Research Article

Linear Precoding Scheme Design for MIMO Two-Way Relay Systems with Imperfect Channel State Information

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The aim of this paper is to investigate a linear precoding scheme design for a multiple-input multiple-output two-way relay system with imperfect channel state information. The scheme design is simplified as an optimal problem with precoding matrix variables, which is deduced with the maximum power constraint at the relay station based on the minimum mean square error criterion. With channel feedback delay at both ends of the channel and the channel estimation errors being taken into account, we propose a matrix decomposition scheme and a joint iterative scheme to minimize the average sum mean square error. The matrix decomposition method is used to derive the closed form of the relay matrix, and the joint iterative algorithm is used to optimize the precoding matrix and the processing matrix. According to numerical simulation results, the matrix decomposition scheme reduces the system bit error rate (BER) effectively and the joint iterative scheme achieves the best performance of BER against existing methods.

1. Introduction

Multiple-input multiple-output (MIMO) technology has become the key technology owing to its ability to increase system capacity and improve spectral efficiency without increasing the bandwidth [1]. The relay technology is a common space diversity method that can effectively reduce the effects of channel fading. The combination of MIMO with relay technology can further improve the capacity and system performance of the relay network. There has been extensive theoretical research on the performance improvement of unidirectional MIMO relay system [2, 3]. In fact, the MIMO two-way relay system has received great deal of attention recently owing to its high spectrum efficiency [4, 5].

Some existing studies have investigated for MIMO relay systems. In the actual communication system, the reliability of the communication link could be ensured by minimizing the mean square error (MSE) of the receiver. In references [6, 7] analysis under the condition of perfect channel state information (CSI), the precoding algorithm regards the MSE at two source nodes as the optimization target; the study [8] presents the complete CSI and the joint optimization method based on the criterion of MSE duality of two-way MIMO relay system. In [9], the precoding algorithm of amplify-and-forward MIMO two-way relay system based on channel capacity analysis is studied under limited channel feedback condition. However, the precoding scheme only considered the power constraints of the relay node without considering the power constraints of the transmitter. The paper [10] details the design of the combined precoding scheme for the two-way relay system, using the iterative method to optimize the combined precoding matrix. However, due to the time-varying wireless channel noise, the existence of the desired CSI is not realistic. References [11–13] researched for the amplify-and-forward two-way MIMO relay system, assuming that, in the relay terminal known source-relay, relay destination channel from the channel estimation errors and transmit antenna correlation proposed a joint transceiver design scheme based on the imperfect CSI. The study [14] presents the precoding algorithm with consideration channel feedback delay and channel estimation errors; however, it is only suitable for the MIMO system.

Although some excellent works about precoding design have been done for MIMO two-way relay networks [15–17], most of them have not taken the channel estimation errors and feedback delay into account. However, in practical
networks, due to the limitation of the channel estimation method and the existence of channel feedback delay, the ideal CSI is difficult to obtain. This motivates us to investigate the problem of optimal precoding design in MIMO two-way relay networks in the presence of channel estimation errors and feedback delay. In this paper, a MIMO two-way relay system is considered where the channel estimation error and feedback delay between relay node and the destination exist. At the relay node, the linear precoding design method is produced based on the minimum mean square error (MMSE) rule. Finally, the optimal precoding matrix and the detection matrix are derived using the theoretical proof.

2. System Model and Channel Model

2.1. System Model. Considering MIMO two-way relay system as shown in Figure 1, which consists of two source nodes $S_1$ and $S_2$ with one relay node denoted by $F$, both $S_1$ and $S_2$ are equipped with $n_s$ antennas. Relay node $F$ has $n_r$ antennas. At present, the research on the performance of MIMO two-way relay system is based on the model of time division duplex (TDD). During the first phase, source nodes $S_1$ and $S_2$ transmit their information to relay node $F$. In the second time slot, relay node multiplies its received signal by a linear precoding matrix $F \in \mathbb{C}^{n_r \times n_s}$ and forwards it to $S_1$ and $S_2$, respectively. It is assumed that all nodes are operating in half-duplex mode due to the hardware complexity and the relay node transmits and receives in two orthogonal time slots.

In the first time slot, source nodes $S_i$ $(i = 1, 2)$ transmit information $x_i \in \mathbb{C}^{n_s \times 1}$ to relay node, $x_i$ is satisfies with covariance matrix $R_{x_i} = \mathbb{E}[x_i x_i^H]$, $\mathbb{E}[*]$ donates the statistical expectation, and then the signal vector $y_i \in \mathbb{C}^{n_r \times 1}$ received at relay node is

$$y_i = H_i x_i + H_i x_j + n_i.$$  

Here $H_i \in \mathbb{C}^{n_r \times n_s}$ represents the channel matrix of $S_i$ to $F$ and $n_i \in \mathbb{C}^{n_r \times 1}$ denotes the additive white Gaussian noise (AWGN) vector with covariance matrix $R_{n_i} = \mathbb{E}[n_i n_i^H]$.

In the second time slot, relay node multiplies its received signal and forwards it to $S_1$ and $S_2$, respectively. The received signal $y_i \in \mathbb{C}^{n_r \times 1}$ at $S_i$ can be expressed as

$$y_i = H_i^H F H_j x_j + H_i^H F H_i x_i + H_i^H F n_r + n_i,$$  

where $j = 2$ if $i = 1$ and $j = 1$ if $i = 2$. $n_i \in \mathbb{C}^{n_r \times 1}$ denotes the AWGN vector with covariance matrix $R_{n_i} = \mathbb{E}[n_i n_i^H]$.

2.2. Channel Model. Considering the channel estimation error and channel feedback delay, the channel matrix at time instant $t$ can be expressed as $[18, 19]

$$H_i = \bar{H}_i + T_i + \Xi_i,$$  

where $\Xi_i$ denotes the channel estimation error matrix and $T_i$ is the delay estimation error matrix. With a certain amount of estimation error the channel matrix $H_i$ changes to be $\bar{H}_i$. In addition, after experiencing a period of time $\tau$ for feedback delay, $\bar{H}_i$ turns to be $\sqrt{\rho} \bar{H}_{i-\tau}$. Then the channel matrix $H_i$ can be written as $[19]

$$H_i = \sqrt{\rho} \bar{H}_{i-\tau} + T_i + \Xi_i,$$  

where $\rho$ is the channel correlation coefficient formulated as in [19]. For Clarke’s fading spectrum, $\rho$ could be obtained as $\rho = j_0(2\pi f_d \tau)$, and here $j_0(\cdot)$ is the zeroth-order Bessel function of the first kind and $f_d$ is the Doppler frequency $[20]$. In this paper, we discuss the channel matrix of source-relay and relay-source at a certain time, and correlation coefficient $\rho$ is related to the delay coefficient $\tau$. For the convenience of expression, $H_{ij}$ and $\bar{H}_{i-\tau,j}$ could be written as $H_i$ and $\bar{H}_i$. Therefore, our channel matrix could be further expressed as $[21]

$$H_i = \sqrt{\rho} \bar{H}_i + T_i + \Xi_i.$$  

Each element of $\Xi_i$ is independent of $T_i$ and its elements are subjected to $\mathbb{C}N(0, \sigma_i^2)$. $\mathbb{C}N(u, \sigma^2)$ represents the complex Gaussian distribution with a mean value of $u$ and a variance of $\sigma^2$. Let $\Sigma_i = T_i + \Xi_i$ and $H_i$ can be noted as

$$H_i = \sqrt{\rho} \bar{H}_i + \Sigma_i.$$  

where $\Sigma_i = (1 - \rho^2)(1 - \sigma_i^2)$ subject to $\mathbb{C}N(0, (1 - \rho^2)(1 - \sigma_i^2) + \sigma_i^2)$.

Let $\Sigma_i = (1 - \rho^2)(1 - \sigma_i^2) + \sigma_i^2$. The receiver signal $\bar{y}_j$ at $S_j$ is

$$\bar{y}_j = y_j - \rho \bar{H}_j H f \bar{H}_i x_i = H_j^H \bar{F} H_i x_i + H_j^H F n_r + n_i + k_i,$$  

where $k_i = (H_j^H \bar{F} H_i - \rho \bar{H}_j H f \bar{H}_i x_i$ is the residual self-interference (SI) with covariance matrix $R_k = \mathbb{E}[k_i k_i^H]$. When the detection matrix $Q_i$ is employed by $S_i$, the mean squared error (MSE) of the detected signal at $S_i$ can be expressed as below:

$$\text{MSE}_\Sigma[i, Q_i] = \mathbb{E} \left( \left\| Q_i \bar{y}_j - x_j \right\|_2^2 \right).$$  

3. Problem Formulation

In this paper, a joint design of the relay precoding matrix $F$ and the detection matrices $Q_1$ and $Q_2$ is proposed with the maximum transmission power constraint based on the MMSE criterion $[22]$. Taking the channel feedback delay and
estimation errors into account, the MSE expression of $S_i$ can be further represented as [23]

$$\text{MSE}_i (F, Q) = \text{Tr} \left\{ \varepsilon_{S_i} \left[ \| Q \tilde{Y}_i - x_i \|^2 \right] \right\}$$

$$= \text{Tr} \left\{ \varepsilon_{S_i} \left[ \left( Q \bar{H}^H F H \right) R_{x_i} \left( Q \bar{H}^H F H \right)^H \right] \right\}$$

$$+ \text{Tr} \left\{ \varepsilon_{S_i} \left( Q R_{x_i} Q^H \right) \right\} + \text{Tr} \left( R_{x_i} \right)$$

$$+ \text{Tr} \left( Q R_{x_i} Q^H \right),$$

where $\text{Tr}[*]$ is the trace of the matrix and $\text{Re}[*]$ denotes the real part. It is assumed that each of the elements in a random matrix $x \in C^{m \times n}$ is an independent and identically distributed zero mean complex Gaussian variable, $x$ is satisfied with $\text{Tr}(x x^H) = \sigma_x^2 I_n$, and here $I_n$ is an $n \times n$ unit matrix. We assume the variance of $S_i$ is much smaller than one. Hence, we have

$$\text{Tr} \left\{ \varepsilon_{S_i} \left( Q R_{x_i} Q^H \right) \right\}$$

$$= \rho_i \text{Tr} \left\{ \varepsilon_{S_i} \left[ Q \left( \bar{H}^H F \Sigma \right) R_{x_i} \left( \bar{H}^H F \Sigma \right)^H \right] \right\}$$

$$+ \Sigma^H \bar{H}^H F \Sigma R_{x_i} \Sigma^H \bar{H}^H F \Sigma + \bar{H}^H F \Sigma R_{x_i} \bar{H}^H F \Sigma$$

$$+ \bar{H}^H F \Sigma R_{x_i} \bar{H}^H F \Sigma Q^H \text{Tr} (R_{x_i})$$

$$+ \text{Tr} \left( F \bar{H}^H F \Sigma R_{x_i} \bar{H}^H F \Sigma Q^H \right) Q^H.$$

Here $Z_i = \sigma_x^2 I_n$. In the same way, some simplifications can be written as

$$\text{Tr} \left\{ \varepsilon_{S_i} \left[ \left( Q \bar{H}^H F H \right) R_{x_i} \left( Q \bar{H}^H F H \right)^H \right] \right\}$$

$$= \rho_i \rho_j \text{Tr} \left\{ Q \bar{H}^H F \Sigma R_{x_i} \bar{H}^H F \Sigma Q^H \right\}$$

$$+ \rho_i \text{Tr} \left\{ Q \bar{H}^H F \Sigma R_{x_i} \bar{H}^H F \Sigma Q^H \text{Tr} (R_{x_i}) \right\}$$

$$+ \rho_i \text{Tr} \left\{ Q \bar{H}^H F \Sigma R_{x_i} \bar{H}^H F \Sigma Q^H \right\}$$

$$+ \rho_i \text{Tr} \left\{ Q \bar{H}^H F \Sigma R_{x_i} \bar{H}^H F \Sigma Q^H \right\}$$

$$= \rho_i \left( \bar{H}^H F \Sigma R_{x_i} \bar{H}^H F \Sigma Q^H \right).$$

$$+ \text{Tr} \left\{ Q Z_i Q^H \right\} \text{Tr} (F \bar{H}^H F \Sigma R_{x_i} \bar{H}^H F \Sigma Q^H \right\}$$

$$= \text{Tr} \left\{ \varepsilon_{S_i} \left[ Q Z_i Q^H \right] \right\}$$

where $\Pi_j = \bar{H}^H R_{x_i} \bar{H}^H + \text{Tr} (R_{x_i}) Z_i$; substituting (10)–(11) into (9), the original expression (9) can be simplified as

$$\text{MSE}_i (F, Q) = \rho_i \text{Tr} \left\{ Q \bar{H}^H F \Sigma R_{x_i} \bar{H}^H F \Sigma Q^H \right\} + \text{Tr} (R_{x_i})$$

$$+ \text{Tr} \left( Q \Phi_i Q^H \right)$$

$$- 2 \sqrt{\rho_i} \text{Tr} \left\{ \text{Re} \left( Q \bar{H}^H F \Sigma R_{x_i} \bar{H}^H F \Sigma Q^H \right) \right\},$$

where

$$\Theta_i = \Pi_j + R_{x_i} + \text{Tr} (R_{x_i}) Z_i,$$

$$\Phi_i = \text{Tr} \left\{ F \left( \Pi_j + R_{x_i} + \rho_i \bar{H}^H F \Sigma R_{x_i} \bar{H}^H \right) F \right\} Z_i + R_{x_i}.$$

We are now ready to design detection matrix $Q_i$ and precoding matrix $F$ to minimize the average sum MSE of $S_i$. The problem can be formulated as

$$\min_{Q, Q_i, F} \text{MSE} = \text{MSE}_1 (F, Q_1) + \text{MSE}_2 (F, Q_2)$$

s.t. $\text{Tr} (F R_i F^H) \leq P_r$

and here

$$R_r = \varepsilon_{S_i} \left[ Y_i Y_i^H \right] = \Pi_1 + \Pi_2 + R_{x_i}.$$

4. The MMSE-Based Joint Precoding Scheme Design

4.1. Matrix Decomposition Algorithm Design. Since the joint iterative algorithm is being proposed, this paper introduces a kind of matrix decomposition algorithm with relatively low computational complexity. For (12), when the precoding matrix $F$ is determined, the optimal linear detection matrix $Q_i$ can be uniquely determined.

From the following condition ($\partial \text{MSE} / \partial Q_i |_{Q_i = Q_e} = 0$), $Q_e$ can be written as

$$Q_e = \rho_i \bar{H}^H F \Sigma R_{x_i} \bar{H}^H F \Sigma \left( \rho_i \bar{H}^H F \Sigma \right)^{-1}.$$

(16)
Substituting (16) into (12), then (12) can be expressed as

\[
\text{MSE}_{i}(F, Q_i) = \text{Tr} \left( R_{x_i} \right) - \rho_i \rho_{i}' \text{Tr} \left[ \left( R_{x_i} H_i^H F_i^H F_i H_i + \Theta_i \right)^{-1} \cdot \left( H_i^H F_i H_i + \Phi_i \right) \right] - \rho_i \text{Tr} \left( R_{x_i} H_i^H \Theta_i^{-1} H_i^H R_{x_i} \right) + \rho_i(F),
\]

where

\[
\rho_i(F) = \rho_i \text{Tr} \left[ R_{x_i} H_i^H \Theta_i^{-1/2} \left( \rho_i Q_i^{1/2} F_i^H \Theta_i^{-1/2} H_i^H F_i Q_i^{1/2} + I_{n_i} \right)^{-1} \cdot I_{n_i} \right]^{-1/2} H_i^{-1/2} R_{x_i} \right].
\]

Observing that only the term \( \rho_i(F) \) is related to \( F \), the optimization problem in (14) is equivalent to minimizing \( \rho_i(F) + \rho_i(F) \). Assuming \( \rho_i(F) \leq \overline{\rho}_i(F) \) by contraction method, where

\[
\overline{\rho}_i(F) = \rho_i \text{Tr} \left[ R_{x_i} H_i^H \Theta_i^{-1/2} \left( \rho_i Q_i^{1/2} F_i^H \Theta_i^{-1/2} H_i^H F_i Q_i^{1/2} + I_{n_i} \right)^{-1} \cdot I_{n_i} \right]^{-1/2} H_i^{-1/2} R_{x_i} \right] + \rho_i(F),
\]

and here \( \overline{\Phi}_i = P_i Z_i + R_{x_i} \), hence the original optimization problem (14) can be simplified as

\[
\min_{F} \text{MSE} = \overline{\rho}_i(F) + \overline{\rho}_i(F)
\]

subject to \( \text{Tr} \left( F R_{x_i} F^H \right) \leq P_r \).

Next, using matrix decomposition method to simplify the structure of \( \overline{\rho}_i(F) \), it can be rewritten as

\[
\overline{\rho}_i(F) = \text{Tr} \left[ X_i \left( \Theta_i^{1/2} F_i^H \Xi F_i \Theta_i^{1/2} + I_{n_i} \right)^{-1} \right]
\]

and here \( \Xi = H_i^H \overline{\Phi}_i^{-1} H_i \) and \( X_i = \Theta_i^{-1/2} R_{x_i} \Theta_i H_i^H \Theta_i^{-1/2} \). \( \Xi \) can be expressed as \( X_i = U_X A_X U_{X_i}^H \) by EVD decomposition, \( U_X \) is unitary matrix, and \( A_X \) is diagonal matrix. Expression (21) can be described as

\[
\overline{\rho}_i(F) = \rho_i \text{Tr} \left[ A_X Y_i Y_i^H \left( \Theta_i^{1/2} F_i^H \Xi F_i \Theta_i^{1/2} + I_{n_i} \right)^{-1} \cdot U_X \right],
\]

and here \( Y_i = \Theta_i^{1/2} U_X \). We propose the following structure:

\[
F = U_X A_F U_Y^{-1}.
\]

Here, \( U_X \) is unitary, \( A_F \) is \( n_r \times n_r \) diagonal, and \( U_Y \) is the nonsingular matrix obtained by the generalized singular value decomposition (GSVD) from \( Y_1 \) and \( Y_2 \). \( Y_i^H \) can be written as

\[
Y_i^H = V_Y A_i U_Y^H,
\]

where \( V_Y \) is unitary and \( A_i \) is diagonal. \( U_Y^H \) is obtained from the following singular value decomposition (SVD):

\[
\Xi_i^{-1/2} = \left( V_{i_1} V_{Y} \right) \left( \begin{array}{c} A_i I_{n_1} B_i U_{Y_i} \end{array} \right)^{-1} U_Y^H
\]

and here \( A_i = n_r \times n_r \) diagonal, from (25), and \( \Xi_i^{-1/2} = V_{i_1} A_i U_{Y_i} \). Substituting (23)–(25) into (22) gives

\[
\overline{\rho}_i(F) = \rho_i \text{Tr} \left[ A_i \left( \rho_i Y_i^H \Xi_i Y_i \right)^{1/2} \left( \rho_i Y_i^H \Xi_i Y_i + I_{n_i} \right)^{-1} \cdot I_{n_i} \right]^{-1/2} H_i^{-1/2} R_{x_i} \right] + \rho_i(F),
\]

and here \( A_i = V_Y A_i V_Y \). In order to achieve some simple solutions, we further relax the problem by only considering the main diagonal of \( V_{i_1} A_i V_Y \). Finally, the problem in (20) is simplified to the following problem:

\[
\min_{A_{i_1}} \rho_i \text{Tr} \left[ A_i \left( I_{n_1} + U_{Y_i} A_i U_{Y_i}^H \right)^{-1} \right]
\]

subject to \( \text{Tr} \left( D A_i^H \right) \leq P_r \),

where \( U_{Y_i} = \text{diag}(V_{i_1} V_Y) \) and \( D = U_{Y_i}^H R_{i_1} U_{i_1}^H \); the optimization problem (27) can be rewritten in scalar form as

\[
\min_{n=n_1} \sum_{n=1}^{n_2} \rho_i \rho_i \mu_{n_1} \leq P_r \leq \sum_{n=1}^{n_2} d_{n_1} \eta_{n_1} \eta_{n_1}
\]

where \( \eta_{n_1} \), \( \mu_{n_1} \), \( \alpha_{n_1} \), and \( d_{n_1} \) are the diagonal entries of \( A_{i_1}^H \), \( U_{i_1} A_{i_1}^H A_{i_1} \), \( A_i \), and \( D \). It is easy to recognize that (28) is a convex optimization problem [24,25]; translating the nonconvex problem of formula (28) into a convex optimization problem by Lagrange multiplier method, \( \lambda \) is the Lagrange multiplier, and the Lagrange function is constructed for

\[
L \left( \eta_{n_1}, \lambda \right) = \sum_{n=1}^{N} \rho_i \rho_i \mu_{n_1}^{\alpha_{n_1}} + \lambda \left( \sum_{n=1}^{N} d_{n_1} \eta_{n_1} - P_r \right).
\]

\[
\frac{\partial L}{\partial \eta_{n_1}} = 0 \text{ gives}
\]

\[
\eta_{n_1} = -\frac{2 \lambda d_{n_1} \mu_{n_1}}{4 \lambda \nu_{n_1} + 4 \lambda d_{n_1} \mu_{n_1} \nu_{n_1} - 4 \lambda d_{n_1} \mu_{n_1} \nu_{n_1}} - \lambda \left( \sum_{n=1}^{N} \rho_i \rho_i \mu_{n_1} \right)
\]

\[
= -\frac{1}{\sum_{n=1}^{N} \rho_i \rho_i \mu_{n_1}} + \sqrt{\frac{\sum_{n=1}^{N} \rho_i \rho_i \mu_{n_1} \nu_{n_1}}{\sum_{n=1}^{N} \rho_i \rho_i \mu_{n_1} \nu_{n_1}}}
\]

\[
= -\frac{1}{\sum_{n=1}^{N} \rho_i \rho_i \mu_{n_1}} + \sqrt{\frac{\sum_{n=1}^{N} \rho_i \rho_i \mu_{n_1} \nu_{n_1}}{\sum_{n=1}^{N} \rho_i \rho_i \mu_{n_1} \nu_{n_1}}}
\]
\[ \lambda \text{ is in accord with} \]
\[
\lambda = \sum_{i=1}^{2} \frac{\rho \rho^T_\text{in} d_n}{1 + \mu \eta_n} \leq \frac{\lambda}{\delta} \frac{\rho \rho^T_\text{in} d_n}{1 + \mu \eta_n} 
\]
\[
\leq \max \left\{ \frac{\sum_{i=1}^{2} \rho \rho^T_\text{in} d_n}{1 + \mu \eta_n}, \forall = 1, 2, \ldots, n \right\} \quad (31)
\]
and then
\[
0 \leq \lambda \leq \max \left\{ \frac{\sum_{i=1}^{2} \rho \rho^T_\text{in} d_n}{1 + \mu \eta_n}, \forall = 1, 2, \ldots, n \right\} \quad (32)
\]
\[ \lambda \text{ should be chosen from bisection algorithm and satisfied with} \]
\[ \sum_{i=1}^{n} \delta \delta_i = P_x, \eta_i \text{ could be obtained by substituting} \lambda \]
\[ \text{into (30)}, \text{and then} \Lambda_1^T, F, Q_1, \text{and} Q_2 \text{will be gotten as follows.} \]

### 4.2. Joint Iterative Algorithm Design

When the detection matrix \( Q \) is determined, the optimization problem of (14) can be translated into a convex optimization problem of relay precoding matrix \( F \). The Lagrangian function of (14) is formulated as
\[
L(F, \lambda) = \min_{Q_1, Q_2, F} \lambda \left[ \text{Tr} \left( FRF^H \right) - P_r \right]. \quad (33)
\]
Seeking partial derivative of \( F^* \)
\[
\frac{\partial L}{\partial F} |_{F=\tilde{F}} = \sum_{i=1}^{2} \rho \tilde{H}_i Q_i^H \tilde{Q}_i^H F \Theta_i, \quad (34)
\]
\[
\lambda \left[ \text{Tr} \left( FRF^H \right) - P_r \right] = 0, \quad (36)
\]
\[
\text{Tr} \left( FRF^H \right) \leq P_r \quad (37)
\]
and from (34) and (35) one has
\[
\text{vec}(F) = \Gamma^{-1} \text{vec}(Q), \quad (38)
\]
where
\[
\Gamma = \sum_{i=1}^{2} \Theta_i^T \left( \rho \tilde{H}_i Q_i^H Q_i \right) H_i^T + \sum_{i=1}^{2} \Theta_i^T \text{vec}(Q_i) F \Theta_i^T \quad (39)
\]
\[
B = \sum_{i=1}^{2} \sqrt{\rho \rho_i} \tilde{H}_i Q_i^H R_{x_i} \tilde{H}_i^T \quad (40)
\]
The Lagrange multiplier is determined by (36) and (37). It satisfies [26]
\[
0 \leq \lambda \leq \lambda_1 \left( \text{vec}(B) \text{vec}^T(B) \right) \lambda_1 \left( R_r \right) \quad (40)
\]
and here \( \odot \) denotes Kronecker product, \( \lambda_1(A) \) represents the maximum eigenvalue of the matrix \( A \), and \( \lambda \) can be obtained by bisection algorithm with its upper and lower bounds. By (17) and (38), we can see that when the signal matrix \( x \) of the source node is certain, the relay matrix and the detection matrix are correlated. A joint iterative design algorithm is proposed as shown in Algorithm 1.

### 5. Simulation Results and Analysis

In this section, simulation results are provided to evaluate the performance of the proposed approaches in a flat Rayleigh fading environment. We consider that the source nodes \( S_1 \) and \( S_2 \) are equipped with \( n_s = 4 \) antennas. Relay node \( F \) is equipped with \( n_e = 4 \) antennas. The source node transmits independent uncoded QPSK symbol streams to the corresponding receiver. The covariance of the transmit signal at source nodes \( S_1 \) and \( S_2 \) is \( R_r = R_{s_i} = (P_s/n_i)I_{n_i} \) and here \( P_s \) is the transmit power at each source node; let \( P_s = n_i \). The covariance of the AWGN is satisfied with \( R_n = \sigma_n^2 I_{n_e} \) and \( R_{n_i} = R_{n_i} = \sigma_n^2 I_{n_i} \). We investigate the scenarios with
various SNR$_1 = P_s/n_s \sigma^2$, while fixing SNR$_2 = P_r/n_r \sigma^2$. SNR$_1$ and SNR$_2$ are the average signal-to-noise ratio (SNR) for the first phase and the second phase of the two-way communication protocol. The results are averaged over 5000 independent channel realizations, and in each realization 2000 QPSK symbols are transmitted for each data stream. In order to verify the superiority of the proposed algorithm, the proposed methods are compared with the existing methods as follows:

(i) Amplify-and-forward (AF) relay approach [27] is

$$F_{AF} \propto \sqrt{P_r} \text{Tr}(R_x) I_{n_r}. \quad (41)$$

(ii) Zero-forcing (ZF) relay approach [28] is

$$F_{ZF} = \left( \sum_{i=1}^{2} \rho_i \tilde{H} Q^H H \Theta_i + \lambda R_x \right)^{-1} \cdot \left( \sum_{i=1}^{2} \sqrt{\rho_i \rho_j} \tilde{H} Q^H R_x \tilde{H}^H \right), \quad Q \propto I_{n_r}. \quad (42)$$

Figures 2 and 3 are given the performance comparison of BER while fixing estimated errors $\sigma^2 = \sigma^2 = \sigma^2$ and delay coefficients $f_d \tau_1 = f_d \tau_2 = f_d \tau$. Simulation parameters are as follows: iterations $\text{Iter}_{\max} = 30$ and convergence threshold $\xi = 10^{-4}$.

Figure 4 is given the performance comparison of BER with different iteration numbers. Simulation parameters are as follows: $\sigma^2 = 0.002$, $f_d \tau = 0.02$, and convergence threshold $\xi = 10^{-4}$.

Figures 5 and 6 are the BER performance comparisons in the two kinds of SNR while fixing feedback delay coefficient $f_d \tau = 0.02$ and estimation error variance $\sigma^2 = 0.02$, respectively. The simulation parameters are as follows: iterations $\text{Iter}_{\max} = 30$ and convergence threshold $\xi = 10^{-4}$.

From the above simulations, the following conclusions can be drawn:

(1) From Figures 2 and 3, the joint iterative algorithm achieved 3.2 dB and 4 dB performance gain compared to the ZF scheme and AF scheme while fixing $f_d \tau = 0.05$, $\sigma^2 = 0.005$, and BER = 0.05, respectively. The matrix decomposition algorithm could obtain 2 dB and 3 dB performance gain compared to the ZF scheme and AF scheme. And it is obvious that matrix decomposition algorithm could get the optimal system BER performance when the channel estimation error and feedback delay coefficient are certain in low SNR conditions. As the SNR increases to 10 dB, the joint iterative algorithm obtains the best BER.
Amplify-and-forward
Zero-forcing
Matrix decomposition
Joint Iterative optimization

Figure 5: BER performance comparisons with different $\sigma^2$.

Figure 6: BER performance comparisons with different $f_d \tau$.

(2) As shown in Figure 4, the system BER performance could be improved by 1-2 dB with the increase of the number of iterations. However, this increase is limited. When the iteration number is more than 30 times, BER remains almost constant. By defining the threshold value of iterations as 30, the joint iterative method can obtain the best performance.

(3) From Figures 5 and 6, we find that the system BER performance has a downward trend with the increase of the feedback delay coefficient or the channel estimation error; however, the proposed scheme achieves the best BER performance because it has a certain extent compensated with feedback delay coefficient and estimation error.

6. Conclusion

This paper investigates a linear precoding scheme design for the MIMO two-way relay system with imperfect CSI. The transceiver design is simplified as an optimal problem with precoding matrix variables, which is deduced with the maximum power constraint at the relay station based on the MMSE criterion. With channel feedback delay at both ends of the channel and the channel estimation errors being taken into account, a matrix decomposition scheme and a joint iterative scheme are proposed to minimize the BER. Numerical simulation results show that the matrix decomposition scheme can improve the system BER effectively and the joint iterative scheme can achieve the best BER performance with the increase of SNR against existing methods.

Conflicts of Interest

The authors declared that they have no conflicts of interest to this work. We declare that we do not have any commercial or associative interest that represents conflicts of interest in connection with the work submitted.

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