3) + g_3 u \\ y &= x_1,\end{aligned} \tag{68}$$

where $f_2(x_1, x_2) = (-N\sin(x_1) - Bx_2)/D$, $g_2 = 1/D$, $f_2(x_1, x_2, x_3) = (-K_m x_2 - HDx_3)/M$, and $g_3 = 1/M$.

The dead-zone model is assumed as

$$u = D(v) = \begin{cases} 1.3(v+4), & v \le -4 \\ 0, & -4 < v < 1.5 \\ 1.4(v-1.5), & v \ge 1.5. \end{cases} \tag{69}$$

The control goal is to derive the output to follow the desired trajectory $y_d = 0.5\pi\sin(1 - e^{-0.1t^2})$ with dead-zone input nonlinearity shown in (69) while ensuring predefined time-varying output constraints (5) with $\underline{y}(t) = y_d + \underline{\varphi}_1(t)$ and $\overline{y}(t) = y_d + \overline{\varphi}_1(t)$. Here, we define $\rho(t) = \coth(t+0.01) - 1 + 0.5$, $\sigma_1(t) = (1-0.2)e^{-10t} + 0.2$, and $\sigma_2(t) = (1-0.2)e^{-10t} + 0.2$.

System (68) satisfies Assumptions 1, 2, and 3. Neural network $\widehat{W}\Phi(X_{en})$ contains 128 nodes (i.e., $l_1 = 128$), $X_{en} = [Y_{d4}, \overline{x}_3, \overline{Y}_4, \underline{Y}_4]$, with centers μ_l evenly spaced in their corresponding scopes and widths $\eta_l = 5$. The design parameters of the controller are chosen as $k_1 = 2$, $k_2 = 10$, $k_3 = 2$, $\gamma = 10$, and $\sigma = 0.02$. The parameters of HGO are selected as $\lambda = 0.05$, $d_1 = 10$, $d_2 = 8$, and $d_3 = 6$. In the simulation, the initial conditions are set as $[x_1(0), x_2(0), x_3(0)] = [0, 0, 0]$, $\widehat{W}(0) = 0$, $[\varsigma_1(0), \varsigma_2(0), \varsigma_3(0), \varsigma_4(0)]^T = [0, 0, 0, 0]^T$.

To verify the proposed controller, a comparative simulation study [49] has been conducted in this section. For fair comparison, the control gains and initial conditions of control scheme in [49] are the same with this approach. The comparative simulation results are depicted in Figures 1, 2, 3, and 4, where y_N, z_N, u_N, and $\|\widehat{W}_N\|$ denote the simulation results of comparative study [49]. In Figures 1 and 2, we plot the controller tracking performance and tracking errors trajectory along with their bounds. Figures 1 and 2 imply that the proposed control method can ensure the output and its tracking error both confined within their bounds. However, the tracking performance and tracking error of comparative study are given an unsatisfactory effect. The control input

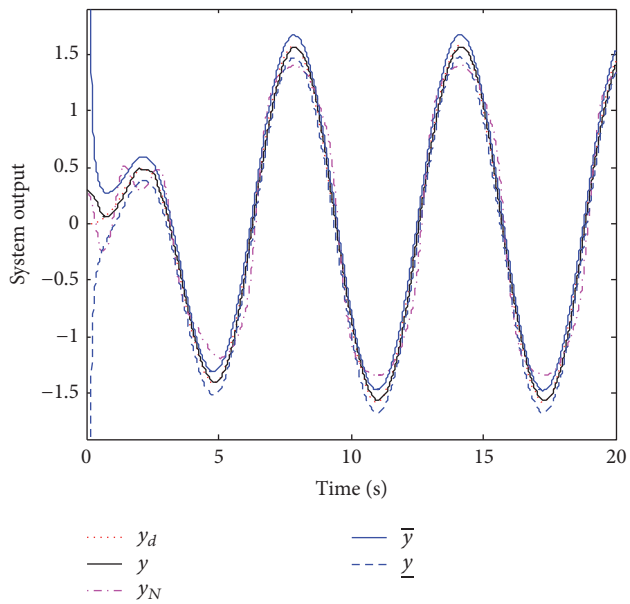

FIGURE 1: System output along with its bounds.

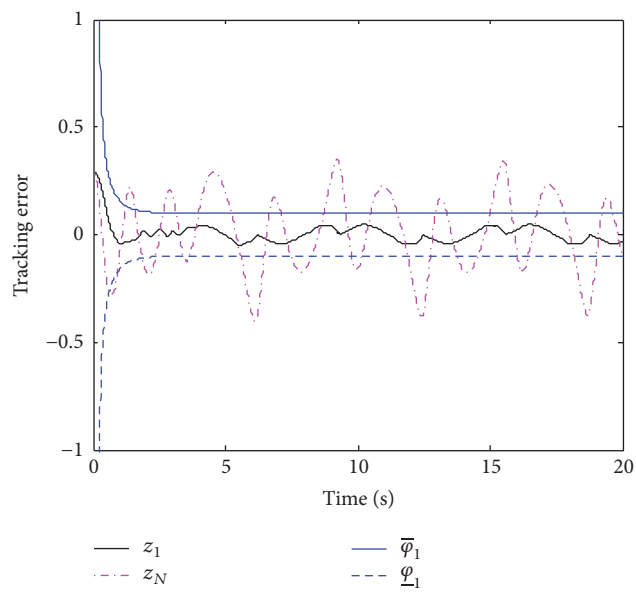

FIGURE 2: Trajectory of error z_1 and its bounds.

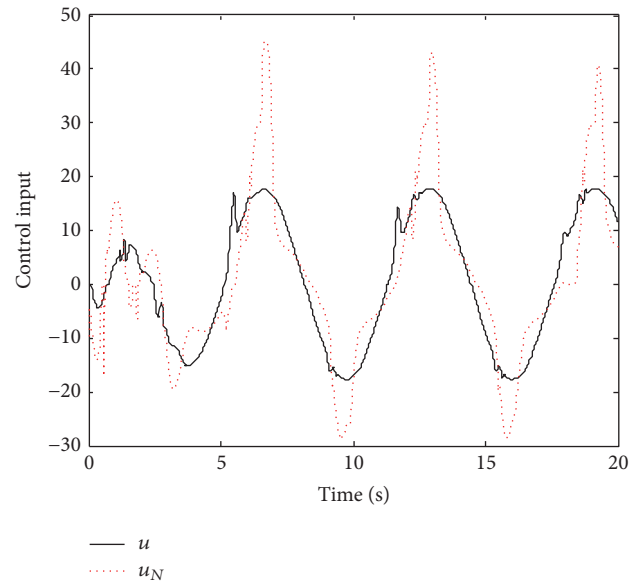

FIGURE 3: Control input.

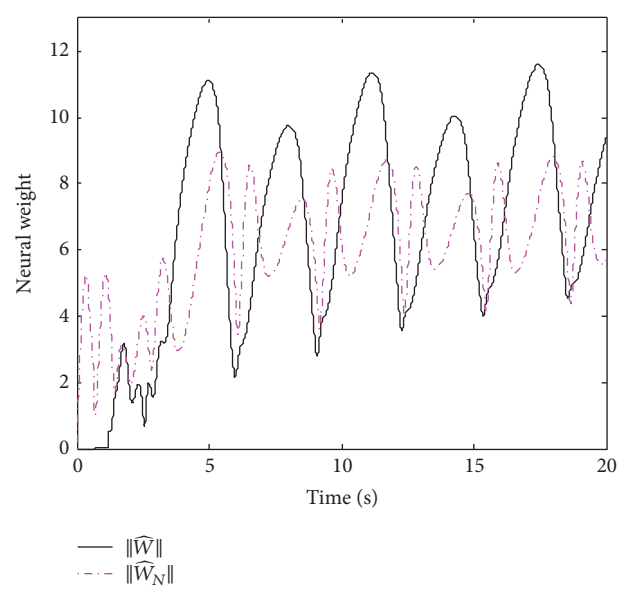

FIGURE 4: Trajectory of $\|\widehat{W}\|$.

and norm of NN weights are depicted in Figures 3 and 4, respectively. It is easy for us to find that the control input and norm of NN weights are bounded. Apparently, as opposed to the output tracking performance achieved by control scheme [49], the proposed control approach has a satisfactory result which can ensure the output and tracking error without overstepping their bounds.

5. Conclusion

In this paper, a new concise adaptive neural tracking control scheme has been developed for a class of strict-feedback system subject to input dead-zone and time-varying output constraint. Firstly, to release the limit condition on previous performance function that the initial tracking error needs to be known, a new modified performance function is constructed. By utilizing a system transformation technique, after we transform the original constrained nonlinear system into an unconstrained one, a composite adaptive neural state-feedback control approach is investigated. All unknown functions from 1 to $n-1$ steps are integrated to the n step such that only one neural network is required to approximate the lump uncertainty, thus deriving a low-computational scheme. Meanwhile, the output constraint and input dead-zone are both considered in this scheme. Finally, the closed-loop stability is proved using Lyapunov technique. Comparative

simulation is used to demonstrate the effectiveness of this scheme.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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