Research Article

Novel Stability Analysis for Uncertain Neutral-Type Lur'e Systems with Time-Varying Delays Using New Inequality

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This paper considers the delay-dependent stability analysis of neutral-type Lur'e systems with time-varying delays and sector bounded nonlinearities. First of all, using constructed function methods, a new Jensen-like inequality is introduced to obtain less conservative results. Second, a new class of Lyapunov-Krasovskii functional (LKF) is constructed according to the characteristic of the considered systems. Third, combining with the new inequality and reciprocal convex approach and some other inequality techniques, the new less conservative robust stability criteria are shown in the form of linear matrix inequalities (LMIs). Finally, three examples demonstrate the feasibility and the superiority of our methods.

1. Introduction

Delay phenomenon is often encountered in many practical systems, such as biological systems, chemical systems, electronic systems, and network control systems. However, time-delay is usually the main cause of instability and bad performance. Hence, many authors devote themselves to studying the stability and many effective methods of the time-varying system to gain less conservative delay-dependent stability criteria [1–11], which include linear systems with the delay-fraction theory [2, 3] and nonlinear systems Lur'e systems [6–8]. As is known to all, delay-dependent stability results are less conservative than the delay-independent ones if delay size is very small. Therefore, a lot of articles were published recently which studied the delay-dependent stability for a class of neutral-type Lur'e systems with time-varying delays and sector bounded nonlinearities, and lots of significant results have been developed [12–30].

Delay-dependent stability criteria were presented for nominal and uncertain neutral-type Lur'e systems with constant time delays and sector bounded nonlinearities in [27]. The robust stability problems for neutral-type Lur'e systems with

time-varying delays were considered because time delays vary always depending on time in [12–21, 27–30]. The free-weighting matrix method was applied to get less conservative stability criteria and to deal with the robust stability problems for neutral-type Lur'e systems with time-varying delays in [15, 16]. However, the free-weighting matrix method brings more variables which make the computation quite complex. So, it is the right time to improve the disadvantage of the free-weighting matrix method and to get less conservative stability criteria for neutral-type Lur'e systems with time-varying delays. Some new robust stability criteria were proposed without using the general free-weighting matrix method which are less conservative and easier to calculate than some previous ones in [15, 19]. Reference [31] can reduce conservatism by reducing the conservatism of the Jensen-like inequality. Motivated by [31], developing the Jensen-like inequality with double-integral term may reduce the conservativeness. As a result, less conservative criteria may be also got by constructing new integral inequalities used in LKF.

This paper studies the stability for a class of neutral-type Lur'e systems with time-varying delays and sector bounded

nonlinearities. To investigate the neutral-type Lur'e system, this paper introduces a new triple-integral inequality used in the following LKFs and gets less conservative criteria. The LKF contains not only double-integral terms but also triple-integral terms. Using some effective techniques, such as a novel integral inequality, a piecewise analysis method, and the reciprocally convex combination inequality, instead of the general free-weighting matrix method, the delay-dependent stability criteria derived in the form of LMIs are less conservative than some existing results in other papers. The effectiveness and the less conservatism of stability criteria proposed in this paper are demonstrated by using numerical examples in Section 4.

Notation. R^n denotes the n -dimensional Euclidean vector space, and $R^{n \times m}$ denotes the set of all $n \times m$ real matrices. For a symmetric matrix P , $P > 0$ (resp., $P < 0$) shows that P is a positive (resp., negative) definite matrix. $\text{diag}\{a_1, a_2, \dots, a_n\}$ represents a diagonal matrix with diagonal elements $a_1, a_2, \dots, a_n$. $*$ denotes a symmetric term in a symmetric matrix.

2. Problem Statement and Preliminaries

Consider a class of Lur'e systems of uncertain neutral type with time-varying delays and sector-bound nonlinearities described by the following equation:

$$\begin{aligned} \dot{x}(t) - C\dot{x}(t - \tau(t)) &= [A + \Delta A(t)]x(t) \\ &\quad + [A_1 + \Delta A_1(t)]x(t - h(t)) \\ &\quad + [B + \Delta B(t)]f(\sigma(t)), \\ \sigma(t) &= H^T x(t), \quad \forall t \geq 0, \\ x(s) &= \phi(s), \\ \dot{x}(s) &= \dot{\phi}(s), \\ s &\in [-\max(h, \tau), 0], \end{aligned} \quad (1)$$

where $\sigma(t) \in R^m$ and $x(t) \in R^n$ stand for the output and state vectors, respectively. $\phi(s) \in R^n$ is a real-valued continuous initial function on $[-\max(h, \tau), 0]$. A , A_1 , B , C , and H are known real constant matrices with appropriate dimensions. $\Delta A(t)$, $\Delta A_1(t)$, and $\Delta B(t)$ represent the time-varying uncertainty parameters. $f(\sigma(t)) \in R^m$ is the nonlinear function such as

$$f(\sigma(t)) = [f_1(\sigma_1(t)) \ f_2(\sigma_2(t)) \ \cdots \ f_m(\sigma_m(t))]^T, \quad (2)$$

where $\sigma_i(t)$ is the i th component of the output vector $\sigma(t)$, and each term $f_i(\sigma_i(t))$ ($i = 1, 2, \dots, m$) satisfies the finite sector condition shown in Figure 1(a) [32]

$$\begin{aligned} f_i(\sigma_i(t)) &\in K_{[0, k_i]} = \{f_i(\sigma_i(t)) \mid f_i(0) = 0, \ 0 < \sigma_i(t) \\ &\quad \cdot f_i(\sigma_i(t)) \leq k_i \sigma_i^2(t), \ \sigma_i(t) \neq 0\} \end{aligned} \quad (3)$$

with known positive scalars k_i or the infinite sector condition shown in Figure 1(b) [32]

$$\begin{aligned} f_i(\sigma_i(t)) &\in K_{[0, \infty)} = \{f_i(\sigma_i(t)) \mid f_i(0) = 0, \ \sigma_i(t) \\ &\quad \cdot f_i(\sigma_i(t)) > 0, \ \sigma_i(t) \neq 0\}. \end{aligned} \quad (4)$$

$\Delta A(t)$, $\Delta A_1(t)$, and $\Delta B(t)$ are assumed to satisfy the following condition:

$$[\Delta A(t) \ \Delta A_1(t) \ \Delta B(t)] = DF(t) [E_a \ E_{a1} \ E_b], \quad (5)$$

where D , E_a , E_{a1} , and E_b are known constant matrices with appropriate dimensions, and the uncertainty time-varying matrix $F(t)$ satisfies

$$F^T(t)F(t) \leq I, \quad \forall t \geq 0. \quad (6)$$

The time-varying delays $h(t)$ and $\tau(t)$ are continuous functions satisfying the following conditions:

$$\begin{aligned} 0 &\leq h(t) \leq h, \\ \dot{h}(t) &\leq h_d, \\ 0 &\leq \tau(t) \leq \tau, \\ \dot{\tau}(t) &\leq \tau_d < 1, \\ \forall t &\geq 0. \end{aligned} \quad (7)$$

This paper investigates the delay-dependent stability of Lur'e system (1) satisfying conditions (3) (or (4)) and (5)–(7) to gain less conservative robust stability criteria by using a new inequality and constructing a new LKF. Throughout this paper, the results will be acquired by assuming that all the eigenvalues of C are inside the unit circle [33]. The following lemmas are useful in deriving criteria.

Lemma 1 (see [34] (reciprocally convex combination)). *Let $f_1, f_2, \dots, f_N : R^m \mapsto R$ have positive values in an open subset D of R^m . Then, the reciprocally convex combination of f_i over D satisfies*

$$\begin{aligned} \min_{\{\alpha_i \mid \alpha_i > 0, \sum_i \alpha_i = 1\}} \sum_i \frac{1}{\alpha_i} f_i(t) &= \sum_i f_i(t) + \max_{g_{i,j}(t)} \sum_{i \neq j} g_{i,j}(t) \\ \text{subject to } \left\{ g_{i,j} : R^m \mapsto R, g_{j,i}(t) \triangleq g_{i,j}(t), \begin{bmatrix} f_i(t) & g_{i,j}(t) \\ g_{i,j}(t) & f_j(t) \end{bmatrix} \geq 0 \right\}. \end{aligned} \quad (8)$$

FIGURE 1: Two sector bounded nonlinearity.

Lemma 2 (see [31]). For a given matrix $R > 0$, any differentiable function ω in $[a, b] \rightarrow \mathbb{R}^n$, the following inequality holds:

$$\begin{aligned} \int_a^b \dot{\omega}^T(u) R \dot{\omega}(u) du &\geq \frac{1}{b-a} (\omega(b) - \omega(a))^T R \\ &\quad \cdot (\omega(b) - \omega(a)) \\ &\quad + \frac{3}{b-a} \Omega_1^T R \Omega_1, \end{aligned} \quad (9)$$

where

$$\Omega_1 = \omega(b) + \omega(a) - \frac{2}{b-a} \int_a^b \omega(u) du. \quad (10)$$

Remark 3. It is clear that this new inequality encompasses the Jensen inequality. It is also worth noting that it plays an important role in getting the derivative of the LKF. Hence, by using optimization theory and improving the Jensen-like inequality, the double-integral inequality would be got in the following lemma.

Lemma 4. For any matrix $R > 0$, scalars $h_2 \geq h_1 \geq 0$, and any continuous function ω in $\mathbb{R} \rightarrow \mathbb{R}^n$ the following inequality holds:

$$\begin{aligned} & - \int_{-h_2}^{-h_1} \int_{t+\theta}^t \omega^T(u) R \omega(u) du d\theta \\ & \leq - \frac{2}{h_2^2 - h_1^2} \left(\int_{-h_2}^{-h_1} \int_{t+\theta}^t \omega(u) du d\theta \right)^T R \\ & \quad \cdot \left(\int_{-h_2}^{-h_1} \int_{t+\theta}^t \omega(u) du d\theta \right) - \frac{48}{h_2^4 - h_1^4} \Omega_2^T R \Omega_2, \end{aligned} \quad (11)$$

where

$$\begin{aligned} \Omega_2 &= \frac{h_1}{2} \int_{-h_2}^{-h_1} \int_{t+\theta}^t \omega(u) du d\theta \\ & \quad + \frac{1}{2} \int_{-h_2}^{-h_1} \int_{-h_2}^\theta \int_{t+r}^t \omega(u) du dr d\theta \\ & \quad - \int_{-h_2}^{-h_1} \int_{t+\theta}^t \int_{t+\theta}^u \omega(s) ds du d\theta. \end{aligned} \quad (12)$$

Proof. Recall that the objectives of the present paper lemma are to acquire new lower bounds of the integral $\int_{-h_2}^{-h_1} \int_{t+\theta}^t \omega^T(u) R \omega(u) du d\theta$ and to compare with the Jensen-like inequality. For this reason, the appropriate function $z(u)$ is given as the following form:

$$\begin{aligned} z(u) &= \omega(u) - \frac{2}{h_2^2 - h_1^2} \int_{-h_2}^{-h_1} \int_{t+\theta}^t \omega(u) du d\theta \\ & \quad - \frac{48 f(u, \theta)}{h_2^4 - h_1^4} \Omega_2, \end{aligned} \quad (13)$$

where $f(u, \theta) = u - (1/2)\theta - t$ is constructed to satisfy

$$\int_{-h_2}^{-h_1} \int_{t+\theta}^t f(u, \theta) du d\theta = 0 \quad (14)$$

and to get rid of the effect of cross terms. It is easy to obtain

$$\begin{aligned} & \int_{-h_2}^{-h_1} \int_{t+\theta}^t f^2(u, \theta) du d\theta = \frac{h_2^4 - h_1^4}{48}, \\ & \int_{-h_2}^{-h_1} \int_{t+\theta}^t \omega(u) f(u, \theta) du d\theta \\ & = \frac{h_1}{2} \int_{-h_2}^{-h_1} \int_{t+\theta}^t \omega(u) du d\theta \\ & \quad - \int_{-h_2}^{-h_1} \int_{t+\theta}^t \omega(u) (t - u) du d\theta \\ & \quad - \frac{1}{2} \int_{-h_2}^{-h_1} \int_{t+\theta}^t \omega(u) (h_1 + \theta) du d\theta \\ & = \frac{h_1}{2} \int_{-h_2}^{-h_1} \int_{t+\theta}^t \omega(u) du d\theta \\ & \quad + \frac{1}{2} \int_{-h_2}^{-h_1} \int_{-h_2}^\theta \int_{t+r}^t \omega(u) du dr d\theta \\ & \quad - \int_{-h_2}^{-h_1} \int_{t+\theta}^t \int_{t+\theta}^u \omega(s) ds du d\theta = \Omega_2. \end{aligned} \quad (15)$$

Then, based on the formulas above, the following equality holds:

$$\begin{aligned}
& \int_{-h_2}^{-h_1} \int_{t+\theta}^t z^T(u) R z(u) du d\theta \\
&= \int_{-h_2}^{-h_1} \int_{t+\theta}^t \omega^T(u) R \omega(u) du d\theta \\
&- \frac{2}{h_2^2 - h_1^2} \left(\int_{-h_2}^{-h_1} \int_{t+\theta}^t \omega(u) du d\theta \right)^T \\
&\cdot R \left(\int_{-h_2}^{-h_1} \int_{t+\theta}^t \omega(u) du d\theta \right) + \frac{48^2}{(h_2^4 - h_1^4)^2} \\
&\cdot \int_{-h_2}^{-h_1} \int_{t+\theta}^t f^2(u, \theta) du d\theta \Omega_2^T R \Omega_2 \\
&- \frac{2 \cdot 48}{h_2^4 - h_1^4} \left(\int_{-h_2}^{-h_1} \int_{t+\theta}^t \omega(u) f(u, \theta) du d\theta \right)^T R \Omega_2 \quad (16) \\
&+ \frac{4 \cdot 48}{(h_2^4 - h_1^4)(h_2^2 - h_1^2)} \int_{-h_2}^{-h_1} \int_{t+\theta}^t f(u, \theta) du d\theta \\
&\cdot \left(\int_{-h_2}^{-h_1} \int_{t+\theta}^t \omega(u) du d\theta \right)^T R \Omega_2 \\
&= \int_{-h_2}^{-h_1} \int_{t+\theta}^t \omega^T(u) R \omega(u) du d\theta \\
&- \frac{2}{h_2^2 - h_1^2} \left(\int_{-h_2}^{-h_1} \int_{t+\theta}^t \omega(u) du d\theta \right)^T \\
&\cdot R \left(\int_{-h_2}^{-h_1} \int_{t+\theta}^t \omega(u) du d\theta \right) - \frac{48}{h_2^4 - h_1^4} \Omega_2^T R \Omega_2.
\end{aligned}$$

For a given symmetric positive definite matrix $R > 0$, then, the left-hand side of formula (16) above is positive definite. It is obvious that Lemma 4 is correct. $\square$

Remark 5. Compared with Lemma 1 in [35], the inequality in Lemma 4 has two advantages. First, since $R > 0$, the second term in the right side of the inequality in Lemma 4 is definite negative. It thus implies that the Jensen-like inequality in [35] has been included in the inequality proposed above. Second, it is also worth noting that this improvement is allowed by using extra signals $\int_{-h_2}^{-h_1} \int_{t+\theta}^t \omega(u) du dr d\theta$, $\int_{-h_2}^{-h_1} \int_{t+\theta}^t \omega(s) ds du d\theta$, and $\int_{-h_2}^{-h_1} \int_{t+\theta}^t \omega(u) du d\theta$.

Remark 6. It is worth paying attention to the fact that the construction of function f is the key to Lemma 4. $\int_{-h}^0 \int_{t+\theta}^t f du d\theta = 0$ must be sure when we construct the function f to get rid of crossing items and to make the second term in the right side of our inequality be definite negative.

Remark 7. In fact, the result of Lemma 4 is less conservative than formula (3) in literature [36]. When $h_1 \neq 0$, the result of Lemma 4 is different from inequality (16) in literature [37]. What is more, under the special scaling condition of the left side of the inequality in Lemma 4, the more conservativeness of inequality (16) in literature [37] would be easily concluded. To some extent, the conditions of Lemma 4 are wider than inequality (16) in literature [37] in terms of integral domain. Actually, with the help of delay decomposition technology, $\int_{-h/2}^0 \int_{t+\theta}^t f(s) ds d\theta$ and $\int_{-h}^{-h/2} \int_{t+\theta}^t f(s) ds d\theta$ can be scaled properly with our inequality in Lemma 4. In our future research, we will try to use formulas in the literature [37] which may be more flexible than the Lemma 4 to reduce the conservatism of stability criteria in another case.

3. Main Results

To accept easily the robust stability problem of system (1), firstly, we investigate the stability criterion of the nominal form:

$$\begin{aligned}
\dot{x}(t) - C\dot{x}(t - \tau(t)) &= Ax(t) + A_1x(t - h(t)) \\
&+ Bf(\sigma(t)), \\
\sigma(t) &= H^T x(t), \quad \forall t \geq 0, \\
x(s) &= \phi(s), \\
\dot{x}(s) &= \dot{\phi}(s), \\
s &\in [-\max(h, \tau), 0].
\end{aligned} \quad (17)$$

The stability of system (17) can be analyzed via time-independent functions of the form

$$V(x(t)) = \sum_{i=1}^5 V_i(x(t)), \quad (18)$$

where

$$\begin{aligned}
V_1(x(t)) &= \xi_1^T(t) P \xi_1(t) + 2 \sum_{i=1}^m \lambda_i \int_0^{\sigma_i(t)} f_i(\sigma) d\sigma \\
V_2(x(t)) &= \int_{t-h/2}^t \begin{bmatrix} x(s) \\ x\left(s - \frac{h}{2}\right) \end{bmatrix}^T \\
&\cdot \begin{bmatrix} X_{11} & X_{12} \\ * & X_{22} \end{bmatrix} \begin{bmatrix} x(s) \\ x\left(s - \frac{h}{2}\right) \end{bmatrix} ds \\
V_3(x(t)) &= \int_{t-h(t)}^t x^T(s) Q_1 x(s) ds + \int_{t-\tau(t)}^t \dot{x}^T(s) \\
&\cdot Q_2 \dot{x}(s) ds
\end{aligned}$$

$$\begin{aligned}
V_4(x(t)) &= \frac{h^2}{8} \\
&\cdot \int_{-h/2}^0 \int_{t+\theta}^t (s-t-\theta) \dot{x}^T(s) Q_3 \dot{x}(s) ds d\theta + \frac{3h^2}{8} \\
&\cdot \int_{-h}^{-h/2} \int_{t+\theta}^t (s-t-\theta) \dot{x}^T(s) Q_4 \dot{x}(s) ds d\theta
\end{aligned}$$

$$\begin{aligned}
V_5(x(t)) &= \frac{h}{2} \int_{-h/2}^0 \int_{t+\theta}^t \dot{x}^T(s) Q_5 \dot{x}(s) ds d\theta + \frac{h}{2} \\
&\cdot \int_{-h}^{-h/2} \int_{t+\theta}^t \dot{x}^T(s) Q_6 \dot{x}(s) ds d\theta,
\end{aligned}$$

(19)

and define

$$\begin{aligned}
\xi_1^T(t) &= \left[x^T(t) \int_{t-h/2}^t x^T(s) ds \int_{t-h}^{t-h/2} x^T(s) ds \int_{-h/2}^0 \int_{t+\theta}^t x^T(s) ds d\theta \int_{-h}^{-h/2} \int_{t+\theta}^t x^T(s) ds d\theta \right], \\
\xi^T(t) &= \left[x^T(t) \ x^T(t-h(t)) \ x^T\left(t-\frac{h}{2}\right) \ x^T(t-h) \ \dot{x}^T(t-\tau(t)) \ f^T(\sigma(t)) \int_{t-h/2}^t x^T(s) ds \int_{t-h}^{t-h/2} x^T(s) ds \int_{-h/2}^0 \int_{t+\theta}^t x^T(s) ds d\theta \int_{-h}^{-h/2} \int_{t+\theta}^t x^T(s) ds d\theta \right].
\end{aligned}$$

(20)

Remark 8. It is worth noticing that the construction of LKF can be improved by adding

$$\begin{aligned}
&\frac{h}{2} \int_{-h/2}^0 \int_{t+\theta}^t x^T(s) N_1 x(s) ds d\theta, \\
&\frac{h}{2} \int_{-h}^{-h/2} \int_{t+\theta}^t x^T(s) N_2 x(s) ds d\theta
\end{aligned}$$

(21)

to get less conservative stability criteria for any positive definite matrices N_1 and N_2 . However, our methods have reduced the conservatism of the existing results in the instance. Hence, this paper does not add N_1 and N_2 to control the number of variables.

Remark 9. A new Lyapunov-Krasovskii functional is constructed in Section 3 to obtain a new delay-dependent stability criterion, which includes the relationship among the states, the nonlinear function, and the derivative of the states. In order to fully reduce the conservatism of the condition, the constant delay h is decomposed into $h/2$ and thus (18) is obtained. Particularly, the function V_1 is constructed to eliminate the effects among different forms of the states; the function V_2 is constructed to eliminate the relationship among the states with different delays; to consider the time-varying delay the function V_3 is constructed; and our Lyapunov-Krasovskii functional contains double-integral (function V_4) and triple-integral (function V_5) terms which yield less conservative delay-dependent stability criteria. Therefore, more information for the state is employed in constructing the more general Lyapunov-Krasovskii functionals, which may lead to reduced conservatism.

If nominal system (17) satisfies conditions (3) and (7) the following theorem can be got.

Theorem 10. Nominal system (17) satisfying conditions (3) and (7) is asymptotically stable for given values $h \geq 0$, $h_d, \tau_d < 1$, and $k_l > 0$ ($l = 1, 2, \dots, m$), if there exist appropriate dimensional matrices X_1, X_2, X_{12} , positive matrices $P = [P_{ij}]_{5 \times 5}$, Q_i ($i = 1, 2, \dots, 6$), X_{11}, X_{22} positive

semidefinite diagonal matrices $S = \text{diag}\{s_1, s_2, \dots, s_m\}$, and $\Lambda = \text{diag}\{\lambda_1, \lambda_2, \dots, \lambda_m\}$ such that the following LMIs hold:

$$\begin{bmatrix} Q_5 & X_1 \\ * & Q_5 \end{bmatrix} > 0,$$

(22)

$$\begin{bmatrix} Q_6 & X_2 \\ * & Q_6 \end{bmatrix} > 0,$$

(23)

$$\begin{bmatrix} X_{11} & X_{12} \\ * & X_{22} \end{bmatrix} > 0,$$

(24)

$$\Xi_r = \begin{bmatrix} E + \Theta_1 + \Psi_r & \eta^T \Omega \\ * & -\Omega \end{bmatrix} < 0, \quad r = 1, 2, \quad (25)$$

where

$$E = [E_{ij}]_{10 \times 10}, \quad E_{ij} = E_{ji}, \quad i, j = 1, 2, \dots, 10,$$

$$\begin{aligned}
E_{11} &= P_{11}A + P_{12} + \frac{1}{2}hP_{14} + \frac{1}{2}hP_{15} + A^T P_{11} + P_{12}^T \\
&\quad + \frac{1}{2}hP_{14}^T + \frac{1}{2}hP_{15}^T + X_{11} + Q_1 - \frac{37}{40}h^2Q_4 \\
&\quad - \frac{5}{8}h^2Q_3,
\end{aligned}$$

$$E_{12} = P_{11}A_1,$$

$$E_{13} = -P_{12} + P_{13} + X_{12},$$

$$E_{14} = -P_{13},$$

$$E_{15} = P_{11}C,$$

$$E_{16} = P_{11}B + A^T H \Lambda,$$

$$\begin{aligned}
E_{17} &= -P_{14} + A^T P_{12} + P_{22} + \frac{h}{2}P_{24}^T + \frac{h}{2}P_{25}^T - \frac{9h}{10}Q_4 \\
&\quad - hQ_3,
\end{aligned}$$

$$\begin{aligned}
E_{18} &= -P_{15} + A^T P_{13} + P_{23} + \frac{1}{2} h P_{34}^T + \frac{1}{2} h P_{35}^T \\
&\quad - \frac{13}{10} h Q_4, \\
E_{19} &= A^T P_{14} + P_{24} + \frac{1}{2} h P_{44} + \frac{1}{2} h P_{45}^T + 9 Q_3, \\
E_{1,10} &= A^T P_{15} + P_{25} + \frac{1}{2} h P_{45} + \frac{1}{2} h P_{55} + \frac{27}{5} Q_4, \\
E_{22} &= -(1 - h_d) Q_1, \\
E_{26} &= A_1^T H \Lambda, \\
E_{27} &= A_1^T P_{12}, \\
E_{28} &= A_1^T P_{13}, \\
E_{29} &= A_1^T P_{14}, \\
E_{2,10} &= A_1^T P_{15}, \\
E_{33} &= X_{22} - X_{11}, \\
E_{34} &= -X_{12}, \\
E_{37} &= -P_{22} + P_{23}^T, \\
E_{38} &= -P_{23} + P_{33}, \\
E_{39} &= -P_{24} + P_{34}, \\
E_{3,10} &= -P_{25} + P_{35}, \\
E_{44} &= -X_{22}, \\
E_{47} &= -P_{23}^T, \\
E_{48} &= -P_{33}, \\
E_{49} &= -P_{34}, \\
E_{4,10} &= -P_{35}, \\
E_{55} &= -(1 - \tau_d) Q_2, \\
E_{56} &= C^T H \Lambda, \\
E_{57} &= C^T P_{12}, \\
E_{58} &= C^T P_{13}, \\
E_{59} &= C^T P_{14}, \\
E_{5,10} &= C^T P_{15}, \\
E_{66} &= \Lambda H^T B + B^T H \Lambda, \\
E_{67} &= B^T P_{12}, \\
E_{68} &= B^T P_{13}, \\
E_{69} &= B^T P_{14}, \\
E_{6,10} &= B^T P_{15}, \\
E_{77} &= -P_{24} - P_{24}^T - \frac{6}{5} Q_4 - 7 Q_3, \\
E_{78} &= -P_{25} - P_{34}^T - \frac{12}{5} Q_4, \\
E_{79} &= -P_{44} + \frac{36}{h} Q_3, \\
E_{7,10} &= -P_{45} + \frac{36}{5h} Q_4, \\
E_{88} &= -P_{35} - P_{35}^T - \frac{29}{5} Q_4, \\
E_{89} &= -P_{45}^T, \\
E_{8,10} &= -P_{55} + \frac{72}{5h} Q_4, \\
E_{99} &= -\frac{216}{h^2} Q_3, \\
E_{10,10} &= -\frac{216}{5h^2} Q_4, \\
E_{23} &= E_{24} = E_{25} = E_{35} = E_{36} = E_{45} = E_{46} = E_{9,10} \\
&= 0, \\
\Psi_1 &= -(e_3 - e_4)^T Q_6 (e_3 - e_4) \\
&\quad - 3 \left(e_3 + e_4 - \frac{4}{h} e_8 \right)^T Q_6 \cdot \left(e_3 + e_4 - \frac{4}{h} e_8 \right) \\
&\quad - \begin{bmatrix} e_2 - e_3 \\ e_1 - e_2 \end{bmatrix}^T \begin{bmatrix} Q_5 & X_1 \\ * & Q_5 \end{bmatrix} \begin{bmatrix} e_2 - e_3 \\ e_1 - e_2 \end{bmatrix}, \\
\Psi_2 &= -(e_1 - e_3)^T Q_5 (e_1 - e_3) \\
&\quad - 3 \left(e_1 + e_3 - \frac{4}{h} e_7 \right)^T Q_5 \cdot \left(e_1 + e_3 - \frac{4}{h} e_7 \right) \\
&\quad - \begin{bmatrix} e_2 - e_4 \\ e_3 - e_2 \end{bmatrix}^T \begin{bmatrix} Q_6 & X_2 \\ * & Q_6 \end{bmatrix} \begin{bmatrix} e_2 - e_4 \\ e_3 - e_2 \end{bmatrix}, \\
\Theta_1 &= e_1^T H K S e_6 - e_6^T S e_6 + (e_1^T H K S e_6 - e_6^T S e_6)^T, \\
e_i &= \begin{bmatrix} 0 & \cdots & 0 & \underbrace{1}_i & 0 & \cdots & 0 \end{bmatrix}_{1 \times 10}, \\
\eta &= [A \quad A_1 \quad 0 \quad 0 \quad C \quad B \quad 0 \quad 0 \quad 0 \quad 0], \\
\Omega &= Q_2 + \frac{h^4}{64} Q_3 + \frac{9h^4}{64} Q_4 + \frac{h^2}{4} Q_5 + \frac{h^2}{4} Q_6.
\end{aligned} \tag{26}$$

Proof. The time-derivative of functions $V_i(x(t))$ can be calculated, respectively, based on the trajectory of system (17) as follows:

$$\dot{V}_1(x(t)) = 2\xi_1^T P \dot{\xi}_1 + 2f^T(\sigma(t)) \Lambda H^T \dot{x}(t)$$

$$\dot{V}_2(x(t))$$

$$= \begin{bmatrix} x(t) \\ x\left(t - \frac{h}{2}\right) \end{bmatrix}^T \begin{bmatrix} X_{11} & X_{12} \\ * & X_{22} \end{bmatrix} \begin{bmatrix} x(t) \\ x\left(t - \frac{h}{2}\right) \end{bmatrix} - \begin{bmatrix} x\left(t - \frac{h}{2}\right) \\ x(t-h) \end{bmatrix}^T \begin{bmatrix} X_{11} & X_{12} \\ * & X_{22} \end{bmatrix} \begin{bmatrix} x\left(t - \frac{h}{2}\right) \\ x(t-h) \end{bmatrix}$$

$$\dot{V}_3(x(t))$$

$$\leq x^T(t) Q_1 x(t) - (1 - h_d) x^T(t-h(t)) Q_1 x(t-h(t)) + \dot{x}^T(t) Q_2 \dot{x}(t) - (1 - \tau_d) \dot{x}^T(t-\tau(t)) Q_2 \dot{x}(t-\tau(t))$$

$$\dot{V}_4(x(t))$$

$$= \dot{x}^T(t) \left(\frac{h^4}{64} Q_3 + \frac{9h^4}{64} Q_4 \right) \dot{x}(t) - \frac{h^2}{8} \int_{-h/2}^0 \int_{t+\theta}^t \dot{x}^T(s) Q_3 \dot{x}(s) ds d\theta - \frac{3h^2}{8} \int_{-h}^{-h/2} \int_{t+\theta}^t \dot{x}^T(s) Q_4 \dot{x}(s) ds d\theta$$

$$\dot{V}_5(x(t))$$

$$= \frac{h^2}{4} \dot{x}^T(t) (Q_5 + Q_6) \dot{x}(t) - \frac{h}{2} \int_{t-h/2}^t \dot{x}^T(s) Q_5 \dot{x}(s) ds - \frac{h}{2} \int_{t-h}^{t-h/2} \dot{x}^T(s) Q_6 \dot{x}(s) ds.$$

Using Lemma 4 and the method of the exchange of integral order, we get

$$- \frac{h^2}{8} \int_{-h/2}^0 \int_{t+\theta}^t \dot{x}^T(s) Q_3 \dot{x}(s) ds d\theta \leq - \left(\int_{-h/2}^0 \int_{t+\theta}^t \dot{x}(s) ds d\theta \right)^T \cdot Q_3 \left(\int_{-h/2}^0 \int_{t+\theta}^t \dot{x}(s) ds d\theta \right) - \frac{96}{h^2} \left(\frac{1}{2} \int_{-h/2}^0 \int_{-h/2}^\theta \int_{t+r}^t \dot{x}(s) ds dr d\theta \right)^T$$

$$\begin{aligned} & - \int_{-h/2}^0 \int_{t+\theta}^t \int_{t+\theta}^u \dot{x}(s) ds du d\theta \Big)^T \\ & \cdot Q_3 \left(\frac{1}{2} \int_{-h/2}^0 \int_{-h/2}^\theta \int_{t+r}^t \dot{x}(s) ds dr d\theta - \int_{-h/2}^0 \int_{t+\theta}^t \int_{t+\theta}^u \dot{x}(s) ds du d\theta \right) = - \left(\frac{h}{2} x(t) - \int_{t-h/2}^t x(s) ds \right)^T Q_3 \left(\frac{h}{2} x(t) - \int_{t-h/2}^t x(s) ds \right) \\ & - \frac{96}{h^2} \left(\frac{h^2}{16} x(t) + \frac{h}{4} \int_{t-h/2}^t x(s) ds - \frac{3}{2} \int_{-h/2}^0 \int_{t+\theta}^t x(s) ds d\theta \right)^T \cdot Q_3 \left(\frac{h^2}{16} x(t) + \frac{h}{4} \int_{t-h/2}^t x(s) ds - \frac{3}{2} \int_{-h/2}^0 \int_{t+\theta}^t x(s) ds d\theta \right) \end{aligned} \quad (28)$$

$$\begin{aligned} & - \frac{3h^2}{8} \int_{-h}^{-h/2} \int_{t+\theta}^t \dot{x}^T(s) Q_4 \dot{x}(s) ds d\theta \leq - \left(\int_{-h}^{-h/2} \int_{t+\theta}^t \dot{x}(s) ds d\theta \right)^T \cdot Q_4 \left(\int_{-h}^{-h/2} \int_{t+\theta}^t \dot{x}(s) ds d\theta \right) \\ & - \frac{96}{5h^2} \left(\frac{h}{4} \int_{-h}^{-h/2} \int_{t+\theta}^t \dot{x}(s) ds d\theta + \frac{1}{2} \int_{-h}^{-h/2} \int_{-h}^\theta \int_{t+r}^t \dot{x}(s) ds dr d\theta - \int_{-h}^{-h/2} \int_{t+\theta}^t \int_{t+\theta}^u \dot{x}(s) ds du d\theta \right)^T \\ & \cdot Q_4 \left(\frac{h}{4} \int_{-h}^{-h/2} \int_{t+\theta}^t \dot{x}(s) ds d\theta + \frac{1}{2} \int_{-h}^{-h/2} \int_{-h}^\theta \int_{t+r}^t \dot{x}(s) ds dr d\theta - \int_{-h}^{-h/2} \int_{t+\theta}^t \int_{t+\theta}^u \dot{x}(s) ds du d\theta \right) = - \left(\frac{h}{2} x(t) - \int_{t-h}^{t-h/2} x(s) ds \right)^T Q_4 \left(\frac{h}{2} x(t) - \int_{t-h}^{t-h/2} x(s) ds \right) \\ & - \frac{96}{5h^2} \left(\frac{3h^2}{16} x(t) + \frac{h}{4} \int_{t-h/2}^t x(s) ds + \frac{h}{2} \int_{t-h}^{t-h/2} x(s) ds - \frac{3}{2} \int_{-h}^{-h/2} \int_{t+\theta}^t x(s) ds d\theta \right)^T \end{aligned}$$

$$\begin{aligned} & \cdot Q_4 \left(\frac{3h^2}{16} x(t) + \frac{h}{4} \int_{t-h/2}^t x(s) ds \right. \\ & \left. + \frac{h}{2} \int_{t-h}^{t-h/2} x(s) ds - \frac{3}{2} \int_{-h}^{-h/2} \int_{t+\theta}^t x(s) ds d\theta \right). \end{aligned} \quad (29)$$

For $h(t) \in [0, h/2]$, the following formulas can be got using Lemmas 1 and 2:

$$\begin{aligned} & -\frac{h}{2} \int_{t-h/2}^t \dot{x}^T(s) Q_5 \dot{x}(s) ds = -\frac{h}{2} \\ & \cdot \int_{t-h/2}^{t-h(t)} \dot{x}^T(s) Q_5 \dot{x}(s) ds - \frac{h}{2} \\ & \cdot \int_{t-h(t)}^t \dot{x}^T(s) Q_5 \dot{x}(s) ds \\ & \leq -\frac{h/2}{h/2-h(t)} \left(x(t-h(t)) - x\left(t-\frac{h}{2}\right) \right)^T Q_5 \\ & \cdot \left(x(t-h(t)) - x\left(t-\frac{h}{2}\right) \right) - \frac{h/2}{h(t)} (x(t) \\ & - x(t-h(t)))^T \cdot Q_5 (x(t) - x(t-h(t))) \\ & - \frac{3h/2}{h/2-h(t)} \left(x(t-h(t)) + x\left(t-\frac{h}{2}\right) \right) \\ & - \frac{h}{h/2-h(t)} \int_{t-h/2}^{t-h(t)} \dot{x}(s) ds \Big)^T \cdot Q_5 \left(x(t-h(t)) \right. \\ & \left. + x\left(t-\frac{h}{2}\right) - \frac{h}{h/2-h(t)} \int_{t-h/2}^{t-h(t)} \dot{x}(s) ds \right) \\ & - \frac{3h/2}{h(t)} \left(x(t) + x(t-h(t)) \right. \\ & \left. - \frac{h}{h(t)} \int_{t-h(t)}^t \dot{x}(s) ds \right)^T \cdot Q_5 \left(x(t) + x(t-h(t)) \right. \\ & \left. - \frac{h}{h(t)} \int_{t-h(t)}^t \dot{x}(s) ds \right) \\ & \leq -\frac{h/2}{h/2-h(t)} \left(x(t-h(t)) - x\left(t-\frac{h}{2}\right) \right)^T Q_5 \\ & \cdot \left(x(t-h(t)) - x\left(t-\frac{h}{2}\right) \right) - \frac{h/2}{h(t)} (x(t) \\ & - x(t-h(t)))^T \cdot Q_5 (x(t) - x(t-h(t))) \\ & \leq - \begin{bmatrix} x(t-h(t)) - x\left(t-\frac{h}{2}\right) \\ x(t) - x(t-h(t)) \end{bmatrix}^T \begin{bmatrix} Q_5 & X_1 \\ * & Q_5 \end{bmatrix} \\ & \cdot \begin{bmatrix} x(t-h(t)) - x\left(t-\frac{h}{2}\right) \\ x(t) - x(t-h(t)) \end{bmatrix} \end{aligned}$$

$$\begin{aligned} & -\frac{h}{2} \int_{t-h}^{t-h/2} \dot{x}^T(s) Q_6 \dot{x}(s) ds \leq - \left(x\left(t-\frac{h}{2}\right) \right. \\ & \left. - x(t-h) \right)^T Q_6 \left(x\left(t-\frac{h}{2}\right) - x(t-h) \right) \\ & - 3 \left(x\left(t-\frac{h}{2}\right) + x(t-h) - \frac{4}{h} \int_{t-h}^{t-h/2} x(s) ds \right)^T \\ & \cdot Q_6 \left(x\left(t-\frac{h}{2}\right) + x(t-h) - \frac{4}{h} \int_{t-h}^{t-h/2} x(s) ds \right). \end{aligned} \quad (30)$$

At the same time, we can get the following inequation from formula (3):

$$s_i f_i(\sigma_i(t)) [k_i h_i^T x(t) - f_i(\sigma_i(t))] \geq 0. \quad (31)$$

It is equivalent to

$$2 [x^T(t) HKSf(\sigma(t)) - f^T(\sigma(t)) Sf(\sigma(t))] \geq 0, \quad (32)$$

where

$$\begin{aligned} S &= \text{diag} \{s_1, s_2, \dots, s_m\}, \\ K &= \text{diag} \{k_1, k_2, \dots, k_m\}. \end{aligned} \quad (33)$$

According to inequalities (27), (28), (29), (30), and (32), we get

$$\begin{aligned} & \dot{V}(x(t)) \\ & \leq \sum_{i=1}^5 \dot{V}_i(x(t)) \\ & \quad + 2 [x^T(t) HKSf(\sigma(t)) - f^T(\sigma(t)) Sf(\sigma(t))] \\ & \leq \xi^T(t) (E + \Theta_1 + \Psi_1 + \eta^T \Omega \eta) \xi(t). \end{aligned} \quad (34)$$

Obviously, LMI (25) with $r = 1$ suggests that $\dot{V}(x(t)) < 0$ using Schur complement.

For $h(t) \in [h/2, h]$, the following formulas can be got by using Lemmas 1 and 2:

$$\begin{aligned} & -\frac{h}{2} \int_{t-h/2}^t \dot{x}^T(s) Q_5 \dot{x}(s) ds \\ & \leq - \left(x(t) - x\left(t-\frac{h}{2}\right) \right)^T Q_5 \left(x(t) - x\left(t-\frac{h}{2}\right) \right) \\ & - 3 \left(x(t) + x\left(t-\frac{h}{2}\right) - \frac{4}{h} \int_{t-h/2}^t x(s) ds \right)^T \\ & \cdot Q_5 \left(x(t) + x\left(t-\frac{h}{2}\right) - \frac{4}{h} \int_{t-h/2}^t x(s) ds \right) \end{aligned}$$

$$\begin{aligned}
& -\frac{h}{2} \int_{t-h}^{t-h/2} \dot{x}^T(s) Q_6 \dot{x}(s) ds \\
& = -\frac{h}{2} \int_{t-h}^{t-h(t)} \dot{x}^T(s) Q_6 \dot{x}(s) ds \\
& \quad -\frac{h}{2} \int_{t-h(t)}^{t-h/2} \dot{x}^T(s) Q_6 \dot{x}(s) ds \\
& \leq - \begin{bmatrix} x(t-h(t)) - x(t-h) \\ x\left(t - \frac{h}{2}\right) - x(t-h(t)) \end{bmatrix}^T \begin{bmatrix} Q_6 & X_2 \\ * & Q_6 \end{bmatrix} \\
& \quad \cdot \begin{bmatrix} x(t-h(t)) - x(t-h) \\ x\left(t - \frac{h}{2}\right) - x(t-h(t)) \end{bmatrix}.
\end{aligned} \tag{35}$$

According to inequalities (27) and (28), (32), and (35), we have

$$\begin{aligned}
& \dot{V}(x(t)) \\
& \leq \sum_{i=1}^5 \dot{V}_i(x(t)) \\
& \quad + 2 \left[x^T(t) H K S f(\sigma(t)) - f^T(\sigma(t)) S f(\sigma(t)) \right] \\
& \leq \xi^T(t) (E + \Theta_1 + \Psi_2 + \eta^T \Omega \eta) \xi(t).
\end{aligned} \tag{36}$$

Obviously, LMI (25) with $r = 2$ suggests that $\dot{V}(x(t)) < 0$ using Schur complement. In conclusion, the asymptotic stability of nominal system (17) satisfying (3) has been proven. $\square$

Remark 11. If function $f(\sigma(t))$ of normal system (17) satisfies sector condition (4), for any $s_i \geq 0$, $i = 1, 2, \dots, m$, it follows that

$$s_i f_i(\sigma_i(t)) h_i^T x(t) \geq 0, \quad i = 1, 2, \dots, m, \tag{37}$$

which is equivalent to

$$2x^T(t) H S f(\sigma(t)) \geq 0. \tag{38}$$

Corollary 12. Nominal system (17) satisfying conditions (4) and (7) is asymptotically stable for given values $h \geq 0$, h_d , and $\tau_d < 1$, if there exist appropriate dimensional matrices X_1, X_2, X_{12} , positive matrices $P = [P_{ij}]_{5 \times 5}$, Q_i ($i = 1, 2, \dots, 6$), X_{11}, X_{22} positive semidefinite diagonal matrices $S = \text{diag}\{s_1, s_2, \dots, s_m\}$, and $\Lambda = \text{diag}\{\lambda_1, \lambda_2, \dots, \lambda_m\}$, such that the following LMIs hold:

$$\tilde{\Xi}_r = \begin{bmatrix} E + \Theta_2 + \Psi_r & \eta^T \Omega \\ * & -\Omega \end{bmatrix} < 0, \quad r = 1, 2 \tag{39}$$

as well as (22)–(24), where

$$\Theta_2 = 2e_1^T H S e_6. \tag{40}$$

Remark 13. Absolute stability criteria Theorem 10 and Corollary 12 proposed in this paper are less conservative than existing results which are shown by the following examples mentioned in the Section 4. The main reasons are the combination of introducing new inequality in Lemma 4 and the method of the exchange of integral order, based on the new constructed Lyapunov functionals.

The next theorem and corollary will investigate the robust stability for uncertain neutral-type Lur'e system (1) satisfying conditions (3)–(7) following similar methods of the proof for Theorem 10 and Corollary 12.

Theorem 14. System (1) satisfying conditions (3) and (5)–(7) is robustly absolutely stable for given values $h \geq 0$, $h_d, \tau_d < 1$, and $k_l > 0$ ($l = 1, 2, \dots, m$), if there exist appropriate dimensional matrices X_1, X_2, X_{12} , positive matrices $P = [P_{ij}]_{5 \times 5}$, Q_i ($i = 1, 2, \dots, 6$), X_{11}, X_{22} positive semidefinite matrices $S = \text{diag}\{s_1, s_2, \dots, s_m\}$, $\Lambda = \text{diag}\{\lambda_1, \lambda_2, \dots, \lambda_m\}$, and scalars $\varepsilon_r > 0$ such that the following LMIs hold:

$$\begin{bmatrix} E + \varepsilon_r \Delta E^T \Delta E + \Theta_1 + \Psi_r & \eta^T \Omega & J \\ * & -\Omega & 0 \\ * & * & D^T \Omega D - \varepsilon_r \end{bmatrix} < 0, \tag{41}$$

as well as (22)–(24), where

$$\Delta E = [E_a \ E_{a1} \ 0 \ 0 \ 0 \ E_b \ 0 \ 0 \ 0 \ 0],$$

$$J = [J_1^T \ J_2^T \ 0 \ 0 \ J_5^T \ J_6^T \ J_7^T \ J_8^T \ J_9^T \ J_{10}^T]^T,$$

$$J_1 = P_{11} D + A^T \Omega D,$$

$$J_2 = A_1^T \Omega D,$$

$$J_5 = C^T \Omega D,$$

$$J_6 = \Lambda H^T D + B^T \Omega D, \tag{42}$$

$$J_7 = P_{12}^T D,$$

$$J_8 = P_{13}^T D,$$

$$J_9 = P_{14}^T D,$$

$$J_{10} = P_{15}^T D,$$

$$r = 1, 2.$$

Proof. If

$$p(t) = F(t) (E_a x(t) + E_{a1} x(t-h(t)) + E_b f(\sigma(t))), \tag{43}$$

for any $\varepsilon > 0$, we have the following inequality:

$$\begin{aligned}
& -\varepsilon p^T(t) p(t) \\
& \quad + \varepsilon (E_a x(t) + E_{a1} x(t-h(t)) + E_b f(\sigma(t)))^T \\
& \quad \cdot (E_a x(t) + E_{a1} x(t-h(t)) + E_b f(\sigma(t))) \geq 0.
\end{aligned} \tag{44}$$

Using $A + DF(t)E_a$, $A_1 + DF(t)E_{a1}$, $B + DF(t)E_b$ instead of A , A_1 , B in formula (18), respectively, LMIs in (25) can be rewritten as (41). $\square$

Now, if function $f(\sigma(t))$ of system (1) satisfies sector condition (4), we have Corollary 15.

Corollary 15. *System (1) satisfying conditions (4)–(7) is robustly absolutely stable for given values $h \geq 0$, h_d and $\tau_d < 1$, if there exist appropriate dimensional matrices X_1, X_2, X_{12} , positive matrices $P = [P_{ij}]_{5 \times 5}$, Q_i ($i = 1, 2, \dots, 6$), X_{11}, X_{22} positive semidefinite diagonal matrices*

$$\begin{aligned} S &= \text{diag} \{s_1, s_2, \dots, s_m\}, \\ \Lambda &= \text{diag} \{\lambda_1, \lambda_2, \dots, \lambda_m\}, \end{aligned} \quad (45)$$

and scalars $\varepsilon_r > 0$ such that the following LMIs hold:

$$\begin{bmatrix} E + \varepsilon_r \Delta E^T \Delta E + \Theta_2 + \Psi_r & \eta^T \Omega & J \\ * & -\Omega & 0 \\ * & * & D^T \Omega D - \varepsilon_r \end{bmatrix} < 0, \quad (46)$$

as well as (22)–(24), $r = 1, 2$.

Remark 16. Clearly, robust stability criteria Theorem 14 and Corollary 15 proposed in this paper would be less conservative, because of using the new inequality and some other techniques which are similar to Theorem 10 and Corollary 12.

Remark 17. In order to handle the integral term

$$- \int_{-h_2}^{-h_1} \int_{t+\theta}^t \dot{x}^T(s) R \dot{x}(s) ds d\theta \quad (47)$$

Jensen inequality [35] is always presented as

$$\begin{aligned} & - \int_{-h_2}^0 \int_{t+\theta}^t \dot{x}^T(s) R \dot{x}(s) ds d\theta \\ & \leq - \frac{1}{h_2^2} \left(\int_{-h_2}^0 \int_{t+\theta}^t \dot{x}(s) ds d\theta \right)^T \\ & \cdot R \left(\int_{-h_2}^0 \int_{t+\theta}^t \dot{x}(s) ds d\theta \right). \end{aligned} \quad (48)$$

Obviously, Lemma 4 not only can obtain a more accurate bound than the Jensen inequality but also can reduce the number of decision variables. What is more, Lemma 4 can dispose the integral terms with $h_1 \neq 0$. Therefore, Lemma 4 is used to derive our criterion, and the effectiveness of our method will be demonstrated by some existing numerical examples.

4. Examples

In this section, the effectiveness of our approaches will be shown by the following three numerical examples.

Example 1 (see [16, 19–21]). Consider Chua's circuit example discussed in [16, 19–21]. Chua's circuit is a simple nonlinear electronic circuit design. It can show the standard chaos theory. It was published by professor Leo Chua in 1983. The ease of making this circuit makes it a ubiquitous example of chaotic systems in the real world, leading somebody to declare it as a model of a chaotic system. The system equation is given by

$$\begin{aligned} \dot{x}(t) &= \alpha(y(t) - h(x(t))), \\ \dot{y}(t) &= x(t) - y(t) + z(t), \\ \dot{z}(t) &= -\beta y(t), \end{aligned} \quad (49)$$

with nonlinear characteristic

$$\begin{aligned} h(x(t)) &= m_1 x(t) \\ &+ \frac{1}{2} (m_0 - m_1) [|x(t) + c| - |x(t) - c|], \end{aligned} \quad (50)$$

and parameters $m_0 = -1/7$, $m_1 = 2/7$, $\alpha = 9$, $\beta = 14.28$, and $c = 1$. The system can be represented in normal Lur'e system framework (17) with

$$\begin{aligned} A &= \begin{bmatrix} -\alpha m_1 - 1 & \alpha & 0 \\ 1 & -2 & 1 \\ 0 & -\beta & -1 \end{bmatrix}, \\ A_1 &= \begin{bmatrix} -6.0029 & 0 & 0 \\ -1.3367 & 0 & 0 \\ 2.1264 & 0 & 0 \end{bmatrix}, \\ B &= \begin{bmatrix} -\alpha(m_0 - m_1) \\ 0 \\ 0 \end{bmatrix}, \\ H &= \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \\ C &= 0. \end{aligned} \quad (51)$$

The feedback nonlinear function belongs to $K_{[0,1]}$.

The MADBs of h of system (49) for different h_d have been shown in Table 1 with the existing results in [16, 19–21]. Table 1 shows us that the MADBs obtained by using Theorem 10 in this paper are better than the existing results in [16, 19–21].

Example 2 (see [12, 14–20]). Consider the following nominal neutral-type Lur'e systems:

$$\begin{aligned} \dot{x}(t) - C\dot{x}(t - \tau(t)) &= Ax(t) + A_1 x(t - h(t)) \\ &+ Bf(\sigma(t)), \\ \sigma(t) &= H^T x(t), \quad \forall t \geq 0, \end{aligned} \quad (52)$$

TABLE 1: MADBs h for different h_d (Example 1).

h_d	0	0.3	0.6	0.9	>1.0
[21]	0.1622	0.1591	0.1566	0.1541	0.1527
[16]	0.1745	0.1698	0.1698	0.1698	0.1698
[19]	0.1747	0.1710	0.1703	0.1703	0.1703
[20]	0.1771	0.1721	0.1715	0.1715	0.1715
Theorem 10	0.2790	0.2691	0.2553	0.2463	0.2463

FIGURE 2: The dynamical behavior for Example 2.

with

$$\begin{aligned}
 A &= \begin{bmatrix} -2 & 0.5 \\ 0 & -1 \end{bmatrix}, \\
 A_1 &= \begin{bmatrix} 1 & 0.4 \\ 0.4 & -1 \end{bmatrix}, \\
 B &= \begin{bmatrix} -0.5 \\ -0.75 \end{bmatrix}, \\
 C &= \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}, \\
 H &= \begin{bmatrix} 0.2 \\ 0.6 \end{bmatrix}.
 \end{aligned} \tag{53}$$

The maximum allowable delay bounds (MADBs) of h of nominal neutral-type Lur'e system (52) for different h_d and τ_d have been shown in Table 2 along with the existing results in [12, 14–20]. Applying Matlab LMI-toolbox, one can get the simulation result in Figure 1. Table 2 shows us that the MADBs obtained by using Corollary 15 in this paper are better than the existing results in [12, 14–20]. What is more, for all the nonlinear function $f(\sigma(t)) \in R^m$ with condition (3) or condition (4), the simulation study is entirely viable, which has been shown in this example (Figure 2).

TABLE 2: MADBs h for different h_d and τ_d (Example 2).

τ_d	h_d	0.2	0.4	0.6	0.8
0.1	[12]	2.2424	1.5434	0.9888	0.7228
	[14]	2.6062	1.9730	1.6408	1.1854
	[15]	2.6488	2.0055	1.6679	1.2049
	[17]	2.9962	2.1316	1.7138	1.3204
	[16]	2.9997	2.1012	1.7307	1.3359
	[18]	2.6052	1.9538	1.6232	1.1659
	[19]	3.0717	2.1171	1.7310	1.3367
	[20]	3.1743	2.1789	1.7467	1.7153
	Corollary 15	3.4880	2.3787	1.8062	1.4625
	[12]	1.8344	1.3029	0.7793	0.6282
0.5	[14]	2.1537	1.7041	1.3828	1.0095
	[15]	2.1883	1.7322	1.4056	1.0262
	[17]	2.4472	1.8289	1.4551	1.1058
	[16]	2.4563	1.8009	1.4823	1.1157
	[18]	2.1532	1.6986	1.3687	1.0066
	[19]	2.5131	1.8146	1.4951	1.1168
	Corollary 15	2.8538	2.0236	1.5679	1.2425
	[12]	0.1085	0.1019	0.0983	0.0967
	[14]	0.1122	0.1088	0.1086	0.1086
	[15]	0.1122	0.1089	0.1086	0.1086
0.9	[17]	0.1126	0.1102	0.1102	0.1102
	[16]	0.1130	0.1105	0.1105	0.1105
	[18]	0.1221	0.1087	0.1083	0.1083
	[19]	0.1227	0.1197	0.1197	0.1197
	Corollary 15	0.1799	0.1697	0.1660	0.1658

Example 3 (see [15–17, 19, 20]). Consider uncertain system (1) with the following parameters:

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix},$$

$$A_1 = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix},$$

$$B = \begin{bmatrix} -0.2 \\ -0.3 \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$H = \begin{bmatrix} 0.6 \\ 0.8 \end{bmatrix},$$

$$D = E_a = E_{a1} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$

TABLE 3: MADBs h for different K (Example 3).

K	Methods	h
$f(\cdot) \in K_{[0,0.5]}$	[15]	4.3196
	[17]	5.1366
	[16]	5.1666
	[19]	5.3486
	[20]	5.4972
	Theorem 14	5.5513
$f(\cdot) \in K_{[0,100]}$	[15]	4.1792
	[17]	4.9572
	[16]	4.9731
	[19]	5.1828
	Theorem 14	5.3557
$f(\cdot) \in K_{[0,\infty]}$	[15]	4.1581
	[17]	4.9304
	[16]	4.9431
	[19]	5.1654
	[20]	5.2489
	Corollary 15	5.3186

$$\begin{aligned}
 E_b &= \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}, \\
 E_c &= \begin{bmatrix} \sin(\omega(t)) & 0 \\ 0 & \sin(\omega(t)) \end{bmatrix}.
 \end{aligned} \tag{54}$$

The MADBs of h with $h_d = 0$ and $\tau_d < 1$ for different values of $K_{[0,k_i]}$ are listed in Table 3 by using Theorem 14 and Corollary 15 along with the existing results in [15–17, 19, 20]. Table 3 shows us that the MADBs obtained by using Theorem 14 and Corollary 15 in this paper are better than the existing results in [15–17, 19, 20].

5. Conclusions

In this paper, we have studied the stability for a class of neutral-type Lur'e systems with time-varying delays and sector bounded nonlinearities by using a new LKF. The LKF contains not only double-integral terms but also triple-integral terms. Using some effective techniques, such as a novel integral inequality, a piecewise analysis method, and the reciprocally convex combination inequality, instead of the general free-weighting matrix method, the delay-dependent stability criteria derived in the form of LMIs are less conservative than some existing results. The effectiveness and the less conservatism of stability criteria proposed in this paper are demonstrated by using standard numerical examples.

Competing Interests

The authors declare that they have no competing interests.

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