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Research Article

Stabilization of Stochastic Markovian Jump Systems with Partially Unknown Transition Probabilities and Multiplicative Noise

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We are concerned with the problems of stability and stabilization for stochastic Markovian jump systems subject to partially unknown transition probabilities and multiplicative noise, including the continuous- and discrete-time cases. Sufficient conditions guaranteeing systems considered to be asymptotically stable in the mean square are presented in the form of LMIs. Furthermore, the desired state feedback controllers are designed. It is shown that, by introducing the free-weighing matrix method, the results we have obtained not only are less conservative than the existing ones but also can be regarded as extensions of the corresponding results of Markovian jump systems without noise. Numerical examples are finally provided to illustrate the effectiveness of the proposed theoretical results.

1. Introduction

Over the past years, considerable attention has been devoted to the study of a class of stochastic systems governed by Itô's differential equation because of their extensive applications in some practical areas such as economics, finance, biology, and fault detection [1–5]. It was shown that many results related to these systems have been presented for stability [6–8], linear quadratic optimal control [9–12], output feedback control [13, 14], and H_2/H_∞ control [15, 16].

On the other hand, Markovian jump linear systems (MJLS), which are referred to as the stochastic systems with abrupt changes, have come to play an important role in practical applications owing to the powerful modeling capability of Markov chains [17–19]. Up to now, a great number of interesting and important results on this subject have been addressed; see, for example, [20–23] and the references therein. Very recently, the authors in [24] proposed a novel sliding mode observer-based fault tolerant control scheme to investigate the stabilization of nonlinear Markovian jump systems with output disturbances and actuator and sensor faults simultaneously. Furthermore, the state and fault estimation problem for stochastic switched systems with disturbances and sensor and actuator fault was addressed in [25]. It should be pointed out that the obtained results in [24, 25] are very important and useful to study the practical switched systems with sensor and actuator failures. It was shown that most of results on MJLS were based on the assumption that the transition probabilities were accessible. However, in some cases such as networked control systems, it is difficult or even impossible to acquire complete information on the transition probabilities of MJLS. Therefore, there is a strong incentive to further study more general MJLS with incomplete knowledge of transition probabilities. Until now, lots of results on this topic have been addressed, for instance, stability and stabilization [26–28] and H_∞ control [28–31]. Among the aforementioned works, [32] proposed the free-connection weighing matrices method, which obtained less conservative results than those in [26–28, 32]. To the authors' knowledge,

there is little work done on stochastic Markovian jump systems with partially unknown transition rates and multiplicative noise [33]. Such systems are much more advanced and realistic, so the research on this topic should be theoretically interesting and challenging.

In this paper, we will investigate problems of stability and stabilization for stochastic Markovian jump systems with partially unknown transition rates and multiplicative noise, including the continuous- and discrete-time cases. With the aid of the free-weighting matrices, a new stability criterion is established, which is less conservative than that in [33] for the continuous-time case. Moreover, it is theoretically shown that the previous results are special cases if the free-weighting matrices are chosen to be some special forms. Furthermore, the results of stability and stabilization in the discrete-time case are successfully obtained. What we have obtained can be regarded as generations of corresponding results without noise as those in [30–32]. Numerical examples are finally given to illustrate the effectiveness of the proposed theoretical results.

The outline of this paper is listed as follows: Section 2 contains some preliminary results. In Section 3, the problems of stochastic stability and stabilization for the system considered are addressed, including the continuous-time and discrete-time cases. In Section 4, two simulation examples are given to illustrate the effectiveness of the proposed theoretical results. Conclusions are presented in Section 5.

Notations. R^n is the space of all n- dimensional real vectors with usual 2-norm $\|\cdot\|$; $R^{m\times n}$ is the space of all $m\times n$ real matrices; S_n is the set of all $n\times n$ symmetric matrices; $A>0$ (resp., $A<0$): A is a real symmetric positive definite (resp., negative definite) matrix; $A\geq 0$ (resp., $A\leq 0$): A is a real semi-positive definite (resp., semi-negative definite) matrix; A^T is the transpose of A; $\mathscr{E}(\cdot)$ is the expectation operator; I_n is the $n\times n$ identity matrix. M_i is the simple notation of $M(i)$.

2. Preliminaries

Consider the following continuous- and discrete-time stochastic systems subject to Markov jump parameters and multiplicative noise, respectively:

$$\begin{aligned} dx(t) &= \left[A(r_t)x(t)+B(r_t)u(t)\right]dt \\ &\quad + \left[C(r_t)x(t)+D(r_t)u(t)\right]dw(t), \end{aligned} \tag{1}$$

$$\begin{aligned} x(k+1) &= \left[A(r_k)x(k)+B(r_k)u(k)\right] \\ &\quad + \left[C(r_k)x(k)+D(r_k)u(k)\right]w(k), \end{aligned} \tag{2}$$

where $x(t)\in R^n$ (resp., $x(k)$) is the state vector and $u(t)\in R^q$ (resp., $u(k)$) is the control input. For the continuous-time stochastic systems (1), $w(t)$ is one-dimensional, standard Wiener process that is defined on the complete probability space $(\Omega,\mathscr{F},\mathscr{F}_t,\mathscr{P})$ with a filtering $\{\mathscr{F}_t\}_{\{t\geq 0\}}$; $\{r_t,\ t\geq 0\}$ is a right continuous homogeneous Markov chain taking values in a finite state space $S=\{1,\ldots,N\}$ with transition probability matrix $\Lambda=\{\lambda_{ij}\}_{N\times N}$ given by

$$\begin{aligned} \lambda_{ij} &= P_r\{r_{t+h}=j\mid r_t=i\} \\ &= \begin{cases} \lambda_{ij}h+o(h), & i\neq j, \\ 1+\lambda_{ii}h+o(h), & i=j, \end{cases} \end{aligned} \tag{3}$$

where $h>0$, $\lim_{h\to 0}o(h)/h=0$, and $\lambda_{ij}\geq 0$ $(i\neq j)$ represents the transition rate from i to j, which satisfies $\lambda_{ii}=-\sum_{j=1,j\neq i}^{N}\lambda_{ij}$. For the discrete-time stochastic systems (2), $w(k)\in R$ is a wide stationary, second-order process, $\mathscr{E}(w(k))=0$, and $\mathscr{E}(w(k)w(s))=\delta_{ks}$ with δ_{ks} being a Kronecker delta; the parameter r_k denotes a discrete-time Markov chain taking values in a finite set $S=\{1,\ldots,N\}$ with transition probabilities $P_r\{r_{k+1}=j\mid r_k=i\}=\pi_{ij}$, and transition probabilities matrix is given as $\Pi=\{\pi_{ij}\}_{N\times N}$, where $\pi_{ij}\geq 0$ and it satisfies $\sum_{j=1}^{N}\pi_{ij}=1$ for any $i\in S$. In the case of $r_t=i$ (resp., $r_k=i$), the system matrices of the ith mode are given by A_i, B_i, C_i, D_i.

In this paper, the transition rates of Markovian jump process are considered to be partially accessible. For example, the transition rate matrix Λ for system (1) or Π for (2) with N operation modes is described as

$$\begin{gathered} \begin{bmatrix} \lambda_{11} & ? & ? & \cdots & \lambda_{1N} \\ ? & ? & \lambda_{23} & \cdots & \lambda_{2N} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \lambda_{N1} & ? & \cdots & \cdots & \lambda_{NN} \end{bmatrix}, \\ \begin{bmatrix} ? & \pi_{12} & ? & \cdots & \pi_{1N} \\ \pi_{21} & ? & \pi_{23} & \cdots & ? \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \pi_{N1} & ? & \cdots & \cdots & \pi_{NN} \end{bmatrix}, \end{gathered} \tag{4}$$

where the unknown transition rate is represented by ?. For each $i\in S$, we define $S=S_k^i+S_{uk}^i$, where $S_k^i\triangleq\{j:\Lambda_{ij}$ or π_{ij} is known for $j\in S\}$ and $S_{uk}^i\triangleq\{j:\Lambda_{ij}$ or π_{ij} is unknown for $j\in S\}$. Furthermore, in the case of $S_k^i\neq\emptyset$, it is given as $S_k^i=\{k_1^i,k_2^i,\ldots,k_m^i\}$ with $1\leq m\leq N$, and $k_t^i\in Z^+$ $(1\leq t\leq m)$ denotes the sequence number of the tth known element in the ith row of matrix Λ or Π.

Definition 1. Unforced system (1) (resp., (2)) is called asymptotically stable in the mean square if, for any initial condition $x_0\in R^n$, we have

$$\begin{gathered} \lim_{t\to\infty}\mathscr{E}\left\{\|x(t,x_0,r_0)\|^2\mid x_0,r_0\right\}=0, \\ \left(\text{respectively, } \lim_{k\to\infty}\mathscr{E}\left\{\|x(k,x_0,r_0)\|^2\mid x_0,r_0\right\}=0\right). \end{gathered} \tag{5}$$

3. Stochastic Stability and Stabilization Analysis

In this section, the problems of stability and stabilization for stochastic Markovian jump systems with multiplicative noise and partially unknown transition rates in both continuous- and discrete-time cases are investigated. The state feedback controllers guaranteeing systems to be mean square stable are designed.

Before proceeding, let us first recall the stability results for system (1) and (2) with completely known transition rate matrix.

Lemma 2. *System (1) with $u(t) = 0$ is asymptotically stable in the mean square if there are matrices $P_i > 0$ such that the following LMIs hold for each $i \in S$:*

$$A_i^T P_i + P_i A_i + C_i^T P_i C_i + \sum_{j=1}^{N} \lambda_{ij} P_j < 0. \tag{6}$$

Proof. Selecting a stochastic Lyapunov functional candidate

$$V\left(x(t), t, r_t = i\right) = x^T(t) P_i x(t), \tag{7}$$

where P_i is a positive matrix. The infinitesimal operator $\mathscr{L}$ acting on $V(x(t), t, i)$ is given as follows:

$$\begin{aligned} &\mathscr{L}V(x(t), t, i) \\ &\quad = x^T(t)\left(A_i^T P_i + P_i A_i + C_i^T P_i C_i + \sum_{j=1}^{N} \lambda_{ij} P_j\right) x(t). \end{aligned} \tag{8}$$

For arbitrary $x(t) \neq 0$, it is shown from (6) that $\mathscr{E}\{\mathscr{L}V(x(t), t, i)\} < 0$. Therefore, by [6], system (1) with $u(t) \equiv 0$ is asymptotically stable in the mean square. □

Lemma 3. *System (2) with $u(t) \equiv 0$ is asymptotically stable in the mean square if there are matrices $P_i > 0$ for each $i \in S$ such that the following LMIs hold:*

$$A_i^T \left(\sum_{j=1}^{N} \pi_{ij} P_j\right) A_i + C_i^T \left(\sum_{j=1}^{N} \pi_{ij} P_j\right) C_i - P_i < 0. \tag{9}$$

Proof. Following the same line as done in Lemma 2, Lemma 3 can be easily verified. □

3.1. Continuous-Time Case. This section aims to develop a new stability criterion for system (1) by making using of free-weighting matrices. It will be shown that what we have obtained are less conservative than the existing ones in [33].

Theorem 4. *Unforced system (1) with partially unknown transition rates is asymptotically stable in the mean square if there are matrices $P_i > 0$, $Q_i = Q_i^T$ satisfying the following LMIs for each $i \in S$:*

$$A_i^T P_i + P_i A_i^T + C_i^T P_i C_i + \sum_{j \in S_k^i} \lambda_{ij}\left(P_j - Q_i\right) < 0, \tag{10}$$

$$\begin{aligned} &P_j - Q_i \leq 0, \\ &\quad j \in S_{uk}^i, \ j \neq i, \end{aligned} \tag{11}$$

$$\begin{aligned} &P_i - Q_i \geq 0, \\ &\quad j \in S_{uk}^i, \ j = i. \end{aligned} \tag{12}$$

Proof. It is noted that $\sum_{j=1}^{N} \lambda_{ij} = 0$ leads to $\sum_{j=1}^{N} \lambda_{ij} Q_i = 0$ for arbitrary symmetric matrices Q_i. Next, the left side of (6) can be rewritten as

$$\begin{aligned} \psi_i &\triangleq A_i^T P_i + P_i A_i + C_i^T P_i C_i + \sum_{j=1}^{N} \lambda_{ij} P_j - \sum_{j=1}^{N} \lambda_{ij} Q_i \\ &= A_i^T P_i + P_i A_i + C_i^T P_i C_i + \sum_{j \in S_k^i} \lambda_{ij}\left(P_j - Q_i\right) \\ &\quad + \sum_{j \in S_{uk}^i} \lambda_{ij}\left(P_j - Q_i\right). \end{aligned} \tag{13}$$

If $i \in S_k^i$, we have $\lambda_{ij} \geq 0$ ($i \neq j$, $j \in S_u^k$). In this case, inequalities (10) and (11) can result in (6).

On the other hand, if $i \in S_{uk}^i$, ψ_i can be represented as

$$\begin{aligned} \psi_i &\triangleq A_i^T P_i + P_i A_i + C_i^T P_i C_i + \sum_{j \in S_k^i} \lambda_{ij}\left(P_j - Q_i\right) \\ &\quad + \lambda_{ii}\left(P_i - Q_i\right) + \sum_{j \in S_{uk}^i, j \neq i} \lambda_{ij}\left(P_j - Q_i\right). \end{aligned} \tag{14}$$

Obviously, (10), (11), and (12) together with (14) can deduce (6). This completes the proof. □

Below, we further discuss the stabilization problem of system (1). Employing the state feedback controller $u(t) = K(r_t)x(t)$ to system (1), then (1) becomes a close-loop system which is described as

$$\begin{aligned} dx(t) &= \left[A(r_t) + B(r_t) K(r_t)\right] x(t)\, dt \\ &\quad + \left[C(r_t) + D(r_t) K(r_t)\right] x(t)\, dw(t), \end{aligned} \tag{15}$$

where $K(r_t)$ is the controller gain to be determined.

Theorem 5. *The closed-loop system (15) with partially unknown transition rates is asymptotically stable in the mean square if there are matrices $X_i > 0$, $W_i = W_i^T$, and Y_i for each $i \in S$ satisfying the following LMIs:*

$$\begin{bmatrix} \Theta_i + \lambda_{ii} X_i & \left(C_i X_i + D_i Y_i\right)^T & \Gamma_{1i} \\ * & -X_i & 0 \\ * & * & -\Upsilon_{1i} \end{bmatrix} < 0, \quad i \in S_k^i, \tag{16}$$

$$\begin{bmatrix} \Theta_i & (C_iX_i + D_iY_i)^T & \Gamma_{2i} \\ * & -X_i & 0 \\ * & * & -\Upsilon_{2i} \end{bmatrix} \le 0, \quad i \in S_{uk}^i, \tag{17}$$

$$\begin{bmatrix} -W_i & X_i \\ * & -X_j \end{bmatrix} < 0, \quad j \in S_{uk}^i,\ j \ne i, \tag{18}$$

$$X_i - W_i \ge 0, \quad j \in S_{uk}^i,\ i = j, \tag{19}$$

where

$$\begin{aligned} \Theta_i &= A_iX_i + B_iY_i + X_i^TA_i^T + Y_i^TB_i^T - \sum_{j\in S_k^i}\lambda_{ij}W_i, \\ \Gamma_{1i} &= \left[\sqrt{\lambda_{ik_1^i}}X_i, \sqrt{\lambda_{ik_2^i}}X_i, \ldots, \sqrt{\lambda_{ik_{f-1}^i}}X_i, \sqrt{\lambda_{ik_{f+1}^i}}X_i, \ldots, \sqrt{\lambda_{ik_m^i}}X_i\right], \\ \Upsilon_{1i} &= \operatorname{diag}\left(X_{k_1^i}, X_{k_2^i}, \ldots, X_{k_{f-1}^i}, X_{k_{f+1}^i}, \ldots, X_{k_m^i}\right), \\ \Gamma_{2i} &= \left[\sqrt{\lambda_{ik_1^i}}X_i, \sqrt{\lambda_{ik_2^i}}X_i, \ldots, \sqrt{\lambda_{ik_m^i}}X_i\right], \\ \Upsilon_{2i} &= \operatorname{diag}\left(X_{k_1^i}, X_{k_2^i}, \ldots, X_{k_m^i}\right), \end{aligned} \tag{20}$$

with $k_f^i = i$. The stabilizing state feedback controllers are presented as $u(t) = Y_iX_i^{-1}x(t)$.

Proof. Substituting the state feedback controller $u(t) = Y_iX_i^{-1}x(t)$ to (1), we derive the following closed-loop system:

$$\begin{aligned} dx(t) &= \left(A_i + B_iY_iX_i^{-1}\right)x(t)\,dt \\ &\quad + \left(C_i + D_iY_iX_i^{-1}\right)x(t)\,dw(t). \end{aligned} \tag{21}$$

If $i \in S_k^i$, by Schur complement Lemma, the inequality (16) is equivalent to

$$\begin{aligned} &(A_iX_i + B_iY_i) + (A_iX_i + B_iY_i)^T - \sum_{j\in S_k^i}\lambda_{ij}W_i \\ &\quad + (C_iX_i + D_iY_i)^T X_i^{-1}(C_iX_i + D_iY_i) + \lambda_{ii}X_i \\ &\quad + \sum_{j\in S_k^i, j\ne i}\lambda_{ij}X_iX_j^{-1}X_i < 0. \end{aligned} \tag{22}$$

Pre- and postmultiplying (22) and (19) by X_i^{-1}, respectively, we have

$$\begin{aligned} &X_i^{-1}\left(A_i + B_iY_iX_i^{-1}\right) + \left(A_i + B_iY_iX_i^{-1}\right)^T X_i^{-1} \\ &\quad - \sum_{j\in S_k^i}\lambda_{ij}X_i^{-1}W_iX_i^{-1} + \left(C_i + D_iY_iX_i^{-1}\right)^T X_i^{-1} \\ &\quad \cdot \left(C_i + D_iY_iX_i^{-1}\right) + \sum_{j\in S_k^i}\lambda_{ij}X_j^{-1} < 0, \\ &X_i^{-1} - X_i^{-1}W_iX_i^{-1} \ge 0. \end{aligned} \tag{23}$$

Let $P_i = X_i^{-1}$, $Q_i = X_i^{-1}W_iX_i^{-1}$; according to Theorem 4, LMIs (23) imply that the closed-loop system (21) is asymptotically mean square stable.

If $i \in S_{uk}^i$, from Schur complement Lemma, (17) and (18) are, respectively, equivalent to

$$\begin{aligned} &(A_iX_i + B_iY_i) + (A_iX_i + B_iY_i)^T - \sum_{j\in S_k^i}\lambda_{ij}W_i \\ &\quad + (C_iX_i + D_iY_i)^T X_i^{-1}(C_iX_i + D_iY_i) \\ &\quad + \sum_{j\in S_k^i}\lambda_{ij}X_iX_j^{-1}X_i < 0, \\ &-W_i + X_iX_j^{-1}X_i \le 0. \end{aligned} \tag{24}$$

Similar to the obtained procedures of (23), inequalities (17), (18), and (19) deduce that the closed-loop system (21) is asymptotically stable in the mean square by Theorem 4. □

Remark 6. It can be seen from Theorem 4 that a new stability criterion for system (1) has been established by introducing free-weighting matrices Q_i, which is less conservative than Theorem 1 of [33]. In the case of $Q_i = A_i^TP_i + P_i^TA_i + C_i^TP_iC_i$, Theorem 4 coincides with Theorem 1 of [33]. Obviously, Q_i provides more degrees of freedom for the scope of variables. In addition, it should be mentioned that Theorem 2 of [33] is incorrect, because the condition that $1 + \pi_k^i > 0$ (which appeared in (14) of [33]) may not be true.

3.2. Discrete-Time Case. In this section, we focus our attention on the stability and stabilization problems for discrete-time stochastic Markovian jump systems subject to incomplete knowledge of transition probability and multiplicative noise. Sufficient conditions for the stability and stabilization of systems under consideration are formulated as LMIs.

Theorem 7. *System (2) with $u(k) \equiv 0$ is asymptotically stable in the mean square if there exist matrices $P_i > 0$, $Q_i = Q_i^T$, such that the following inequalities hold:*

$$\begin{bmatrix} \left(1 - \sum_{j\in S_k^i}\pi_{ij}\right)Q_i - P_i & A_i^T\Lambda_{1i} & C_i^T\Lambda_{1i} \\ * & -\Sigma_{1i} & 0 \\ * & * & -\Sigma_{1i} \end{bmatrix} < 0, \tag{25}$$

$$\begin{bmatrix} -Q_i & A_i^TP_j & C_i^TP_j \\ * & -P_j & 0 \\ * & * & -P_j \end{bmatrix} \le 0, \quad j \in S_{uk}^i, \tag{26}$$

where

$$\Lambda_{1i} = \left[\sqrt{\pi_{ik_1^i}} P_{ik_1^i}, \sqrt{\pi_{ik_2^i}} P_{ik_2^i}, \ldots, \sqrt{\pi_{ik_m^i}} P_{ik_m^i}\right],$$
$$\Sigma_{1i} = \operatorname{diag}\left(P_{ik_1^i}, P_{ik_2^i}, \ldots, P_{ik_m^i}\right). \tag{27}$$

Proof. Because of $\sum_{j=1}^{N} \pi_{ij} = 1$ for each $i \in S$, the following equality holds for arbitrary symmetric matrix $Q_i = Q_i^T$

$$\left(1 - \sum_{j=1}^{N} \pi_{ij}\right) Q_i = 0. \tag{28}$$

Note that the left side of (9) can be expressed as

$$\Delta_i \triangleq A_i^T \left(\sum_{j=1}^{N} \pi_{ij} P_j\right) A_i + C_i^T \left(\sum_{j=1}^{N} \pi_{ij} P_j\right) C_i - P_i + \left(1 - \sum_{j=1}^{N} \pi_{ij}\right) Q_i. \tag{29}$$

Considering $S = S_k^i + S_{uk}^i$, (29) can be rewritten as

$$\begin{aligned}\Delta_i \triangleq{}& \sum_{j\in S_k^i} \pi_{ij} A_i^T P_j A_i + \sum_{j\in S_k^i} \pi_{ij} C_i^T P_j C_i \\ &+ \left(1 - \sum_{j\in S_k^i} \pi_{ij}\right) Q_i - P_i + \sum_{j\in S_{uk}^i} \pi_{ij} A_i^T P_j A_i \\ &+ \sum_{j\in S_{uk}^i} \pi_{ij} C_i^T P_j C_i - \sum_{j\in S_{uk}^i} \pi_{ij} Q_i.\end{aligned} \tag{30}$$

It is easy to see that $\Delta_i < 0$ holds if the following inequalities are satisfied:

$$\begin{aligned}&- P_i + \sum_{j\in S_k^i} \pi_{ij} A_i^T P_j A_i + \sum_{j\in S_k^i} \pi_{ij} C_i^T P_j C_i \\ &+ \left(1 - \sum_{j\in S_k^i} \pi_{ij}\right) Q_i < 0,\end{aligned} \tag{31}$$

$$A_i^T P_j A_i + C_i^T P_j C_i - Q_i < 0, \quad j \in S_{uk}^i. \tag{32}$$

From Schur complement lemma, it can be verified that (31) and (32) are, respectively, equivalent to (25) and (26). Therefore, it is shown from Lemma 3 that system (2) with $u(t) \equiv 0$ is asymptotically stable in the mean square. This completes the proof. □

Next, we are set about to investigate the stabilization problem of system (2) with partially unknown transition rates. The state feedback controller is given in the form of $u(k) = K(r_k)x(k)$. Applying this controller to system (2) results in the following closed-loop system:

$$\begin{aligned}&x(k+1) \\ &\quad = \left[A(r_k)x(k) + B(r_k)K(r_k)\right]x(k) \\ &\qquad + \left[C(r_k)x(k) + D(r_k)K(r_k)\right]x(k)w(k).\end{aligned} \tag{33}$$

Theorem 8. *The closed-loop system (33) is asymptotically stable in the mean square if there exist matrices $X_i > 0$, $W_i = W_i^T$, Y_i, such that the following LMIs hold:*

$$\begin{bmatrix} \left(1 - \sum_{j\in S_k^i} \pi_{ij}\right) W_i - X_i & \left(X_i A_i^T + Y_i^T B_i^T\right) \Lambda_{2i} & \left(X_i C_i^T + Y_i^T D_i^T\right) \Lambda_{2i} \\ * & -\Sigma_{2i} & 0 \\ * & * & -\Sigma_{2i} \end{bmatrix} < 0, \tag{34}$$

$$\begin{bmatrix} -W_i & X_i A_i^T + Y_i^T B_i^T & X_i C_i^T + Y_i^T D_i^T \\ * & -X_j & 0 \\ * & * & -X_j \end{bmatrix} \le 0, \quad j \in S_{uk}^i, \tag{35}$$

where

$$\Lambda_{2i} = \left[\sqrt{\pi_{ik_1^i}} I_{k_1^i}, \sqrt{\pi_{ik_2^i}} I_{k_2^i}, \ldots, \sqrt{\pi_{ik_m^i}} I_{k_m^i}\right],$$
$$\Sigma_{2i} = \operatorname{diag}\left(X_{k_1^i}, X_{k_2^i}, \ldots, X_{k_m^i}\right). \tag{36}$$

The desired state feedback controller gains are given by $K_i = Y_i X_i^{-1}$.

Proof. Applying the state feedback controller $u(k) = Y_i X_i^{-1} x(k)$ to system (2), we derive the following closed-loop system:

$$\begin{aligned}x(k+1) = {}& \left[A_i + B_i Y_i X_i^{-1}\right] x(k) + \left[C_i + D_i Y_i X_i^{-1}\right] \\ &\cdot x(k) w(k).\end{aligned} \tag{37}$$

Pre- and postmultiply the left sides of (34) and (35) by

$$\begin{gathered}\operatorname{diag}\left(X_i^{-1}, X_{k_1^i}^{-1}, \ldots, X_{k_m^i}^{-1}, X_{k_1^i}^{-1}, \ldots, X_{k_m^i}^{-1}\right), \\ \operatorname{diag}\left(X_i^{-1}, X_j^{-1}, X_j^{-1}\right),\end{gathered} \tag{38}$$

respectively, and let

$$\begin{aligned} P_i &= X_i^{-1}, \\ Q_i &= X_i^{-1} W_i X_i^{-1}; \end{aligned} \tag{39}$$

then (34) and (35) are, respectively, equivalent to

$$\begin{bmatrix} \left(1 - \sum_{j \in S_k^i} \pi_{ij}\right) Q_i - P_i & \left(A_i + B_i Y_i X_i^{-1}\right)^T \Lambda_{1i} & \left(C_i + Y_i X_i^{-1}\right)^T \Lambda_{1i} \\ * & -\Sigma_{1i} & 0 \\ * & * & -\Sigma_{1i} \end{bmatrix} < 0, \tag{40}$$

$$\begin{bmatrix} -Q_i & \left(A_i + B_i Y_i X_i^{-1}\right)^T P_j & \left(C_i + D_i Y_i X_i^{-1}\right)^T P_j \\ * & -P_j & 0 \\ * & * & -P_j \end{bmatrix} \leq 0. \tag{41}$$

It is easy from Theorem 7 to see that the closed-loop system (33) is asymptotically stable in the mean square. □

4. Examples

In this section, two numerical examples are proposed to demonstrate the effectiveness of our presented approaches, including the continuous- and discrete-time cases.

Consider the continuous-time stochastic Markov jumping system in the form of (15) with the following matrices:

$$\begin{aligned}
A_1 &= \begin{bmatrix} 0 & 1 \\ -4.90 & -2 \end{bmatrix}, \\
A_2 &= \begin{bmatrix} 0 & 1 \\ -4.90 & -1.60 \end{bmatrix}, \\
A_3 &= \begin{bmatrix} 0 & 1 \\ -1.625 & -1 \end{bmatrix}, \\
A_4 &= \begin{bmatrix} 0 & 1 \\ -4.41 & -0.8 \end{bmatrix}, \\
B_1 &= \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \\
B_2 &= \begin{bmatrix} 1 \\ 0.8 \end{bmatrix}, \\
B_3 &= \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}, \\
B_4 &= \begin{bmatrix} 1 \\ 0.4 \end{bmatrix}, \\
C_1 &= \begin{bmatrix} 0.15 & 0.20 \\ 0.48 & 0.21 \end{bmatrix}, \\
C_2 &= \begin{bmatrix} 0.19 & -0.11 \\ 0.58 & 0.11 \end{bmatrix}, \\
C_3 &= \begin{bmatrix} -0.31 & 0.25 \\ 0.31 & -0.20 \end{bmatrix}, \\
C_4 &= \begin{bmatrix} -0.17 & 0.48 \\ 0.19 & 0.27 \end{bmatrix}, \\
D_1 &= \begin{bmatrix} 0.27 \\ 0.23 \end{bmatrix}, \\
D_2 &= \begin{bmatrix} 0.1 \\ -0.29 \end{bmatrix}, \\
D_3 &= \begin{bmatrix} 0.14 \\ 0.29 \end{bmatrix}, \\
D_4 &= \begin{bmatrix} 0 \\ 0.17 \end{bmatrix}.
\end{aligned} \tag{42}$$

The transition rate matrix Λ with the partially unknown elements is presented as

$$\begin{bmatrix} -1.5 & 0.1 & ? & ? \\ ? & ? & 0.2 & 0.4 \\ 0.5 & ? & -1.6 & ? \\ 0.3 & ? & ? & ? \end{bmatrix}. \tag{43}$$

Based on Theorem 5 the controller gains for system (15) are given as follows:

$$\begin{aligned} K_1 &= \begin{bmatrix} 0.7187 & 2.0915 \end{bmatrix}, \\ K_2 &= \begin{bmatrix} -1.6273 & 2.1038 \end{bmatrix}, \\ K_3 &= \begin{bmatrix} -3.8298 & 3.4187 \end{bmatrix}, \\ K_4 &= \begin{bmatrix} 0.9722 & 2.8755 \end{bmatrix}. \end{aligned} \tag{44}$$

Consider discrete-time stochastic Markov jumping system (33) with four operation modes and the following system matrices:

$$A_1 = \begin{bmatrix} 0.35 & -0.30 \\ 0.48 & 0.81 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -0.29 & -0.11 \\ 1.48 & 0.21 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 0.11 & -0.35 \\ 0.31 & -0.10 \end{bmatrix},$$

$$A_4 = \begin{bmatrix} 0.17 & -1.48 \\ 1.59 & -0.27 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} -0.27 \\ 1.23 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 0.78 \\ -0.49 \end{bmatrix},$$

$$B_3 = \begin{bmatrix} 1.34 \\ 0.39 \end{bmatrix},$$

$$B_4 = \begin{bmatrix} -0.38 \\ 1.07 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0.05 & 0.30 \\ 0.48 & 0.01 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 0.19 & -0.11 \\ 1.58 & 0.11 \end{bmatrix},$$

$$C_3 = \begin{bmatrix} -0.61 & 0.25 \\ 0.31 & 0.20 \end{bmatrix},$$

$$C_4 = \begin{bmatrix} 0.17 & 0.48 \\ 0.39 & 0.27 \end{bmatrix},$$

$$D_1 = \begin{bmatrix} 0.27 \\ 0.23 \end{bmatrix},$$

$$D_2 = \begin{bmatrix} 0.38 \\ -0.29 \end{bmatrix},$$

$$D_3 = \begin{bmatrix} 0.34 \\ 0.29 \end{bmatrix},$$

$$D_4 = \begin{bmatrix} 0.1 \\ 0.17 \end{bmatrix}. \tag{45}$$

The transition rate matrix Π with the partially unknown elements is presented as

$$\begin{bmatrix} 0.2 & 0.1 & ? & ? \\ ? & ? & 0.2 & 0.3 \\ 0.2 & ? & 0.4 & ? \\ 0.3 & ? & ? & ? \end{bmatrix}. \tag{46}$$

By Theorem 8, the controller gains for system (33) are given as

$$\begin{aligned} K_1 &= \begin{bmatrix} -0.2275 & -0.7036 \end{bmatrix}, \\ K_2 &= \begin{bmatrix} 1.0951 & 0.1754 \end{bmatrix}, \\ K_3 &= \begin{bmatrix} 0.3090 & 0.1018 \end{bmatrix}, \\ K_4 &= \begin{bmatrix} 1.8828 & -5.4904 \end{bmatrix}. \end{aligned} \tag{47}$$

Remark 9. It was shown that the random packet loss and channel delay in the network control system are often modeled as Markov chains and the variation of delays and packet dropouts may be random in the different period of networks [34]. Therefore, it is difficult to obtain complete elements of the transition probabilities matrix. The same problems may arise in a single-link robot arm in [31, 35], whose dynamic equation is presented as

$$\ddot{\theta}(t) = \frac{-MgL}{J} \sin(\theta(t)) - \frac{D(t)}{J}\dot{\theta}(t) + \frac{1}{J}u(t), \tag{48}$$

where $\theta(t)$ is the angle position of the arm, $u(t)$ is the control input, g is the acceleration of gravity, L is the length of the arm, $D(t)$ is the coefficient of viscous friction which is assumed to be time invariant, M is the mass of the payload, and J is the moment of inertia. Let $x_1(t) = \theta(t)$, $x_2(t) = \dot{\theta}(t)$, and $-\pi/2 \le \theta(t) \le -\pi/2$. Under this condition, $\sin(\theta(t))$ is usually denoted as $\theta(t)$ when $\theta(t)$ is about 0 rad. Next, consider that system (48) can be modeled as a Markovian jump system with 4 subsystems:

$$\dot{x}(t) = A_i x(t) + B_i u(t), \tag{49}$$

where

$$\begin{aligned} A_i &= \begin{bmatrix} 0 & 1 \\ \dfrac{-M_i g L}{J_i} & \dfrac{-D}{J_i} \end{bmatrix}, \\ B_i &= \begin{bmatrix} 1 \\ \dfrac{1}{J_i} \end{bmatrix}, \\ & i \in \{1, 2, 3, 4\}. \end{aligned} \tag{50}$$

However, it might occur that A_i, B_i is subject to some random environmental noise effects [1, 2]. In this case, (49) becomes stochastic Markovian jump systems with partially unknown transition probabilities and multiplicative noise

$$dx(t) = [A_i x(t) + B_i u(t)]\, dt + [C_i x(t) + D_i u(t)]\, dw(t). \tag{51}$$

If the parameters are taken as $g = 9.8$, $L = 0.5$, $D(t) = D = 2$, $M_1 = 1$, $M_2 = 1.25$, $M_3 = 2.5$, $M_4 = 2.25$, $J_1 = 1$, $J_2 = 1.25$, $J_3 = 2$, $J_4 = 2.5$, then the controller gains for system (15) are provided in (44).

5. Conclusion

In this paper, the stability and stabilization problems for a class of stochastic Markovian jump linear systems (MJLS) with partly unknown transition rates have been studied. The LMI-based sufficient conditions ensuring systems considered to be stable are given in the continuous- and discrete-time cases. Numerical examples are provided to show the validness and applicability of the developed results.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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