Hindawi
Mathematical Problems in Engineering
Volume 2017, Article ID 3493687, 12 pages
https://doi.org/10.1155/2017/3493687

Research Article

Joint Optimization of Preventive Maintenance and Spare Parts Inventory with Appointment Policy

Jing Cai,[1] Yibing Yin,[1] Li Zhang,[1] and Xi Chen[2]

[1] *College of Civil Aviation, Nanjing University of Aeronautics and Astronautics, Nanjing 211106, China*
[2] *Shanghai Aircraft Customer Service Co., Ltd., Shanghai 200241, China*

Correspondence should be addressed to Jing Cai; caijing@nuaa.edu.cn

Received 2 September 2016; Accepted 12 February 2017; Published 20 March 2017

Academic Editor: Inmaculada T. Castro

Under the background of the wide application of condition-based maintenance (CBM) in maintenance practice, the joint optimization of maintenance and spare parts inventory is becoming a hot research to take full advantage of CBM and reduce the operational cost. In order to avoid both the high inventory level and the shortage of spare parts, an appointment policy of spare parts is first proposed based on the prediction of remaining useful lifetime, and then a corresponding joint optimization model of preventive maintenance and spare parts inventory is established. Due to the complexity of the model, the combination method of genetic algorithm and Monte Carlo is presented to get the optimal maximum inventory level, safety inventory level, potential failure threshold, and appointment threshold to minimize the cost rate. Finally, the proposed model is studied through a case study and compared with both the separate optimization and the joint optimization without appointment policy, and the results show that the proposed model is more effective. In addition, the sensitivity analysis shows that the proposed model is consistent with the actual situation of maintenance practices and inventory management.

1. Introduction

The critical unit in complex systems has an important impact on the system utilization, total operating costs, and so on, and both the procedure and criterion of how to judge the critical unit are presented by Godoy et al. [1]. Based on the procedure and criterion, the air cycle machine (ACM) is judged as the critical unit of the environmental control system (ECS) used in aircraft, so more preventive maintenances (PM) should be implemented to enhance the safety and availability of ACM. More PM mean more spare parts consumption [2]; therefore, a higher inventory level is required and then more capital fund would be tied up for a long time. In order to reduce the inventory level with the requirement of service level, there has been a lot of research on the joint optimization models of maintenance and inventory. Van Horenbeek et al. [3] reviewed the pertinent literatures and concluded that the joint optimization of maintenance and inventory seemed to be more beneficial than the separate optimization.

The joint optimization models are developing with the change of maintenance policies from age-based, periodic/block to condition-based maintenance (CBM). The age-based policy has been applied in the joint optimization models, such as [4–9]. And the periodic/block policy is widely adopted in the joint optimization models; for example, Acharva et al. [10] found a jointly optimal block preventive replacement and spare provisioning policy for a system consisting of several like units. Brezavšček and Hudoklin [11] developed a stochastic mathematical model to determine the jointly optimal "block replacement" and "periodic review spare provisioning policy." Ali Ilgin and Tunali [12] proposed a simulation optimization approach using genetic algorithm (GA) in joint models about block replacement and continuous review inventory policies. Xie and Wang [13] adopted a continuous review ordering policy (s, S) combined with an inspection period to obtain the optimal (T, s, S). Regattieri et al. [14] proposed an approach that integrated the failure and reparation processes, such as modeling, optimization algorithms, and simulation methods, were proposed to define the best maintenance strategies for complex systems. With CBM widely applied in maintenance practice, monitoring information is more and more integrated into joint optimization models. Wang et al. [15, 16] presented a condition-based order-replacement policy for a single-unit system and then proposed a condition-based replacement and spare provisioning policy for deteriorating systems with a number of identical units to optimize inspection interval, maximum stock level, the reorder level, and the preventive replacement threshold. Li and Ryan [17] developed a framework for incorporating real-time condition monitoring information into inventory decisions for spare parts. Romeijnders et al. [18] proposed a two-step method for forecasting spare parts demand using information of component repairs. Louit et al. [19] presented a model to determine the ordering decision for a spare part and assumed that a lead time for spares is random. Tracht et al. [20] developed an enhanced forecast model with the information of both supervisory control and data acquisition to more accurately forecast spare part demand. Wang et al. [21] proposed a prognostics-based spare part ordering and system replacement policy based on the real-time health condition of a deteriorating system subjected to a random lead time. Wang et al. [22] utilized in situ sensor data to predict mechanism of the remaining useful lifetime (RUL) to update the integrated decisions. In addition to the above studies, from the perspective of the whole logistics, repair capacity and deterioration inventory are considered in joint optimization [23–25].

To summarize, increasing studies have focused on maintenance and inventory together, and it is the tendency to integrate the monitoring information into the joint optimization, in order to optimize maintenance decision, inventory level, and so on. The joint optimization based on monitoring information makes just-in-time inventory possible for a single component; however, it almost does not change the existing inventory policies of many identical components, for instances, (s, S) policy and (s, Q) policy. Because the demand of spare parts can be forecasted through prediction of remaining useful life (RUL) based on the monitoring information, an order for spare parts can be placed correspondingly in advance instead of just when the stock drops to the safety inventory level in the existing inventory policies. So an appointment policy of spare parts based on (s, S) policy is first proposed in this paper to place an order in advance. And then a corresponding joint optimization model of preventive maintenance and spare parts inventory is established. GA has strong robustness and fast convergence, and it is easy for GA to combine with Monte Carlo (MC) method, so the combination method of GA and MC is presented to solve the joint optimization model. The rest of this paper is organized as follows: Section 2 describes the joint strategy with the appointment policy in detail. Section 3 estimates parameters and predicts RUL. Section 4 presents the joint optimization model and its algorithm. And then a case study is developed and the sensitivity to the optimal result is analyzed in Section 5. Finally, the conclusions from the work presented in this study and suggestions for future research are given in Section 6.

2. Joint Strategy with Appointment Policy

2.1. Notion. The main notations that will be used throughout the paper are summarized in Notations.

2.2. Description of Joint Strategy. A system consists of n identical critical units that are subject to Wiener deterioration process. In order to minimize the cost rate of maintenance and spare parts inventory, the proposed joint strategy with the appointment policy based on the prediction of RUL is as follows.

(1) Except for the units that have been known as being in the functional/potential failure state through previous inspections, each unit should be inspected periodically at the times $t_k = k \cdot T$ $(k \in \mathbb{N})$ and the inspection time can be neglected compared with the inspection interval. The observed deterioration level of the unit i $(i = 1, 2, \ldots, n)$ at the time t_k is denoted as $X_i^{t_k}$.

(2) If $X_i^{t_k}$ is greater than L_f, CM for the unit i would be implemented. If $X_i^{t_k}$ is between L_p and L_f, PM for the unit i would be implemented. The unit is as good as new after PM/CM, and the PM/CM time can also be neglected compared with the inspection interval. If $X_i^{t_k}$ is less than L_p, the unit i keeps on operating and its RUL ($\mathrm{RUL}_i^{t_k}$) needs to be predicted, and if $\mathrm{RUL}_i^{t_k}$ is less than t_b, one spare part is appointed for the unit i from the stock, so the number ($N_{a\text{-spare}}^{t_k}$) of the available spare parts reduces by one. After the inspections and PM/CM, the time when the units are put into operation again is denoted as t_k^+, and the corresponding deterioration level of the unit i is denoted as $X_i^{t_k^+}$.

(3) An order is placed when the number of the available spare parts is less than or equal to s. However, if the ordered spares have not been delivered, no new order could be made. The ordering cost per order is C_o and the order lead time is $t_l = l \cdot T$ $(l \in \mathbb{N})$.

(4) If no spare unit is available in the stock, the unit in potential failure state would keep on operating and the unit in functional failure state would stop operating. The shortage cost per unit time per unit is c_d.

(5) When the ordered spare units have been delivered, the unit in functional failure state has priority in maintenance. And the remaining spare parts should be put into the stock, and the holding cost and capital charge per unit time per spare part are c_k.

A hypothetical system with two critical units as an example is given to illustrate the joint strategy, as shown in Figure 1, where $S = 3$, $s = 1$, and $t_l = 3T$. The joint decision process of two- unit system is as follows:

(1) At the time t_1, both $X_1^{t_1}$ and $X_2^{t_1}$ are less than L_p, and both $\mathrm{RUL}_1^{t_1}$ and $\mathrm{RUL}_2^{t_1}$ are more than t_b, so no appointment or maintenance is needed.

(2) At the time t_2, $\mathrm{RUL}_2^{t_2}$ is less than t_b, and one spare part needs to be appointed for the unit 2 from the stock, so $N_{\mathrm{app}}^{t_2} = 1$ and $N_{a\text{-spare}}^{t_2}$ reduce from 3 to 2, but the total number of spare parts in the stock remains unchanged; that is, $N_{\mathrm{stock}}^{t_2}$ remains unchanged.

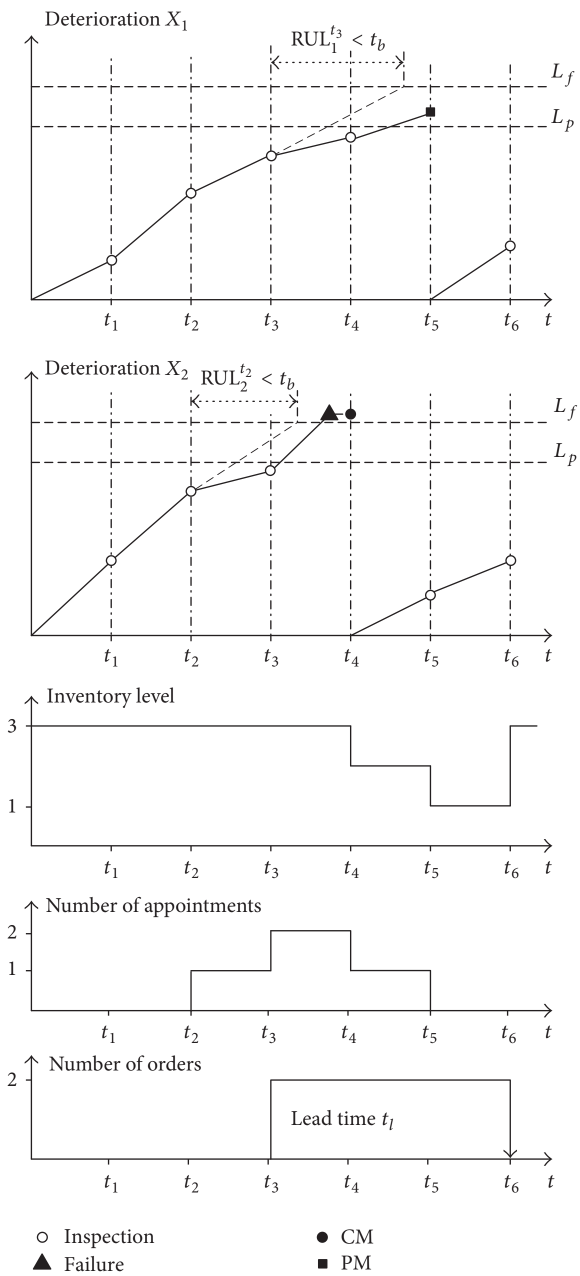


Figure 1: Joint decision process of two-unit system.

(3) At the time t_3, $\mathrm{RUL}_1^{t_3}$ is less than t_b, and one spare part needs to be appointed for the unit 1 from the stock, so $N_{\text{app}}^{t_3} = N_{\text{app}}^{t_2} + 1 = 2$ and $N_{a\text{-spare}}^{t_3} = 1$. Then an order for ($N_{\text{order}}^{t_3} = 2$) spare parts is placed.

(4) At the time t_4, $X_2^{t_4}$ is greater than L_f, and CM for the unit 2 needs to be implemented, so $N_{\text{stock}}^{t_4}$ reduces from 3 to 2; then $N_{\text{app}}^{t_4} = N_{\text{app}}^{t_3} - 1 = 1$ and $N_{a\text{-spare}}^{t_4} = N_{\text{stock}}^{t_4} - N_{\text{app}}^{t_4} = 1$.

(5) At the time t_5, $X_1^{t_5}$ is between L_p and L_f, so PM for the unit 1 needs to be implemented, and $N_{\text{stock}}^{t_5}$ reduces from 2 to 1, $N_{\text{app}}^{t_5} = 0$, and $N_{a\text{-spare}}^{t_5} = 1$.

(6) At the time t_6, the ordered spare units have been delivered, so both $N_{\text{stock}}^{t_6} = N_{a\text{-spare}}^{t_6} = 3$.

The rest can be done in the same manner.

3. Parameter Estimation and RUL Prediction

Wiener process can be used to describe a variety of performance degradation process of typical unit and has been applied in many fields like unit corrosion, mechanical vibration, and so forth [26]. So it is assumed that the unit is subject to Wiener deterioration process in this paper.

3.1. The Basic Model of Wiener Process. According to Wiener process, $X_i^{t_k}$ can be characterized by the following:

$$X_i^{t_k} = X(0) + \mu t_k + \sigma W(t_k), \tag{1}$$

where $X(0)$ is the initial state, μ and σ are the drift coefficient and diffusion coefficient, respectively, and $W(t_k)$ is a standard Brownian motion; that is, $W(t_k) \sim N(0, t_k)$.

So $\Delta X_i^{t_k}$ is subject to normal distribution and can be described as

$$\Delta X_i^{t_k} \sim N(\mu T, \sigma^2 T). \tag{2}$$

3.2. Parameter Estimation. μ and σ can be estimated using maximum likelihood estimate and the likelihood function is

$$L(\mu,\sigma) = \prod_{i=1}^{n}\prod_{k=1}^{m_i} \frac{1}{\sqrt{2\sigma^2 \pi T}} \exp\left[-\frac{\left(\Delta X_i^{t_k} - \mu T\right)^2}{2\sigma^2 T}\right]. \tag{3}$$

Solve the following equations:

$$\begin{aligned} \frac{\partial \ln(L(\mu,\sigma))}{\partial \mu} &= 0, \\ \frac{\partial \ln(L(\mu,\sigma))}{\partial \sigma} &= 0. \end{aligned} \tag{4}$$

$\hat{\mu}$ and $\hat{\sigma}^2$ can be obtained:

$$\begin{aligned} \hat{\mu} &= \frac{\sum_{i=1}^{n}\sum_{k=1}^{m_i} \Delta X_i^{t_k}}{\sum_{i=1}^{n} m_i T}, \\ \hat{\sigma}^2 &= \frac{1}{\sum_{i=1}^{n} m_i}\left[\sum_{i=1}^{n}\sum_{k=1}^{m_i} \frac{\left(\Delta X_i^{t_k}\right)^2}{T} - \frac{\left(\sum_{i=1}^{n}\sum_{k=1}^{m_i} \Delta X_i^{t_k}\right)^2}{\sum_{i=1}^{n} m_i T}\right]. \end{aligned} \tag{5}$$

3.3. RUL Prediction. Functional failure state threshold is L_f, so the $\mathrm{RUL}_i^{t_k}$ is

$$\begin{aligned} \mathrm{RUL}_i^{t_k} &= \inf\left\{t : X_i^{t_k+t} \ge L_f \mid X_i^{t_k} < L_f,\ t \ge 0\right\} \\ &= \inf\left\{t : X_i^{t_k+t} - X_i^{t_k} \ge L_f - X_i^{t_k},\ t \ge 0\right\} \\ &= \inf\left\{t : X_i^{t} \ge L_f - X_i^{t_k},\ t \ge 0\right\}. \end{aligned} \tag{6}$$

According to [27], the cumulative distribution function and probability density function of $\mathrm{RUL}_i^{t_k}$ can be obtained as follows:

$$F(t) = \Phi\left(\frac{\mu t - \left(L_f - X_i^{t_k}\right)}{\sigma\sqrt{t}}\right) + \exp\left(\frac{2\mu\left(L_f - X_i^{t_k}\right)}{\sigma^2}\right)\Phi\left(\frac{-\left(L_f - X_i^{t_k}\right) - \mu t}{\sigma\sqrt{t}}\right), \quad (7)$$

$$f(t) = \frac{L_f - X_i^{t_k}}{\sqrt{2\pi\sigma^2 t^3}} \exp\left[-\frac{\left(\left(L_f - X_i^{t_k}\right) - \mu t\right)^2}{2\sigma^2 t}\right]. \quad (8)$$

If $\alpha = (L_f - X_i^{t_k})/\mu$, $\lambda = (L_f - X_i^{t_k})^2/\sigma^2$, (8) can be transferred to

$$f(t) = \sqrt{\frac{\lambda}{2\pi t^3}} \exp\left[\frac{-\lambda(t-\alpha)^2}{2\alpha^2 t}\right]. \quad (9)$$

So $f(t)$ is an inverse Gaussian distribution, and based on its definition, the mean and variance of $\mathrm{RUL}_i^{t_k}$ can be obtained:

$$E\left(\mathrm{RUL}_i^{t_k}\right) = \frac{L_f - X_i^{t_k}}{\mu}, \qquad \mathrm{var}\left(\mathrm{RUL}_i^{t_k}\right) = \frac{\left(L_f - X_i^{t_k}\right)\cdot\sigma^2}{\mu^3}. \quad (10)$$

4. Joint Optimization Model and Solution

4.1. Establishment of Joint Optimization Model. According to the joint strategy, the average cost rate (EC) of maintenance and inventory can be represented as a function of T, S, s, L_p, and t_b, that is, $f_{\mathrm{EC}}(T, S, s, L_p, t_b)$, as follows:

$$\begin{aligned} \mathrm{EC} = \min \quad & f_{\mathrm{EC}}\left(T, S, s, L_p, t_b\right) \\ \text{s.t.} \quad & 0 < T < t_b; \\ & 0 < L_p < L_f; \\ & 0 \le s < S. \end{aligned} \quad (11)$$

Due to the complexity of the joint strategy, it is difficult to derive the analytical formulation of the function $f_{\mathrm{EC}}(T, S, s, L_p, t_b)$. However, the average cost rate of the maintenance and inventory over an infinite time span can also be simply represented as

$$f_{\mathrm{EC}}\left(T, S, s, L_p, t_b\right) = \lim_{t_m \to \infty} \frac{C_p \cdot N_p + C_c \cdot N_c + C_{\mathrm{ins}} \cdot N_{\mathrm{ins}} + C_o \cdot N_o + c_k \cdot \sum_{k=1}^{m} N_{\mathrm{stock}}^{t_k} + c_d \cdot \sum_{k=1}^{m} N_d^{t_k}}{t_m \cdot n}, \quad (12)$$

where N_p, N_c, N_{ins}, N_o, $N_{\mathrm{stock}}^{t_k}$, and $N_d^{t_k}$ can be obtained as follows by finishing the simulation of all events over the simulation time span t_m.

(1) N_p, N_c. According to the definitions, N_p and N_c can be expressed as

$$N_p = \sum_{k=1}^{m}\sum_{i=1}^{n} \mathrm{IPR}_i^k, \qquad N_c = \sum_{k=1}^{m}\sum_{i=1}^{n} \mathrm{ICR}_i^k. \quad (13)$$

The value of ICR_i^k and IPR_i^k depends on whether PM/CM for the unit i is implemented or not, and then whether PM/CM is implemented depends on two factors: (1) the observed deterioration level $X_i^{t_k}$ (2) whether there is a spare part or not in the stock. So ICR_i^k and IPR_i^k can be identified, respectively:

$$\mathrm{ICR}_i^k = \begin{cases} 1, & X_i^{t_k} \ge L_f \cap N_{\mathrm{stock}}^{t_k} > 0 \\ 0, & \text{others}, \end{cases}$$

$$\mathrm{IPR}_i^k = \begin{cases} 1, & L_p \le X_i^{t_k} < L_f \cap N_{\mathrm{stock}}^{t_k} > 0 \\ 0, & \text{others}. \end{cases} \quad (14)$$

(2) N_{ins}. According to the definition, N_{ins} can be expressed as

$$N_{\mathrm{ins}} = \sum_{k=1}^{m}\sum_{i=1}^{n} I_i^k. \quad (15)$$

As described in the joint strategy, if the unit i has been known as being in the functional/potential failure state, the inspection of the unit i does not need to be implemented regardless of whether there are spare parts or not. So I_i^k can be identified:

$$I_i^k = \begin{cases} 0, & X_i^{t_{k-1}+} \ge L_p \\ 1, & \text{others}. \end{cases} \quad (16)$$

The unit becomes as good as new if PM/CM is implemented after an inspection; otherwise the deterioration level remains the same, so the deterioration level $X_i^{t_k^+}$ can be obtained:

$$X_i^{t_{k-1}^+ +} = \begin{cases} 0, & \mathrm{IPR}_i^{k-1} = 1 \cup \mathrm{ICR}_i^{k-1} = 1 \\ X_i^{t_{k-1}}, & \text{others.} \end{cases} \quad (17)$$

(3) N_o, $N_{stock}^{t_k}$. According to the definition, N_o can be expressed as

$$N_o = \sum_{k=1}^{m} \mathrm{OC}^{t_k}. \quad (18)$$

OC^{t_k} depends on whether an order is placed at the time t_k, so OC^{t_k} can be identified:

$$\mathrm{OC}^{t_k} = \begin{cases} 1, & N_{\text{order}}^{t_k} \geq 1 \\ 0, & \text{others.} \end{cases} \quad (19)$$

According to the joint strategy, $N_{\text{order}}^{t_k}$ is equal to $S - N_{a\text{-spare}}^{t_k}$. However, only one order is permitted at the same time; that is, one order is permitted when there is no undelivered order, so $N_{\text{order}}^{t_k}$ can be obtained:

$$N_{\text{order}}^{t_k} = \begin{cases} S - N_{a\text{-spare}}^{t_k}, & N_{a\text{-spare}}^{t_k} \leq s \cap \mathrm{Undel} = 0 \\ 0, & \text{others.} \end{cases} \quad (20)$$

Since $N_{a\text{-spare}}^{t_k} = N_{\text{stock}}^{t_k} - N_{\text{app}}^{t_k}$, $N_{\text{stock}}^{t_k}$ and $N_{\text{app}}^{t_k}$ should be obtained first.

(a) $N_{stock}^{t_k}$. $N_{\text{stock}}^{t_k}$ depends on two things, the number of remaining spare parts ($N_{\text{stock}}^{t_{k-1}^+}$) and the number of the delivered spare parts ($N_{\text{order}}^{t_k - t_l}$), so $N_{\text{stock}}^{t_k}$ can be obtained:

$$N_{\text{stock}}^{t_k} = \begin{cases} N_{\text{stock}}^{t_{k-1}^+} + N_{\text{order}}^{t_k - t_l}, & \mathrm{OC}^{t_k - t_l} = 1 \\ N_{\text{stock}}^{t_{k-1}+}, & \text{others.} \end{cases} \quad (21)$$

The number of the units whose deterioration levels are greater than L_p is $\sum_{i=1}^{n}(\mathrm{ICR}_i^{k-1} + \mathrm{IPR}_i^{k-1})$ at the time t_{k-1}, so $N_{\text{stock}}^{t_{k-1}^+}$ is equal to $N_{\text{stock}}^{t_{k-1}} - \sum_{i=1}^{n}(\mathrm{ICR}_i^{k-1} + \mathrm{IPR}_i^{k-1})$. However, $N_{\text{stock}}^{t_{k-1}^+}$ does not allow being negative value, so $N_{\text{stock}}^{t_{k-1}^+}$ should be expressed as

$$N_{\text{stock}}^{t_{k-1}^+} = \max \left(N_{\text{stock}}^{t_{k-1}} - \sum_{i=1}^{n} \left(\mathrm{ICR}_i^{k-1} + \mathrm{IPR}_i^{k-1} \right), 0 \right). \quad (22)$$

(b) $N_{app}^{t_k}$. $N_{\text{app}}^{t_k}$ can be obtained:

$$N_{\text{app}}^{t_k} = \sum_{i=1}^{n} \mathrm{app}_i^k. \quad (23)$$

At the time t_k, even if $\mathrm{RUL}_i^{t_k}$ is less than t_b and $X_i^{t_k}$ is less than L_p, no new appointment could be made for the unit i if an appointment for the unit i has been made at the time t_{k-1}; that is, only one appointment for the unit i is permitted at the same time. So app_i^k can be identified:

$$\mathrm{app}_i^k = \begin{cases} 1, & X_i^{t_k} < L_p \cap \mathrm{RUL}_i^{t_k} < t_b \cap \mathrm{app}_i^{k-1} = 0 \\ 0, & \text{others.} \end{cases} \quad (24)$$

(4) $N_d^{t_k}$. According to the definition, $N_d^{t_k}$ can be expressed as

$$N_d^{t_k} = \sum_{i=1}^{n} F_i^{t_{k-1}^+}. \quad (25)$$

Because $X_i^{t_{k-1}^+} \geq L_f$ means that $F_i^{t_{k-1}^+} = 1$, $F_i^{t_{k-1}^+}$ can be identified:

$$F_i^{t_{k-1}^+} = \begin{cases} 1, & X_i^{t_{k-1}^+} \geq L_f \\ 0, & \text{others.} \end{cases} \quad (26)$$

4.2. Algorithm of Model. The combination method of GA and MC is adopted to obtain an approximate optimization result $\theta^* = (T^*, S^*, s^*, L_p{}^*, t_b{}^*)$. The flow diagram for the combination method is given in Figure 2.

The main steps of the flow diagram are as follows.

Step 1. "Initialize population (N)" to obtain N groups of initial parameters (T, S, s, L_p, t_b).

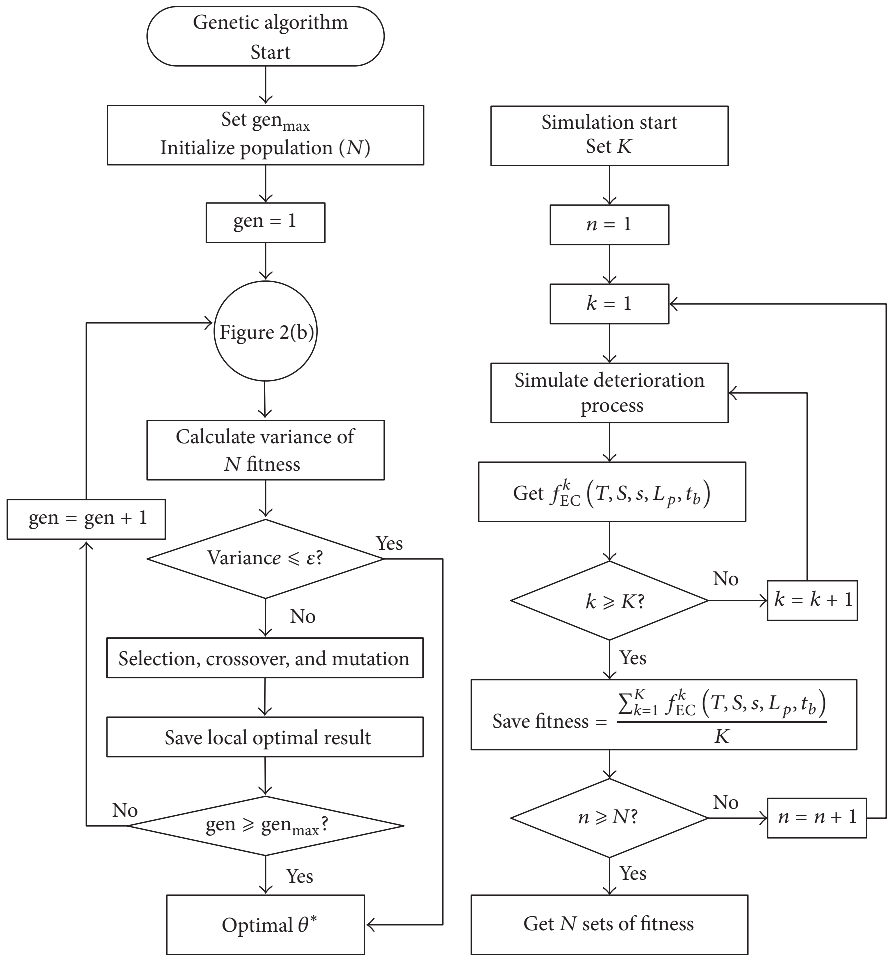
Step 2. Evaluate the fitness $f_{\mathrm{EC}}(T, S, s, L_p, t_b)$ of each group of the parameters by simulation. In order to eliminate the randomness of simulation, t_m is taken as a big enough value, and the mean value of K simulation results is taken as the fitness of each population; that is, the fitness of each population is equal to $\sum_{k=1}^{K} f_{\mathrm{EC}}^k(T, S, s, L_p, t_b)/K$, where $f_{\mathrm{EC}}^k(T, S, s, L_p, t_b)$ is evaluated through number k simulation described in Figure 2(b).

Step 3. If the variance of all population's fitness obtained based on Step 2 is less than a sufficiently small value ε, it means that it is unnecessary to optimize further. The parameters whose fitness is the minimum are the optimal result $\theta^* = (T^*, S^*, s^*, L_p{}^*, t_b{}^*)$; otherwise, go to Step 4.

Step 4. The new populations are obtained through selection, crossover, and mutation based on GA method and go to Step 2 if the iteration does not end.

5. Case Study

5.1. Estimation of Deterioration Parameter. ACM is an important refrigeration unit of the ECS used in pressurized gas turbine-powered aircraft, and the outlet temperature of ACM rises with its performance deterioration. When the outlet temperature rises up to the functional failure threshold ($L_f = 10^\circ$C), ACM would be removed and overhauled. The outlet temperatures of 20 ACMs in AIR CHINA have been inspected very 1000 flight hours (FH), that is, $T = 1000$ FH, and the data of those outlet temperatures are described in Figure 3.



(a) Flow diagram for GA (b) Flow diagram for MC

Figure 2: Flow diagram for the combination method.

According to Section 3.1, the outlet temperatures can be seen as subject to Wiener deterioration process if the deterioration increment of the outlet temperatures can be proven to be subject to normal distribution. Based on the deterioration increment of the outlet temperatures of 20 ACMs, the parameters can be estimated as shown in Figure 4. With the Kolmogorov-Smirnov test, the P value is 0.2 greater than 0.05 when the level of significance is 5%, so the deterioration increment of the outlet temperatures fellows normal distribution $N(0.333, 0.314)$, and the deterioration parameters can be obtained: $\widehat{\mu} = 3.33 \times 10^{-4}$; $\widehat{\sigma} = 0.0099$. Because the initial outlet temperature is usually 2°C, the outlet temperatures can be expressed as

$$X(t) = 2 + 0.000333t + 0.0099W(t). \tag{27}$$

5.2. Joint Optimization Simulation. According to the preliminary statistics, $C_{ins} = 1{,}000$, $C_p = 100{,}000$, $C_c = 400{,}000$, $C_o = 5{,}000$ (RMB), $c_d = 100$, $c_k = 10$ (RMB/FH), and $t_l = 2{,}000$ (FH). In the GA, roulette wheel selection and elite strategy are used in selection. Crossover probability is 0.8. Mutation probability is 0.05. Population size is 80. Maximum generation is 300. In order to eliminate the randomness of simulation, the simulation time span t_m is 100,000 (FH) and the mean value of 50 simulation results is taken as the fitness of each population; that is, fitness $= \sum_{k=1}^{50} f_{EC}^{k}(T, S, s, L_p, t_b)/50$. The iterative process of the GA is shown in Figure 5, and the optimal result $[S^*, s^*, L_p^*, t_b^*]$ is $[4, 1, 9.17, 3391]$, and $f_{EC}(S^*, s^*, L_p{}^*, t_b{}^*) = 116.03$ (RMB/FH).

With the optimal result $[S^*, s^*, L_p^*, t_b^*]$, the change of the inventory level over the simulation time span is shown in Figure 6, where the actual maximum inventory level is 5 greater than $S^* = 4$, which is caused by applying the appointment policy. For example, at time $t_{43} = 43000$ FH, $N_{stock}^{t_{43}} = 1$ and $N_{app}^{t_{43}} = 2$, as shown with the red point, respectively, in Figures 6 and 7. Therefore, based on (20), $N_{a\text{-}spare}^{t_{43}} = N_{stock}^{t_{43}} - N_{app}^{t_{43}} = -1$ and $N_{order}^{t_{43}} = S^* - N_{a\text{-}spare}^{t_{43}} = 5$, as shown with the red point, respectively, in Figures 8 and 9. So an order for 5 spare parts is placed.

The order for 5 spare parts will arrive at time $t_{45} = 45000$ FH, there is one PM at time $t_{44} = 44000$ FH as shown with the red point in Figure 10, and there is no CM at time

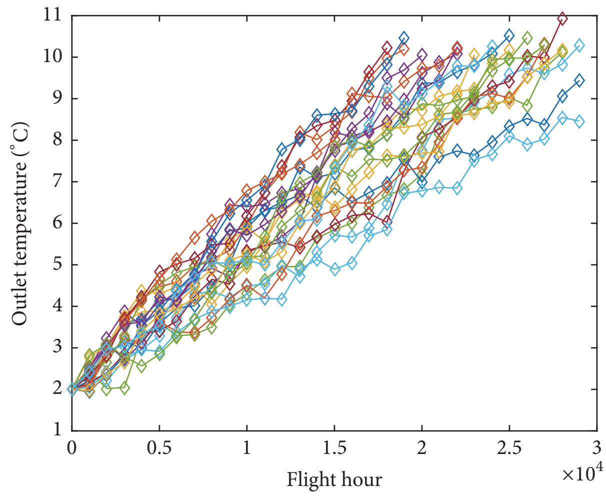


FIGURE 3: The collected outlet temperature data of the ACM.

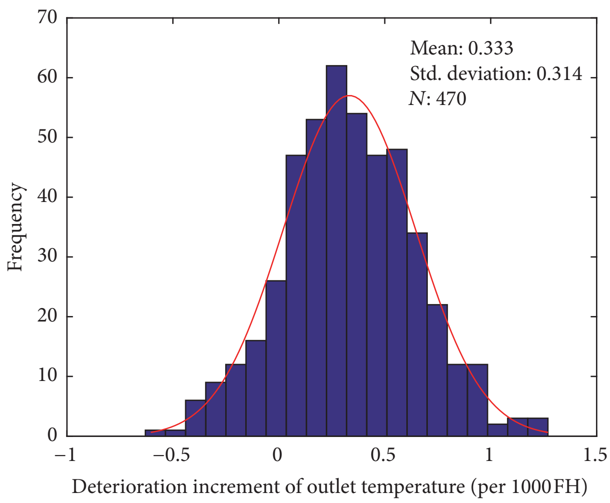


FIGURE 4: The distribution of the deterioration increment.

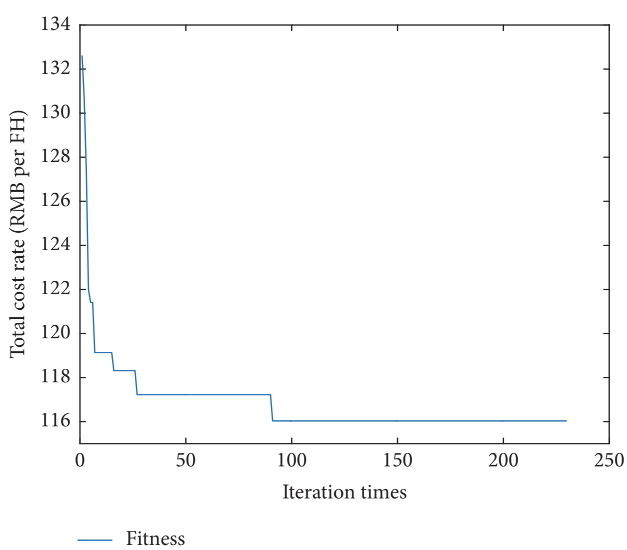


FIGURE 5: The change of the fitness with iteration times (with the appointment policy).

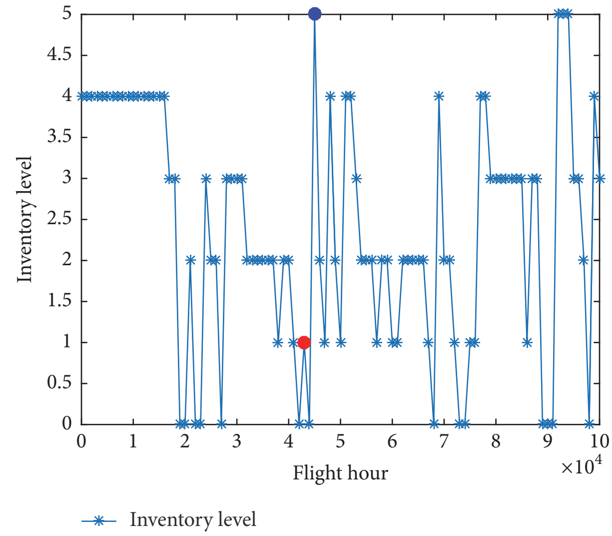


FIGURE 6: The change of the inventory level.

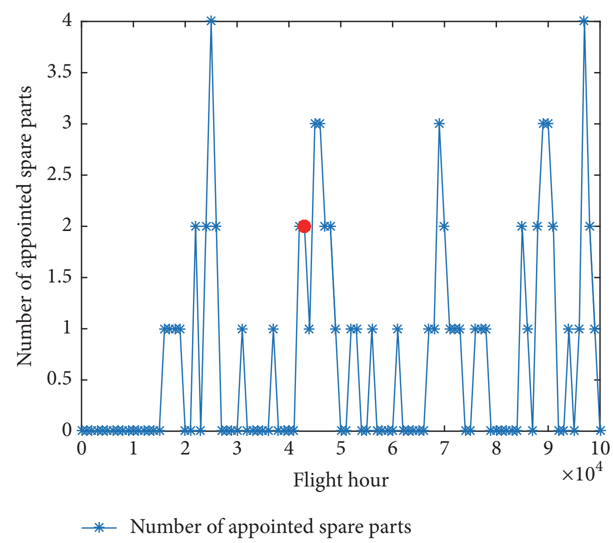


FIGURE 7: The change of number of appointed spare parts.

$t_{44} = 44000$ FH and $t_{45} = 45000$ FH as shown in Figure 11, so one spare part is used and the inventory level becomes 5 at time $t_{45} = 45000$ FH, as shown with a blue point in Figure 6.

5.3. Comparison with the Separate Optimization Method. According to the data in Figure 6, the average required number of spare parts per 1000 FH over the simulation time span is about 2.38. The ACM normally operates about 3000 FH each year; therefore, the average annual required number (D) of spare parts is about 7. In most airlines, according to the calculation method of Boeing and Airbus, it is assumed that D is subject to Poisson distribution. In this paper it is assumed that the required shortage rate (FR) of ACM is less than 0.1%; therefore, S and s can be obtained

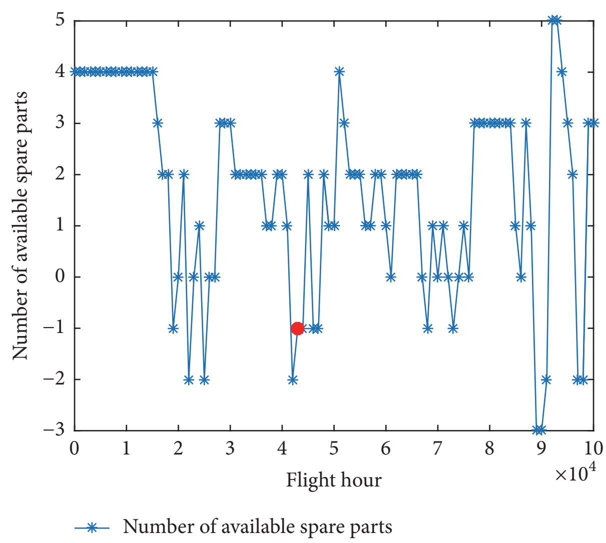


FIGURE 8: The change of number of available spare parts.

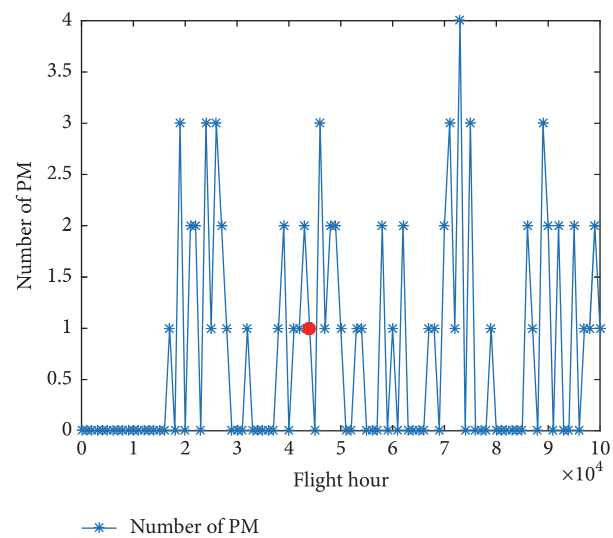


FIGURE 10: The change of number of PM.

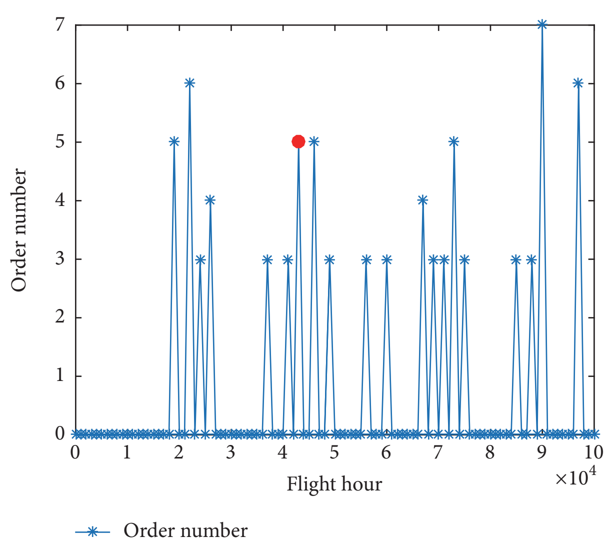


FIGURE 9: The change of order number.

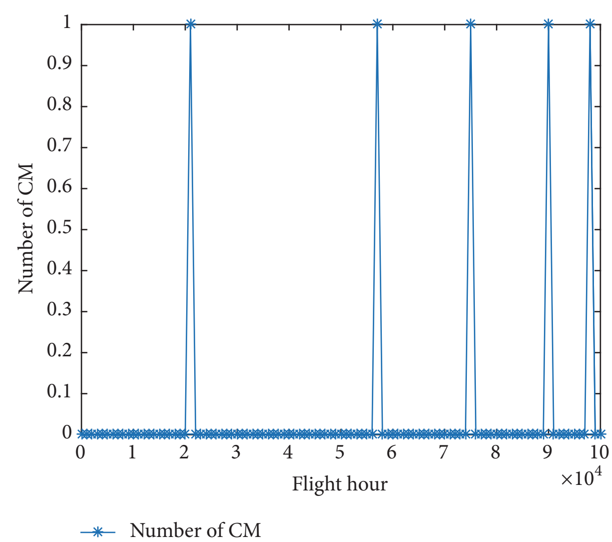


FIGURE 11: The change of number of CM.

in accordance with the calculation method of Boeing and Airbus as follows:

$$\mathrm{FR} = \sum_{s} \frac{D^{(D+s)} \cdot e^{-D}}{(D+s)!}. \tag{28}$$

By (28), it can be obtained that s is equal to 3, and the maximum inventory S is equal to $D + s = 7 + 3 = 10$. So $[S, s, L_p^*, t_b^*] = [10, 3, 9.17, 3391]$ is taken as an input to the simulation model, the average cost rate is 168.66 (RMB/FH) that is 45.36% higher than the above optimal result 116.03 (RMB/FH), and the average inventory level without the joint optimization is 6.5 that is far greater than 2.37. The inventory difference of whether the joint optimization is adopted or not is shown in Figure 12.

5.4. Comparison with the Traditional Joint Optimization. In the same example, without the appointment policy in the optimization model, the optimal result is $[S^*, s^*, L_p^*] = [4, 1, 9.10]$ and $f_{EC}(S^*, s^*, {L_p}^*) = 120.95$ (RMB/FH) that is 4.24% higher than the above optimal result 116.03 (RMB/FH) with the appointment policy. The iterative process of the GA is shown in Figure 13, and the change of inventory level with the appointment policy or not over the simulation time span is shown in Figure 14.

From Figure 14, it can be found that the average required number of spare parts per 1000 FH per unit without the appointment policy is about 0.132 that is 10.92% higher than with the appointment policy. In addition, without the appointment policy, the shortage happens at time $t_{26} = 26000$ FH; however the shortage never happens over the simulation time span with the appointment policy. In summary,

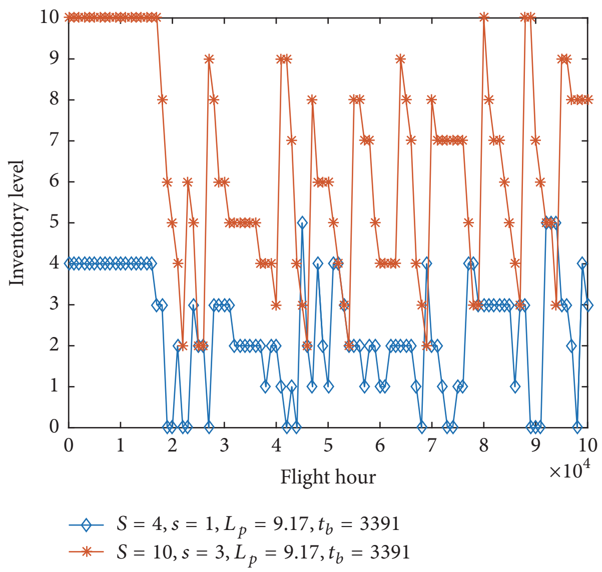


FIGURE 12: Inventory difference of whether joint optimization is adopted or not.

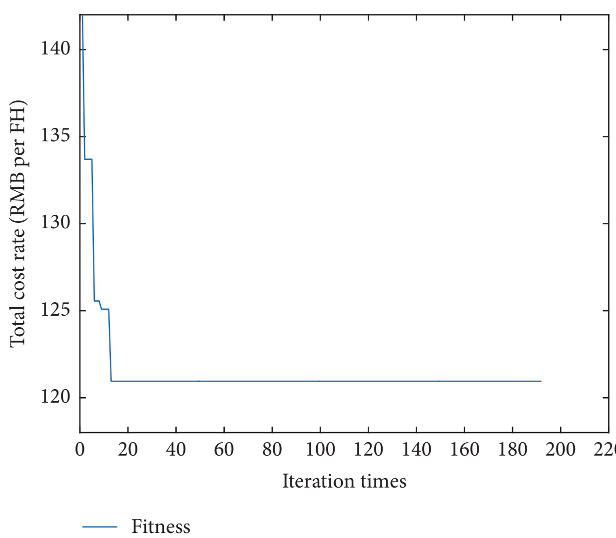


FIGURE 13: The change of the fitness with iteration times (without the appointment policy).

the appointment policy reduces not only the average cost rate but also the probability of shortage.

5.5. Sensitivity Analysis. The input parameters of the joint optimization model may not be absolutely accurate; for example, the shortage loss or CM cost of ACM is very difficult to be estimated. Therefore, it is necessary to analyze the parameter sensitivity to the optimal result.

5.5.1. Sensitivity of CM Cost C_c. Table 1 shows the different optimal results under the different C_c.

From Table 1, it can be obtained that when C_c increases, L_p decreases; however t_b increases, which means that more

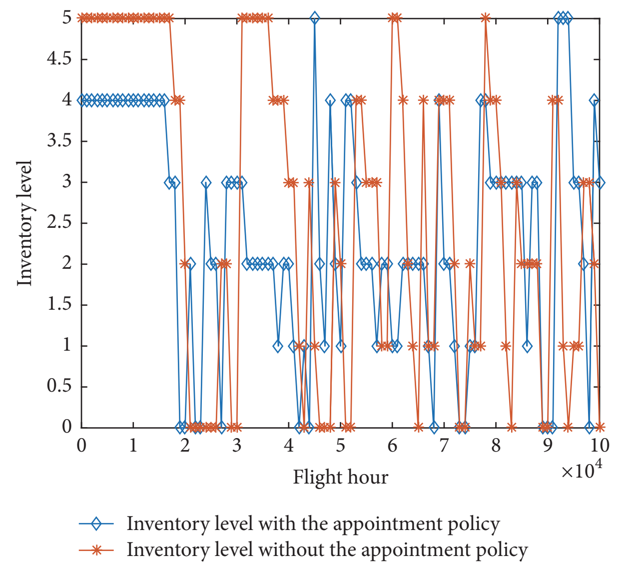


FIGURE 14: The change of the inventory level (with the appointment policy or not).

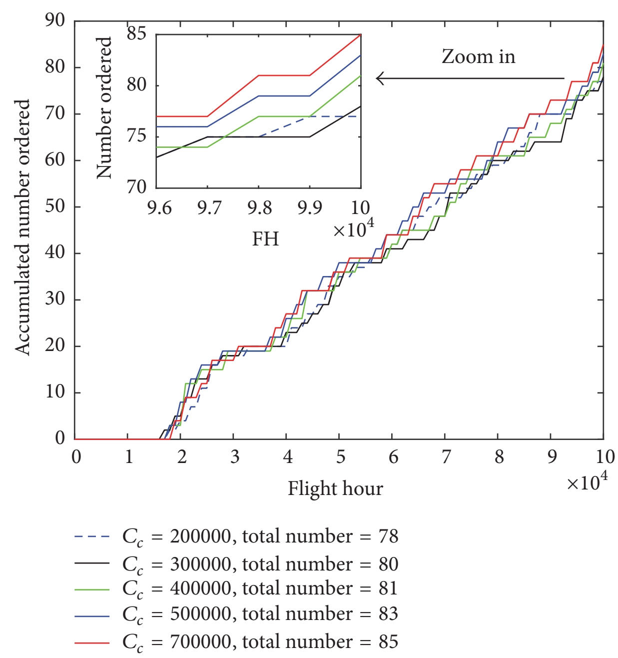


FIGURE 15: Accumulated ordered number of spare parts in different C_c.

PM should be implemented to avoid the functional failures. So the corresponding required number of spare parts increases, as shown in Figure 15. For example, the accumulated required number of spare parts over the simulation time span is 78 when $C_c = 200{,}000$ RMB; however, it is 85 when $C_c = 700{,}000$ RMB. In addition, Table 1 shows that the sharp increase of C_c does not result in a substantial increase of EC, for which the reason is that the CM will be almost avoided when C_c is taken as a big value.

Table 1: The different optimal results under the different C_c.

C_c	S	s	L_p	t_b	EC
200000	3	1	9.27	2943	114.57
300000	3	1	9.22	3254	115.82
400000	4	1	9.17	3391	116.03
500000	4	1	9.11	3721	117.08
700000	4	1	9.02	3982	118.98

Table 2: The different optimal results under the different t_l.

t_l	S	s	L_p	t_b	EC_∞
1000	1	0	9.37	3482	110.22
2000	4	1	9.17	3391	116.03
3000	5	2	9.13	3132	126.13
4000	7	2	9.06	3721	131.93
6000	9	3	9.16	3693	153.24

Table 3: The different optimal results under the different C_{ins}.

C_{ins}	S	s	L_p	t_b	EC_∞
200	4	1	9.03	3576	98.66
500	4	1	9.10	3422	103.34
1000	4	1	9.17	3391	116.03
2000	4	1	9.21	3292	131.90
4000	4	1	9.11	3591	161.09

Table 4: The different optimal results under the different C_o.

C_o	S	s	L_p	t_b	EC_∞
1000	4	1	9.17	3382	115.22
2000	4	1	9.19	3366	115.43
5000	4	1	9.17	3391	116.03
7000	4	1	9.16	3421	116.93
10000	4	1	9.12	3439	123.24

Both c_d and C_c are associated with the functional failure; therefore, they have the similar influence on the optimal result. So the sensitivity of c_d to the optimal result does not need to be further discussed.

5.5.2. Sensitivity of Lead Time t_l. Table 2 shows the different optimal results under the different t_l.

From Table 2, it can be obtained that the safety inventory level is equal to 0 when the lead time is 1000 FH. With the increase of the lead time, an order for spare parts should be placed in advance to ensure a timely supply of spare parts, or a high inventory level should be kept, which is in accordance with the fact that higher inventory level is needed if spare parts cannot be delivered immediately.

5.5.3. Sensitivity of Lead Times C_{ins}, C_o, and C_k. Tables 3 and 4 show the different optimal results under the different C_{ins} and C_o, respectively. Tables 3 and 4 show that C_{ins} has almost no impact on the optimal result, for which the reason is that the total number of orders over the simulation time span is relatively steady. C_o also has almost no impact on the optimal result, for which the reason is that C_o are too small compared with C_c and C_p.

Table 5: The different optimal results under the different C_k.

C_k	S	s	L_p	t_b	EC_∞
1	6	2	8.91	3692	94.78
5	4	2	9.21	3497	107.82
10	4	1	9.17	3391	116.03
20	3	0	9.23	3543	132.88
40	2	0	9.27	3738	151.63

Table 5 shows the different optimal results under the different C_k, and in Table 5, with the increase of C_k, the inventory level decreases significantly, which can save the holding cost and capital charge of spare parts. But the corresponding appointment and PM need to be carried out in advance in order to make up the low inventory level.

6. Conclusion

In this study, a joint optimization model with the appointment policy is first proposed based on the prediction of RUL in order to place an order for spare parts in advance and minimize the cost rate of maintenance and inventory, and the algorithm has been developed and described in detail. In the case study, the proposed model and its optimal results are analyzed, compared with both the model without joint optimization and the joint optimization without the appointment policy. Finally the parameter sensitivity to the optimal result is analyzed.

Through the case study, the conclusions are as follows:

(1) Adopting the appointment policy in the optimization model reduces not only the cost rate but also the probability of shortage.

(2) The proposed optimization model saves 45.36% of the cost compared with the model without joint optimization and saves 4.24% of the cost compared with the joint optimization without the appointment policy, which means that the proposed optimization model is effective.

(3) The results of sensitivity analysis show that the proposed optimization model is consistent with the actual situation of maintenance practices and inventory management.

In reality the inventory management is always classified into the initial provisioning phase and ongoing provisioning phase. The initial provisioning phase is called a "maintenance honeymoon" with limited demand for spare parts, differing from the ongoing provisioning phase, so the different spare provisioning phases needed to be considered in the joint optimization model in the further research.

Notations

n: Number of identical critical units, $n \in \mathbb{N}$
i: Number of units, $i = 1, 2, \ldots, n$
T: Inspection interval
k: The serial number of inspections, $k \in \mathbb{N}$
t_k: The kth inspection time
t_k^+: Time of putting into operation after the kth inspection, and between t_k and t_k^+, PM/CM may be implemented or not
$X_i^{t_k}$: Deterioration level of the unit i at the time t_k
$X_i^{t_k^+}$: Deterioration level of the unit i at the time t_k^+
$\Delta X_i^{t_k}$: Deterioration increment of the unit i from t_k^+ to t_k, $\Delta X_i^{t_k} = X_i^{t_k} - X_i^{t_{k-1}^+}$
t_l: Order lead time
t_m: Simulation time span, $t_m = m \cdot T$ $(m \in \mathbb{N})$
L_f: Functional failure threshold, and if $X_i^{t_k}$ is greater than L_f, CM for the unit i should be implemented
L_p: Potential failure threshold ($L_p < L_f$), and if $X_i^{t_k}$ is between L_p and L_f, PM for the unit i should be implemented
$\mathrm{RUL}_i^{t_k}$: Predicted RUL of the unit i at the time t_k
t_b: Appointment threshold, and if $\mathrm{RUL}_i^{t_k}$ is less than t_b, one spare part needs to be appointed for the unit i from the stock
C_{ins}: Cost of inspection (per unit)
N_{ins}: Total number of inspections over the time span t_m
C_p: Cost of PM (per unit)
N_p: Total number of PM over the time span t_m
C_c: Cost of corrective maintenance (CM) (per unit)
N_c: Total number of CM over the time span t_m
C_o: Ordering cost per order
N_o: Total number of orders over the time span t_m
OC^{t_k}: Code to signify whether an order is placed at the time t_k ("$OC^{t_k} = 1$" means that an order is placed and "$OC^{t_k} = 0$" otherwise)
c_d: Shortage cost per unit time per unit
$N_d^{t_k}$: Number of the units in functional failure state between t_{k-1}^+ and t_k
c_k: Holding cost and capital charge per unit time per spare part
$N_{\mathrm{stock}}^{t_k}$: Number of spare parts between t_{k-1}^+ and t_k
$F_i^{t_k}$: Code to signify whether the unit i is in functional failure state the time t_k ("$F_i^{t_k} = 1$" means that the unit i is in functional failure state and "$F_i^{t_k} = 0$" otherwise)
IPR_i^k: Code to signify whether PM of the unit i is implemented at the time t_k ("$\mathrm{IPR}_i^k = 1$" means that PM is implemented and "$\mathrm{IPR}_i^k = 0$" otherwise)
ICR_i^k: Code to signify whether CM of the unit i is implemented at the time t_k ("$\mathrm{ICR}_i^k = 1$" means that CM is implemented and "$\mathrm{ICR}_i^k = 0$" otherwise)
I_i^k: Code to signify whether an inspection of the unit i is implemented at the time t_k ("$I_i^k = 1$" means that an inspection is implemented and "$I_i^k = 0$" otherwise)
Undel: Code to signify whether all orders until the current time has been delivered t_k ("Undel = 0" means that all orders have been delivered and "Undel = 1" otherwise)
S: Maximum inventory level
s: Safety inventory level
app_i^k: Code to signify whether an appointment for the unit i is made at the time t_k ("$\mathrm{app}_i^k = 1$" means that an appointment is made and "$\mathrm{app}_i^k = 0$" otherwise)
$N_{\mathrm{app}}^{t_k}$: Total number of appointed parts at the time t_k
$N_{\mathrm{stock}}^{t_k}$: Actual inventory level at time t_k
$N_{a\text{-spare}}^{t_k}$: Number of the available spare parts at the time t_k, $N_{a\text{-spare}}^{t_k} = N_{\mathrm{stock}}^{t_k} - N_{\mathrm{app}}^{t_k}$
$N_{\mathrm{order}}^{t_k}$: Number of the ordered spare parts at time t_k, $N_{\mathrm{order}}^{t_k} = S - N_{a\text{-spare}}^{t_k}$.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

This study was cosupported by the Fundamental Research Funds for the Central Universities (no. NS2015072) and the National Natural Science Foundation of China (nos. 61079013 and U1233114).

References

[1] D. R. Godoy, R. Pascual, and P. Knights, "Critical spare parts ordering decisions using conditional reliability and stochastic lead time," *Reliability Engineering and System Safety*, vol. 119, pp. 199–206, 2013.

[2] Y. Jiang, M. Chen, and D. Zhou, "Joint optimization of preventive maintenance and inventory policies for multi-unit systems subject to deteriorating spare part inventory," *Journal of Manufacturing Systems*, vol. 35, pp. 191–205, 2015.

[3] A. Van Horenbeek, J. Buré, D. Cattrysse, L. Pintelon, and P. Vansteenwegen, "Joint maintenance and inventory optimization systems: a review," *International Journal of Production Economics*, vol. 143, no. 2, pp. 499–508, 2013.

[4] A. B. M. Z. Kabir and A. S. Al-Olayan, "A stocking policy for spare part provisioning under age based preventive replacement," *European Journal of Operational Research*, vol. 90, no. 1, pp. 171–181, 1996.

[5] A. B. M. Zohrul Kabir and S. H. A. Farrash, "Simulation of an integrated age replacement and spare provisioning policy using SLAM," *Reliability Engineering & System Safety*, vol. 52, no. 2, pp. 129–138, 1996.

[6] M.-C. Chen, C.-M. Hsu, and S.-W. Chen, "Optimizing joint maintenance and stock provisioning policy for a multi-echelon

spare part logistics network," *Journal of the Chinese Institute of Industrial Engineers*, vol. 23, no. 4, pp. 289–302, 2006.

[7] D. Q. Nguyen, C. Brammer, and M. Bagajewicz, "New tool for the evaluation of the scheduling of preventive maintenance for chemical process plants," *Industrial and Engineering Chemistry Research*, vol. 47, no. 6, pp. 1910–1924, 2008.

[8] N. Z. Kontrec, G. V. Milovanović, S. R. Panić, and H. Milošević, "A reliability-based approach to nonrepairable spare part forecasting in aircraft maintenance system," *Mathematical Problems in Engineering*, vol. 2015, Article ID 731437, 7 pages, 2015.

[9] Z. Yaobao, H. Ping, and Y. Shu, "An improved particle swarm optimization for the automobile spare part warehouse location problem," *Mathematical Problems in Engineering*, vol. 2013, Article ID 726194, 6 pages, 2013.

[10] D. Acharva, G. Nagabhushanam, and S. S. Alam, "Jointly optimal block-replacement and spare provisioning policy," *IEEE Transactions on Reliability*, vol. 35, no. 4, pp. 447–451, 1986.

[11] A. Brezavšček and A. Hudoklin, "Joint optimization of block-replacement and periodic-review spare-provisioning policy," *IEEE Transactions on Reliability*, vol. 52, no. 1, pp. 112–117, 2003.

[12] M. Ali Ilgin and S. Tunali, "Joint optimization of spare parts inventory and maintenance policies using genetic algorithms," *The International Journal of Advanced Manufacturing Technology*, vol. 34, no. 5-6, pp. 594–604, 2007.

[13] J. Xie and H. Wang, "Joint optimization of condition-based preventive maintenance and spare ordering policy," in *Proceedings of the International Conference on Wireless Communications, Networking and Mobile Computing (WiCOM '08)*, Dalian, China, October 2008.

[14] A. Regattieri, A. Giazzi, M. Gamberi, and R. Gamberini, "An innovative method to optimize the maintenance policies in an aircraft: general framework and case study," *Journal of Air Transport Management*, vol. 44-45, pp. 8–20, 2015.

[15] L. Wang, J. Chu, and W. Mao, "A condition-based order-replacement policy for a single-unit system," *Applied Mathematical Modelling. Simulation and Computation for Engineering and Environmental Systems*, vol. 32, no. 11, pp. 2274–2289, 2008.

[16] L. Wang, J. Chu, and W. Mao, "A condition-based replacement and spare provisioning policy for deteriorating systems with uncertain deterioration to failure," *European Journal of Operational Research*, vol. 194, no. 1, pp. 184–205, 2009.

[17] R. Li and J. K. Ryan, "A Bayesian inventory model using real-time condition monitoring information," *Production and Operations Management*, vol. 20, no. 5, pp. 754–771, 2011.

[18] W. Romeijnders, R. Teunter, and W. van Jaarsveld, "A two-step method for forecasting spare parts demand using information on component repairs," *European Journal of Operational Research*, vol. 220, no. 2, pp. 386–393, 2012.

[19] D. Louit, R. Pascual, D. Banjevic, and A. K. S. Jardine, "Condition-based spares ordering for critical components," *Mechanical Systems and Signal Processing*, vol. 25, no. 5, pp. 1837–1848, 2011.

[20] K. Tracht, G. Goch, P. Schuh, M. Sorg, and J. F. Westerkamp, "Failure probability prediction based on condition monitoring data of wind energy systems for spare parts supply," *CIRP Annals—Manufacturing Technology*, vol. 62, no. 1, pp. 127–130, 2013.

[21] Z. Wang, C. Hu, W. Wang, X. Kong, and W. Zhang, "A prognostics-based spare part ordering and system replacement policy for a deteriorating system subjected to a random lead time," *International Journal of Production Research*, vol. 53, no. 15, pp. 4511–4527, 2015.

[22] W. Wang, Z. Wang, C. Hu, and X. Liu, "An integrated decision model for critical component spare parts ordering and condition-based replacement with prognostic information," *Chemical Engineering Transactions*, vol. 33, pp. 1063–1068, 2013.

[23] D. R. Godoy, R. Pascual, and P. Knights, "A decision-making framework to integrate maintenance contract conditions with critical spares management," *Reliability Engineering and System Safety*, vol. 131, pp. 102–108, 2014.

[24] T. Jin, Z. Tian, and M. Xie, "A game-theoretical approach for optimizing maintenance, spares and service capacity in performance contracting," *International Journal of Production Economics*, vol. 161, pp. 31–43, 2015.

[25] W. Wang and S. Yue, "An inventory pooling model for spare units of critical systems that serve multi-companies," *Transportation Research Part E: Logistics and Transportation Review*, vol. 76, no. 4, pp. 34–44, 2015.

[26] X.-S. Si, W. Wang, C.-H. Hu, M.-Y. Chen, and D.-H. Zhou, "A Wiener-process-based degradation model with a recursive filter algorithm for remaining useful life estimation," *Mechanical Systems and Signal Processing*, vol. 35, no. 1-2, pp. 219–237, 2013.

[27] R. K. Jain and S. Jain, "Inverse Gaussian distribution and its application to reliability," *Microelectronics Reliability*, vol. 36, no. 10, pp. 1323–1335, 1996.