

Research Article

Support Recovery of Greedy Block Coordinate Descent Using the Near Orthogonality Property

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In this paper, using the near orthogonal property, we analyze the performance of greedy block coordinate descent (GBCD) algorithm when both the measurements and the measurement matrix are perturbed by some errors. An improved sufficient condition is presented to guarantee that the support of the sparse matrix is recovered exactly. A counterexample is provided to show that GBCD fails. It improves the existing result. By experiments, we also point out that GBCD is robust under these perturbations.

1. Introduction

Greedy block coordinate descent (GBCD) algorithm was presented by [1] for direction of arrival (DOA) estimation. In the work of [1], the DOA estimation is treated as the multiple measurement vectors (MMV) model that recovers a common support shared by multiple unknown vectors from multiple measurements. The authors provided a sufficient condition, based on mutual coherence, to guarantee that GBCD exactly recover the nonzero supports with noiseless measurements.

Recently, the work of [2] discussed the following method:

\[
\begin{align*}
\min_X & \quad \|X\|_{2,1} \\
\text{s.t.} & \quad \hat{Y} = AX + N, \\
& \quad \hat{A} = A + E,
\end{align*}
\]

with inputs \( \hat{Y} \in \mathbb{R}^{m \times L} \) and \( \hat{A} \in \mathbb{R}^{m \times N} \). \( N \) denotes the measurement noise and \( E \) denotes the system perturbation. The perturbations \( E \) and \( N \) are quantified with the following relative bounds:

\[
\begin{align*}
\frac{\|N\|_F}{\|Y\|_F} & \leq \varepsilon, \\
\frac{\|E\|_{2,(K)}^2}{\|A\|_{2,(K)}^2} & \leq \varepsilon_A,
\end{align*}
\]

where \( \|A\|_{2,(K)}^2 \) and \( \|Y\|_F \) are nonzero. Here, \( \|A\|_{2,(K)}^2 \) denotes the largest spectral norm taken over all \( K \)-column submatrices of \( A \). Throughout the paper, we are only interested in the case where \( \varepsilon \) and \( \varepsilon_A \) are far less than 1. In (1), \( X \) is a \( K \)-group sparse matrix; that is, it has no more than \( K \) nonzero rows, and \( \|X\|_{2,1} = \sum_{i=1}^{N} \|x_i\|_2 \), \( x_i \) is the \( i \)th row of \( X \). It is assumed that all columns of \( \hat{A} \) are normalized to be of unit-norm [3]. Both \( Y = AX \) and \( A \) are totally perturbed in (1). This case can be found in source separation [4], radar [5], remote sensing [6], and countless other problems. In addition, the total perturbations have also been discussed in [7–9].

One of the most commonly known conditions is the restricted isometry property (RIP). A matrix \( A \) satisfies RIP of the order \( K \) if there exists a constant \( \delta \in (0, 1) \) such that

\[
(1 - \delta) \|h\|_2^2 \leq \|Ah\|_2^2 \leq (1 + \delta) \|h\|_2^2
\]

for all \( K \)-sparse vector \( h \). In particular, the minimum of all constants \( \delta \) satisfying (3) is called the restricted isometry constant (RIC) \( \delta_K \).

There are many papers [8, 10–14] discussing the sufficient condition for orthogonal matching pursuit (OMP) that is one of the widely greedy algorithms for sparse recovery. In [3], using the near orthogonality property, the authors improved the sufficient condition of OMP. As cited in [3], the near orthogonality property can further develop the orthogonality characterization of columns in \( A \); it will play a fundamental role in the study of the signal reconstruction performance in
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\[ (1 - \delta_K) \sum_{i=1}^L \| A x_i \|_F^2 \leq \| A x \|_F^2 \]
\[ (1 - \delta_K) \| A X \|_F^2 . \]

Lemma 1 (near orthogonality property, see [3]). Let \( u \) and \( v \) be two orthogonal sparse vectors with supports \( T_u \) and \( T_v \) fulfilling \( |T_u \cup T_v| \leq K \). Suppose that \( A \) satisfies RIP of order \( K \) with RIC \( \delta_K \). Then we have
\[ |\cos \angle (A u, A v)| \leq K, \]
where \( \angle (A u, A v) \) denotes the angle between \( A u \) and \( A v \).

Lemma 2 (see [3]). Under the same assumptions as in Lemma 1, we have
\[ |\langle A u, A v \rangle| \leq \delta \| A u \|_2 \| A v \|_2 . \]

Lemma 3. For finite sets \( \Gamma \) and \( \Gamma \), let \( \text{supp}(X) = \Gamma \) and \( \text{supp}(\bar{X}) = \bar{\Gamma} \). Here, \( \Gamma \cap \bar{\Gamma} = \emptyset \), and \( |\Gamma \cup \bar{\Gamma}| \leq K \). If \( A \) satisfies the RIP condition (3) with \( \delta_K \in (0, 1) \), then we have
\[ |\langle A \bar{X}, A \bar{X} \rangle_F| \leq \delta_K \| A \bar{X} \|_F \| A \bar{X} \|_F . \]

Proof. Note that the Frobenius norm of \( A \) is derived from the Frobenius inner product.

\[ |\langle A \bar{X}, A \bar{X} \rangle_F| \leq \delta_K \| A \bar{X} \|_F \| A \bar{X} \|_F . \]

where (15) and (17) follow from Lemma 2 and Cauchy-Schwarz inequality, respectively. \( \square \)

3. RIP Based Recovery Condition

In this section, we firstly present the upper bound of the noise matrix \(-EX + N\) and provide the recovery condition for GBCD.

Lemma 4 (see [2]). Suppose that \( \tilde{A} \) satisfies the Kth order RIC \( \delta_K \in (0, 0.5) \). Then we have
\[ \| -EX + N \|_F < \left( \frac{\varepsilon_A}{1/\sqrt{3} - (1 + 1/\sqrt{3})\varepsilon_A} + \varepsilon \right) \frac{\| Y \|_F}{1 - \varepsilon} . \]
Input: $\tilde{A}$, $\hat{Y}$, $X(0) = 0$, $n = 1$, $\lambda > 0$, $\beta > 0$.
(1) Repeat until "stopping criterion" is met
(2) for $i = 1 : N$
(3) $p(n-1) = x(n-1) - \beta \hat{a}_i \tilde{A}x(n-1) - \hat{Y}$
(4) $x(n) = (p(n-1)/\|p(n-1)\|_2) \max(0, \|p(n-1)\|_2 - \lambda \beta)$
(5) comp(i) = $\|x(n) - x(n-1)\|$
(6) end for
(7) Choose the index $i_0$ such that $\text{comp}(i_0) = \max(\text{comp})$
(8) $X(n) \leftarrow [x^t(n-1), \ldots, x^t(1), x^t(0)]$
(9) $X(n) = [x^t(n-1), \ldots, x^t(1)]$
(10) $n \leftarrow n + 1$
(11) End Repeat

Algorithm 1: GBCD: greedy block coordinate descent algorithm [1].

According to steps (7) and (8) of Algorithm 1, at the $n$th iteration, GBCD can obtain a correct index if
$$\max_{i \in \Gamma} \|X^i(n) - X^i(n-1)\|_2 > \max_{j \in \Gamma} \|X^j(n) - X^j(n-1)\|_2.$$  
(20)

**Theorem 5.** Consider model (4). Let $t_0 = \min_{i \in \Gamma} \|x^i\|_2$. If the matrix $\tilde{A}$ satisfies RIP of order $K + 1$ with
$$\tilde{\delta}_{K+1} < \frac{\sqrt{K+1} - 1}{2K},$$  
(21)
and
$$t_0 > \frac{\left(1 + \tilde{\delta}_{K+1} + \sqrt{K} \epsilon_0\right)}{1 - \tilde{\delta}_{K+1} - \sqrt{1 - \tilde{\delta}_{K+1} \sqrt{K} \tilde{\delta}_{K+1}},}$$  
(22)

where $\epsilon_0 = (\epsilon_A/\sqrt{3} - (1 + 1/\sqrt{3}) \epsilon_A + \epsilon)(\|	ilde{Y}\|_F/(1-\epsilon))$, then GBCD can exactly recover the support set $\Gamma$.

Proof. Consider $n = 1$. The initial value is $X(0) = 0$. In order to guarantee that GBCD selects a correct index $i_0 \in \Gamma$, combining step (4) of Algorithm 1 and (20), we should verify the following inequality:
$$\max_{i \in \Gamma} \|x^i(1) - x^i(0)\|_2$$  
(23)
$$= \max_{i \in \Gamma} \frac{\|p^i(0)\|_2}{\|p^i(0)\|_2} \max(0, \|p^i(0)\|_2 - \lambda \beta)$$  
(24)
$$> \max_{i \in \Gamma} \|x^i(1) - x^i(0)\|_2$$  
(25)
$$= \max_{i \in \Gamma} \frac{\|p^i(0)\|_2}{\|p^i(0)\|_2} \max(0, \|p^i(0)\|_2 - \lambda \beta).$$  
(26)

If $\|p^i(0)\|_2 - \lambda \beta \leq 0$ ( $j \in \Gamma$), the right-hand-side is 0. Then inequality (26) holds. Thus, we only consider $\|p^i(0)\|_2 - \lambda \beta > 0$. Using Remark 1 in [2], inequality (26) is true when
$$\max_{i \in \Gamma} \|p^i(0)\|_2 > \max_{i \in \Gamma} \|p^i(0)\|_2.$$  
(27)

Now, it is sufficient to verify (27). Let us construct an upper bound for $\max_{i \in \Gamma} \|p^i(0)\|_2$. By step (3) of Algorithm 1, we have
$$\max_{i \in \Gamma} \|p^i(0)\|_2$$  
(28)
$$= \max_{i \in \Gamma} \|x^i(0) - \beta \hat{a}_i \tilde{A}x(0) - \hat{Y}\|_2$$  
(29)
$$= \max_{i \in \Gamma} \|\beta \hat{a}_i \tilde{A}x - Y\|_2$$  
(30)
$$\leq \max_{i \in \Gamma} \|\beta \hat{a}_i \tilde{A}x\|_2 + \beta \|	ilde{A}x\|_2 \|E - N\|_F$$  
(31)
$$= \beta \max_{i \in \Gamma} \|\tilde{A}x\|_2 \|E - N\|_F$$  
(32)
$$\leq \beta \delta_{K+1} \|\tilde{A}x\|_2 + \beta \epsilon_0,$$  
(33)

where (32) is from the property of norm and (34) follows from each column of $\tilde{A}$ which is of unit-norm, Lemmas 3 and 4.

To prove (27), we only need to prove
$$\max_{i \in \Gamma} \|p^i(0)\|_2 > \beta \delta_{K+1} \|\tilde{A}x\|_2 + \beta \epsilon_0.$$  
(35)

We then go on to show by contradiction that (35) is true. For all $i \in \Gamma$, assume that
$$\|p^i(0)\|_2 \leq \beta \delta_{K+1} \|\tilde{A}x\|_2 + \beta \epsilon_0.$$  
(36)

Then we have
$$\|p^i(0)\|_F = \sqrt{\sum_{i \in \Gamma} \|p^i(0)\|_2^2} \leq \beta \sqrt{K} \left(\delta_{K+1} \|\tilde{A}x\|_F + \epsilon_0\right).$$  
(37)
Using the triangle inequality, we can get
\[
\|P^T (0)\|_F = \|X^T (0) - \beta \tilde{A}_1^T (\tilde{A} X (0) - \tilde{Y})\|_F \\
= \|\beta \tilde{A}_1^T \tilde{Y}\|_F \\
\geq \beta \|\tilde{A}_1^T \tilde{A} X\|_F - \beta \|\tilde{A}_1^T (-EX + N)\|_F \\
\geq \beta \left( \sqrt{1 - \delta_{K+1}} \|\tilde{A} X\|_F - \sqrt{1 + \delta_{K+1} \epsilon_0} \right),
\]
where (40) is from (10) and the property of norm.

After straightforward manipulations, we have
\[
\|P^T (0)\|_F \\
\geq \beta \sqrt{K} (\delta_{K+1} \|\tilde{A} X\|_F + \epsilon_0) \\
+ \beta \left( \sqrt{1 - \delta_{K+1} - \sqrt{K} \delta_{K+1}} \right) \|\tilde{A} X\|_F \\
- \beta \left( \sqrt{1 + \delta_{K+1} + \sqrt{K}} \right) \epsilon_0 \\
\geq \beta \sqrt{K} (\delta_{K+1} \|\tilde{A} X\|_F + \epsilon_0) \\
+ (K - 1) \sqrt{1 + \delta_{K+1} \epsilon_0} \\
+ (K - \sqrt{K}) \epsilon_0 \\
\geq \beta \sqrt{K} (\delta_{K+1} \|\tilde{A} X\|_F + \epsilon_0),
\]
where (41) follows from (21) and (43) follows from \(\|X\|_F \geq \sqrt{\delta_{K+1}}\) and (22).

Obviously, (44) contradicts (37), so this fact guarantees (27).

Assume that GBCD always picks up indices from the support \(\Gamma\) for \(n \leq k\) (\(k \geq 1\) is an integer). Consider \(n = k+1\). In order to prove that GBCD can choose a correct index \(i_0 \in \Gamma\), analogous to [2], inequality (46) should be verified.
\[
\max_{j \in \Gamma^*} \|p^j (n-1) - x^j (n-1)\|_2 \geq \max_{j \in \Gamma^*} \|p^j (n-1) - x^j (n-1)\|_2.
\]
Combining step (3) of Algorithm 1 with (46) yields
\[
\max_{j \in \Gamma} \|\tilde{a}_i^j (\tilde{A} X (n-1) - \tilde{Y})\|_2 \\
> \max_{j \in \Gamma^*} \|\tilde{a}_i^j (\tilde{A} X (n-1) - \tilde{Y})\|_2.
\]

It is sufficient to prove that (48) holds. Note that \(\text{supp}(X(n-1)) \subseteq \Gamma\); we have
\[
\max_{j \in \Gamma} \|\tilde{a}_i^j (\tilde{A} X (n-1) - \tilde{Y})\|_2 \leq \delta_{K+1} \|\tilde{A} (X (n-1) - X)\|_F + \epsilon_0.
\]

Now, we only need to prove
\[
\max_{j \in \Gamma^*} \|\tilde{a}_i^j (\tilde{A} X (n-1) - \tilde{Y})\|_2 > \delta_{K+1} \|\tilde{A} (X (n-1) - X)\|_F + \epsilon_0.
\]
We then show that (52) is true by contradiction. For all \(i \in \Gamma^*\), assume that
\[
\|\tilde{a}_i^j (\tilde{A} X (n-1) - \tilde{Y})\|_2 \leq \delta_{K+1} \|\tilde{A} (X (n-1) - X)\|_F + \epsilon_0.
\]
Using the definition of Frobenius norm, we have
\[
\|\tilde{A}^j (\tilde{A} X (n-1) - \tilde{Y})\|_F = \sqrt{\sum_{i \in \Gamma} \|\tilde{a}_i^j (\tilde{A} X (n-1) - \tilde{Y})\|_2^2} \leq \sqrt{K} (\delta_{K+1} \|\tilde{A} (X (n-1) - X)\|_F + \epsilon_0).
\]
Combining \(\|X(n-1) - X\|_F \geq t_0\), (21), and (22), we have
\[
\|\tilde{A}^j (\tilde{A} X (n-1) - \tilde{Y})\|_F \geq t_0, (21), (22)
\]
\[
\geq \sqrt{1 - \delta_{K+1} \|\tilde{A} (X (n-1) - X)\|_F} - \sqrt{1 + \delta_{K+1} \epsilon_0} \\
= \sqrt{K} (\delta_{K+1} \|\tilde{A} (X (n-1) - X)\|_F + \epsilon_0) \\
+ \left( \sqrt{1 - \delta_{K+1} - \sqrt{K} \delta_{K+1}} \right) \|\tilde{A} (X (n-1) - X)\|_F \\
- \left( \sqrt{1 + \delta_{K+1} + \sqrt{K}} \right) \epsilon_0 \\
\geq \sqrt{K} (\delta_{K+1} \|\tilde{A} (X (n-1) - X)\|_F + \epsilon_0),
\]
where (59) follows from
\[
\left( \sqrt{1 - \delta_{K+1} - \sqrt{K} \delta_{K+1}} \right) \|\tilde{A} (X (n-1) - X)\|_F \\
+ \left( \sqrt{1 + \delta_{K+1} + \sqrt{K}} \right) \epsilon_0 \\
> \sqrt{K} (\delta_{K+1} \|\tilde{A} (X (n-1) - X)\|_F + \epsilon_0).
\]
This contradicts (53). Thus, (48) is true.

\(\square\)

Remark 6. The weaker the RIC bound is, the less required number of measurements we need, and the improved RIC results can be used in many CS-based applications [16]. In the work of [2], the authors provided that the condition for GBCD is \(\delta_{K+1} < 1/(\sqrt{K} + 1)\). Obviously, it is smaller than the bound \((\sqrt{4K + 1})/2K\) in (21).
4. The Counterexample

Consider the measurements
\[ \hat{Y} = (\hat{A} - E)X + N = \hat{A}X - EX + N. \] (62)

In this section, giving a matrix $\hat{A}$, whose RIC is a slight relaxation of $1/\sqrt{K+1}$, we will verify that GBCD can fail to recover the support of sparse matrix from (62).

Let
\[
X = \begin{pmatrix}
\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \cdots & \frac{\sqrt{2}}{2} \\
\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \cdots & \frac{\sqrt{2}}{2} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \cdots & \frac{\sqrt{2}}{2}
\end{pmatrix}_{(K+1) \times 2},
\]
\[
E = \begin{pmatrix}
0 & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & 0
\end{pmatrix}_{(K+1) \times (K+1)},
\]
\[
N = \begin{pmatrix}
\frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4} & \cdots & \frac{\sqrt{2}}{4} \\
\frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4} & \cdots & \frac{\sqrt{2}}{4} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4} & \cdots & \frac{\sqrt{2}}{4}
\end{pmatrix}_{(K+1) \times 2},
\]
where $\text{supp}(X) = \{1, 2, \ldots, K\} = \Gamma$, $\Gamma^c = \{K+1\}$ and $t_0/e > 1 + 1/K$ (the value of $e$ is far less than 1; this is reasonable).

The matrix $\hat{A}$ is constructed as
\[
\hat{A} = \begin{pmatrix}
a & 0 & 0 & \cdots & 0 & s \\
a & 0 & 0 & \cdots & 0 & s \\
a & 0 & 0 & \cdots & 0 & s \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & a & s \\
0 & 0 & 0 & \cdots & 0 & 1
\end{pmatrix}_{(K+1) \times (K+1)},
\]
where
\[
s = \frac{\delta}{\sqrt{K}}, \quad a = \sqrt{1 - \delta^2}.
\] (65)

Set
\[ \delta = \frac{\sqrt{1 - e/t_0}}{\sqrt{K+1}}. \] (66)

The eigenvalues $\{\lambda_i\}_{i=1}^{K+1}$ of $\hat{A}^* \hat{A}$ are
\[ \lambda_i = 1 - \delta^2, \quad 1 \leq i \leq K - 1, \]
\[ \lambda_K = 1 - \delta, \]
\[ \lambda_{K+1} = 1 + \delta. \] (67)

Thus, the RIC of $\hat{A}$ is $\delta_{K+1}(\hat{A}) = \delta$.

Recall that condition (27) is the criterion of recovery for GBCD. Note that
\[ \|p'(0)\|_2 = \|\beta a \hat{Y}\|_2. \]
One can obtain
\[ \max_{i \in \Gamma} \|\hat{a}_i \hat{Y}\|_2 = \min \left( \left\| \left( \frac{\sqrt{2}}{2} t_0 a^2, \frac{\sqrt{2}}{2} t_0 a^2 \right) \right\|_2, a^2 t_0 \right). \] (68)

On the other hand, we have
\[ \max_{j \in \Gamma^c} \|\hat{a}_j \hat{Y}\|_2 = \min \left( \left\| \left( \frac{\sqrt{2}}{2} t_0 K a s + \frac{\sqrt{2}}{2} t_0 K a s + \frac{\sqrt{2}}{2} \right) \right\|_2, K a s t_0 + \epsilon \right). \] (69)

It can be derived that
\[ K a s t_0 + \epsilon - t_0 a^2 = t_0 \left( a K s - a^2 \right) + \epsilon \]
\[ = t_0 \left( 1 - \delta^2 \sqrt{K} \delta - 1 + \delta^2 \right) + \epsilon \]
\[ > 0, \] (70)
where (71) and (72) follow from (65) and (66).

It is obviously in contradiction to (27). Thus, GBCD fails to recover support $\Gamma$.

Remark 7. In the work of [2], the authors presented a matrix $\hat{A}$ whose RIC is $\delta_{K+1}(\hat{A}) = 1/\sqrt{K} - \epsilon^2/\sqrt{K} (t_0^2 K + e^2)$. They showed that the GBCD algorithm fails when using $\hat{A}$ as measurement matrix. After a simple calculation, we can get
\[ \frac{\sqrt{1 - e/t_0}}{\sqrt{K+1}} < \frac{1}{\sqrt{K}} - \frac{\epsilon^2}{\sqrt{K} (t_0^2 K + e^2)}. \] (73)
Thus, our result improves this existing result.

5. Experimental Results

In this section, under the total perturbations, we test the performance of the GBCD algorithm for solving the DOA estimation problem.

Consider $K$ narrowband far-field point source signals impinging on an $m$-element uniform linear array. The steering vector of the matrix $A$ is
\[ a_i = \left[ 1, e^{-j \pi \cos \theta_{i-1}}, \ldots, e^{-j(m-1) \pi \cos \theta_{i-1}} \right]^T, \] (74)
where $1 \leq i \leq N$. $L$ is the number of snapshots.
Using the sparse optimization approach in [1], the DOA estimation problem can be rewritten as model (1). Then the aim is hence to find out which row of the matrix $X$ is nonzero, that is, the support of the matrix $X$.

Analogous to the simulation of [1], we have the following assumptions:

(i) The number of the array elements is $m = 11$.
(ii) The number of snapshots is $L = 200$.
(iii) The grid spacing is $1^\circ$ from $0^\circ$ to $180^\circ$. Then $N = 181$.
(iv) Five ($K = 5$) uncorrelated signals impinge from $\theta_{\ell} = 30^\circ$, $\theta_{\ell} = 80^\circ$, $\theta_{\ell} = 100^\circ$, $\theta_{\ell} = 120^\circ$, and $\theta_{\ell} = 145^\circ$.
(v) Both the signals and the noise are white and follow Gaussian distributions. The power of nonzero entries of $X$ is $\sigma^2$, and the power of each entry of $N$ is $\sigma_N^2$.
(vi) Use the following SNR1 and SNR2 to measure noises $E$ and $N$, respectively:

$$\text{SNR}_1 = 10 \log_{10} \left( \frac{\sigma^2}{\sigma_N^2} \right). \quad (75)$$

$$\text{SNR}_2 = \frac{\|A\|_F}{\|E\|_F}. \quad (76)$$

Define the root mean square error (RMSE) of 500 Monte Carlo trials as the performance index:

$$\text{RMSE} = \sqrt{\frac{1}{500K} \sum_{\ell=1}^{K} \sum_{k=1}^{K} (\hat{\theta}_{\ell}(k) - \theta_{\ell}(k))^2}, \quad (77)$$

where $\hat{\theta}_{\ell}(k)$ is the estimate of $\theta_{\ell}$ at the $k$th trial.

Figure 1, fixing matrix $E$, describes the performance of GBCD. The results show that RMSE decreases as SNR1 increases. Figure 2, fixing matrix $N$, describes the performance of GBCD. The results show that RMSE decreases as SNR2 increases. Thus, the performance of GBCD still is robust under the total perturbations.

6. Conclusion

In this paper, using the near orthogonality property, we provide a recovery condition for GBCD under the total perturbations. A counterexample is presented to show that GBCD fails. By experiments, we point out that GBCD is robust under the total perturbations.

Conflicts of Interest

The author declares that there are no conflicts of interest regarding the publication of this paper.

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