Research Article

A Consistent Relationship between the Stress and Plastic Strain Components and Its Application in Deep Drawing Process

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As von Mises yield criterion and associated flow rule (AFR) are widely applied in metal forming field, a semitotal deformation consistent relationship between the stress and plastic strain components and the rule of dimensional changes of metal forming processes in a plane-stress state are obtained on the basis of them in this paper. The deduced consistent relationship may be easily used in forming interval of the workpiece. And the rule of dimensional changes can be understood through three plastic strain incremental circles on which the critical points can be easily determined on the same basis. Analysis of stress and plastic strain evolution of aluminum warm deep drawing process is conducted, and the advantage of nonisothermal warm forming process is revealed, indicating that this method has the potential in practical large deformation applications.

1. Introduction

The rule of metal forming process has been investigated since several decades ago. Two main theories were proposed, the law of minimum resistance [1] and the consistent relationship [2]. The former pointed out that the particle of metal material will move to the direction which has the minimum resistance during deformation, and this direction is the direction of dimensional increasing. However, this theory lacks experimental support and is hard to obtain its practical application due to its too simple form of expression. As for the latter one, the relationship between order of total principle strain components and the associated principle stress components was deduced on the basis of Levy-Mises equation. This theory reveals the dramatic relationship between dimensional change of metal and the external forces added to the metal during deformation process and received its applications in qualitative and quantitative analysis [3, 4].

However, it is better to develop a rule of metal forming in a form of plastic strain components. On the one hand, as for the nature of deformation of metal materials, plastic strain components should be highlighted because the elastic strain components will change themselves after unloading. On the other hand, in many practical situations, such as

forging and forming, the plastic strains may be 1000 times of elastic strains, and the elastic strain components are always neglected. In this paper, a semitotal deformation consistent relationship between stress and plastic strain components is proposed on the basis of the AFR and von Mises yield criterion. With regard to the plane-stress situation, the critical points on the three plastic strain incremental circles can be easily determined due to the physical meaning of the AFR.

In recent years, the studies of aluminum warm forming are being more and more popular due to the problems of energy shortage and environmental damage. However, the rule of the dimensional changes of this sheet metal in the warm forming process is rarely investigated. In this paper, the analysis of stress and plastic strain evolution of the blank in this potential process is fully conducted.

2. Consistent Relationship and Rule of Dimensional Changes of Metal Forming Process

The existing consistent relationship between the stress and total strain components is on the basis of Levy-Mises equation. In this section, a similar consistent relationship between

stress and plastic strain components is deduced on the basis of von Mises yield criterion and the AFR, and the rule of dimensional changes of metal forming processes in plane-stress state is also discussed.

2.1. Instantaneous Deformation Tendency. At a moment during forming process, generally $\sigma_1 > \sigma_2 > \sigma_3$ can be acquired, where σ_1, σ_2 , and σ_3 are the principal stress components. So the order of the deviatoric stress components can be obtained:

$$\sigma_1 - \sigma_m > \sigma_2 - \sigma_m > \sigma_3 - \sigma_m, \quad (1)$$

where σ_m is the average stress.

As is given by [5], the AFR can be expressed as

$$d\epsilon^p = d\lambda \frac{\partial f}{\partial \sigma}, \quad (2)$$

where $d\epsilon^p$ denotes the increment in the plastic strain tensor, $d\lambda$ is plastic multiplier, and f is the yield criterion. It is clear that the increment plastic strain tensor $d\epsilon^p$ is in a direction which is normal to the tangent to the yield surface at the load point. For a von Mises yield criterion, (2) can be rewritten as

$$d\epsilon^p = \frac{3}{2} d\lambda \frac{\sigma'}{\sigma_e}, \quad (3)$$

where σ_e means the effective stress and σ' means the deviatoric stress tensor. Equation (3) can be rewritten as

$$\frac{d\epsilon_1^p}{\sigma_1 - \sigma_m} = \frac{d\epsilon_2^p}{\sigma_2 - \sigma_m} = \frac{d\epsilon_3^p}{\sigma_3 - \sigma_m} = \frac{3d\lambda}{2\sigma_e}. \quad (4)$$

Combine (1) and (4), as $d\lambda$ and σ_e are positive, the following expression can be achieved:

$$d\epsilon_1^p > d\epsilon_2^p > d\epsilon_3^p. \quad (5)$$

So the order of the increment principle plastic strain components is the same as the order principal stress components. In other words, at a moment of forming process if $\sigma_1 > \sigma_2 > \sigma_3$, then order of the associated instantaneous plastic strain is $d\epsilon_1^p > d\epsilon_2^p > d\epsilon_3^p$.

As

$$d\epsilon_1^p + d\epsilon_2^p + d\epsilon_3^p = 0 \quad (6)$$

so $d\epsilon_1^p > 0$ and $d\epsilon_3^p < 0$.

From (4), the following equation can be obtained:

$$d\epsilon_2^p = (\sigma_2 - \sigma_m) \frac{3d\lambda}{2\sigma_e}. \quad (7)$$

So if $\sigma_2 - \sigma_m > 0$, then $d\epsilon_2^p > 0$. Similarly, if $\sigma_2 - \sigma_m < 0$, then $d\epsilon_2^p < 0$, and when $\sigma_2 - \sigma_m = 0$, $d\epsilon_2^p = 0$. It can be found clearly that the sign of $d\epsilon_2^p$ is identical to the sign of $\sigma_2 - \sigma_m$; if $\sigma_2 - \sigma_m = 0$, then $d\epsilon_2^p = 0$.

2.2. Semitotal Deformation Consistent Relationship. The above-mentioned derivation is on the basis of instantaneous deformation. And the obtained conclusions can only reflect the instant tendency of deformation. As for a period, if $\sigma_1 > \sigma_2 > \sigma_3$ and the principle directions of stress tensor σ do not change, the same conclusions can be achieved. The derivation using incremental theory is as follows.

A certain time period $t_0 < t < t_n$ of an arbitrary deformation can be divided into several small intervals, such as $t_0 \sim t_1, t_1 \sim t_2, \dots, t_{n-1} \sim t_n$, where t is the deformation time. So the following equation for interval $t_0 \sim t_1$ can be obtained due to (3) as

$$\begin{aligned} \frac{d\epsilon_1^p|_{t_0 \sim t_1}}{(\sigma_1 - \sigma_m)|_{t_0 \sim t_1}} &= \frac{d\epsilon_2^p|_{t_0 \sim t_1}}{(\sigma_2 - \sigma_m)|_{t_0 \sim t_1}} = \frac{d\epsilon_3^p|_{t_0 \sim t_1}}{(\sigma_3 - \sigma_m)|_{t_0 \sim t_1}} \\ &= \frac{3d\lambda_1}{2\sigma_{e1}}. \end{aligned} \quad (8)$$

As for interval $t_1 \sim t_2$,

$$\begin{aligned} \frac{d\epsilon_1^p|_{t_1 \sim t_2}}{(\sigma_1 - \sigma_m)|_{t_1 \sim t_2}} &= \frac{d\epsilon_2^p|_{t_1 \sim t_2}}{(\sigma_2 - \sigma_m)|_{t_1 \sim t_2}} = \frac{d\epsilon_3^p|_{t_1 \sim t_2}}{(\sigma_3 - \sigma_m)|_{t_1 \sim t_2}} \\ &= \frac{3d\lambda_2}{2\sigma_{e2}}. \end{aligned} \quad (9)$$

During interval $t_{n-1} \sim t_n$,

$$\begin{aligned} \frac{d\epsilon_1^p|_{t_{n-1} \sim t_n}}{(\sigma_1 - \sigma_m)|_{t_{n-1} \sim t_n}} &= \frac{d\epsilon_2^p|_{t_{n-1} \sim t_n}}{(\sigma_2 - \sigma_m)|_{t_{n-1} \sim t_n}} \\ &= \frac{d\epsilon_3^p|_{t_{n-1} \sim t_n}}{(\sigma_3 - \sigma_m)|_{t_{n-1} \sim t_n}} = \frac{3d\lambda_n}{2\sigma_{en}}. \end{aligned} \quad (10)$$

As $\sigma_1 > \sigma_2 > \sigma_3$ is kept unchanged during this period, so (1) can also be achieved in this section, because the increment plastic multiplier $d\lambda_1, d\lambda_2, \dots, d\lambda_n$ and the effective stress $\sigma_{e1}, \sigma_{e2}, \dots, \sigma_{en}$ are always positive. Then from (8), (9), and (10),

$$\begin{aligned} d\epsilon_1^p|_{t_0 \sim t_1} &> d\epsilon_2^p|_{t_0 \sim t_1} > d\epsilon_3^p|_{t_0 \sim t_1} \\ d\epsilon_1^p|_{t_1 \sim t_2} &> d\epsilon_2^p|_{t_1 \sim t_2} > d\epsilon_3^p|_{t_1 \sim t_2} \\ &\vdots \end{aligned} \quad (11)$$

$$d\epsilon_1^p|_{t_{n-1} \sim t_n} > d\epsilon_2^p|_{t_{n-1} \sim t_n} > d\epsilon_3^p|_{t_{n-1} \sim t_n}.$$

As the principle directions of stress tensor σ do not change during this period, so the principle directions of σ' and $d\epsilon^p$ will not change too. The proof is as follows.

It is well known that the principle directions of stress tensor σ are the same as the eigenvectors of the matrix σ . The eigenvalues of matrix σ can be obtained by the following equation:

$$|\lambda \mathbf{I} - \sigma| = 0, \quad (12)$$

where $\mathbf{I}$ is the unit matrix and λ denote the eigenvalues of matrix $\boldsymbol{\sigma}$. The eigenvalues of matrix $\boldsymbol{\sigma}'$ can also be obtained in the same way:

$$|\mu\mathbf{I} - (\boldsymbol{\sigma} - \sigma_m)| = 0, \quad (13)$$

where μ denote the eigenvalues of matrix $\boldsymbol{\sigma}'$. As $|\mu\mathbf{I} - (\boldsymbol{\sigma} - \sigma_m)| = |(\mu - \sigma_m)\mathbf{I} - \boldsymbol{\sigma}|$, so the eigenvalues of matrix $\boldsymbol{\sigma}'$ will be $(\lambda - \sigma_m)$ by comparing (12). The eigenvectors of the matrix $\boldsymbol{\sigma}$ can be achieved by solving the following equation:

$$(\lambda\mathbf{I} - \boldsymbol{\sigma})\mathbf{x} = \mathbf{0}, \quad (14)$$

where $\mathbf{x}$ is the eigenvector of the matrix $\boldsymbol{\sigma}$. The eigenvectors of matrix $\boldsymbol{\sigma}'$ can also be obtained in the same way:

$$[\mu\mathbf{I} - (\boldsymbol{\sigma} - \sigma_m)]\mathbf{y} = \mathbf{0}, \quad (15)$$

where $\mathbf{y}$ is the eigenvector of the matrix $\boldsymbol{\sigma}'$. As $[\mu\mathbf{I} - (\boldsymbol{\sigma} - \sigma_m)]\mathbf{y} = [\lambda\mathbf{I} - \boldsymbol{\sigma}]\mathbf{y}$, so the eigenvectors of matrices $\boldsymbol{\sigma}'$ and $\boldsymbol{\sigma}$ will be the same by comparing (14). It can be shown by (3) that the principle directions of $\boldsymbol{\sigma}'$ and $d\boldsymbol{\varepsilon}^p$ are the same; that is, the principle directions of $\boldsymbol{\sigma}'$, $d\boldsymbol{\varepsilon}^p$, and $\boldsymbol{\sigma}$ are the same, so if the principle directions of stress tensor $\boldsymbol{\sigma}$ do not change, the principle directions of $\boldsymbol{\sigma}'$ and $d\boldsymbol{\varepsilon}^p$ will not change too.

From (11), if the principle direction of $\boldsymbol{\sigma}$ (the principle directions of $\boldsymbol{\sigma}'$ and $d\boldsymbol{\varepsilon}^p$ are the same as that of $\boldsymbol{\sigma}$) is kept unchanged during the time period $t_0 < t < t_n$, the corresponding increment principle plastic strain components in (11) can be summed up, and then the following inequation can be obtained:

$$\Delta\varepsilon_1^p > \Delta\varepsilon_2^p > \Delta\varepsilon_3^p. \quad (16)$$

So during a time period $t_0 < t < t_n$, if $\sigma_1 > \sigma_2 > \sigma_3$ and the principle directions of $d\boldsymbol{\varepsilon}^p$ or $\boldsymbol{\sigma}'$ are kept unchanged, then $\Delta\varepsilon_1^p > \Delta\varepsilon_2^p > \Delta\varepsilon_3^p$ can be achieved.

As

$$\Delta\varepsilon_1^p + \Delta\varepsilon_2^p + \Delta\varepsilon_3^p = 0. \quad (17)$$

Then $\Delta\varepsilon_1^p > 0$ and $\Delta\varepsilon_3^p < 0$.

$\Delta\varepsilon_2^p$ can be calculated by combining (8), (9), and (10):

$$\begin{aligned} \Delta\varepsilon_2^p = & (\sigma_2 - \sigma_m)|_{t_0 \sim t_1} \frac{3d\lambda_1}{2\sigma_{e1}} + (\sigma_2 - \sigma_m)|_{t_1 \sim t_2} \frac{3d\lambda_2}{2\sigma_{e2}} \\ & + \dots + (\sigma_2 - \sigma_m)|_{t_{n-1} \sim t_n} \frac{3d\lambda_n}{2\sigma_{en}}. \end{aligned} \quad (18)$$

So during the time period $t_0 < t < t_n$, if $\sigma_2 - \sigma_m > 0$, then $\Delta\varepsilon_2^p > 0$; if $\sigma_2 - \sigma_m = 0$, $\Delta\varepsilon_2^p = 0$, and when $\sigma_2 - \sigma_m < 0$, $\Delta\varepsilon_2^p < 0$.

The application of the consistent relationship between stress and plastic strain components has been divided into two types. However, only semitotal deformation has the value of practical application. The reason why the instantaneous deformation is also pointed out is for a general understanding of deformation, and semitotal deformation is on the basis of instantaneous deformation. The consistent relationship of the

TABLE 1: Consistent relationship of two types of deformation.

Application condition	Magnitude of σ_2	Plastic strain state
instantaneous deformation	$\sigma_2 - \sigma_m > 0$	$d\varepsilon_1^p > 0, d\varepsilon_2^p > 0, d\varepsilon_3^p < 0$
$\sigma_1 > \sigma_2 > \sigma_3, t$	$\sigma_2 - \sigma_m = 0$	$d\varepsilon_1^p > 0, d\varepsilon_2^p = 0, d\varepsilon_3^p < 0$
	$\sigma_2 - \sigma_m < 0$	$d\varepsilon_1^p > 0, d\varepsilon_2^p < 0, d\varepsilon_3^p < 0$
semi-total deformation	$\sigma_2 - \sigma_m > 0$	$\Delta\varepsilon_1^p > 0, \Delta\varepsilon_2^p > 0, \Delta\varepsilon_3^p < 0$
$\sigma_1 > \sigma_2 > \sigma_3, t_1 < t < t_2$	$\sigma_2 - \sigma_m = 0$	$\Delta\varepsilon_1^p > 0, \Delta\varepsilon_2^p = 0, \Delta\varepsilon_3^p < 0$
$t < t_2$	$\sigma_2 - \sigma_m < 0$	$\Delta\varepsilon_1^p > 0, \Delta\varepsilon_2^p < 0, \Delta\varepsilon_3^p < 0$
principle directions of $\boldsymbol{\sigma}$ unchanged		

above-mentioned two types of deformation can be concluded in Table 1.

What should be pointed out is that only von Mises yield criterion is used here to account for the derivation. The result may be the same if other yield criterions are taken into consideration. The following summary can be obtained: during a period $t_1 < t < t_2$, if $\sigma_1 > \sigma_2 > \sigma_3$ do not change and the principle direction of $\boldsymbol{\sigma}$ is kept unchanged, then $\Delta\varepsilon_1^p > \Delta\varepsilon_2^p > \Delta\varepsilon_3^p$ do not change too; during this period, if the sign of $\sigma_2 - \sigma_m$ does not change, then the sign of $\Delta\varepsilon_2^p$ is the same as $\sigma_2 - \sigma_m$, and if $\sigma_2 - \sigma_m = 0$, then $\Delta\varepsilon_2^p = 0$.

2.3. Rule of Dimensional Changes of Metal Forming Processes in Plane-Stress State. In this subsection, three plastic strain incremental circles are used in predicting the tendency of the dimensional changes of a forming part in plane-stress state. It is well known that tubular and sheet forming process can be modeled into plane-stress deformation. For some particular conditions, the associated yield criterion of von Mises can be written as

$$f = f(\sigma_\theta, \sigma_\rho, \sigma_s) = \sigma_\theta^2 - \sigma_\rho\sigma_\theta + \sigma_\rho^2 - \sigma_s^2 = 0. \quad (19)$$

In (19), f means the yield criterion, σ_θ is circumferential stress, σ_ρ is radial stress, and σ_s is flow stress. According to (2), $d\boldsymbol{\varepsilon}^p$ is in a direction normal to the tangent to the yield surface at the load point, as is shown in Figure 1.

Different from the method given by [1], the critical points on the plastic strain incremental circle can be independently determined based on von Mises criterion and the AFR. As for the thickness plastic strain incremental circle, the critical points on it can be achieved by (6), that is, through $d\varepsilon_t^p = -(d\varepsilon_\rho^p + d\varepsilon_\theta^p) = 0$, as is shown in Figure 1, by drawing line OB_2 and OB_4 and then expanding them to meet the thickness plastic strain incremental circle. The algebraic sign of $d\varepsilon_t^p$ can also be determined by the algebraic sign of $-(d\varepsilon_\rho^p + d\varepsilon_\theta^p)$. For example, when the stress state locates at the top side of quadrant I, obviously $d\varepsilon_t < 0$.

In order to find the critical points on the circumferential and radial plastic strain incremental circles, two methods will be discussed here.

2.3.1. Graphical Method. There are two intersection points of the yield surface and its horizontal tangent, as is shown in

FIGURE 1: Tendency of the dimensional changes in plane-stress state.

Figure 1, B_5 and B_6 . The circumferential components of incremental plastic strain on these two points are zero. So the critical points on the circumferential plastic strain incremental circle can be found on the basis of these two points. The critical points on the radial plastic strain incremental circle can also be found by using this method. The algebraic sign of $d\epsilon_\theta^p$ and $d\epsilon_\rho^p$ can be determined by this graphical method.

2.3.2. Analytical Method. As to the critical points on the circumferential plastic strain incremental circle, the following equation can be established:

$$\frac{\partial f}{\partial \sigma_\theta} = 2\sigma_\theta - \sigma_\rho = 0. \quad (20)$$

Then the coordinate of critical points on the yield locus can be determined analytically from (19) and (20). So the critical points on the circumferential plastic strain incremental circle can also be found on the basis of these two points. The algebraic sign of $d\epsilon_\theta^p$ can be determined by combining $\partial f / \partial \sigma_\theta = 2\sigma_\theta - \sigma_\rho > 0$ or < 0 and (19). The critical points on the radial plastic strain incremental circle and the algebraic sign of $d\epsilon_\rho^p$ can also be found by using the above-mentioned method.

The dimensional changes of the blank can be determined by the following steps [2]: (i) determining the size of the yield locus by the value of σ_s ; (ii) determining location of the loading point P on the yield locus; (iii) drawing line OP and extending it to meet the three plastic strain incremental circles; (iv) determining the tendency of the dimensional changes of this forming part in plane-stress state.

3. Insight Analysis of Stress and Plastic Strain Evolution in Warm Deep Drawing Condition

This warm forming process can be modeled as plane-stress deformation because most zone of the blank is in a stress free surface. Mises cylinder yield surface and the AFR are applied here, and the yield surface for plane-stress conditions is as shown in Figure 3.

Due to its better formability, only nonisothermal cup deep drawing of aluminum alloy is considered here. The authors in [6, 7] proved this phenomenon through experiment and simulation methods. In Figure 2, the blank holder and die are heated, and the punch is cooled by cyclic cold water. So the blank regions of I and II have a higher temperature, V and VI have a lower temperature, and III (from material flow of I and II) and IV (from the middle region before forming) have a medium temperature; however, the temperature of II, IV, and VI will be lower than that of regions I, III, and V, respectively, due to heat exchange of the blank. So during the deep drawing process, the temperature of different regions will be $I > II > III > IV > V > VI$. As aluminum alloy sheet metals have a warm, soft flow property demonstrated by [8], so the size-dependent yield loci of the above-mentioned regions can be described in Figure 3 (only four yield loci are given to simply describe the notion: i.e., temperature gradient between I and II is neglected and so is that of regions V and VI, and the medium temperature will replace the temperature of each region simply).

This technology promotes the material of outskirt to flow into the wall of the cup due to the warm, soft property of aluminum alloy. Regions I and II have a higher temperature so their yield loci are the inner one a . Regions V and VI occupy the outer one d , region III occupies the yield locus b , and region IV occupies the yield locus c . The authors in [9] also gave a similar result of these temperature-dependent yield loci. In Figure 2, the stress and strain states are also given. It is clear that region II has two stress states due to prevalent of tension near point C. In Figure 3, the stress states of deformation zones on the above-mentioned yield loci and tendency of blank dimensional changes are also given.

The following gives an insight into analysis of stress and plastic strain evolution according to the proposed consistent relationship and the rule of dimensional changes of metal forming processes in plane-stress state. In warm cup deep drawing, initially the stress state at the entry point A is $\sigma_\rho = 0$ and $\sigma_\theta < 0$, as is shown by the inner yield locus a due to higher temperature in Figure 3. In zone I, the maximum principle stress $\sigma_1 = \sigma_\rho > 0$, the medium one $\sigma_2 = \sigma_t = 0$ (near to zero), and the minimum one $\sigma_3 = \sigma_\theta < 0$; accordingly $\Delta\epsilon_1^p = \Delta\epsilon_\rho^p > 0$, $\Delta\epsilon_2^p = \Delta\epsilon_t^p > 0$ ($\sigma_t > \sigma_m$ or according to Figure 3), and $\Delta\epsilon_3^p = \Delta\epsilon_\theta^p < 0$. It can be concluded that the blank thickness in this region is increased.

In zone II, there are two different stress states. The principle stress states close to point B in zone II are as follows: $\sigma_1 = \sigma_\rho > 0$, $\sigma_2 = \sigma_t = 0$, and $\sigma_3 = \sigma_\theta < 0$, obviously this stress state only occupies quadrant II, and then $\Delta\epsilon_1^p = \Delta\epsilon_\rho^p > 0$, $\Delta\epsilon_2^p = \Delta\epsilon_t^p < 0$ ($\sigma_t < \sigma_m$ or according to Figure 3),

FIGURE 2: The warm deep drawing process and stress strain states of different blank regions.

FIGURE 3: Yield loci due to different temperatures and tendency of the dimensional changes under different loading state.

and $\Delta\epsilon_3^p = \Delta\epsilon_\theta^p < 0$. The blank thickness is decreased at the position close to point B on the fillet. The order of principle stress components close to point C in zone II will be changed as is shown in quadrant I in Figure 3: $\sigma_1 = \sigma_\rho > 0$, $\sigma_2 = \sigma_\theta > 0$, and $\sigma_3 = \sigma_t = 0$; accordingly $\Delta\epsilon_1^p = \Delta\epsilon_\rho^p > 0$, $\Delta\epsilon_2^p = \Delta\epsilon_\theta^p < 0$, and $\Delta\epsilon_3^p = \Delta\epsilon_t^p < 0$. The blank thickness is also

decreased at the position close to point C on the fillet. So the blank thickness in region II will always be decreased. In zones I and II, deformation is easy to take place because of their lower flow stress as is shown in Figure 3, so warm blank holder/die could improve the material flow from the flange into the cup wall.

In zones III and IV, the plane strain occurs; they respectively occupy a point on the yield loci *b* and *c* due to their medium temperature. Their stress states can be expressed as $\sigma_1 = \sigma_\rho > 0$, $\sigma_2 = \sigma_\theta = 1/2\sigma_\rho > 0$ (due to von Mises yield criterion), and $\sigma_3 = \sigma_t = 0$; then $\Delta\epsilon_1^p = \Delta\epsilon_\rho^p > 0$, $\Delta\epsilon_2^p = \Delta\epsilon_\theta^p = 0$, and $\Delta\epsilon_3^p = \Delta\epsilon_t^p < 0$. In these two regions, the blank thickness will also be decreased.

The stress and plastic strain components in zone VI are discussed next because the material of this region has the tendency to flow into zone V in warm deep drawing process. In zone VI, the biaxial tension occurs, so $\sigma_1 = \sigma_\rho = \sigma_2 = \sigma_\theta > 0$; as is shown in Figure 3 the yield state only occupies a point on yield locus *d* due to its lowest temperature as a consequence of heat exchange with the cooled punch during warm deep drawing process, and $\sigma_3 = \sigma_t = 0$; accordingly $\Delta\epsilon_1^p = \Delta\epsilon_\rho^p = \Delta\epsilon_2^p = \Delta\epsilon_\theta^p > 0$ and $\Delta\epsilon_3^p = \Delta\epsilon_t^p < 0$. Clearly in this region the blank thickness is decreased. In zone V, $\sigma_1 = \sigma_\rho > 0$, $\sigma_2 = \sigma_\theta > 0$, and $\sigma_3 = \sigma_t = 0$; then $\Delta\epsilon_1^p = \Delta\epsilon_\rho^p > 0$, $\Delta\epsilon_2^p = \Delta\epsilon_\theta^p > 0$, and $\Delta\epsilon_3^p = \Delta\epsilon_t^p < 0$. So the blank thickness is also decreased in this region. However, in these two regions, deformation is difficult to take place because of their higher flow stress as is shown in Figure 3, so cooled punch could hold the material from flowing into the wall too much and improve the blank thickness in the cup corner. This phenomenon was also verified by [10, 11].

4. Conclusions

- (1) A semitotal deformation consistent relationship and the rule of dimensional changes in metal forming processes are independently deduced based on von Mises yield criterion and associated flow rule.
- (2) Stress and plastic strain evolution in warm deep drawing condition could be analyzed by the semitotal deformation consistent relationship or the rule of dimensional changes. It is found that the thickness evolution of warm deep drawing process may be explained by these methods. The nonisothermal warm forming process helps to improve the formability of the deformed cup.

Competing Interests

The authors declare that they have no competing interests.

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