Research Article

Stochastic versus Deterministic Approach to Coordinated Supply Chain Scheduling

Tadeusz Sawik

Department of Operations Research, AGH University of Science and Technology, Al. Mickiewicza 30, 30-059 Kraków, Poland

Correspondence should be addressed to Tadeusz Sawik; ghawik@cyf-kr.edu.pl

Received 9 January 2017; Accepted 11 May 2017; Published 19 June 2017

The purpose of this paper is to consider coordinated selection of supply portfolio and scheduling of production and distribution in supply chains under regional and local disruption risks. Unlike many papers that assume the all-or-nothing supply disruption pattern, in this paper, only the regional disruptions belong to the all-or-nothing disruption category, while for the local disruptions all disruption levels can be considered. Two biobjective decision-making models, stochastic, based on the wait-and-see approach, and deterministic, based on the expected value approach, are proposed and compared to optimize the trade-off between expected cost and expected service. The main findings indicate that the stochastic programming wait-and-see approach with its ability to handle uncertainty by probabilistic scenarios of disruption events and the much simpler expected value problem, in which the random parameters are replaced by their expected values, lead to similar expected performance of a supply chain under multilevel disruptions. However, the stochastic approach, which accounts for all potential disruption scenarios, leads to a more diversified supply portfolio that will hedge against a variety of scenarios.

1. Introduction

Unexpected disruptions of material flows have become a major source of concerns in global supply chains over the recent years and coordinated decision-making of supplies, production, and distribution operations under disrupted flows appears to be a crucial issue (e.g., Blackhurst et al. [1] and Hoffmann et al. [2]). While the probability of flow disruptions is very low, their business impact can be huge. For example, flow disruptions in the electronics supply chains due to the great East Japan earthquake of 11 March 2011 and then the catastrophic October flooding in Thailand, where many component manufacturers were concentrated, resulted in huge losses of major electronics producers (e.g., Park et al. [3] and Haraguchi and Lall [4]). Similar losses were experienced by the automotive industry (e.g., Fujimoto and Park [5], Matsuo [6], and Marszew ska [7]). For example, two months after the earthquake, Toyota North America, which received up to 15% of its parts from Japan, experienced a shortage of 150 critical parts and was forced to operate at only 30% of its capacity.

The business practices of many companies (e.g., Zeng and Xia [8]) provide a typical decision-making environment of supply chains under disruptions. Suppliers are often located in different geographic regions and differ in wholesale prices, delivery lead time, and reliability, while their disruption profiles contain parameters such as disruption probability and fulfillment rate or the percentage of an order that is actually delivered. A popular approach to decision-making in supply chains with disrupted flows is stochastic programming, which is capable of incorporating probabilistic disruption scenarios and finding supply chain coordinated schedules for all potential scenarios with respect to various conflicting objective functions. In this paper, we present an application of stochastic mixed integer programming (stochastic MIP) to coordinated selection of supply portfolio and scheduling of production and distribution in supply chains with partially or fully disrupted supplies. The suppliers are located in different geographic regions and the supplies are subject to partial (multilevel) local disruptions of each supplier individually and to all-or-nothing (two-level) regional disruptions of all suppliers in the same region. Two equally important and
conflicting objectives are simultaneously optimized: expected cost and expected service level. In this paper, the stochastic MIP approach is compared with a deterministic MIP approach, in which all potential disruption scenarios are replaced by a single scenario, which is obtained by replacing the stochastic parameters by their expected values.

The paper is organized as follows. The review of relevant literature is presented in Section 2. The problem of coordinated decision-making in a supply chain subject to partial local disruptions and all-or-nothing regional disruptions is described in Section 3. The stochastic mixed integer program with the objective of minimizing the weighted sum of expected cost and expected service level is developed in Section 4 and the corresponding deterministic mixed integer program is proposed in Section 5. Numerical examples, computational results, and some comparison of the two solution approaches are provided in Section 6. Finally, conclusions and managerial implications as well as directions for further research are presented in Section 7.

2. Literature Review

The literature on coordinated decision-making in production-distribution planning and scheduling is mostly limited to deterministic models with the supply operation considered separately (e.g., Erenç et al. [9]). For example, an integrated production, inventory, and distribution routing problem and a MIP approach combined with a heuristic routing algorithm to coordinate the production, inventory, and transportation operations was considered by Lei et al. [10]. Kaur et al. [11] proposed a graph theoretic approach for supply chain coordination to model various mechanisms of coordination and their interdependencies. A digraph representing the supply chain coordination is converted to its adjacency matrix whose permanent function gives a composite index of coordination. In Choi et al. [12], a single-or multisupplier and single-manufacturer supply chain scheduling and coordination problem was formulated as a two-machine common-due-window flow shop scheduling problem. The authors developed scheduling algorithms for both the decentralized (with the manufacturer as a decision-maker) and centralized supply chains. Liu and Papageorgiou [13] developed a multiobjective MIP approach to address production, distribution, and capacity planning of global supply chains considering cost, responsiveness, and service level simultaneously. Chen [14] presented a review of existing models that integrate production and outbound distribution operations at the detailed scheduling level. The models aim at optimizing detailed order-by-order production and delivery scheduling jointly by taking into account relevant revenues, costs, and service levels at the individual order level. Sawik [15] proposed a MIP approach for integrated scheduling of material manufacturing, material supply, and product assembly in a customer driven supply chain. A monolithic approach, where the manufacturing, supply, and assembly schedules are determined simultaneously, was compared with a hierarchical approach. Numerical examples modeled after real-world scheduling in the electronics supply chain were reported.

Mitigation and contingency/recovery actions were studied by Tomlin [16] in a dual sourcing setting, one unreliable supplier and another reliable and more expensive one. A buyer that suffers a supply shortage can buy from a more expensive alternate supplier or produce less and its decision depends on its inventory. The reliable supplier has volume flexibility, contingent rerouting by temporarily increasing its production may prove to be an effective way to speed up recovery process. The author established that, along with cost, percentage of supplier uptime, disruption length, capacity, and flexibility play an important role in determining a buyer's disruption management strategy. The recent developments in the field of supply chain disruption management from a multidisciplinary perspective were summarized by Ivanov et al. [17–19] who studied the Ripple effect in supply chains. They emphasized that the Ripple effect can consolidate research in supply chain disruption management, similar to the bullwhip effect regarding demand and lead time fluctuations.

In the literature on supply uncertainty, the supply is subject to either complete disruptions or yield uncertainty. Yield uncertainty occurs when the quantity of supply delivered is a random variable, modeled as either a random additive or multiplicative quantity, whereas disruptions occur when supply is subject to partial or complete failure. Typically disruptions are modeled as events which occur randomly and may have a random length. Schmitt and Snyder [20] considered inventory systems subject to both supply disruptions and yield uncertainty. They compared single-period versus multiperiod models and showed that the former can lead to selecting the wrong strategy for mitigating supply risk. Schmitt et al. [21] investigated optimal system design in a multilocation system under supply disruptions. They examined the expected costs and cost variances of the system in both centralized and decentralized inventory systems. They showed that when demand is deterministic and supply is disrupted, a decentralized inventory reduces cost variance through the risk diversification effect and that a decentralized inventory may also be selected when supply is disrupted and demand is stochastic. A recent literature review on OR/MS models for supply chain disruptions was presented by Snyder et al. [22]. They discussed 180 scholarly works on the topic, organized into six categories: evaluating supply disruptions, strategic decisions, sourcing decisions, contracts and incentives, inventory, and facility location.

Sawik [23, 24] proposed a new stochastic MIP approach to integrated selection of supply portfolio and scheduling of customer orders in a supply chain under all-or-nothing disruption risks. The stochastic MIP formulations were further enhanced by Sawik [25] to jointly optimize supply portfolio and production and distribution of finished products. For the distribution of products, three shipping methods were considered and compared.

This paper differs from the previous research in the following two aspects. First, unlike many articles that assume the all-or-nothing supply disruption pattern, in this paper, only the regional disruptions belong to the all-or-nothing disruption category. For the local disruptions, however, all disruption levels can be considered within three categories: minor disruption, major disruption, and complete shutdown.
(e.g., [8, 26]). Disruption profiles contain parameters such as probability of disruption at all levels and fulfillment rate or the percentage of an order that is actually delivered. Second, in this paper, a stochastic programming wait-and-see approach with its ability to handle uncertainty by probabilistic scenarios of disruption events is compared with a deterministic programming approach, in which the random parameters are replaced by their corresponding expected values to achieve the so-called expected value problem (e.g., Kall and Mayer [27]). The expected value problem is a MIP and is often used in practice as the related stochastic mixed integer program is in general much harder to solve, since it considers multiple scenarios (e.g., Durbach and Stewart [28] and Maggioni and Wallace [29]). The objective of both the wait-and-see approach and the expected value approach is to optimize expected performance of a supply chain under the two types of disruptions with respect to two conflicting objective functions, expected cost and expected service. While the stochastic approach aims at optimizing the expected performance of a supply chain over all possible disruption scenarios, the deterministic approach accounts only for a single scenario representing the expected disruption conditions. The stochastic programming approach determines a subset of nondominated solutions for all disruption scenarios, whereas the deterministic approach produces a nondominated solution for a single scenario only. The solution of the expected value problem does not take into account any distribution information and remains the same as long as the expectations do not change. Unlike the expected value problem, stochastic programming provides a recommendation for selection of supply portfolio that will hedge against a variety of disruption scenarios. The two approaches and the corresponding solutions are compared and some managerial insights derived.

3. Problem Description

Consider a three-echelon supply chain (see Figure 1), in which a single producer of one product type assembles and delivers products to multiple distribution centers to meet customer demand, using a critical part type that can be manufactured and provided by many suppliers.

Let $I = \{1, \ldots, I\}$ be the set of $I$ suppliers, let $J = \{1, \ldots, J\}$ be the set of $J$ customers, let $K = \{1, \ldots, K\}$ be the set of $K$ distribution centers, and let $T = \{1, \ldots, T\}$ be the set of $T$ planning periods (for notations, see Notations).

The orders for parts are assumed to be placed at the beginning of the planning horizon, and the parts ordered from supplier $i$ are delivered in period $\sigma_i$. Each customer is supplied with the ordered products via exactly one distribution center. The products for each customer $j \in I_k$ are delivered to the distribution center $k$ in a single delivery, which cannot be scheduled before all customer orders $j \in I_k$ have been completed. The products shipped in period $t$ to distribution center $k$ are delivered in period $t+\tau_k-1$.

The suppliers of parts are located in $K$ geographic regions. The supplies are subject to random local disruptions of different levels, $l \in L = \{0, \ldots, \bar{L}\}$, where the disruption level refers to the fraction of an order that can be delivered (fulfillment rate). Level $l = 0$ represents complete shutdown of a supplier, that is, no order delivery, while level $l = \bar{L}$ represents normal conditions with no disruption, that is, full order delivery. The intermediate disruption levels $l = 1, \ldots, \bar{L} - 1$ represent different fractions of an order that can be delivered. The smaller $l < L$, the smaller portion of an order that can be delivered due to the smaller fraction of the supplier capacity available. The fraction of an order that can be delivered by supplier $i$ under disruption level $l$ is described by the associated fulfillment rate, $y_{il}$:

$$y_{il} = \begin{cases} 0, & \text{if } l = 0, \\ \epsilon \in (0, 1), & \text{if } l = 1, \ldots, \bar{L} - 1, \\ 1, & \text{if } l = \bar{L}. \end{cases}$$

Denote by $p_{il}$ the probability of disruption level $l \in L$ for supplier $i$; that is, the parts ordered from supplier $i$ are delivered fully with probability $p_{il}^r$, partially at different levels of supplier output, $y_{il}$, with probability $p_{il}, l = 1, \ldots, \bar{L} - 1$, or not at all with probability $p_{il}^n$.

In addition to independent local disruptions of each supplier, there are potential regional disasters that may result in complete shutdown of all suppliers in the same region simultaneously. For example, regional disaster events may include an earthquake and flooding. Let $p_i^s$ be the probability of regional disruptions of all suppliers $i \in I'$ in region $r \in R$.

Denote by $S = \{1, \ldots, \bar{S}\}$ the index set of all disruption scenarios, where each scenario $s \in S$ can be represented by an integer-coded vector $\lambda_s = (\lambda_{1s}, \ldots, \lambda_{\bar{R}s})$, where $\lambda_{is} \in L$ is the disruption level of an order delivery from supplier $i$ under scenario $s$. All potential disruption scenarios will be considered; that is, $S = (\bar{L} + 1)^\bar{R}$.

The probability $P_s$ for disruption scenario $s \in S$ with the subset $I_s$ of nonshutdown suppliers (that can deliver parts under scenario $s$) is [26]

$$P_s = \prod_{r \in R} P_s^r,$$

where $P_s^r$ is the probability of realizing of disruption scenario $s$ for suppliers in $I'$:

$$P_s^r = \begin{cases} (1 - p_i^s)^{\prod_{j \in I_s} (p_{jls})}, & \text{if } I' \cap I_s \neq \emptyset, \\ p_i^s + (1 - p_i^s) \prod_{j \in I_s} P_{00}, & \text{if } I' \cap I_s = \emptyset, \end{cases}$$

and $p_{jls}$ is the probability of occurrence of the disruption at level $l = \lambda_{is}$ of an order delivery from supplier $i$ under scenario $s$.

The objective of the coordinated decision-making in a supply chain under multilevel disruptions is to allocate the total demand for parts among a subset of selected suppliers (i.e., to determine the supply portfolio) and to schedule for each disruption scenario the customer orders for products and the delivery of products to distribution centers to optimize the trade-off between expected cost and expected service level.
4. Problem Formulation: Stochastic Approach

In this section, a stochastic MIP model WCS is presented for the coordinated decision-making in the presence of supply chain under multilevel disruption risks. The following decisions are jointly made using the proposed model [25, 26]:

(i) Supply portfolio selection: \( u_i = 1 \), if supplier \( i \) is selected; otherwise \( u_i = 0 \), and \( v_i \in [0, 1] \), the fraction of total demand for parts ordered from supplier \( i \).

(ii) Production scheduling: \( w_{stj} = 1 \), if under disruption scenario \( s \) customer order \( j \) is scheduled for period \( t \); otherwise \( w_{stj} = 0 \).

(iii) Distribution scheduling: \( x_{skt} = 1 \), if under disruption scenario \( s \) as shipped to distribution center \( k \) is scheduled for period \( t \); otherwise \( x_{skt} = 0 \).

(iv) Customer order nondelayed delivery: \( y_{sj} = 1 \), if under disruption scenario \( s \) customer order \( j \) is delivered by its due date; otherwise \( y_{sj} = 0 \).

The demand allocation vector \( (v_1, \ldots, v_T) \), where \( \sum_{i \in I} v_i = 1 \) and \( 0 \leq v_i \leq 1, i \in I \), defines the supply portfolio, introduced by Sawik [30].

Let \( E_1 \) be the minimized expected cost per product and let \( E_2 \) be the maximized expected service level:

\[
E_1 = \sum_{i \in I} e_i u_i + \sum_{s \in S} P_s \left( \sum_{i \in I} B_0 \gamma_{i,s} v_i + \sum_{j \in J} g_j b_j (\sum_{t \in T} w_{stj} - y_{sj}) + \sum_{j \in J} h_j b_j (1 - \sum_{t \in T} w_{stj}) \right),
\]

\[
E_2 = \frac{\sum_{s \in S} \sum_{j \in J} b_j y_{sj}}{B},
\]

where \( \lambda_i \) is disruption level of supplier \( i \) under scenario \( s \) and \( \gamma_{i,s} \) is the corresponding fulfillment rate, that is, the fraction of an order delivered by supplier \( i \) under disruption scenario \( s \).

The expected cost \( E_1 \) (see (4)) constitutes fixed ordering cost, \( \sum_{i \in I} e_i u_i \), expected purchasing cost for delivered parts, \( \sum_{s \in S} P_s \sum_{i \in I} B_0 \gamma_{i,s} v_i \), expected penalty for delayed customer demand, \( \sum_{s \in S} P_s \sum_{j \in J} g_j b_j (\sum_{t \in T} w_{stj} - y_{sj}) \), and expected penalty for unsatisfied (rejected) customer demand, \( \sum_{s \in S} P_s \sum_{j \in J} h_j b_j (1 - \sum_{t \in T} w_{stj}) \).

Denote by

\[
f_1 = \frac{E_1 - E_1}{E_1 - E_2},
\]

the normalized expected service level \( (E_2, E_2) \) are the minimum and the maximum values of \( E_2 \), resp.).

Model WCS. It consists in supplier selection, customer order, and distribution scheduling to minimize Weighted sum of normalized expected Cost and expected Service level.

Minimize

\[
f_1 + (1 - \alpha) f_2,
\]

where \( 0 \leq \alpha \leq 1 \), subject to (4)–(7).

Supply Portfolio Selection Constraints

(i) The total demand for parts must be fully allocated among the selected suppliers.

(ii) Demand for parts cannot be assigned to nonselected suppliers:

\[
\sum_{i \in I} v_i = 1 \quad u_i = 0, i \in I.
\]
Customer Order Scheduling Constraints

(i) For each disruption scenario \( s \), each customer order \( j \) is either scheduled during the planning horizon \( (\sum_{t \in \mathcal{T}}^t w_{jt} = 1) \) or unscheduled and rejected \( (\sum_{j \in \mathcal{J}} w_{jt} = 0) \).

(ii) For any period \( t \) and each disruption scenario \( s \), the total demand on capacity of all customer orders scheduled in period \( t \) must not exceed the producer capacity:

\[
\sum_{j \in \mathcal{J}} w_{jt} \leq 1; \quad j \in \mathcal{J}, \ s \in S,
\]

\[
\sum_{j \in \mathcal{J}} b_j w_{jt} \leq C; \quad t \in \mathcal{T}, \ s \in S. \tag{10}
\]

Supply-Production-Distribution Coordinating Constraints

(i) For each disruption scenario \( s \) and each planning period \( t \), the cumulative demand for parts of all customer orders scheduled in period \( 1 \) through \( t \) cannot exceed the cumulative deliveries of parts in period \( 1 \) through \( t-1 \), from the nonshutdown suppliers \( i \in \mathcal{I}_{\text{r}} \).

(ii) For each disruption scenario, shipment to distribution center \( k \) can be scheduled only after the latest completion period of scheduled customer orders \( j \in \mathcal{J}_k \):

\[
\sum_{t \in \mathcal{T}} \sum_{j \in \mathcal{J}_t} b_j w_{jt}^s \leq B \sum_{t \in \mathcal{T}, \sigma_t \leq t-1} y_i \lambda_i, \quad t \in \mathcal{T}, \ s \in S, \tag{11}
\]

\[
\sum_{t \in \mathcal{T}}^t x_{kt}^s \geq \sum_{t \in \mathcal{T}}^t (t+1) w_{jt}^s; \tag{12}
\]

\[
k \in \mathcal{K}, \quad j \in \mathcal{J}_k, \ s \in S,
\]

where \( \mathcal{T} = \{ \min_{\mathcal{I}, \mathcal{I}_r} \sigma_i + 2, \ldots, T + 1 \} \) is the set of shipping periods.

Shipping Constraints

(i) For each disruption scenario, at most one shipment can be scheduled to each distribution center:

\[
\sum_{t \in \mathcal{T}}^t x_{kt}^s \leq 1; \quad k \in \mathcal{K}, \ s \in S. \tag{13}
\]

Customer Due Date Meeting Constraints

(i) For each disruption scenario \( s \in S \), customer order \( j \in \mathcal{J}_j \) can be delivered without delay (i.e., \( y_j^s = 1 \)), if it is scheduled not later than \( d_j - \tau_k \) and shipped to distribution center \( k \) not later than \( d_j - \tau_k + 1 \); otherwise the customer order is delayed or unscheduled (i.e., \( y_j^s = 0 \)):

\[
y_j^s \leq \sum_{t \in \mathcal{T} : t \leq d_j - \tau_k} w_{jt}^s; \quad k \in \mathcal{K}, \quad j \in \mathcal{J}_k, \ s \in S,
\]

\[
y_j^s \leq \sum_{t \in \mathcal{T} : t \leq d_j - \tau_k + 1} x_{kt}^s, \tag{14}
\]

Note that \( y_j^s \) does not need to be restricted to being binary, since, for any feasible solution satisfying constraints (14), \( y_j^s \) is always binary.

Nonnegativity and Integrality Conditions

\[
u_i \in [0, 1]; \quad i \in \mathcal{I}, \tag{15}
\]

\[
u_j \in [0, 1]; \quad i \in \mathcal{I}, \tag{16}
\]

\[
w_{jt} \in [0, 1]; \quad j \in \mathcal{J}, \ t \in \mathcal{T}, \ s \in S, \tag{17}
\]

\[
x_{kt}^s \in [0, 1]; \quad k \in \mathcal{K}, \ t \in TK, \ s \in S, \tag{18}
\]

\[
y_j^s \geq 0; \quad j \in \mathcal{J}, \ s \in S. \tag{19}
\]

5. Problem Formulation: Deterministic Approach

In this section, the expected value problem EWCS is presented for the coordinated supply chain scheduling under...
expected supply conditions. In model WCS, where the randomness is characterized by a set of disruption scenarios, the only random parameters are suppliers fulfillment rates, $\gamma_{i,\lambda,s}$, which appear both in the objective function (4) and in constraints (11).

In model EWCS, suppliers probabilistic fulfillment rates defined for each disruption scenario, $\gamma_{i,\lambda,s}$, or equivalently for each disruption level, $\gamma_{i,l}$, have been replaced by the expected fulfillment rates of each supplier:

$$\Gamma_i = \sum_{s \in S} P_s \gamma_{i,\lambda,s}; \quad i \in I,$$

or equivalently

$$\Gamma_i = (1 - p_r) \sum_{l \in L} p_i \gamma_{i,l}; \quad i \in I, \quad r \in R. \quad (21)$$

Accordingly, stochastic binary decision variables, $w_{s,j,t}, x_{s,k,t}, y_{s,j}$, (17)–(19), defined for each disruption scenario $s \in S$ have been replaced by their deterministic equivalents $W_{j,t}, X_{k,t}, Y_j$.

Now, the expected cost per product, $E_1$ (see (22)), and the expected service level, $E_2$ (see (23)), are defined as follows:

$$E_1 = \sum_{i \in I} e_i u_i + \sum_{i \in I} B_0 \Gamma_i v_i + \sum_{j \in J} b_j (\sum_{t \in T} W_{j,t} - Y_j) + \sum_{j \in J} h_j (1 - \sum_{t \in T} W_{j,t}), \quad (22)$$

$$E_2 = \sum_{j \in J} b_j Y_j; \quad (23)$$

Model EWCS is presented below.

**Model EWCS**

Minimize

subject to

$$\sum_{t \in T} W_{j,t} \leq 1; \quad j \in J,$$

$$\sum_{j \in J} b_j W_{j,t} \leq C; \quad t \in T,$$

$$\sum_{j \in J} b_j W_{j,t} \leq B \sum_{i \in I} \Gamma_i v_i; \quad t \in T,$$

$$\sum_{t \in T} X_{k,t} \geq \sum_{t \in T} (t + 1) W_{j,t}; \quad k \in K, \quad j \in J_k,$$

$$\sum_{t \in T} X_{k,t} \leq 1; \quad k \in K,$$

$$Y_j \leq \sum_{t \in T, \tau_j \leq t} W_{j,t}; \quad k \in K, \quad j \in J_k,$$

$$Y_j \leq \sum_{t \in T \cap \tau_k \leq \tau_j + 1} X_{k,t}; \quad k \in K, \quad j \in J_k,$$

$$\sum_{t \in T, \tau_k \leq \tau_j + 1} W_{j,t} + \sum_{t \in T \cap \tau_k \leq \tau_j + 1} X_{k,t} - 1 \leq Y_j; \quad k \in K, \quad j \in J_k,$$

$$u_i \in [0,1]; \quad i \in I,$$

$$v_i \in [0,1]; \quad i \in I$$

Notice that, unlike the stochastic programming model WCS which is formulated to determine optimal schedules for all potential disruption scenarios, model EWCS accounts for a single scenario only, representing the expected supplies. Except for the expected values of the random parameters, this model does not take into account any distribution information and the solution remains the same as long as the expectations do not change. In contrast to model WCS, where the selection of supply portfolio, $(v_1, \ldots, v_I)$, is combined with supply chain scheduling for all disruption scenarios considered, now the portfolio is determined along with a single schedule.

If random parameters appear only in the constraints, then [27]

$$E \mathcal{V} \leq \mathcal{W} S, \quad (25)$$

where $E \mathcal{V}$ is the optimal solution value of the expected value problem $EWCS$ and $\mathcal{W} S$ is the optimal solution value of the wait-and-see problem WCS. On the other hand, when uncertainty is limited to the objective function of the problem, the solution obtained by simply replacing the random parameters with their expected values provides already a robust alternative (Delage et al. [31]).

**6. Computational Examples**

In this section, some computational examples are presented to illustrate possible applications of the proposed MIP models and to compare the wait-and-see and the expected value
and the probability for disruption scenario \( s \in S \) is given by \( p_s = p_I^I p_{s I}^I \).

Figure 2 presents basic characteristics of all suppliers: probability of complete shutdown, \( p_I^I \), expected fulfillment rate, \( \Gamma_r \), probability of disruption occurrence, \( \gamma_r \), for all \( r \in R \). The solution shows that the service-oriented supply portfolio (\( \alpha \) close to 0) is more diversified than the cost-oriented portfolio (\( \alpha \) close to 1). Table 2 also shows the expected fraction of fulfilled demand, \( E_\gamma = \sum_{s \in S} \sum_{r \in R} \sum_{i \in I} p_{s I}^I p_i \), that is, demand fulfilled on time or delayed. The solution results demonstrate that a large expected service level is sometimes associated with a small expected fraction of fulfilled demand. Thus, the probability of realizing of disruption scenario \( s \) is calculated as follows:

\[
p_s^I = \left(1 - p_I^I \right) \left( \prod_{i \in I^I} \left(1 - p_{ij} \right) \right) \left( \prod_{i \in I^I} 0.1 \left(1 - p_{ij} \right) \right) \left( \prod_{i \in I^I} \left( 0.6 \left(1 - p_{ij} \right) \right) \right) \left( \prod_{i \in I^I} p_{ij} \right) , \quad \text{if } \sum_{i \in I^I} \lambda_i > 0, \quad (26)
\]

and the probability for disruption scenario \( s \in S \) is given by \( p_s = p_I^I p_{s I}^I \).

Figure 2 presents basic characteristics of all suppliers: probability of complete shutdown, \( p_I^I \), expected fulfillment rate, \( \Gamma_r \), probability of disruption occurrence, \( \gamma_r \), for all \( r \in R \). The solution shows that the service-oriented supply portfolio (\( \alpha \) close to 0) is more diversified than the cost-oriented portfolio (\( \alpha \) close to 1). Table 2 also shows the expected fraction of fulfilled demand, \( E_\gamma = \sum_{s \in S} \sum_{r \in R} \sum_{i \in I} p_{s I}^I p_i \), that is, demand fulfilled on time or delayed. The solution results demonstrate that a large expected service level is sometimes associated with a small expected fraction of fulfilled demand. Thus, the probability of realizing of disruption scenario \( s \) is calculated as follows:

\[
p_s^I = \left(1 - p_I^I \right) \left( \prod_{i \in I^I} \left(1 - p_{ij} \right) \right) \left( \prod_{i \in I^I} 0.1 \left(1 - p_{ij} \right) \right) \left( \prod_{i \in I^I} \left( 0.6 \left(1 - p_{ij} \right) \right) \right) \left( \prod_{i \in I^I} p_{ij} \right) , \quad \text{if } \sum_{i \in I^I} \lambda_i > 0, \quad (26)
\]

and the probability for disruption scenario \( s \in S \) is given by \( p_s = p_I^I p_{s I}^I \).

Figure 2 presents basic characteristics of all suppliers: probability of complete shutdown, \( p_I^I \), expected fulfillment rate, \( \Gamma_r \), probability of disruption occurrence, \( \gamma_r \), for all \( r \in R \). The solution shows that the service-oriented supply portfolio (\( \alpha \) close to 0) is more diversified than the cost-oriented portfolio (\( \alpha \) close to 1). Table 2 also shows the expected fraction of fulfilled demand, \( E_\gamma = \sum_{s \in S} \sum_{r \in R} \sum_{i \in I} p_{s I}^I p_i \), that is, demand fulfilled on time or delayed. The solution results demonstrate that a large expected service level is sometimes associated with a small expected fraction of fulfilled demand. Thus, the probability of realizing of disruption scenario \( s \) is calculated as follows:

\[
p_s^I = \left(1 - p_I^I \right) \left( \prod_{i \in I^I} \left(1 - p_{ij} \right) \right) \left( \prod_{i \in I^I} 0.1 \left(1 - p_{ij} \right) \right) \left( \prod_{i \in I^I} \left( 0.6 \left(1 - p_{ij} \right) \right) \right) \left( \prod_{i \in I^I} p_{ij} \right) , \quad \text{if } \sum_{i \in I^I} \lambda_i > 0, \quad (26)
\]
Figure 2: Suppliers.

Figure 3: Expected schedules for model WCS.
<table>
<thead>
<tr>
<th>$s$</th>
<th>$i = 1$</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>11</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>12</td>
<td>0</td>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>13</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>14</td>
<td>0</td>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>15</td>
<td>0</td>
<td>3</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>16</td>
<td>0</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>17</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>18</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>19</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>20</td>
<td>1</td>
<td>0</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>21</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>22</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>23</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>24</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>25</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>26</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>27</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>28</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>29</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>30</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>31</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>32</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>33</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>34</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>35</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>36</td>
<td>2</td>
<td>0</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>37</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>38</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>39</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>40</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>41</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>42</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>43</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>44</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>45</td>
<td>2</td>
<td>3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>46</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>47</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>48</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>49</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$s$</th>
<th>$i = 1$</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>51</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>52</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>53</td>
<td>0</td>
<td>3</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>54</td>
<td>0</td>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>55</td>
<td>0</td>
<td>3</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>56</td>
<td>0</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>57</td>
<td>0</td>
<td>3</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>58</td>
<td>0</td>
<td>3</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>59</td>
<td>0</td>
<td>3</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>60</td>
<td>0</td>
<td>3</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>61</td>
<td>0</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>62</td>
<td>0</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>63</td>
<td>0</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>64</td>
<td>0</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>65</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>66</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>67</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>68</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>69</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>70</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>71</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>72</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>73</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>74</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>75</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>76</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>77</td>
<td>1</td>
<td>0</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>78</td>
<td>1</td>
<td>0</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>79</td>
<td>1</td>
<td>0</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>80</td>
<td>1</td>
<td>0</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>81</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>82</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>83</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>84</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>85</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>86</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>87</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>88</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>89</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>90</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>91</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>92</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>93</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>94</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>95</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>96</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>97</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>98</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$s$</td>
<td>$i = 1$</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>------</td>
<td>---------</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>99</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>100</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>101</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>102</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>103</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>104</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>105</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>106</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>107</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>108</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>109</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>110</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>111</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>112</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>113</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>114</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>115</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>116</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>117</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>118</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>119</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>120</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>121</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>122</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>123</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>124</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>125</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>126</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>127</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>128</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>129</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>130</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>131</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>132</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>133</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>134</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>135</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>136</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>137</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>138</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>139</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>140</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>141</td>
<td>2</td>
<td>0</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>142</td>
<td>2</td>
<td>0</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>143</td>
<td>2</td>
<td>0</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>144</td>
<td>2</td>
<td>0</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>145</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>146</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>147</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>148</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>149</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>150</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$s$</th>
<th>$i = 1$</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>151</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>152</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>153</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>154</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>155</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>156</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>157</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>158</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>159</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>160</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>161</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>162</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>163</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>164</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>165</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>166</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>167</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>168</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>169</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>170</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>171</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>172</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>173</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>174</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>175</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>176</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>177</td>
<td>2</td>
<td>3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>178</td>
<td>2</td>
<td>3</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>179</td>
<td>2</td>
<td>3</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>180</td>
<td>2</td>
<td>3</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>181</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>182</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>183</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>184</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>185</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>186</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>187</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>188</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>189</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>190</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>191</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>192</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>193</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>194</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>195</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>196</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>197</td>
<td>3</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>198</td>
<td>3</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>199</td>
<td>3</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>200</td>
<td>3</td>
<td>0</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>201</td>
<td>3</td>
<td>0</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>202</td>
<td>3</td>
<td>0</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>
maximum expected service level), $\alpha = 0.5$, and $\alpha = 1$ (i.e., for the minimum expected cost). The expected schedules were computed using the formulae presented below:

(i) Expected schedule of supplies of parts to the producer:
$$\sum_{s \in S} \sum_{i \in I} s_i \cdot \gamma^i = t \cdot P_s \cdot B_i \cdot \lambda^s_i \cdot V_i; t \in T$$  \hspace{1cm} (27)

(ii) Expected production schedule:
$$\sum_{s \in S} \sum_{j \in J} s_j \cdot B_j \cdot \omega^s_j; t \in T$$  \hspace{1cm} (28)

(iii) Expected schedule of shipping of products from the producer to the distribution centers:
$$\sum_{s \in S} \sum_{k \in K} \sum_{j \in J} s_k \cdot B_j \cdot \sum_{t' \in T^s_{t,t'} < t} \omega^s_{t'} \cdot x^s_{t,t'}; t \in T$$  \hspace{1cm} (29)

As $\alpha$ increases, that is, the decision-maker’s priority shifts from the maximum service level to minimum cost and more parts are ordered from less reliable and lower cost suppliers, the expected supply schedules and the corresponding production schedules are more delayed as well as the delivery of products to the customers. Note that, despite constraint (13) that ensures a feasible schedule with at most one shipment of products to each distribution center for each disruption scenario, the expected shipping schedule (29) may be split into more smaller size shipments (cf. Figure 3, where two major and one small-size shipments are indicated for each confidence level).

For comparison, Table 3 presents a subset of non-dominated solutions obtained for the expected value problem $EWCS$, and Figure 4 shows supply, production, and shipping schedules. Unlike the stochastic programming approach which accounts for all potential disruption scenarios to optimize an expected performance of the supply chain, the solution obtained using the deterministic approach is based on aggregate information on suppliers expected fulfillment. In general, the results are similar for both models and the corresponding optimal solution values are close to each other, which indicates that the expected value problem can be used in practice, when stochastic mixed integer programs are hard to solve. The optimal solution values for the expected value problem frequently outperform the corresponding solution values of the wait-and-see problem, which is in line with the proposition $EV \leq WS$, in [27]; compare Section 5, for example, the minimum cost $E_1$ for $EWCS$ (see Table 3),
Table 2: Nondominated solutions for model WCS.

<table>
<thead>
<tr>
<th>α</th>
<th>0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5 and 0.6</th>
<th>0.7</th>
<th>0.8 and 0.9</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Var. = 38077; Bin. = 33076; Cons. = 28769; Nonz. = 348325&lt;sup&gt;a&lt;/sup&gt;</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exp. cost</td>
<td>17.31</td>
<td>16.20</td>
<td>15.47</td>
<td>12.06</td>
<td>12.00</td>
<td>11.76</td>
<td>10.70</td>
<td>10.65</td>
<td>10.51</td>
</tr>
<tr>
<td>Exp. service level&lt;sup&gt;b&lt;/sup&gt;</td>
<td>85.52</td>
<td>84.87</td>
<td>84.52</td>
<td>77.19</td>
<td>76.76</td>
<td>75.95</td>
<td>58.63</td>
<td>58.71</td>
<td>38.22</td>
</tr>
<tr>
<td>Exp. fulfilled demand&lt;sup&gt;c&lt;/sup&gt;</td>
<td>85.52</td>
<td>85.30</td>
<td>84.52</td>
<td>93.70</td>
<td>93.61</td>
<td>93.53</td>
<td>91.34</td>
<td>91.20</td>
<td>90.68</td>
</tr>
<tr>
<td>Suppliers selected&lt;sup&gt; (%) of total demand&lt;/sup&gt;</td>
<td>1 (76)</td>
<td>1 (35)</td>
<td>2 (24)</td>
<td>2 (55)</td>
<td>3 (10)</td>
<td>3 (13)</td>
<td>3 (22)</td>
<td>3 (32)</td>
<td>3 (88)</td>
</tr>
</tbody>
</table>

<sup>a</sup>Var.: number of variables; Bin.: number of binary variables; Cons.: number of constraints; Nonz.: number of nonzero coefficients. <sup>b</sup>\(\sum_{s \in S} \sum_{j \in J} P_{s} b_{j} y_{s} / B \times 100\). <sup>c</sup>\(\sum_{s \in S} \sum_{t \in T} P_{s} b_{j} w_{s} / B \times 100\%\).

Table 3: Nondominated solutions for model EWCS.

<table>
<thead>
<tr>
<th>α</th>
<th>0</th>
<th>0.1, 0.2 and 0.3</th>
<th>0.4 and 0.5</th>
<th>0.6 and 0.7</th>
<th>0.8, 0.9 and 1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Var. = 162; Bin. = 136; Cons. = 125; Nonz. = 1430&lt;sup&gt;a&lt;/sup&gt;</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exp. cost</td>
<td>16.91</td>
<td>14.27</td>
<td>12.14</td>
<td>11.67</td>
<td>10.45&lt;sup&gt; (Ε&lt;sub&gt;1&lt;/sub&gt;)&lt;/sup&gt;</td>
</tr>
<tr>
<td>Exp. service level&lt;sup&gt;b&lt;/sup&gt;</td>
<td>87&lt;sup&gt; (Ε&lt;sub&gt;2&lt;/sub&gt;)&lt;/sup&gt;</td>
<td>86</td>
<td>81</td>
<td>79</td>
<td>62&lt;sup&gt; (Ε&lt;sub&gt;2&lt;/sub&gt;)&lt;/sup&gt;</td>
</tr>
<tr>
<td>Exp. fulfilled demand&lt;sup&gt;c&lt;/sup&gt;</td>
<td>87</td>
<td>90</td>
<td>91</td>
<td>93</td>
<td>91</td>
</tr>
<tr>
<td>Suppliers selected&lt;sup&gt; (%) of total demand&lt;/sup&gt;</td>
<td>1 (89)</td>
<td>2 (94)</td>
<td>2 (49)</td>
<td>2 (50)</td>
<td>2 (5)</td>
</tr>
<tr>
<td></td>
<td>3 (6)</td>
<td>3 (51)</td>
<td>3 (50)</td>
<td>3 (95)</td>
<td></td>
</tr>
</tbody>
</table>

<sup>a</sup>Var.: number of variables; Bin.: number of binary variables; Cons.: number of constraints; Nonz.: number of nonzero coefficients. <sup>b</sup>\(\sum_{j \in J} b_{j} Y_{j} / B \times 100\%\). <sup>c</sup>\(\sum_{j \in J} \sum_{t \in T} b_{j} W_{j} / B \times 100\%\).

Figure 4: Schedules for model EWCS.
10.45 ≤ 10.51 for WCS (see Table 2), or the maximum service level $E_{x}$% for EWCS (see Table 3), 87.00 ≥ 85.52 for WCS (see Table 2).

The stochastic wait-and-see approach, however, leads to a more diversified supply portfolio. For the deterministic approach, most of non-dominated supply portfolios consist of two suppliers only: $i = 2, 3$, while suppliers $i = 1,4$ are selected only for the maximum service level objective (that is, for $\alpha = 0$). Comparison of Figures 3 and 4 indicates that the expected schedules for model WCS, computed as expectations over all schedules for all potential disruption scenarios, (27)--(29), are similar to the corresponding single schedules determined by model EWCS. The main differences observed are more delayed expected production and shipping schedules for model WCS when minimization of cost is considered (that is, for $\alpha = 1$). Finally, it is interesting to note that, in the multiple sourcing environment considered, both the wait-and-see approach and, in particular, the expected value approach frequently select a dual sourcing supply portfolio, with one main supplier and one supporting supplier.

The computational experiments were performed using the AMPL programming language and the CPLEX 12.6.2 solver on a MacBook Pro laptop with Intel Core i7 processor running at 2.8 GHz and with 16 GB RAM. The solver was capable of finding proven optimal solution for all examples with CPU time ranging from several minutes to several hours for the stochastic model WCS and fraction of a second for the deterministic model EWCS. Since the stochastic programming model WCS needs to determine nondominated schedules for all potential disruption scenarios, while model EWCS deals with a single scenario only, the difference in the computational effort required is obvious.

7. Conclusions

In this paper, two biobjective MIP formulations, stochastic and deterministic, have been proposed and compared for the coordinated decision-making in supply chains under partial local disruptions and all-or-nothing regional disruptions. The problem objective has been to jointly schedule supplies, production, and distribution to optimize the trade-off between expected cost and expected service level. While the stochastic programming approach aims at optimizing the expected performance of a supply chain over all possible disruption scenarios, the deterministic approach accounts only for a single scenario representing average disruption conditions. As a result, the stochastic programming approach determines nondominated solutions for all disruption scenarios, whereas the deterministic approach produces a single solution only. In particular, in the stochastic programming approach, the selection of supply portfolio is combined with supply chain scheduling for all disruption scenarios considered. In contrast, the deterministic expected value approach provides the portfolio along with a single expected schedule of production and distribution.

The expected schedules obtained for the stochastic programming model WCS, as expectations over all schedules for all disruption scenarios, have been compared with the corresponding schedules determined by the deterministic model EWCS, based on an expected disruption scenario. The comparison has demonstrated that the two approaches lead to similar expected solutions.

The main findings are in line with other research and are listed as follows:

(i) The two decision-making approaches, stochastic and deterministic, lead to similar expected performance of a supply chain under multilevel disruptions.

(ii) The optimal solution values for the expected value problem frequently outperform the corresponding solution values of the wait-and-see problem.

(iii) Despite the multiple sourcing environment considered, both the wait-and-see approach and, in particular, the expected value approach frequently select a dual sourcing supply portfolio, with one main supplier and one supporting supplier.

(iv) The stochastic approach, which accounts for all potential disruption scenarios, may lead to a more diversified supply portfolio that will hedge against a variety of scenarios.

(v) The expected schedules are more delayed for the stochastic approach.

(vi) The service-oriented supply portfolio is more diversified and may combine both high-cost, reliable suppliers and low-cost unreliable suppliers, while the cost-oriented portfolio depends mainly on low-cost and less reliable suppliers.

Overall, the results of computational experiments indicate that the proposed approach and developed MIP models are flexible and efficient tools for coordinated supply chain scheduling. The portfolio approach leads to MIP formulations with strong LP relaxations and has been proven to be computationally very efficient. CPU time required to find proven optimal solutions for realistic size examples, using commercially available software for MIP, is acceptable for a real-world supply chain disruption management (see Sawik [33]).

Since the probability distribution of supply disruptions from each supplier is usually unknown (e.g., [8]), local multilevel disruptions in the stochastic programming model WCS have a multinomial discrete distribution, while the two-level regional disruptions have a binomial discrete distribution, and all disruption events are independent. As part of future research, we propose to enhance the stochastic programming model for more general scenarios with finitely many elements singled out and all the probability concentrated in them. For example, rather than complete shutdown, suppliers within a region may have correlated disruptions. Another important stream of future research is the study of robustness and sensitivity in relation to input data changes in supply chain scheduling under disruption risks (e.g., [33]). Future research can also focus on the following improvement of the proposed models: to consider the minimization of time and cost of recovery (e.g., Whitney et al. [34]).
Notations

Indices

\( i \): Supplier, \( i \in I \)

\( j \): Customer, \( j \in J \)

\( k \): Distribution center, \( k \in K \)

\( l \): Disruption level, \( l \in L \)

\( r \): Region, \( r \in R \)

\( s \): Disruption scenario, \( s \in S \)

\( t \): Planning period, \( t \in T \)

Input Parameters

\( b_j \): Size (number of products) of customer order \( j \)

\( B \): Total demand for parts/products, \( B = \sum_{j \in J} b_j \)

\( C \): Capacity of producer

\( d_j \): Due date for customer order \( j \)

\( e_i \): Fixed cost of ordering parts from supplier \( i \)

\( g_{ij} \): Per unit penalty cost of delayed customer order \( j \)

\( h_{ij} \): Per unit penalty cost of unfulfilled customer order \( j \)

\( \Gamma_k \): Subset of suppliers in region \( r \)

\( \delta_k \): Subset of customers serviced by distribution center \( k \)

\( p_{ij} \): Per unit price of parts purchased from supplier \( i \)

\( p_r \): Regional disruption probability for region \( r \)

\( \gamma_i \): Fraction of an order delivered by supplier \( i \) under disruption level \( l \) (fulfillment rate)

\( \Gamma_l \): Expected fraction of an order delivered by supplier \( i \) (expected fulfillment rate)

\( \sigma_i \): Delivery lead time from supplier \( i \)

\( \tau_k \): Transportation time to distribution center \( k \).

Conflicts of Interest

The author declares that there are no conflicts of interest.

Acknowledgments

This work has been supported by NCN research grant (no. DEC-2013/11/B/ST8/04458) and by AGH (no. 11.11.200.324).

References


