Extended VIKOR Method for Intuitionistic Fuzzy Multiattribute Decision-Making Based on a New Distance Measure

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1. Introduction

Multiattribute decision-making (MADM), which has been increasingly studied and is of concern to researchers and administrators, is one of the most important parts of decision theory. It aims to provide a comprehensive solution by evaluating and ranking alternatives based on conflicting attributes with respect to decision-makers' (DMs') preferences and has widely been used in engineering, economics, and management. Several classical MADM methods have been proposed by researchers in literature, such as the TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) method [1], the VIKOR (VIseKriterijumska Optimizacija I Kompromisno Resenje in Serbian, meaning multiattribute optimization and compromise solution) method [2], the PROMETHEE (Preference Ranking Organization Method for Enrichment Evaluations) method [3], and the ELECTRE (Elimination and Choice Translating reality) method [4]. Among these methods, VIKOR is shown to have some advantages over others by several researchers. In [5], Opricovic and Tzeng compared VIKOR method and TOPSIS method from the perspective of aggregation function and found that although both methods calculate the distance of an alternative to the ideal solution, the former method provides a compromise solution by mutual concessions, while the latter method obtains a solution with the farthest distance from the negative-ideal solution and the shortest distance from the positive ideal solution without considering the relative importance degrees of these distances. Furthermore, Opricovic and Tzeng compared an extension of the VIKOR method comprehensively with the TOPSIS, ELECTRE, and PROMETHEE method and revealed that the VIKOR method is superior in meeting conflicting and noncommensurable attributes [6]. At present, the VIKOR method has wide application in many areas, such as design, mechanical engineering, and manufacturing [7–9], logistics and supply chain management [10, 11], structural, construction, and transportation engineering [12, 13], and business management [14].

Since Zadah put forward the concept of fuzzy sets in 1965, fuzzy sets, especially fuzzy numbers (e.g., triangular fuzzy numbers and trapezoidal fuzzy numbers), have been widely researched and applied in MADM problems to deal
with uncertainty in actual decision-making process. Sayadi et al. extended the concept of VIKOR method to solve MADM problems with interval fuzzy numbers [15]. Kaya and Kahrarnan developed an integrated VIKOR-AHP method based on triangular fuzzy numbers in forestation district selection scenario [16]. Ashhtiani and Abdollahi Azgomi constructed a trust modeling based on a combination of fuzzy analytic hierarchy process and VIKOR method with triangular fuzzy numbers [17]. Liu et al. applied the VIKOR method to find a compromise priority ranking of failure modes according to the risk factors, in which the weights of attributes and ratings of alternatives are linguistic variables represented by trapezoidal or triangular fuzzy numbers [18]. Ju and Wang proposed a new method to solve multiattribute group decision-making (MAGDM) problems based on the traditional idea of VIKOR method with trapezoidal fuzzy numbers [19]. However, none of the above-mentioned studies considered hesitancy. Due to the increasing complexity and uncertainty of economic environment, decision-makers' perception of things involves not only affirmation and negation, but also hesitation. Therefore, hesitation, as one of the most important pieces of uncertain information, may have great impact on the final decision and should be considered in decision-making process. To deal with this, Atanassov extended traditional fuzzy sets to intuitionistic fuzzy sets (IFSs) which consider membership, nonmembership, and degree of hesitancy at the same time. In practical application, the use of IFSs can depict the fuzziness and nonspecificity of problems by employing both membership function and nonmembership function. Therefore, it is considered to be more effective than classical FSs with merely a membership function. Recently, intuitionistic fuzzy VIKOR (IF-VIKOR) method becomes a popular research topic because it aims to deal with the widespread uncertainty in decision-making process, and some researchers focus on the IF-VIKOR method to solve MADM problems. For instance, Devi [20] first extended VIKOR method in intuitionistic fuzzy environment to deal with robot selection problem, in which the weights of attributes and ratings of alternatives are represented by triangular intuitionistic fuzzy sets (TIFSS). Lu and Tang [21] employed the VIKOR method to evaluate auto parts supplier based on IFSs. Roostae et al. extended VIKOR method for group decision-making to solve the supplier selection problem under intuitionistic fuzzy environment [22]. Wan et al. put forward a new VIKOR method for MADM problem with TIFSS [23], in which the weights of attributes and DMs are completely unknown. Park et al. extended VIKOR method for dynamic intuitionistic fuzzy MADM concerning university faculty evaluation problem [24]. Based on similarity measure between IFSs, Peng et al. proposed a novel IF-VIKOR method to deal with multiresponse optimization problem, in which the importance of each response is given by an engineer as IFSs [25]. Hashemi et al. proposed a new compromise method based on classical VIKOR model with interval-valued intuitionistic fuzzy sets (IVIFSS), illustrated by an application in reservoir flood control operation [26]. Mousavi et al. presented a new group decision-making method for MADM problems based on IF-VIKOR, in which the ratings of alternatives concerning each attribute and the weights of criterions provided by DMs are represented as linguistic variables, characterized by IFSs with multijudge [27].

Distance is an important fundamental concept of IFSs and plays a significant role in VIKOR method. The classical compromise programing is based on the distance measure which is determined by the closeness of a specific solution to the ideal/infeasible solution, maximizing the group utility and minimizing the individual regret simultaneously [28, 29]. Although some existing methods employ intuitionistic fuzzy subtraction and division operator to calculate “group utility” and “individual regret,” the result is in IFSs form and needs to be further sorted according to IFSs’ ranking rule. To some extent, the amount of computation is increased, and the results are not concise as the crisp value obtained by distance measure. Also, some researchers [30–32] reviewed existing distance and similarity measures between IFSs and tested them with an artificial benchmark. The result showed that many existing distance measures generate counterintuitive cases and fail to capture waver, the lack of knowledge, which should have been an advantage of IFSs. On the one hand, existing distance measures are shown to have some limitations with hindered effectiveness, while, on the other hand, the measure choice is crucial to IF-VIKOR method due to its significant influence on the result. Therefore, this paper’s objective is to construct a new distance measure, which can capture IFS information effectively. Based on this, an IF-VIKOR method is further proposed to solve MADM problems with completely unknown weights of DMs and attributes. The main features and novelties of this paper are as follows:

1. Different from the VIKOR method based on classical fuzzy sets, such as interval fuzzy numbers [15], triangular fuzzy numbers [16–18], and trapezoidal fuzzy numbers [18, 19], the method proposed in this paper is based on IFSs, which adds a nonmembership on the basis of the traditional membership and is able to describe support, opposition, and neutrality in human cognition. Such an extended definition helps more adequately to represent situations when decision maker abstains or hesitates from expressing their assessments.

2. Compared with the method [20, 27] employing intuitionistic fuzzy operator to calculate “group utility” and “individual regret,” our method generates crisp value result in a more intuitive way with less computation. Although Roostae et al. [22] extended the VIKOR method to intuitionistic fuzzy environment based on hamming distance, the method sometimes generates counterintuitive cases and takes no consideration of waver in IFSs. Instead, our method based on a new distance measure could reflect intuitionistic fuzzy information effectively. Also, the new distance does not generate counterintuitive cases in the artificial benchmark test proposed by [30].

3. Compared with the VIKOR method [20, 23, 26] based on those extensions of IFSs, IFSs possess sound theoretical foundation such as basic definition and operations, comparison rules, information fusion method, and measures of IFS.
The proposed IF-VIKOR method can take advantage of these theories in construction and verification, which is favorable to practical application.

(4) Although IFs have the advantage of being able to consider waver, most of existing IF-VIKOR methods are applied to environment with less nonspecificity information [27, 33]. Through an example of ERP system selection, our paper illustrates the effectiveness of proposed method in such situation. Moreover, an example of material handling selection indicates that our method can obtain desired ranking result, superior in environment with low specificity.

(5) In the MADM problem with intuitionistic fuzzy numbers (IFNs), subjective randomness exists because the weights of DMs or attributes are usually given artificially [22, 27]. To avoid this issue, these weights are determined objectively using intuitionistic fuzzy entropy in the proposed method.

Compared with similar methods used for MADM [1, 3, 4], the proposed method based on the decision principle of the classical VIKOR method can maximize the group utility and minimize the individual regret simultaneously, making the decision result more reasonable. Also, the coefficient of decision mechanism can be changed according to actual requirements to balance group utility and individual regret, which can increase the flexibility of decision-making.

The remainder of this paper is organized as follows. Section 2 reviews some basic concepts of IFs. In Section 3, a new distance measure for IFs is proposed and a comparative analysis is conducted. In Section 4, a new distance based IF-VIKOR method is developed to deal with MADM problem. In Section 5, two application examples are demonstrated to highlight the applicability and superiority of proposed IF-VIKOR method. The final section summarizes the main work of this paper with a discussion of implications for future research.

2. Preliminaries

In this section, we briefly review some basic notions and theories related to IFs.

2.1. Intuitionistic Fuzzy Set

Definition 1. An intuitionistic fuzzy set A in the universe of discourse X is defined as follows [34]:

\[ A = \{ (x, u_A(x), v_A(x)) \mid x \in X \} \]  

where \( u_A : X \to [0, 1] \) and \( v_A : X \to [0, 1] \) are called membership function and nonmembership function of \( x \) to \( A \), respectively. \( u_A(x) \) represents the lowest bound of degree of membership derived from proofs of supporting \( x \) while \( v_A(x) \) represents the lowest bound of degree of nonmembership derived from proofs of opposing \( x \). For any \( x \in X \), \( 0 \leq u_A + v_A \leq 1 \). The function \( \pi_A(x) = 1 - u_A(x) - v_A(x) \) is called hesitance index, and \( \pi_A(x) \in [0, 1] \) represents the degree of hesitancy of \( x \) to \( A \). Specially, if \( \pi_A(x) = 0 \), where \( x \in X \) is known absolutely, the intuitionistic fuzzy set \( A \) degenerates into a fuzzy set.

Definition 2. For convenience of computation, let \( \bar{a} = (u_a, v_a) \) be an intuitionistic fuzzy number (IFN). Then the score function of \( \bar{a} \) is defined as follows [35]:

\[ s (\bar{a}) = (u_a - v_a) \]  

And the accuracy function of \( \bar{a} \) is defined as follows [36]:

\[ h (\bar{a}) = (u_a + v_a) \]  

Let \( \bar{a}_1 \) and \( \bar{a}_2 \) be two IFNs; then Xu and Yager [37] proposed the following rules for ranking of IFNs:

(1) \( s(\bar{a}_1) < s(\bar{a}_2) \), then \( \bar{a}_1 \) is smaller than \( \bar{a}_2 \), denoted by \( \bar{a}_1 < \bar{a}_2 \);

(2) \( s(\bar{a}_1) > s(\bar{a}_2) \), then \( \bar{a}_1 \) is larger than \( \bar{a}_2 \), denoted by \( \bar{a}_1 > \bar{a}_2 \);

(3) \( s(\bar{a}_1) = s(\bar{a}_2) \), then

(i) \( h(\bar{a}_1) = h(\bar{a}_2) \), then \( \bar{a}_1 \) is equal to \( \bar{a}_2 \), denoted by \( \bar{a}_1 = \bar{a}_2 \);

(ii) \( h(\bar{a}_1) < h(\bar{a}_2) \), then \( \bar{a}_1 \) is smaller than \( \bar{a}_2 \), denoted by \( \bar{a}_1 < \bar{a}_2 \);

(iii) \( h(\bar{a}_1) > h(\bar{a}_2) \), then \( \bar{a}_1 \) is larger than \( \bar{a}_2 \), denoted by \( \bar{a}_1 > \bar{a}_2 \).

Definition 3. Let \( \bar{a}_i \) (\( i = 1, 2, \ldots, n \)) be the set of IFNs, where \( \bar{a}_i = (u_{a_i}, v_{a_i}) \). The intuitionistic fuzzy weighted averaging (IFWA) operator can be defined as follows [33]:

\[ \text{IFWA}_w (\bar{a}_1, \bar{a}_2, \ldots, \bar{a}_n) = w_1 \bar{a}_1 \oplus w_2 \bar{a}_2 \oplus \cdots \oplus w_n \bar{a}_n \]  

\[ = \left( 1 - \prod_{i=1}^{n} (1 - u_{\bar{a}_i}) \right) \frac{w_1}{\sum_{i=1}^{n} w_i} u_{\bar{a}_1} v_i + \frac{w_1}{\sum_{i=1}^{n} w_i} u_{\bar{a}_1} + \pi_{\bar{a}_1} \]  

where \( w_i \) is the weight of \( \bar{a}_i \) (\( i = 1, 2, \ldots, n \)), \( w_i \in [0, 1] \), and \( \sum_{i=1}^{n} w_i = 1 \).

Definition 4. Let \( A \) be an intuitionistic fuzzy set in the universe of discourse \( X \). Intuitionistic fuzzy entropy is given as follows [38]:

\[ E (A) = \frac{1}{n} \sum_{i=1}^{n} \min (u_A(x_i), v_A(x_i)) + \pi_A (x_i) \]  

2.2. Distance Measure between IFs

Definition 5. For any \( A, B, C \in \text{IFS}(X) \), let \( d \) be a mapping \( d : \text{IFS}(X) \times \text{IFS}(X) \to [0, 1] \). If \( d(A, B) \) satisfies the following properties [39]:

(1) \( 0 \leq d(A, B) \leq 1 \);

(2) \( d(A, B) = 0 \) if and only if \( A = B \);

(3) \( d(A, B) = d(B, A) \);
Therefore, they proposed several new distances as follows.

$$d_{H}(A, B) = \frac{1}{2} \sum_{i=1}^{n} \left[ |u_{A}(x_{i}) - u_{B}(x_{i})| + |v_{A}(x_{i}) - v_{B}(x_{i})| \right]$$

$$d_{nH}(A, B) = \frac{1}{2n} \sum_{i=1}^{n} \left[ |u_{A}(x_{i}) - u_{B}(x_{i})| + |v_{A}(x_{i}) - v_{B}(x_{i})| \right]$$

$$d_{E}(A, B) = \sqrt{\sum_{i=1}^{n} \left[ (u_{A}(x_{i}) - u_{B}(x_{i}))^2 + (v_{A}(x_{i}) - v_{B}(x_{i}))^2 \right]}$$

$$d_{nE}(A, B) = \frac{1}{2n} \sum_{i=1}^{n} \left[ (u_{A}(x_{i}) - u_{B}(x_{i}))^2 + (v_{A}(x_{i}) - v_{B}(x_{i}))^2 \right]$$

Wang and Xin [39] pointed out that Szmidt and Kacprzyk’s distance measures [40] have some good geometric properties, but there are some limitations in the application. Therefore, they proposed several new distances as follows.

$$d_{1}(A, B) = \frac{1}{n} \sum_{i=1}^{n} \left[ \frac{|u_{A}(x_{i}) - u_{B}(x_{i})| + |v_{A}(x_{i}) - v_{B}(x_{i})|}{2} \right]$$

$$d_{2}(A, B) = \frac{1}{\sqrt{n}} \left( \sum_{i=1}^{n} \left[ \frac{|u_{A}(x_{i}) - u_{B}(x_{i})| + |v_{A}(x_{i}) - v_{B}(x_{i})|}{2} \right] \right)^p$$

On the basis of the Hausdorff metric, Grzegorzewski [41] put forward some approaches for computing distances between IFSs, and these suggested distances are also generalizations of the well-known hamming distance, Euclidean distance, and their normalized counterparts.

$$d_{h}(A, B) = \sum_{i=1}^{n} \max \left[ |u_{A}(x_{i}) - u_{B}(x_{i})|, |v_{A}(x_{i}) - v_{B}(x_{i})| \right]$$

$$l_{h}(A, B) = \frac{1}{n} \sum_{i=1}^{n} \max \left[ |u_{A}(x_{i}) - u_{B}(x_{i})|, |v_{A}(x_{i}) - v_{B}(x_{i})| \right]$$

$$e_{h}(A, B) = \sqrt{\sum_{i=1}^{n} \max \left[ (u_{A}(x_{i}) - u_{B}(x_{i}))^2, (v_{A}(x_{i}) - v_{B}(x_{i}))^2 \right]}$$

$$q_{h}(A, B) = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \max \left[ (u_{A}(x_{i}) - u_{B}(x_{i}))^2, (v_{A}(x_{i}) - v_{B}(x_{i}))^2 \right]}$$

Yang and Chiclana [42] suggested that the 3D interpretation of IFSs could provide different contradistinction results to the ones obtained with their 2D counterparts [41] and introduced several extended 3D Hausdorff based distances.

$$d_{eh}(A, B) = \sum_{i=1}^{n} \max \left[ |u_{A}(x_{i}) - u_{B}(x_{i})|, |v_{A}(x_{i}) - v_{B}(x_{i})|, |\pi_{A}(x_{i}) - \pi_{B}(x_{i})| \right]$$

$$l_{eh}(A, B) = \frac{1}{n} \sum_{i=1}^{n} \max \left[ |u_{A}(x_{i}) - u_{B}(x_{i})|, |v_{A}(x_{i}) - v_{B}(x_{i})|, |\pi_{A}(x_{i}) - \pi_{B}(x_{i})| \right]$$


\[ e_{ch}(A, B) = \sqrt{\sum_{i=1}^{n} \max \{ (u_A(x_i) - u_B(x_i))^2, (v_A(x_i) - v_B(x_i))^2, (\pi_A(x_i) - \pi_B(x_i))^2 \} } \]

\[ q_{ch}(A, B) = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} \max \{ (u_A(x_i) - u_B(x_i))^2, (v_A(x_i) - v_B(x_i))^2, (\pi_A(x_i) - \pi_B(x_i))^2 \} \].

(9)

3. New Distance Measure between IFSs

3.1. Analysis on Existing Distance Measures between IFSs.

Although various types of distance measures between IFSs were proposed over the past several years and most of them satisfy the distance axioms (Definition 5), some unreasonable cases can still be found [31, 32]. For example, consider three single-element IFSs that satisfy the distance axioms (Definition 5), some unreasonable cases can still be found [31, 32]. For example, consider three single-element IFSs that satisfy the distance axioms (Definition 5), some unreasonable cases can still be found [31, 32]. For example, consider three single-element IFSs that satisfy the distance axioms (Definition 5), some unreasonable cases can still be found [31, 32]. For example, consider three single-element IFSs that satisfy the distance axioms (Definition 5), some unreasonable cases can still be found [31, 32]. For example, consider three single-element IFSs that satisfy the distance axioms (Definition 5), some unreasonable cases can still be found [31, 32]. For example, consider three single-element IFSs that satisfy the distance axioms (Definition 5), some unreasonable cases can still be found [31, 32]. For example, consider three single-element IFSs that satisfy the distance axioms (Definition 5), some unreasonable cases can still be found [31, 32]. For example, consider three single-element IFSs that satisfy the distance axioms (Definition 5), some unreasonable cases can still be found [31, 32]. For example, consider three single-element IFSs that satisfy the distance axioms (Definition 5), some unreasonable cases can still be found [31, 32]. For example, consider three single-element IFSs that satisfy the distance axioms (Definition 5), some unreasonable cases can still be found [31, 32]. For example, consider three single-element IFSs that satisfy the distance axioms (Definition 5), some unreasonable cases can still be found [31, 32]. For example, consider three single-element IFSs that satisfy the distance axioms (Definition 5), some unreasonable cases can still be found [31, 32]. For example, consider three single-element IFSs that satisfy the distance axioms (Definition 5), some unreasonable cases can still be found [31, 32]. For example, consider three single-element IFSs that satisfy the distance axioms (Definition 5), some unreasonable cases can still be found [31, 32]. For example, consider three single-element IFSs that satisfy the distance axioms (Definition 5), some unreasonable cases can still be found [31, 32]. For example, consider three single-element IFSs that satisfy the distance axioms (Definition 5), some unreasonable cases can still be found [31, 32]. For example, consider three single-element IFSs that satisfy the distance axioms (Definition 5), some unreasonable cases can still be found [31, 32].

As a powerful tool in modeling uncertain information, IFSs have the advantage of being able to consider waver mainly because the hesitant index can be used to describe a neutrality state of “both this and that.” Moreover, it should be noted that the ultimate goal of distance measure is to measure the difference of information carried by IFSs, rather than the numerical value of IFSs themselves. Therefore, in our view the distance measure should take into account the characteristics of intuitionistic fuzzy information, both fuzziness and nonspecificity, and have its own target.

3.2. Intuitive Distance for IFSs.

Inspired by the characteristics of intuitionistic fuzzy information, we introduce a notion called intuitive distance to compare the information between two IFSs and compute the degree of difference, and the axiomatic definition of intuitive distance is as follows.

Definition 6. For any \( A, B, C, D \in \text{IFSs}(X) \), let \( d \) be a mapping \( d : \text{IFSs}(X) \times \text{IFSs}(X) \to [0, 1] \). \( d(A, B) \) is said to be an intuitive distance between \( A \) and \( B \) if \( d(A, B) \) satisfies the following properties:

(P1) \( 0 \leq d(A, B) \leq 1 \);

(P2) \( d(A, B) = 0 \) if and only if \( A = B \) and \( \pi_A(x) = \pi_B(x) = 0 \);

(P3) \( d(A, B) = 1 \) if and only if both \( A \) and \( B \) are crisp sets and \( A = B^C \);

(P4) \( d(A, B) = d(B, A) \);

(P5) \( d(A, C) \leq d(A, B) + d(B, C) \) for any \( A, B, C \in \text{IFSs}(X) \);

(P6) if \( A \subseteq B \subseteq C \), then \( d(B, C) \leq d(A, C) \) and \( d(A, B) \leq d(A, C) \);

(P7) if \( A = B \) and \( C = D \), then

(i) \( d(A, B) > d(C, D) \) when \( \pi_A(x) + \pi_B(x) > \pi_C(x) + \pi_D(x) \);

(ii) \( d(A, B) < d(C, D) \) when \( \pi_A(x) + \pi_B(x) < \pi_C(x) + \pi_D(x) \).


(iii) \( d(A, B) = d(C, D) \) when \( \pi_A(x) + \pi_B(x) = \pi_C(x) + \pi_D(x) \).

The above definition is a development for Definition 5. On the one hand, some properties are introduced to make the definition stronger: (P3) is a more precise condition for the endpoint of distance \( d(A, B) = 1 \); (P5) is a new strong property condition, which requires distance to meet triangle inequality. On the other hand, the definition highlights the characteristics of intuitionistic fuzzy information by reinforcing the existing property condition (P2), as well as adding a new property condition (P7). The intuitive distance not only meets the most general properties of traditional distance, but also satisfies a special property: as long as the hesitant index is not zero, even if two IFSs are equal in value, the distance between them is not zero. And the higher the hesitant index, the greater the distance.

Then, we propose a new distance measure. Let \( A = \langle x, u_A(x), v_A(x) \rangle, B = \langle x, u_B(x), v_B(x) \rangle \) be two IFSs in the universe of discourse \( X \), and denote \( X = \{x_1, x_2, \ldots, x_n\} \).

\[
d_L(A, B) = \frac{1}{3n} \sum_{i=1}^{n} \left[ \theta_1(x_i) + \theta_2(x_i) + \theta_3(x_i) \right],
\]

\[
\theta_1(x_i) = \left| u_A(x_i) - u_B(x_i) \right| + \frac{1}{2} \left| v_A(x_i) - v_B(x_i) \right| + \frac{1}{2} \left| \pi_A(x_i) + \pi_B(x_i) \right|,
\]

\[
\theta_2(x_i) = \frac{\pi_A(x_i) + \pi_B(x_i)}{2},
\]

\[
\theta_3(x_i) = \max \left( \left| u_A(x_i) - u_B(x_i) \right|, \left| v_A(x_i) - v_B(x_i) \right|, \frac{\pi_A(x_i) - \pi_B(x_i)}{2} \right).
\]

The structure of \( d_L(A, B) \) is mainly related to two aspects. One is to take the differences of IFS parameters into account, which is similar to the traditional distance. It includes differences between memberships \( u_A(x_i) \) and \( u_B(x_i) \), nonmemberships \( v_A(x_i) \) and \( v_B(x_i) \), hesitance indexes \( \pi_A(x_i) \) and \( \pi_B(x_i) \), and the differences between median values of intervals \( (u_A(x_i) + v_A(x_i))/2 \) and \( (u_B(x_i) + v_B(x_i))/2 \). The second is to satisfy the new requirements of intuitive distance, taking the degree of hesitancy into account. \( \theta_3(x_i) \) aims to reflect the wavering of uncertain information. The higher the degree of nonspecificity of intuitionistic fuzzy information, the greater the possibility of existing differences between them. Then, we have the following theorem.

**Theorem 7.** \( d_L(A, B) \) is an intuitive distance between IFSs \( A \) and \( B \) in the universe of discourse \( X = \{x_1, x_2, \ldots, x_n\} \).

**Proof.** It is easy to see that \( d_L(A, B) \) satisfies (P4) and (P7) of Definition 6. Therefore, we shall prove that \( d_L(A, B) \) satisfies (P1), (P2), (P3), (P5), and (P6).

(P1) Let \( A \) and \( B \) be two IFSs; we can write the relational expression as follows:

\[
0 \leq \theta_1(x_i) + \theta_2(x_i) = \left| u_A(x_i) - u_B(x_i) \right| + \left| v_A(x_i) - v_B(x_i) \right| + \left| u_A(x_i) + 1 - v_A(x_i) \right| - \left| u_B(x_i) + 1 - v_B(x_i) \right| + \frac{1}{2} \left| \pi_A(x_i) + \pi_B(x_i) \right| + \left| v_A(x_i) - v_B(x_i) \right| + \left| u_A(x_i) - u_B(x_i) \right| - \left| v_A(x_i) - v_B(x_i) \right| + 2
\]

\[
\leq \left( \left| u_A(x_i) - u_B(x_i) \right| + \left| v_A(x_i) - v_B(x_i) \right| \right) \times (2)^{-1}
\]

\[
+ \left( \left| u_A(x_i) - u_B(x_i) \right| + \left| v_A(x_i) - v_B(x_i) \right| + 2 \right)
\]

\[
- \left( \left| u_A(x_i) - u_B(x_i) \right| - \left| u_B(x_i) - u_A(x_i) \right| \right) \times (2)^{-1}
\]

\[
= 2 \left( \left| u_A(x_i) - u_B(x_i) \right| + \left| v_A(x_i) - v_B(x_i) \right| + 2 \right)
\]

\[
\times (2)^{-1} \leq 2 \left( \left| u_A(x_i) + u_B(x_i) \right| - \left( v_A(x_i) + v_B(x_i) \right) + 2 \right)
\]

\[
- \left( v_A(x_i) + v_B(x_i) \right) \times (2)^{-1}
\]

\[
\leq \left( \left| u_A(x_i) + u_B(x_i) \right| + \left| v_A(x_i) + v_B(x_i) \right| + 2 \right)
\]

\[
= \frac{1 + 1 + 2}{2} = 2.
\]

(P2) It is known that

\[
0 \leq \theta_3(x_i) = \max \left( \left| u_A(x_i) - u_B(x_i) \right|, \left| v_A(x_i) - v_B(x_i) \right|, \frac{\pi_A(x_i) - \pi_B(x_i)}{2} \right) \leq 1.
\]
Taking (11) and (12) into account, it is not difficult to find that
\[
0 \leq \theta_1(x_i) + \theta_2(x_i) + \theta_3(x_i) \leq 3.
\] (13)
And therefore we have
\[
0 \leq d_L(A, B) = \frac{1}{3n} \sum_{i=1}^{n} [\theta_1(x_i) + \theta_2(x_i) + \theta_3(x_i)] \leq \frac{3}{3} = 1.
\] (14)

(P2) Let \( A \) and \( B \) be two IFSs; the following relational expression can be written:
\[
d_L(A, B) = 0
\]
\[
\theta_1(x_i) + \theta_2(x_i) + \theta_3(x_i) = 2,
\]
\[
\pi_A(x_i) + \pi_B(x_i) = 0
\]
\[
A(x) = B(x)
\]
\[
\pi_A(x) = \pi_B(x) = 0.
\]
Thus, \( d_L(A, B) \) satisfies (P2) of Definition 6.

(P3) Let \( A \) and \( B \) be two IFSs. Taking (11) and (12) into account, the following relational expressions can be written:
\[
d_L(A, B) = 1
\]
\[
\theta_1(x_i) + \theta_2(x_i) = 2,
\]
\[
\theta_3(x_i) = 1
\]
\[
\frac{u_A(x_i) + u_B(x_i) + v_A(x_i) + v_B(x_i) + 2}{2} = 2,
\]
\[
\max\left(\frac{|u_A(x_i) - u_B(x_i)|}{2}, \frac{|v_A(x_i) - v_B(x_i)|}{2}\right) = 1
\]
\[
\frac{\pi_A(x_i) - \pi_B(x_i)}{2} = 0
\]
\[
A(x) = (1, 0, 0),
\]
\[
B(x) = (0, 1, 0)
\]
or \( A(x) = (0, 1, 0), \)
\[
B(x) = (1, 0, 0).
\]
Thus, \( d_L(A, B) \) satisfies (P3) of Definition 6.

(P5) Let \( A, B, \) and \( C \) be three IFSs; the distances between \( A \) and \( B, B \) and \( C, \) and \( A \) and \( C \) are the following:
\[
d_L(A, B) = \frac{1}{3n} \sum_{i=1}^{n} \left[ |u_A(x_i) - u_B(x_i)| + |v_A(x_i) - v_B(x_i)| + \frac{|u_A(x_i) + 1 - v_A(x_i) - (u_B(x_i) + 1 - v_B(x_i))|}{2} \right]
\]
\[
+ \frac{\pi_A(x_i) + \pi_B(x_i)}{2} + \max\left(\frac{|u_A(x_i) - u_C(x_i)|}{2}, \frac{|v_A(x_i) - v_C(x_i)|}{2}, \frac{|\pi_A(x_i) - \pi_C(x_i)|}{2}\right)
\]
\[
d_L(B, C) = \frac{1}{3n} \sum_{i=1}^{n} \left[ |u_B(x_i) - u_C(x_i)| + |v_B(x_i) - v_C(x_i)| + \frac{|u_B(x_i) + 1 - v_B(x_i) - (u_C(x_i) + 1 - v_C(x_i))|}{2} \right]
\]
\[
+ \frac{\pi_B(x_i) + \pi_C(x_i)}{2} + \max\left(\frac{|u_B(x_i) - u_C(x_i)|}{2}, \frac{|v_B(x_i) - v_C(x_i)|}{2}, \frac{|\pi_B(x_i) - \pi_C(x_i)|}{2}\right)
\]
\[
d_L(A, C) = \frac{1}{3n} \sum_{i=1}^{n} \left[ |u_A(x_i) - u_C(x_i)| + |v_A(x_i) - v_C(x_i)| + \frac{|u_A(x_i) + 1 - v_A(x_i) - (u_C(x_i) + 1 - v_C(x_i))|}{2} \right]
\]
\[
+ \max\left(\frac{|u_A(x_i) - u_C(x_i)|}{2}, \frac{|v_A(x_i) - v_C(x_i)|}{2}, \frac{|\pi_A(x_i) - \pi_C(x_i)|}{2}\right)
\]
It is obvious that

\[ |u_A(x_i) - u_B(x_i)| + |u_B(x_i) - u_C(x_i)| \geq |u_A(x_i) - u_C(x_i)| \]

\[ - u_B(x_i) + u_B(x_i) - u_C(x_i) = |u_A(x_i) - u_C(x_i)| , \]

\[ |v_A(x_i) - v_B(x_i)| + |v_B(x_i) - v_C(x_i)| \geq |v_A(x_i) - v_C(x_i)| , \]

\[ - v_B(x_i) + v_B(x_i) - v_C(x_i) = |v_A(x_i) - v_C(x_i)| , \]

\[ |u_A(x_i) + 1 - v_A(x_i) - (u_B(x_i) + 1 - v_B(x_i))| \]

\[ + |(u_B(x_i) + 1 - v_B(x_i)) - (u_C(x_i) + 1 - v_C(x_i))| \]

\[ - (u_C(x_i) + 1 - v_C(x_i)) = |u_A(x_i) + v_B(x_i) - u_B(x_i) - v_B(x_i) - u_C(x_i) - v_C(x_i)| \]

\[ - v_A(x_i) + v_C(x_i) - u_C(x_i)| \]

\[ = |(u_A(x_i) + 1 - v_A(x_i))| \]

\[ - (u_C(x_i) + 1 - v_C(x_i))| \]

\[ |\pi_A(x_i) - \pi_B(x_i)| + |\pi_B(x_i) - \pi_C(x_i)| \geq |\pi_A(x_i) - \pi_B(x_i)| \]

\[ - \pi_B(x_i) + \pi_B(x_i) - \pi_C(x_i)| = |\pi_A(x_i) - \pi_B(x_i)| \]

\[ - \pi_C(x_i)| , \]

\[ \frac{\pi_A(x_i) + \pi_B(x_i) + \pi_B(x_i) + \pi_C(x_i)}{2} \geq \frac{\pi_A + \pi_C}{2} . \]

(18)

Therefore, we have \( d_L(A, B) + d_L(B, C) \geq d_L(A, C) \).

(P6) Let \( A, B, \) and \( C \) be three IFSs; if \( A \subseteq B \subseteq C \), then we have \( u_A(x_i) \leq u_B(x_i) \leq u_C(x_i) \), \( v_A(x_i) \geq v_B(x_i) \geq v_C(x_i) \). The following equations are given out:

\[ d_L(A, B) = \sum_{i=1}^{n} \frac{\sum_{i=1}^{n} \left[ |u_B(x_i) - u_A(x_i) + v_A(x_i) - v_B(x_i) + u_B(x_i) - u_A(x_i) + v_A(x_i) - v_B(x_i) + \pi_A(x_i) + \pi_B(x_i)| \right]}{2} \]

\[ + \max \left( u_B(x_i) - u_A(x_i) , v_A(x_i) - v_B(x_i) , \frac{|\pi_A(x_i) - \pi_B(x_i)|}{2} \right) = \frac{1}{3n} \]

\[ \cdot \sum_{i=1}^{n} \left[ |u_B(x_i) - u_A(x_i) + v_A(x_i) - v_B(x_i) + u_B(x_i) - u_A(x_i) + v_A(x_i) - v_B(x_i) + 2 - u_A(x_i) - u_B(x_i) - v_A(x_i) - v_B(x_i)| \right] \]

\[ + \max \left( u_B(x_i) - u_A(x_i) , v_A(x_i) - v_B(x_i) , \frac{|\pi_A(x_i) - \pi_B(x_i)|}{2} \right) = \frac{1}{3n} \]

\[ \cdot \sum_{i=1}^{n} \left[ |u_B(x_i) - 3u_A(x_i) + 3v_A(x_i) - 3v_B(x_i) + \max (u_B(x_i) - u_A(x_i) , v_A(x_i) - v_B(x_i)) \right] \]

\[ d_L(A, C) = \sum_{i=1}^{n} \frac{\sum_{i=1}^{n} \left[ |u_C(x_i) - u_A(x_i) + v_A(x_i) - v_C(x_i) + u_A(x_i) - u_C(x_i) + v_A(x_i) - v_C(x_i) + \pi_A(x_i) + \pi_C(x_i)| \right]}{2} \]

\[ + \max \left( u_C(x_i) - u_A(x_i) , v_A(x_i) - v_C(x_i) , \frac{|\pi_A(x_i) - \pi_C(x_i)|}{2} \right) = \frac{1}{3n} \]

\[ \cdot \sum_{i=1}^{n} \left[ |u_C(x_i) - u_A(x_i) + v_A(x_i) - v_C(x_i) + u_A(x_i) - u_C(x_i) + v_A(x_i) - v_C(x_i) + 2 - u_A(x_i) - u_C(x_i) - v_A(x_i) - v_C(x_i)| \right] \]

\[ + \max \left( u_C(x_i) - u_A(x_i) , v_A(x_i) - v_C(x_i) , \frac{|\pi_A(x_i) - \pi_C(x_i)|}{2} \right) = \frac{1}{3n} \]

\[ \cdot \sum_{i=1}^{n} \left[ |u_C(x_i) - 3u_A(x_i) + 3v_A(x_i) - 3v_C(x_i) + \max (u_C(x_i) - u_A(x_i) , v_A(x_i) - v_C(x_i)) \right] \].

We know that

\[ u_C(x_i) - 3v_C(x_i) \geq u_B(x_i) - 3v_B(x_i) , \]

\[ u_C(x_i) - u_A(x_i) \geq u_B(x_i) - u_A(x_i) , \]

\[ v_A(x_i) - v_C(x_i) \geq v_A(x_i) - v_B(x_i) \]

It means that

\[ d_L(A, B) \leq d_L(A, C) . \]

Similarly, it is easy to prove that

\[ d_L(B, C) \leq d_L(A, C) . \]

Thus, \( d_L(A, B) \) satisfies (P6) of Definition 6.
IFSs is equal to or greater than the distance of the 7th test IFSs when $d_{ah}, d_{ae}, l_{h}, l_{eh}$, and $d_{e}^{p}$, $p = 1$, are used, which does not seem to be reasonable since IFSs $A$, $B$, and $C$ are ordered as $C > B > A$ according to the score function and accuracy function given in Definition 2, indicating that the distance between $A$ and $B$ is smaller than that between $A$ and $C$. Furthermore, all of these existing distances claim that the distance of the 6th test IFSs is equal to zero, which does not seem to be reasonable. As a mathematical tool, IFSs can describe the uncertain information greatly, because it adds a hesitant index to describe the state of “both this and that.” In this case, we indeed cannot confirm that there is no difference between the information carried by IFSs $A$ and $B$, because the hesitant index includes some nonspecificity information and the proportion of support and opposition is not sure.

Based on analysis in Table 2, it is deduced that the existing distance measures with their own measuring focus can meet all or most of properties condition of distance measure between IFSs; however, most distance measures show counterintuitive cases and may fail to distinguish IFSs accurately in some practical applications. Besides, the proposed distance measure is the only one that has no aforementioned counterintuitive cases as illustrated in Table 2. Furthermore, the proposed distance conforms to all the property requirements of the intuitive distance and the potential difference brought by hesitant index is considered.

### 4. IF-VIKOR Method for MADM

On the basis of new distance measure, this section present stepwise algorithm for proposed IF-VIKOR method.

For a MADM problem with $n$ alternatives $A_i$ ($i = 1, 2, \ldots, n$), the performance of the alternative $A_i$ concerning the attribute $C_j$ ($j = 1, 2, \ldots, l$) is assessed by a decision organization with several decision-makers $D_q$ ($q = 1, 2, \ldots, l$). The corresponding weights of attributes are denoted by $w_j$ ($j = 1, 2, \ldots, n$), $0 \leq w_j \leq 1$, $\sum_{j=1}^{n} w_j = 1$, and the weights of DMs are denoted by $\lambda_q$ ($q = 1, 2, \ldots, l$), $0 \leq \lambda_q \leq 1$, respectively.
\[ \sum_{q=1}^{l} \lambda_q = 1. \] Inspired by the classical VIKOR method and its extensions, the intuitive distance based VIKOR method can be given for MADM problem with intuitionistic fuzzy information; it includes seven steps.

**Step 1.** Generate assessment information. Assume that DMs \( D_q (q = 1, 2, \ldots, l) \) provide their opinion of the alternatives \( A_j \) concerning each attribute \( C_j \) by using IFNs \( x_{ij}^q = (u_{ij}^q, v_{ij}^q, \pi_{ij}^q) \) or linguistic values represented by IFNs. Then, the assessments given by \( D_q \) can be expressed as

\[
D^{(q)} = \left[ \begin{array}{cccc}
 x_{11}^{(q)} & x_{12}^{(q)} & \cdots & x_{1n}^{(q)} \\
 x_{21}^{(q)} & x_{22}^{(q)} & \cdots & x_{2n}^{(q)} \\
 & \vdots & \ddots & \vdots \\
 x_{m1}^{(q)} & x_{m2}^{(q)} & \cdots & x_{mn}^{(q)} 
\end{array} \right]. \tag{23}
\]

**Step 2.** Acquire the weights of DMs. According to the degree of fuzziness and nonspecificity of assessments provided by DMs, in this step, DM weight \( \lambda_q \) \( (q = 1, 2, \ldots, l) \) can be acquired by intuitionistic fuzzy entropy measure objectively. The lower the degree of fuzziness and nonspecificity is, the smaller the entropy is and the bigger the weight of DM is, and vice versa. By using (5), the intuitionistic fuzzy entropy of assessments provided by \( D_q \) can be obtained as follows:

\[
E_q = \sum_{i=1}^{m} \min_{j=1}^{n} \left( \frac{u_{ij}^q}{u_{ij}^q + v_{ij}^q} \right) + \pi_{ij}^q. \tag{24}\]

Then, the weight of DM \( D_q \) can be defined as follows:

\[
\lambda_q = \frac{1 - E_q}{l - \sum_{q=1}^{l} E_q}, \tag{25}\]

where \( l \) is the number of DMs.

**Step 3.** Establish the aggregated intuitionistic fuzzy decision matrix. By using (4), all individual decision matrices \( D^{(q)} \) can be converted into an aggregated decision matrix as follows:

\[
D = \left[ \begin{array}{cccc}
 x_{11} & x_{12} & \cdots & x_{1n} \\
 x_{21} & x_{22} & \cdots & x_{2n} \\
 & \vdots & \ddots & \vdots \\
 x_{m1} & x_{m2} & \cdots & x_{mn} 
\end{array} \right], \tag{26}\]

where \( x_{ij} = (u_{ij}, v_{ij}, \pi_{ij}) \), \( u_{ij} = 1 - \prod_{q=1}^{l} (1 - u_{ij}^q)^{\lambda_q}, v_{ij} = \prod_{q=1}^{l} (v_{ij}^q)^{\lambda_q}, \pi_{ij} = 1 - u_{ij} - v_{ij}. \)

**Step 4.** Acquire the weights of attributes. Similar to Step 2, the unknown weight of attribute \( u_j \) \( (j = 1, 2, \ldots, n) \) can be determined by entropy measure to effectively reduce the subjective randomness. By using (5), we can obtain the entropy with respect to \( C_j \);

\[
E_j = \frac{1}{m} \sum_{i=1}^{m} \min_{j=1}^{n} \left( \frac{u_{ij} + v_{ij}}{u_{ij} + v_{ij} + \pi_{ij}} \right). \tag{27}\]

Then, the weight of attribute \( C_j \) can be defined as follows:

\[
w_j = \frac{1 - E_j}{n - \sum_{j=1}^{n} E_j}, \tag{28}\]

where \( n \) is the number of attributes.

**Step 5.** Find the best and worst value. The best value \( x_j^+ \) and the worst value \( x_j^- \) for each attribute \( C_j \) can be defined as follows:

\[
\begin{align*}
x_j^+ &= \begin{cases} 
\max_{i=1,2,\ldots,m} x_{ij}, & \text{for benefit attribute } C_j, \\
\min_{i=1,2,\ldots,m} x_{ij}, & \text{for cost attribute } C_j, 
\end{cases} \\
& \quad (j = 1, 2, \ldots, n). \tag{29}
\end{align*}
\]

**Step 6.** Compute the values \( S_j, R_j, \) and \( Q_j \). Three key values of IF-VIKOR method, the group utility value \( S_j \), the individual regret value \( R_j \), and the compromise value \( Q_j \), are computed in light of the intuitive distance measure for each alternative:

\[
\begin{align*}
S_j &= \frac{1}{n} \sum_{j=1}^{n} \left( \frac{d(x_j^+, x_{ij})}{d(x_j^+, x_j^-)} \right), \\
R_j &= \max_j \left( \frac{d(x_j^+, x_{ij})}{d(x_j^+, x_j^-)} \right), \tag{30}
\end{align*}
\]

\[
Q_j = \gamma \left( \frac{S_j - S^*}{S_j - S^*} + (1 - \gamma) \left( \frac{R_j - R^*}{R_j - R^*} \right) \right),
\]

where \( S^* = \max \{ S_j \}, S_j^* = \min \{ S_j \}, R^* = \max \{ R_j \}, R_j^* = \min \{ R_j \} \), \( \gamma \) is the coefficient of decision mechanism. The compromise solution can be elected by majority \((\gamma > 0.5)\), consensus \((\gamma = 0.5)\), or veto \((\gamma < 0.5)\).

**Step 7.** Rank the alternatives and derive the compromise solution. Sort \( S_j, R_j, \) and \( Q_j \) in ascending order and generate three ranking lists \( S_{[1]}, R_{[1]}, \) and \( Q_{[1]} \). Then, the alternative \( A^{(1)} \) that ranks the best in \( Q_{[1]} \) (minimum value) and fulfills following two conditions simultaneously would be the compromise solution.

**Condition 1** (acceptable advantage). One has

\[
Q(A^{(2)}) - Q(A^{(1)}) \geq \frac{1}{m-1}, \tag{31}\]

where \( A^{(1)} \) and \( A^{(2)} \) are the top two alternatives in \( Q_j \).
Condition 2 (acceptable stability). The alternative \( A(1) \) should also be the best ranked by \( S_i \) and \( R_j \).

If the above conditions cannot be satisfied simultaneously, there exist multiple compromise solutions:

1. alternatives \( A(1) \) and \( A(2) \) if only Condition 2 is not satisfied;
2. alternatives \( A(1), A(2), \ldots, A(ω) \) if Condition 1 is not satisfied, where \( A(ω) \) is established by the relation 
   \[ Q(A(ω)) - Q(A(1)) < 1/(m - 1) \] 
   for the maximum.

5. Application Examples

5.1. ERP Selection Problem. In recent years, enterprise resource planning (ERP) system has become a powerful tool for enterprises to improve their operating performance and competitiveness. However, ERP projects report an unusually high failure rate and sometimes imperil implementers’ core competitiveness. However, ERP projects report an unusually high failure rate and sometimes imperil implementers’ core competitiveness. ERP projects report an unusually high failure rate and sometimes imperil implementers’ core competitiveness.

**Step 1.** Each DM assesses alternative \( A_i \) concerning attribute \( C_j \) with linguistic rating variables in Table 3. Table 4 shows the assessments by three decision-makers.

**Step 2.** By using (24) and (25), the weights of DMs can be obtained as \( \lambda_1 = 0.3220, \lambda_2 = 0.3244, \lambda_3 = 0.3536 \).

**Step 3.** By using (26), we can establish the aggregated decision matrix as follows.

\[
D = \begin{bmatrix}
(0.6689, 0.2193) & (0.8000, 0.1000) & (0.3500, 0.5500) & (0.7081, 0.1857) & (0.5592, 0.3388) \\
(0.6062, 0.2717) & (0.6495, 0.2331) & (0.6071, 0.2912) & (0.5000, 0.4000) & (0.7562, 0.1383) \\
(0.5542, 0.3438) & (0.7308, 0.1568) & (0.8355, 0.1223) & (0.8465, 0.1077) & (0.7128, 0.1808) \\
(0.6495, 0.2331) & (0.6719, 0.2168) & (0.6074, 0.2908) & (0.5546, 0.3434) & (0.5423, 0.3491) \\
\end{bmatrix}
\]  

**Step 4.** By using (27) and (28), the weights of attributes are obtained as \( w_1 = 0.1950, w_2 = 0.2129, w_3 = 0.1980, w_4 = 0.1966, w_5 = 0.1976 \).

**Step 5.** By using (29), the best and the worst values of all attribute ratings can be calculated and we have \( x^*_1 = (0.6689, 0.2193), x^*_2 = (0.8000, 0.1000), x^*_3 = (0.8355, 0.1223), x^*_4 = (0.8465, 0.1077), x^*_5 = (0.7562, 0.1383), x^-_1 = (0.5542, 0.3438), x^-_2 = (0.6495, 0.2331), x^-_3 = (0.3500, 0.5500), x^-_4 = (0.5000, 0.4000), x^-_5 = (0.5423, 0.3491) \).
Liu et al. [18] developed the VIKOR method for MADM problem based on trapezoidal fuzzy numbers, which is one of the most commonly used fuzzy numbers. In this method, the weights of DMs or attributes are given artificially and the linguistic variables are represented by trapezoidal fuzzy numbers shown in Table 6. A trapezoidal fuzzy number \( \tilde{A} \) can be denoted as \((a_1, a_2, a_3, a_4)\), where \(a_1 \) and \(a_4\) are called lower and upper limits of \( \tilde{A} \) and \(a_2 \) and \(a_3\) are two most promising values. When \(a_2 \) and \(a_3\) are the same value, the trapezoidal fuzzy number degenerated a triangular fuzzy number. We apply the method [18] in the ERP selection problem to explain the difference between the proposed method and traditional VIKOR method with trapezoidal fuzzy numbers. Assume that the weights of DMs and attributes are \(A_1 = 0.3220, \lambda_2 = 0.3244, \lambda_3 = 0.3536\) and \(w_1 = 0.1950, w_2 = 0.2129, w_3 = 0.1980, w_4 = 0.1966, w_5 = 0.1976\), respectively. Through calculation of linguistic assessment information and trapezoidal fuzzy numbers shown in Tables 4 and 5, the values of \(S, R,\) and \(Q\) for all alternatives are obtained as in Table 7. The result shows that the ranking order of alternatives obtained by the method [18] is \(A_3 > A_1 > A_4 > A_2\), which is in harmony with our proposed method. Although the results obtained from the two VIKOR methods are consistent, the trapezoidal fuzzy numbers are only characterized by a membership function, while IFNs are characterized by a membership function and a nonmembership function, which closely resembles the thinking habit of human beings under the situation of being uncertain and hesitant. Furthermore, the trapezoidal fuzzy numbers need four values to determine the membership distribution, while the IFNs only need two values: membership and nonmembership, and the degree of hesitancy can be automatically generated by 1 minus membership and nonmembership. In general, the computation of trapezoidal fuzzy numbers is large, and the application of IFNs is relatively simple. In addition, the method presented by [18] needs the weight information of DMs and attributes predetermined, whereas these weights are determined objectively using intuitionistic fuzzy entropy in the proposed method, which avoids subjective randomness to some extent.

In [22], Roostaei et al. put forward a hamming distance based IF-VIKOR method. To further illustrate its effectiveness, the proposed method is compared to the IF-VIKOR method presented by [22]. Computational results for the hamming distance based IF-VIKOR method are shown in Table 8. The result indicates that these two methods reach a consensus that the third ERP system should be implemented by the high-tech manufacturing enterprise. In this example, it should be noted that the assessments provided by DMs are in low degree of hesitancy. As shown in Tables 3 and 4, the maximum hesitance index with respect to linguistic values is 0.1, denoting the low degree of nonspecificity (lack of knowledge) for assessments. In this case, both methods can effectively evaluate and sort the alternatives. However, the hamming distance based IF-VIKOR method is not always capable of obtaining valid results especially in MADM problem with high degree of nonspecificity. It might generate some counterintuitive cases so that unreasonable results might be obtained. The following example further illustrates such cases.

5.2. Material Handling Selection Problem. To illustrate the superiority of the proposed methods, a comparison between the proposed IF-VIKOR method and the hamming distance based IF-VIKOR method in a material handling selection problem is made.
Table 7: The values of S, R, and Q for all alternatives by the trapezoidal fuzzy numbers based VIKOR method.

<table>
<thead>
<tr>
<th>Value</th>
<th>A₁</th>
<th>A₂</th>
<th>A₃</th>
<th>A₄</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>0.3827</td>
<td>0.4629</td>
<td>0.3231</td>
<td>0.5632</td>
</tr>
<tr>
<td>R</td>
<td>0.2135</td>
<td>0.2204</td>
<td>0.1973</td>
<td>0.1996</td>
</tr>
<tr>
<td>Q</td>
<td>0.2736</td>
<td>0.7911</td>
<td>0.0000</td>
<td>0.5052</td>
</tr>
</tbody>
</table>

Table 8: The values of S, R, and Q for all alternatives by the hamming distance based IF-VIKOR method.

<table>
<thead>
<tr>
<th>Value</th>
<th>A₁</th>
<th>A₂</th>
<th>A₃</th>
<th>A₄</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>0.4495</td>
<td>0.5896</td>
<td>0.3295</td>
<td>0.6569</td>
</tr>
<tr>
<td>R</td>
<td>0.1980</td>
<td>0.2129</td>
<td>0.1950</td>
<td>0.1976</td>
</tr>
<tr>
<td>Q</td>
<td>0.2671</td>
<td>0.8972</td>
<td>0.0000</td>
<td>0.5726</td>
</tr>
</tbody>
</table>

Table 9: Rating of the alternatives from decision organization.

<table>
<thead>
<tr>
<th>A₁</th>
<th>C₁</th>
<th>C₂</th>
<th>C₃</th>
<th>C₄</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0.1, 0.2)</td>
<td>(0.4, 0.2)</td>
<td>(0.5, 0.2)</td>
<td>(0.2, 0.4)</td>
<td></td>
</tr>
<tr>
<td>A₂</td>
<td>(0.2, 0.1)</td>
<td>(0.3, 0.3)</td>
<td>(0.3, 0.5)</td>
<td>(0.5, 0.5)</td>
</tr>
<tr>
<td>A₃</td>
<td>(0.0, 0.3)</td>
<td>(0.3, 0.4)</td>
<td>(0.2, 0.3)</td>
<td>(0.5, 0.2)</td>
</tr>
<tr>
<td>A₄</td>
<td>(0.4, 0.4)</td>
<td>(0.5, 0.2)</td>
<td>(0.2, 0.4)</td>
<td></td>
</tr>
</tbody>
</table>

Table 10: The compromise values and ranking results obtained by the hamming distance based IF-VIKOR method and the proposed method in this paper.

<table>
<thead>
<tr>
<th>Alternative</th>
<th>Hamming distance based IF-VIKOR method</th>
<th>The values of Qᵢ</th>
<th>Ranking orders</th>
<th>The values of Qᵢ</th>
<th>Ranking orders</th>
</tr>
</thead>
<tbody>
<tr>
<td>A₁</td>
<td></td>
<td>0</td>
<td>1</td>
<td>0.3099</td>
<td>2</td>
</tr>
<tr>
<td>A₂</td>
<td></td>
<td>0.4029</td>
<td>2</td>
<td>0.7286</td>
<td>3</td>
</tr>
<tr>
<td>A₃</td>
<td></td>
<td>0.5046</td>
<td>3</td>
<td>1.0000</td>
<td>4</td>
</tr>
<tr>
<td>A₄</td>
<td></td>
<td>0.6312</td>
<td>4</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Suppose that a manufacturing company is considering implementing a material handling system. After preliminary screening, four alternatives of A₁, A₂, A₃, and A₄ remain to be further evaluated. Several DMs from the company’s technical committee are arranged to evaluate and select the appropriate alternative. They assess the four alternatives according to four conflicting attributes, including investment cost C₁, operation time C₂, expansion possibility C₃, and closeness to market demand C₄. Due to the lack of experience, time constraints, and other factors, the ratings of alternatives concerning each attribute provided by DMs are represented as IFNs with high degree of hesitancy, listed in Table 9. By using (27) and (28), the weights of attributes are obtained as \( w₁ = 0.2426, w₂ = 0.2490, w₃ = 0.2593, w₄ = 0.2490 \). Without losing generality, let \( γ = 0.5 \). Then, the values of \( Sᵢ, Rᵢ, \) and \( Qᵢ \) for each alternative can be calculated. Computational results obtained by the hamming distance based IF-VIKOR method and the proposed method in this paper are listed in Table 10. The ranking of alternatives calculated by the hamming distance based IF-VIKOR method is \( A₁ > A₂ > A₃ > A₄ \), reaching a conclusion that \( A₁ \) (minimum value) is the best choice and \( A₄ \) is the worst choice for the material handling selection problem. However, according to the score function and accuracy function, each attribute rating of \( A₁ \) is lower than or equal to that of \( A₄ \), indicating that \( A₄ \) is superior to \( A₁ \), which is contradicted to the ranking results obtained by hamming distance based IF-VIKOR method. Instead, our proposed method can overcome the drawback of traditional method and obtain a valid ranking result as \( A₄ > A₁ > A₂ > A₃ \), which means that material handling system \( A₄ \) is the best choice for the manufacturing company.

5.3. Sensitivity Analysis. In intuitionistic fuzzy VIKOR method, \( γ \), the coefficient of decision mechanism is critical to the ranking results. Hence, a sensitivity analysis is conducted in order to assess the stability of our method in these examples. For each \( γ \) from 0 to 1 at 0.1 intervals, we calculate the corresponding compromise solution to investigate the influence of different \( γ \) on the ranking result. Table 11 shows the sensitivity analysis of ERP selection example. For all the tested values of \( γ \), three different ranking results are generated including \( A₃ > A₄ > A₁ > A₂, A₃ > A₁ > A₄ > A₂, \) and \( A₃ > A₁ > A₂ > A₄ \). While the ranking result is indeed affected by \( γ \), \( A₃ \) is always the optimal solution. Table 12 shows the sensitivity analysis of material handling system selection example. For all the tested values of \( γ \), the ranking result remains \( A₄ > A₁ > A₂ > A₃ \) and
The optimal solution is $A_4$. The sensitivity analysis illustrates how the decision-making strategy would affect the result, and also indicates that the conclusion arrived from our method is stable and effective.

### 6. Conclusion

Since the VIKOR method is an effective MADM method to reach a compromise solution, and IFSs are an effective tool to depict fuzziness and nonspecificity in assessment information, we combine them and develop a new intuitive distance based IF-VIKOR method. This new method aims at MADM problems with unknown weights of both the DMs and attributes in intuitionistic fuzzy environment. It aggregates assessment information by intuitionistic fuzzy weighted averaging operator, generates weights of DMs and attributes by intuitionistic fuzzy entropy objectively, calculates the group utility and individual regret based on intuitive distance measure, and finally reaches the compromise solution. Two application examples of ERP and material handing selection problem further illustrate each step of this method. Compared with the hamming distance measure used in traditional IF-VIKOR method, the new intuitive distance measure in this method focuses on the fuzziness and nonspecificity of intuitionistic fuzzy information, reflecting not only the difference among the values of intuitionistic fuzzy sets, but also the waver of intuitionistic fuzzy sets. Both the artificial benchmark test and application examples demonstrate its effectiveness and superiority to traditional method. Also, the determination of weights of DMs and attributes using intuitionistic fuzzy entropy can avoid subjective randomness, and sensitivity analysis shows the stability of the proposed method. For future work, the comparison of the proposed VIKOR method with other MADM methods under intuitionistic fuzzy environment, such as the TOPSIS method, the PROMETHEE method, and the ELECTRE method, is worthy of further study and exploration. It would also be interesting to apply the proposed VIKOR method to other MADM problems, such as investment decision and supplier selection.

### Conflicts of Interest

The authors declare that they have no conflicts of interest.

### Acknowledgments

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