Research Article

A Two-Stage Queue Model to Optimize Layout of Urban Drainage System considering Extreme Rainstorms

Xinhua He¹ and Wenfa Hu²

¹Shanghai Maritime University School of Economics and Management, Shanghai 201306, China

²Tongji University School of Economics and Management, Shanghai 200092, China

Correspondence should be addressed to Wenfa Hu; wenfahu@tongji.edu.cn

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Extreme rainstorm is a main factor to cause urban floods when urban drainage system cannot discharge stormwater successfully. This paper investigates distribution feature of rainstorms and draining process of urban drainage systems and uses a two-stage single-counter queue method $M/M/1 \rightarrow M/D/1$ to model urban drainage system. The model emphasizes randomness of extreme rainstorms, fuzziness of draining process, and construction and operation cost of drainage system. Its two objectives are total cost of construction and operation and overall sojourn time of stormwater. An improved genetic algorithm is redesigned to solve this complex nondeterministic problem, which incorporates with stochastic and fuzzy characteristics in whole drainage process. A numerical example in Shanghai illustrates how to implement the model, and comparisons with alternative algorithms show its performance in computational flexibility and efficiency. Discussions on sensitivity of four main parameters, that is, quantity of pump stations, drainage pipe diameter, rainstorm precipitation intensity, and confidence levels, are also presented to provide guidance for designing urban drainage system.

1. Introduction

A storm drainage system is a critical infrastructure for a city to support its development, but many cities in China are suffering severe flood problems because of inadequate investment in their drainage system [1, 2]. An urban flood may seriously ruin buildings, block traffic, and even threaten resident's life. Although there are many factors to cause urban floods, inadequate drainage capacity and improper drainage layout are main factors. Human behavior as well as economy varies in different districts, whose hydrological characteristics change, and a rainstorm would cause different consequences [3].

An urban drainage system usually consists of ground-water convergence points, drainage pipes, and stormwater pump stations and outlets. Its drainage capacity is determined by complicated combination of the above components. The draining operation process can be regarded as a serial two-stage queuing system with three main activities: in the first queue stage, the drainage system is designed to

collect stormwater runoff from the road surface and right-of-way. Stormwater collection is here accommodated through the use of drainage inlets, which serve as the mechanism whereby surface water enters underground storm drainage pipes. Drainage inlet locations are often established by the roadway and on side streets to reduce the spread of water onto the roadway surface. Upon reaching the drainage pipes, stormwater is conveyed along to its discharge point. In the second queue state, the drainage system is to discharge stormwater from the drainage pipes by stormwater pump station to an adequate receiving body such as a stream, creek, river, lake, or other piped systems. A stormwater pump station is often required to drain depressed sections of drainage pipes where gravity drainage is impossible or not economically justifiable.

Many researchers have studied the queuing theory of serial multiple service stations. Paper [4] presented a two-stage queue $M/G/1$ model in solving the state probabilities of the multiqueue problem. Paper [5] developed an $M^{[x]}/(G_1, G_2)/1$ retrial queue model to solve a repeated

drainage problem. Paper [6] focused on a two-stage queuing network for outfitting process in shipbuilding. Paper [7] extended the cascaded queuing system to infinite servers queue system to solve logistics industry problems. It is a reasonable approach to adopt serial multiple queue model for this urban drainage system layout problem. Many researchers also developed queue models to optimize and design facility layout. Paper [8] presented an innovative approach to solve a layout problem with a multiobjective nonlinear mixed integer programming model considering $M/M/m$ queuing congestion. Paper [9] developed a queuing model of ambulance service to calculate key performance indicators which both managers and patients focused on. Paper [10] proposed a multipriority coverage layout model based on queuing theory.

But a facility layout system often faces uncertain factors which might result in stochastic demands; how to use queue models to design a facility layout in an uncertain environment attracts many researchers. Papers [11], [12], and [13] considered influences of stochastic factors on the multiobjective queuing location-allocation problem, respectively. Papers [14–16] discussed the uncertain problems of fuzzy factors on location by fuzzy queuing models.

When designing a stochastic facility system, its objectives and constraints are stochastic; how to solve this kind of nonlinear programming problem is a challenge which few literatures observed. Paper [17] developed a queuing model considering stochastic custom demands and unmovable service counter in a facility layout problem and then solved it by genetic algorithm with expectation value. Paper [18] established a biobjective model for bidirectional facility network under uncertain supplies and then presented a solution method combined with queuing theory and fuzzy programming.

However, previous studies mainly focus on such problems as optimization of queuing layout routines or assignment of customs; they have never tried to solve complex problems such as the following: (1) effects of extreme demands from customers, for example, extreme climate; (2) facility layout as a strategic decision problem because a drainage facility often operates for a long time; (3) priority of custom demands in various areas where a rainstorm might have different damage and losses in different urban neighborhoods.

This paper considers characteristics of extreme rainstorm and urban flood, stochastic demand for drainage, drainage system capability, fuzzy sojourn time of the system, and draining priority in different areas and then discusses the problem of drainage facility layout under extreme rainstorms. A biobjective, drainage system cost and drainage sojourn time model will be developed, and a stochastic and fuzzy generic algorithm is presented to optimize stormwater impacts on drainage facilities.

The remainder is structured as follows. The mathematical modeling of urban drainage system layout under extreme rainstorm is introduced in Section 2, including the identification of parameters and decision variables, assumptions, methodology for stochastic and fuzzy problems, and constraints. A solution method of improved genetic algorithm for nondeterministic problems with hybrid constraints of

randomness and fuzziness is described in Section 3. An example illustrating the parameter setting and flexibility of the proposed algorithm is provided in Section 4. Sensitivities of several parameters and their patterns are discussed in Section 5. Finally, some conclusions are provided in Section 6.

2. Mathematical Modeling

An urban drainage system is designed to collect stormwater runoff from the roadway surface through inlets, to convey it through drainage pipes, and to discharge it to a receiving body by pump stations. The draining process is regarded as a stochastic two-stage queuing system in this paper, where stormwater, as customers, arrives at the drainage system in uncertain time and stochastic quantity. The stochastic and fuzzy features increase complexity in designing urban drainage system. A mathematical model of urban drainage system with stochastic and fuzzy parameters is developed as follows.

2.1. Parameters and Decision Variables. The model parameters and their notations and decision variables are listed in Notations.

Both I and J are set of discrete points in a draining district. Z is determined by a_j , b_j , c_{ij} , and h_r . A smaller Z means the construction and operation cost of urban drainage system is less. d_{ij} is drainage pipe length which is an equivalent distance of drainage pipe from point i to pump station j . s_r is water capacity of drainage pipe whose diameter is r . T is total sojourn time of stormwater collected, conveyed, and discharged by urban drainage system, determined by the longest sojourn time $\max\{t_{ij}\}$. The smaller $\tilde{t}_{ij}$ is, the more efficient the drainage system is. Sojourn time $\tilde{t}_{ij}$ is a fuzzy value, and its mean value is affected by θ_i , $\bar{\lambda}_j$, and u_j . λ_i is demand rate at which stormwater at point i receives service from drainage pipe, and it is relevant to precipitation intensity f_i at point i .

There are three decision variables, m_j , n_{ij} , and x_j , in this model. m_j and n_{ij} are 0-1 variables, and x_j is a continuous variable. m_j indicates whether pump station j is constructed at point j or not, n_{ij} indicates whether pump station j provides service to point i or not, and x_j is service ability, namely, draining capability, of pump station j .

2.2. Assumptions. An extreme rainstorm often shows such characteristics as outburst, intensive precipitation, uneven geographical distribution, and short duration. If such intensive stormwater is not drained out in time, flood or waterlog will appear. When designing a drainage system, all rainfall, equivalent to precipitation in its covered area, is a critical factor. Although precipitation in urban district can be obtained from historical weather data, it is hardly predicted accurately because of its uncertain and random weather conditions. The precipitation duration and intensity are also regarded as stochastic variables. Here are main assumptions in modeling urban drainage system in this paper.

Assumption 1. Precipitation intensity of an extreme rain-storm in an urban district is subject to a stationary Poisson distribution and the duration is stochastic [19, 20].

When precipitation intensity f_i at point i is subject to a Poisson distribution, its distribution function P of precipitation intensity f_i is

$$P(f_i = k) = \frac{\tau^k}{k!} e^{-\tau}, \quad k = 0, 1, 2, \dots; \quad \tau > 0. \quad (1)$$

The parameter τ in the distribution function P could be determined by the maximum likelihood estimation. If there is a population X and sample $x_1, x_2, \dots, x_n$, then its maximum likelihood function [21] is

$$L(\tau) = \prod_{i=1}^n P(X = x_i) = \prod_{i=1}^n \frac{\tau^{x_i}}{x_i!} e^{-\tau}. \quad (2)$$

The parameter τ would be replaced by the maximum likelihood estimation $\hat{\tau}$:

$$\hat{\tau} = \frac{1}{n} \sum_{i=1}^n x_i. \quad (3)$$

This distribution function adaptability then can be tested by chi-square distribution method which has been broadly applied in fit test [22, 23]. The test hypotheses are as follows:

(H_0) The population X is subject to τ Poisson distribution.

(H_1) The population X is not subject to τ Poisson distribution.

All values Ω from X are divided into m subsets: $A_1, A_2, \dots, A_m$, which are pairwise disjoint. Observation of samples $x_1, x_2, \dots, x_m$ reveals that g_i ($i = 1, 2, \dots, m$) belongs to A_i and its occurrence frequency is g_i/n , where a_{i-1} and a_i are, respectively, the lower and upper bounds of group A_i . Then, the test probability $\hat{p}_i$ and the fitting value χ^2 are, respectively, obtained:

$$\hat{p}_i = \hat{P}(A_i) = \sum_{k=a_{i-1}}^{a_i} \frac{\tau^k}{k!} e^{-\tau}, \quad i = 1, 2, \dots, m \quad (4)$$

$$\chi^2 = \sum_{i=1}^m \frac{g_i^2}{n\hat{p}_i} - n.$$

There are w parameters to be estimated, and their confidence level is α . If $\chi^2 \leq \chi_{\alpha}^2(m-w-1)$, then the hypothesis H_0 will be accepted and its expectation and variance are $\mu_{f_i} = \tau$ and $\sigma_{f_i}^2 = \tau$, respectively; otherwise, the hypothesis H_1 will be accepted.

Assumption 2. All stormwater which is drained by drainage pipes comes from inlets, and its volume is equal to quantity of all precipitation.

Stormwater quantity at point i is multiple of its precipitation intensity, precipitation area, and precipitation duration; that is,

$$q_i = f_i o_r t_d, \quad (5)$$

where o_r is the rainfall area covered by drainage pipes (pipe diameter is r_r) and t_d is precipitation duration.

Assumption 3. Stormwater flows in separate queues inside drainage pipes and its flowing sequence will not change until arriving at its pump station. The arrival time of stormwater which flows from drainage pipes to its pump station is stochastic, and only the nearest pump station will provide service to it.

Assumption 4. Urban drainage system consists of several drainage pipes and pump stations, and the whole draining procedure in which stormwater is processed by drainage system is regarded as a serial two-stage queue system, namely, $M/M/1 \rightarrow M/D/1$ queue system, subject to first-come first-served (FCFS) order.

The service counter in the first stage of this queue system is the drainage pipe, and its serviced object is stormwater runoff collected by inlets and its service capacity is drainage pipe's water capacity. In the second stage, the service counter is the pump station and its serviced object is stormwater conveyed inside drainage pipes which finally is discharged into rivers or other bigger pipes, and its service capacity is pump station's operation capacity.

The service time of drainage pipe is regarded as an independent random variable which is subject to a negative exponential distribution [24]. The service capacity of drainage pipe j is u_j , which is relevant to its diameter:

$$u_j = \frac{\pi}{4} r^2 v_j, \quad (6)$$

where r is the pipe diameter and v_j is the average flowing velocity inside drainage pipe j .

A drainage pipe j only receives service from one pump station j . If service capacity of the pump station j is x_j , the service time and the arrival time in two stages are independent, the demand rate of stormwater at point i is λ_i , and the average arrival rate of stormwater arriving at pump station j is $\bar{\lambda}_j$, then, service intensities in two stages are ρ_1 and ρ_2 , respectively:

$$\begin{aligned} \rho_1 &= \frac{\lambda_i}{u_j}, \\ \rho_2 &= \frac{\bar{\lambda}_j}{x_j}. \end{aligned} \quad (7)$$

Assumption 5. Each pump station in the urban drainage system is independent but each one has an independent drainage capacity. The sojourn time of a pump station discharging stormwater is fuzzy.

A deterministic sojourn time θ_i of stormwater collected from point i , conveyed by drainage pipe and discharged by pump station j in urban drainage system, can be derived:

$$\theta_i = \frac{\rho_1 \rho_2 - \rho_1 + 1}{u_j (1 - \rho_1)} + \frac{1}{x_j (1 - \rho_2)}. \quad (8)$$

However, there are many factors affecting the sojourn time; it is regarded as a fuzzy variable in this paper. Since triangular fuzzy number has been proven to be excellent by previous literatures in solving fuzzy simulation problems [25, 26], the sojourn time is regarded as a triangular fuzzy number and it can be transferred to a possibilistic value by this method. The total sojourn time of stormwater from point i processed by pump station j can be represented by a triangular fuzzy number $\tilde{t}_{ij}$:

$$\tilde{t}_{ij} = (\underline{\theta}_i, \theta_i, \overline{\theta}_i). \quad (9)$$

Its fuzzy membership function is

$$f_{\tilde{t}_{ij}}(t) = \begin{cases} \frac{t - \underline{\theta}_i}{\theta_i - \underline{\theta}_i}, & \underline{\theta}_i \leq t \leq \theta_i \\ \frac{\overline{\theta}_i - t}{\overline{\theta}_i - \theta_i}, & \theta_i \leq t \leq \overline{\theta}_i \\ 0, & t \leq \underline{\theta}_i \text{ or } t \geq \overline{\theta}_i. \end{cases} \quad (10)$$

Considering confidence level of fuzzy number, its γ -cut set is $[\tilde{t}_{ij}]^\gamma = [t_{ij,\gamma}^-, t_{ij,\gamma}^+]$, $\forall \gamma \in [0, 1]$:

$$\begin{aligned} t_{ij,\gamma}^- &= \underline{\theta}_i + \gamma(\theta_i - \underline{\theta}_i) \\ t_{ij,\gamma}^+ &= \overline{\theta}_i - \gamma(\overline{\theta}_i - \theta_i). \end{aligned} \quad (11)$$

The upper possibilistic mean value $M_U(\tilde{t}_{ij})$ and the lower possibilistic mean value $M_L(\tilde{t}_{ij})$ of drainage sojourn time $\tilde{t}_{ij}$ in γ -cut set are obtained:

$$\begin{aligned} M_L(\tilde{t}_{ij}) &= \frac{\int_0^1 \gamma t_{ij,\gamma}^- d\gamma}{\int_0^1 \gamma d\gamma} \\ M_U(\tilde{t}_{ij}) &= \frac{\int_0^1 \gamma t_{ij,\gamma}^+ d\gamma}{\int_0^1 \gamma d\gamma}. \end{aligned} \quad (12)$$

The clear possibilistic mean value of the fuzzy numbers is obtained from $M_U(\tilde{t}_{ij})$ and $M_L(\tilde{t}_{ij})$ [27]. Therefore the possibilistic mean value $\overline{M}(\tilde{t}_{ij})$ of drainage sojourn time $\tilde{t}_{ij}$ is as follows:

$$\overline{M}(\tilde{t}_{ij}) = \frac{M_L(\tilde{t}_{ij}) + M_U(\tilde{t}_{ij})}{2} = \int_0^1 \gamma (t_{ij,\gamma}^- + t_{ij,\gamma}^+) d\gamma. \quad (13)$$

The possibilistic variance $\overline{V}(\tilde{t}_{ij})$ of the total drainage sojourn time is as follows:

$$\begin{aligned} \overline{V}(\tilde{t}_{ij}) &= \int_0^1 \gamma \left[(\overline{M}(\tilde{t}_{ij}) - t_{ij,\gamma}^-)^2 + (\overline{M}(\tilde{t}_{ij}) - t_{ij,\gamma}^+)^2 \right] d\gamma. \end{aligned} \quad (14)$$

Assumption 6. If a region at point i is more important than other regions, a smaller restriction value e_i will be applied to reduce its sojourn time of stormwater. e_i is a stochastic

variable and subject to normal distribution. When there are s experts, each one ranks a restricted sojourn time F_{si} of stormwater at point i ; then, its expectation μ_{ei} and variance σ_{ei}^2 can be obtained, and its restriction value e_i of sojourn at point i can be described by this normal distribution function $N(\mu_{ei}, \sigma_{ei})$.

One of the service time constraints is the regional drainage priority based on their importance. The same rain-storm has varied impacts on different districts because their characteristics such as population, traffic, and economics are totally different. In order to reduce potential flood or waterlog in key regions such as commercial centers, major communities, busy roads, and political and culture centers, a regional priority is introduced to adjust their drainage sojourn time as well as drainage service capability in different regions. An expert ranking method or weighting method is often used in choosing key regions. So several experts can be asked to choose a constraint value e_i to adjust regional sojourn time.

Assumption 7. Total cost of urban drainage system includes (1) construction cost of drainage pipes, (2) construction cost of pump stations, (3) equipment cost of pump stations, and (4) operation cost of drainage system. Among them, construction and equipment cost of pump stations is supposed to be proportional to its drainage capacity, and pipe's construction cost is proportional to its length.

2.3. Methodology for Modeling a Stochastic and Fuzzy System. An urban drainage system incorporates randomness and fuzziness because of its uncertain environment, so its model consists of mixed conditions with a lot of random and fuzzy parameters. After considering this kind of mixed conditions, a methodology to model a stochastic and fuzzy system is presented as follows:

$$\begin{aligned} \min \quad & \overline{F} \\ \text{s.t.} \quad & \text{Pos} \{ \Pr \{ f(x, \xi, \tilde{\eta}) \leq \overline{F} \} \geq \alpha_1 \} \geq \alpha_2 \\ & \text{Pos} \{ \Pr \{ g_j(x, \xi, \tilde{\eta}) \leq 0 \} \geq \beta_{1j} \} \geq \beta_{2j}, \\ & j = 1, 2, \dots, m, \end{aligned} \quad (15)$$

where $\text{Pos}\{\cdot\}$ represents possibility of an event in $\{\cdot\}$ and $\text{Pr}\{\cdot\}$ represents probability of an event in $\{\cdot\}$. x is a decision vector, ξ is a stochastic parameter vector, and $\tilde{\eta}$ is a fuzzy parameter vector. $f(x, \xi, \tilde{\eta})$ is an objective function, and $g_j(x, \xi, \tilde{\eta})$ is a constraint function.

Under mixed conditions, the objective function is $\text{Pos}\{\text{Pr}\{f(x, \xi, \tilde{\eta}) \leq \overline{F}\} \geq \alpha_1\} \geq \alpha_2$ and its value is $\min \overline{F}$. The constraint function is $\text{Pos}\{\text{Pr}\{g_j(x, \xi, \tilde{\eta}) \leq 0\} \geq \beta_{1j}\} \geq \beta_{2j}$, $j = 1, 2, \dots, m$. The position of randomness and fuzziness in the same function can be exchanged. Therefore, $\text{Pos}\{\text{Pr}\{\cdot\}\}$ is equivalent to $\text{Pr}\{\text{Pos}\{\cdot\}\}$.

2.4. A Biobjective Model of Urban Drainage System. The two objectives in this biobjective model of urban drainage system

are total sojourn time T of stormwater and total cost Z . The total sojourn time T is the maximum of sojourn time t_{ij} of stormwater from all points i ($i \in I$). The total cost Z consists of construction cost, equipment cost, and operation cost.

Considering stochastic rainstorm demand, fuzzy sojourn time, and its restriction value, the biobjective model with random and fuzzy constraints for urban drainage system is developed:

$$\min \quad \bar{T} \quad (16)$$

$$\min \quad \bar{Z} \quad (17)$$

$$\text{s.t.} \quad \text{Pos} \left\{ \max_{j \in J} \left(\sum_{i \in I} \tilde{t}_{ij} m_j n_{ij} \right) \leq \bar{T} \right\} \geq \alpha_1 \quad (18)$$

$$\text{Pr} \left\{ \sum_{j \in J} a_j x_j + \sum_{j \in J} b_j m_j + \sum_{j \in I} \sum_{i \in J} h_r d_{ij} n_{ij} + \sum_{j \in I} \sum_{i \in J} c_{ij} d_{ij} f_i o_r \tilde{t}_{ij} n_{ij} \leq \bar{Z} \right\} \geq \alpha_2 \quad (19)$$

$$\text{Pos} \left\{ \Pr \left\{ \tilde{t}_{ij} n_{ij} \leq e_i \right\} \geq \beta_1 \right\} \geq \beta_2, \quad \forall i \in I, j \in J \quad (20)$$

$$d_{ij} = l_{ij}^r o_r \quad (21)$$

$$f_i o_r \tilde{t}_{ij} \leq s_r d_{ij}, \quad \forall i \in I, j \in J \quad (22)$$

$$\text{Pr} \left\{ x_j = \sum_{i \in I} f_i n_{ij} \right\} \geq \beta_3, \quad \forall j \in J \quad (23)$$

$$n_{ij} \leq m_j, \quad \forall i \in I, j \in J \quad (24)$$

$$\sum_{j \in J} m_j n_{ij} = 1, \quad \forall i \in I \quad (25)$$

$$\sum_{k \in J | d_{ik} \leq d_{ij}} n_{ik} \geq m_j, \quad \forall i \in I, j \in J \quad (26)$$

$$\sum_{j \in J} m_j = p \quad (27)$$

$$\bar{\lambda}_j = \sum_{i \in I} \lambda_i n_{ij}, \quad \forall j \in J \quad (28)$$

$$m_j, n_{ij} \in \{0, 1\}, \quad \forall i \in I, j \in J. \quad (29)$$

The objective function (16) is to minimize total sojourn time of stormwater processed by drainage system, and constraint function (18) represents the possibility of total sojourn time including waiting time and service time which is not less than α_1 . The objective function (17) is to minimize the expected value of total cost that the stormwater is processed by drainage system, and constraint (19) is to ensure cost possibility is not less than α_2 and denotes the total cost components.

Constraint (20) is to ensure probability of total sojourn time of drainage system subject to its restriction value is not less than β_1 and its possibility is guaranteed to be not less than β_2 . Constraint (21) denotes a method to estimate length of

drainage pipes. Constraint (22) is to ensure stormwater will not exceed the maximum water capacity of drainage pipes. Constraint (23) ensures that probability of balance between service ability of pump stations and drainage demands is not less than β_3 . Constraint (24) denotes that stormwater demanding service of drainage system appears only after establishment of a pump station at location j is finished. Constraints (25) and (26) indicate that stormwater at point i is only served by its nearest drainage pump station. Constraint (27) is a given quantity of pump stations to be constructed in this system. Constraint (28) denotes the average arrival rate of stormwater to pump station j in unit time. Constraint (29) is property of two decision variables.

3. An Improved Genetic Algorithm for Stochastic and Fuzzy System

Since a stochastic and fuzzy system demonstrates complex nonlinear characteristics in its objectives and constraints which are not easy to be converted into deterministic variables, it is difficult to solve a large-scale nonlinear problem by traditional algorithms. Fortunately, many heuristic algorithms such as genetic algorithm have been developed by researchers to solve deterministic problems [28] as well as nonlinear problems [29].

In order to apply genetic algorithm in stochastic and fuzzy urban drainage system, its nondeterministic variables would be sampled based on probability and possibility distribution method and then a simulation model could be developed [30]. After improvement of genetic algorithm, nonlinear problems in urban drainage system would be solved.

3.1. Method of Simulating Nondeterministic Variables. For a given decision vector x , $\text{Pos}\{\text{Pr}\{g_j(x, \xi, \bar{\eta}) \leq 0, j = 1, 2, \dots, m\} \geq \beta_1\} \geq \beta_2$ is equal to

$$\begin{aligned} \mu(\eta^0) &\geq \beta_2, \\ \text{Pr}\{g_j(x, \xi, \bar{\eta}) \leq 0, j = 1, 2, \dots, m\} &\geq \beta_1. \end{aligned} \quad (30)$$

In order to verify the above equation, a fuzzy vector $\bar{\eta}$ generates evenly an η^0 which satisfies $\mu(\eta^0) \geq \beta_2$; namely, η^0 is extracted from level γ -cut set of $\bar{\eta}$, or η^0 is extracted from a super geometry body Ω which includes level γ -cut set. The condition $\mu(\eta^0) \geq \beta_2$ determines whether η^0 is accepted or not.

After η^0 is selected, $\text{Pr}\{g_j(x, \xi, \eta^0) \leq 0, j = 1, 2, \dots, m\} \geq \beta_1$ is verified in the following procedure. There are N individual stochastic variables $\xi_1, \xi_2, \dots, \xi_N$ generated from probability distribution $\Phi(\xi)$. After N times tests of $g_j(x, \xi, \eta^0) \leq 0, j = 1, 2, \dots, m$, if N' times are observed and $N'/N \geq \beta_1$, then $\text{Pos}\{\text{Pr}\{g_j(x, \xi, \bar{\eta}) \leq 0, j = 1, 2, \dots, m\} \geq \beta_1\} \geq \beta_2$ is accepted. Otherwise, η^0 should be selected again and the procedure would be repeated. After K times of selecting η^0 , $\text{Pr}\{g_j(x, \xi, \eta^0) \leq 0, j = 1, 2, \dots, m\} \geq \beta_1$, and then the given vector x is regarded as unacceptable.

When calculating result of $\text{Pos}\{F \geq \beta_1 \mid F = \text{Pr}\{g_j(x, \xi, \eta^0) \leq 0, j = 1, 2, \dots, m\}\}$, if $N'/N \geq \beta_1$ and $G < \mu(\eta^0)$, then $F = N'/N$ and $G = \mu(\eta^0)$.

The objective function $\text{Pos}\{\text{Pr}\{f(x, \xi, \bar{\eta}) \leq \bar{F}\} \geq \alpha_1\} \geq \alpha_2$ in urban drainage system is to maximize $\bar{F}$ value in its inequality when the decision vector x is given. Its solution procedure is described as follows. η^0 is evenly generated from $\bar{\eta}$ and $\mu(\eta^0) \geq \alpha_2$. There are N independent stochastic variables, $\xi_1, \xi_2, \dots, \xi_N$, generated from $\Phi(\xi)$ and a sequence $\{f_1, f_2, \dots, f_N\}$ is obtained, where $f_i = f(x, \xi_i, \eta^0), i = 1, 2, \dots, N$.

It is supposed that the integer part of $\alpha_1 N$ is N' , $N' = \text{int}(\alpha_1 N)$, and the N' th biggest element of $\{f_1, f_2, \dots, f_N\}$ is f_0 . If $f_0 \leq \bar{F}$, then $\bar{F} = f_0$. After repeating K times of choices, $\bar{F}$ is regarded as the objective value at point x .

3.2. Improved Genetic Algorithm for Urban Drainage System. Genetic algorithm is a high parallel, random, and adaptive intelligent optimization algorithm [31]. An improved genetic algorithm can simulate stochastic and fuzzy characteristics in its process of producing population, crossover, and mutation operations and calculating value of chromosome [32]. An improved genetic algorithm is redesigned to solve urban drainage system as follows.

(1) *Coding.* The decision variables n_{ij} and x_j are coded by a mixed coding method which is combined with floating point and binary code. The code length is $J + J \times I$, where I, J represent amount of stormwater inlets points and the amount of pump stations, respectively.

The first J digits in the code are a floating point number, representing decision variable $x_j, x_j \in [0, +\infty)$. If $x_j = 0$, then $m_j = n_{ij} = 0, i = 1, 2, \dots, I, j = 1, 2, \dots, J$, which denotes that in the point j a pump station will not be constructed. If $x_j > 0$, then $m_j = 1$, which denotes that a pump station will be constructed in point j .

The $(J+1)$ th to the $(J+J \times I)$ th digits in the code are binary numbers, which represent decision variable n_{ij} . If $n_{ij} = 1$, it denotes that stormwater will be discharged by pump station j . If $n_{ij} = 0$, it denotes that stormwater will not be discharged by pump station j . Each chromosome can be decoded, and its genotype and phenotype can be exchanged.

(2) *Initialization.* Several random points are generated from the feasible domain of decision vectors when chromosome V_k is initialized. Then, its feasibility would be tested by a random and fuzzy simulation method. If it is feasible, the chromosome will be selected. Otherwise, a new random point should be generated until a feasible solution is obtained. After repeating *popsizes* times, there are *popsizes* initial feasible chromosomes V_k obtained. Target values of all chromosomes would be obtained by the random and fuzzy simulation method.

(3) *Setting Fitness Function.* A probability value is used in each chromosome V_k which is determined based on order of an evaluation function $\text{eval}(V_k)$. Roulette wheel selection method is applied to make the possibility of each selected chromosome be proportional to the adaptability of other chromosomes in the same population, meaning that a more adaptable chromosome would be selected firstly and to produce possibly more offspring.

The evaluation function $\text{eval}(V_k)$ is

$$\text{eval}(V_k) = \alpha(1 - \alpha)^{k-1}, \quad k = 1, 2, \dots, \text{popsizes}, \quad (31)$$

where $\alpha \in (0, 1)$ and usually $\alpha = 0.05$. k is ordered from good to bad according to values of objective functions. A better chromosome is obviously assigned a smaller number.

(4) *Selection.* Cumulative probability is calculated based on evaluation function $\text{eval}(V_k)$ of each chromosome V_k .

$$q_k = \sum_{j=1}^k \text{eval}(V_j), \quad k = 1, 2, \dots, \text{popsizes}. \quad (32)$$

A random number q' is generated from the interval $[0, q_{\text{popsize}}]$. If $q_{k-1} < q' \leq q_k$, then the k th chromosome V_k is selected, and popsize times of duplicated chromosomes are available after repeating popsize times.

(5) *Crossover*. The probability of crossover operation is p_c and the following process is repeated from $k = 1$ to popsize : A random number p' is generated from the interval $[0, 1]$. If $p' < p_c$, chromosome V_k is selected as a parent to participate in crossover operation. After selection, the parents are named V'_k , $k = 1, 2, \dots$, and they are divided into pairwise groups $(V'_1, V'_2), (V'_3, V'_4), \dots$. If the number of parents is odd, one of them is deleted randomly. A random number c is generated from $(0, 1)$ and crossover is operated in the number of the first J digits of each parent. Therefore, two children are generated from the first J digits. The crossover operation is expressed as follows:

$$\begin{aligned} X^J &= c \cdot V_1^{JJ} + (1 - c) \cdot V_2^{JJ}, \\ Y^J &= c \cdot V_3^{JJ} + (1 - c) \cdot V_4^{JJ}. \end{aligned} \quad (33)$$

The numbers from the $(J + 1)$ th digit to $(J + J \times I)$ th digits of each parent are crossed as follows: A random genetic position is regarded as the cross point, where the same genetic positions at two parents are exchanged. Then, two children X^{J+1}, Y^{J+1} are generated from the $(J + 1)$ th digit to $(J + J \times I)$ th digit, respectively, in probability p_c .

Combination of X^J and X^{J+1} and Y^J and Y^{J+1} will generate two new offspring X and Y . The feasibility of offspring is tested by a random and fuzzy simulation method. If its test is feasible, it will be replaced by its parents. Otherwise, a new crossover operation will repeat until two feasible offspring are obtained or the iteration reaches a limit cycle.

(6) *Mutation*. The probability of mutation operation p_m and following process is repeated from $k = 1$ to popsize : A random number p'' is generated from the interval $[0, 1]$. If $p'' < p_m$, then the chromosome V_k is selected as a parent to participate in mutation operation.

Mutation is operated in the number of first J digits of each parent as follows: A genetic position is randomly selected as the mutation position, its genetic value is replaced by a random value generated from $[0, +\infty)$, and the number of its first J digits is regarded as its offspring X^J .

Mutation is operated at the number of the $(J + 1)$ th digit to $(J + J \times I)$ th digit of each parent as follows: A genetic position is also randomly selected as the mutation position, where 1 is replaced by 0 and 0 is replaced by 1. Then, its offspring X^{J+1} is generated at the $(J + 1)$ th digit to $(J + J \times I)$ th digit.

Combination of X^J and X^{J+1} is to generate a new offspring X . The feasibility of offspring is tested by a random and fuzzy simulation method. If its test is feasible, it will be replaced by its parents. Otherwise, a new mutation operation will repeat until a feasible offspring is obtained or the iteration reaches a limit cycle.

(7) The above process of selection, crossover, and mutation will repeat until they meet a termination criterion.

Finally, all optimal solutions of decision variables, sojourn time, and drainage cost are obtained, respectively.

4. Illustrative Example

Since a stochastic and fuzzy problem is so complicated that it hardly validates the proposed algorithm by any direct approaches, after reviewing the methods in [33, 34], this stochastic and fuzzy problem would be simplified as a simulated deterministic problem. In order to verify this algorithm's validity to solve the large-scale nonlinear complex model, a numerical example is chosen and several tests are validated and discussed.

4.1. *Description of Example and Setting of Parameters*. A square district in Shanghai is chosen as a numerical example to illustrate how to optimize drainage system layout by this algorithm. The district is 3200-meter-long and 3200-meter-wide, but it is suffering frequent extreme rainstorms which result in more losses. In order to reduce urban floods, local authority wish to rezone drainage area and redesign its drainage system soon. It is divided into 64 drainage blocks whose length is 400 meters, shown in Figure 1. Drainage block's centers are supposed to be drainage points and all groundwater inside a block only flows to its drainage point where an inlet is installed. A drainage pipe is laid underground to connect with an inlet and a pump station. All groundwater is collected by inlets, conveyed by underground drainage pipes to pump stations. Any drainage point could be a candidate position of pump station. Stormwater from one drainage point only flows to one pump station through a drainage pipe; its flowing direction would never change.

There will be eight stormwater pump stations to be constructed in this district. The goals of this project are to drain all stormwater within a minimal sojourn time and to minimize construction and operation cost of this project at 95% confidence level when an extreme rainstorm hits this district.

In the drainage system model, several basic parameters are shown as follows: $I = J = 64$, $p = 8$, $\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = \beta_3 = 0.95$.

Other parameters of simulation settings are defined as follows: $c_{ij} = 1$ yuan/m³, $r = 1200$ mm, and $S_r = 1.131$ m³/m. d_{ij} is a linear distance between point i and point j .

Precipitation intensity f_i at point i follows a Poisson distribution, which is estimated by 10-year 24-hour statistical precipitation data of extreme rainstorms. The probability distributions of precipitation intensity at all points $(f_1 \sim f_{64})$ are shown in Table 1.

Then, the distribution of precipitation intensity is tested by chi-square distribution method. Taking the point $i = 1$ as an example, where f_1 obeys a Poisson distribution of its parameter $\tau = 7.60$, all possible values Ω from f_1 are divided into seven groups, marked as $A_1 = [0, 2]$, $A_2 = (2, 4]$, $A_3 = (4, 6]$, $A_4 = (6, 8]$, $A_5 = (8, 10]$, $A_6 = (10, 12]$, and $A_7 = (12, +\infty)$. The chi-square distribution test process includes calculation of g_i , $\hat{p}_i$, $\hat{g}_i$, and $g_i^2/\hat{g}_i$, shown in Table 2.

After test, it is obtained that $\chi^2 = 7.8313$ and χ^2 freedom degree is $m - w - 1 = 5$. At the confidence level $\alpha = 0.05$,

TABLE 1: Probability distributions of precipitation intensity f_i (unit: mm/h).

$f_1 \sim f_8$	7.60	5.23	5.60	9.04	6.68	10.09	8.20	7.91
$f_9 \sim f_{16}$	7.62	7.67	6.15	10.09	9.65	10.01	9.65	7.23
$f_{17} \sim f_{24}$	8.93	8.90	8.06	5.39	5.55	8.85	5.95	7.30
$f_{25} \sim f_{32}$	9.81	7.11	9.35	7.95	7.76	6.45	9.38	8.53
$f_{33} \sim f_{40}$	8.41	9.32	5.10	6.50	7.67	6.09	6.69	10.00
$f_{41} \sim f_{48}$	10.17	8.11	8.08	9.22	8.15	7.22	6.11	6.91
$f_{49} \sim f_{56}$	8.50	6.52	8.70	7.32	8.24	5.25	9.22	5.73
$f_{57} \sim f_{64}$	5.35	7.70	6.99	5.95	6.81	8.14	8.35	9.58

FIGURE 1: A drainage district with 64 blocks.

TABLE 2: Example of chi-square distribution test for precipitation intensity f_1 .

A_i	g_i	$\hat{p}_i$	$\hat{g}_i = n\hat{p}_i$	$g_i^2/\hat{g}_i$
A_1	15	0.0188	9.3785	23.9911
A_2	47	0.1062	53.0908	41.6079
A_3	114	0.2397	119.8412	108.4435
A_4	156	0.2836	141.7853	171.6398
A_5	98	0.2053	102.6606	93.5510
A_6	53	0.1001	50.0266	56.1502
A_7	17	0.0464	23.2170	12.4478
Σ	500	1.0000	500.0000	507.8313

$\chi^2_\alpha(m - w - 1) = 11.070$. Therefore, $\chi^2 < \chi^2_\alpha(m - w - 1)$, which means H_0 is accepted at such a confidence level. So it

is verified that f_1 is subject to a Poisson distribution whose parameter $\tau = 7.60$. Its mean value $\mu_{f_1} = 7.60$ and its variance $\sigma_{f_1}^2 = 7.60$.

Considering some areas are more important in drainage management, eight experts are invited to rank all drainage points. A restriction value e_i at point i obtained after calculating their rankings by AHP. All restriction values are uncertain, and they are described by their mean values and variances, shown in Table 3.

The main parameters of improved genetic algorithm are shown as follows. Population size $popsize = 200$, crossover probability $p_c = 0.7$, mutation probability $p_m = 0.04$, order-based evaluation function $\alpha = 0.05$, and maximum iteration number $Maxgen = 1000$.

The first 64 digits in a chromosome gene are the decision variable x_j . The rest digits, namely, the 65th to $(64 + 64 \times 64)$ th

TABLE 3: Restriction value e_i at each drainage point (mean value and variance) (unit: min).

$e_1 \sim e_8$	(94, 13.5)	(83.9, 15.8)	(96.7, 20.2)	(103.1, 20.6)	(87.6, 26.2)	(110.4, 14.7)	(70.3, 23)	(75.6, 17.5)
$e_9 \sim e_{16}$	(94.6, 10.8)	(82.5, 24)	(95.9, 13.9)	(101.5, 13.7)	(87.1, 18)	(108.1, 13.7)	(73.1, 8.4)	(76.9, 14.7)
$e_{17} \sim e_{24}$	(89.6, 7.3)	(80.4, 24.3)	(96.2, 20.3)	(102.3, 19.3)	(87.7, 11.9)	(111.4, 16.7)	(74.3, 14.3)	(75.4, 6.7)
$e_{25} \sim e_{32}$	(91.7, 14.6)	(86.7, 17)	(95.7, 21.4)	(102.7, 10.1)	(90.1, 13.9)	(110.2, 18.8)	(73.2, 28.4)	(77.3, 14.3)
$e_{33} \sim e_{40}$	(92.9, 12.3)	(82.8, 14)	(96.7, 16.5)	(102.6, 17.7)	(88.2, 8.9)	(107.7, 20)	(71.7, 16.6)	(76.7, 30.2)
$e_{41} \sim e_{48}$	(93.3, 9.8)	(83.4, 14.4)	(96.6, 8.8)	(102.2, 18.2)	(85.4, 5.6)	(109.2, 12)	(72.4, 18.3)	(78.9, 10.4)
$e_{49} \sim e_{56}$	(93, 9.7)	(82.6, 15.1)	(97.7, 16.1)	(99.6, 19.6)	(83.6, 20.6)	(105.1, 24.1)	(73.8, 12.3)	(76.6, 32.1)
$e_{57} \sim e_{64}$	(93.6, 24)	(80.9, 7.5)	(98.5, 18.6)	(104.3, 8.5)	(87.4, 17.9)	(107.7, 11.7)	(72.8, 16.1)	(76.3, 19.3)

FIGURE 2: Optimal layout of the drainage system example.

digits, are decision variable n_{ij} . When chromosome V_k is initialized, a random point is generated from feasible domain of decision vector and its feasibility is verified by the random and fuzzy simulation method.

The evaluation function $\text{eval}(V_k) = 0.05(1 - 0.05)^{k-1}$, $k = 1, 2, \dots, 200$. Each chromosome V_k is then operated in selection, crossover, and mutation with a given probability and its offspring's feasibility is verified by the random and

fuzzy simulation method. The algorithm process will terminate after 1000 iterations. Finally, the optimal solution and the objective function values are obtained.

4.2. Main Results. The optimal locations of eight pump stations in this drainage system are x_{10} , x_{13} , x_{23} , x_{27} , x_{37} , x_{39} , x_{50} , and x_{54} . Its layout of inlets, drainage pipes, and pump stations is shown in Figure 2. Each pump station

TABLE 4: Solutions of decision variable x_j .

Location of pump station x_i	x_{10}	x_{13}	x_{23}	x_{27}	x_{37}	x_{39}	x_{54}	x_{58}
Drainage ability of pump station x_j (m ³ /s)	67.51	66.5	64.47	63.38	59.02	44.33	68.54	62.04

connects with six to nine inlet points by drainage pipes. All drainage inlets, pipes, and pump stations form an urban drainage system which provides optimal draining service for this district.

This drainage system can meet the urban drainage requirements at 95% service level, which means that two objectives, to minimize the total sojourn time and to minimize the total cost, are guaranteed to be fulfilled at not less than 95% level.

The optimal draining capacity of each pump station is shown in Table 4.

The optimal values of two objective functions are $\bar{T} = 1738.6$ min and $\bar{Z} = 4841.74$ million yuan, respectively. Their confidence levels are

$$\begin{aligned} \text{Pos} \left\{ \max_{1 \leq j \leq 64} \left(\sum_{i=1}^{64} \tilde{t}_{ij} m_j n_{ij} \right) \leq \bar{T} \right\} &\geq 0.9536 \\ \text{Pr} \left\{ \sum_{j=1}^{64} a_j x_j + \sum_{j=1}^{64} b_j m_j + \sum_{j=1}^{64} \sum_{i=1}^{64} h_r d_{ij} n_{ij} \right. & \\ \left. + \sum_{j=1}^{64} \sum_{i=1}^{64} c_{ij} d_{ij} f_i o_r \tilde{t}_{ij} n_{ij} \leq \bar{Z} \right\} &\geq 0.9587. \end{aligned} \quad (34)$$

The total sojourn time $\tilde{t}_{ij}$ of stormwater in this drainage system is stochastic, and their results from 64 points are described by three numbers $(\theta_i, \theta_i, \bar{\theta}_i)$, shown in Table 5.

The stochastic and fuzzy characteristics cause this system to vary in searching for optimal results in this case study. After repeating execution of this model for times, it reveals that probability and possibility of all optimal results can be guaranteed at more than 95% level. Therefore, it is verified that this model and improved algorithm are flexible and effective to solve complex nonlinear problems of drainage system.

4.3. Comparing Results by Alternative Algorithms. After repeating 10 times of solving the test by the redesigned genetic algorithm (RGA), their results of average optimal solutions including total time $\bar{T}$, total cost $\bar{Z}$, and running times are listed in Table 6. Its results are compared with alternative algorithms such as simulated annealing genetic algorithm (SA-GA) [35] and Tabu Search (TS) [36]. Their parameters are set as follows: Population size $popsiz$ = 200, crossover probability p_c = 0.75, mutation probability p_m = 0.04, maximum iteration number $Maxgen$ = 200~2000, and TS length L = $\sqrt{A}$, where A is quantity of candidate sets. The performance improvement of RGA is measured by

comparison of searched total time T and total cost Z with alternative algorithms, respectively, as follows:

$$\begin{aligned} &\text{performance improvement (\%)} \\ &= - \frac{\text{RGA value} - \text{SA} - \text{GA value}}{\text{RGA value}} \times 100. \end{aligned} \quad (35)$$

The comparison reveals that RGA is better than SA-GA in searching optimal solutions of total time $\bar{T}$ and total cost $\bar{Z}$ with max performance improvement of 8.43% in $\bar{T}$ and 3.72% in $\bar{Z}$ and average performance improvement of 4.32% in $\bar{T}$ and 2.55% in $\bar{Z}$, and RGA is better than TS with max improvement of 5.77% in $\bar{T}$ and 2.68% in $\bar{Z}$ and average performance improvement of 3.06% in $\bar{T}$ and 1.67% in $\bar{Z}$. It is validated that RGA has the best search ability to find out optimal solutions.

While comparing the running times of algorithms, RGA needs less running time than TS but needs slightly more running time than SA-GA. For example, RGA, SA-GA, and TS needs 173 s, 162 s, and 187 s, respectively, in solving Case 1, which reveals that RGA needs only 11 s more than SA-GA but 14 s less than TS. Because RGA is a single step algorithm redesigned for urban drainage layout system but TS is a two-step algorithm, RGA is generally faster than TS in searching optimal solutions. Although RGA needs more running time, 11 s, this running time is acceptable.

4.4. Validating the Proposed Algorithm. In order to validate the proposed algorithm, RGA is also reasonable and effective in solving fuzzy problems besides their stochastic features; Case 1 is chosen as a numerical test which is modified according to fuzzy features of urban drainage system. It is supposed that the poisson distribution parameter λ of precipitation density is 7.60 mm/h. After review of [37], sojourn time t_{ij} is a triangle fuzzy number by simply extending it at scale β as follows: $\tilde{t}_{ij} = ((1 - \beta)t_{ij}, t_{ij}, (1 + \beta)t_{ij})$.

Parameters of RGA in this test are the same as those in the above cases, and its repeat number is 100. Running time of RGA is nearly 17685 s, 4.9 hours, which is about 100 times of deterministic Case 1. The results after ten operations are shown in Table 7. Since precipitation is stochastic and sojourn time is fuzzy, neither total sojourn time nor total cost of urban drainage system is deterministic.

This demonstrates the following results: (1) Layout of pump stations is exactly the same in all optimal solutions of 10 tests; (2) maximum total cost in this test is 2500.37 million yuan, minimum total cost is 2492.71 million yuan, and mean total cost is 2496.08 million yuan, and cost deviation is 0.31%; (3) maximum total draining time in this test is 3887.5 minutes, minimum total draining time is 3837.4 minutes, and mean

TABLE 5: Total sojourn time $\tilde{t}_{ij}(\underline{\theta}_i, \underline{\theta}_j, \underline{\theta}_i)$ of stormwater in this drainage system (unit: min).

$\tilde{t}_{1,j} \sim \tilde{t}_{8,j}$	(76, 10, 12.9)	(16.5, 20, 22.1)	(25, 30, 32.9)	(30.5, 40, 41.6)	(48.2, 50, 55.1)	(46.4, 60, 66)	(60, 70, 73.2)	(64, 80, 97.3)
$\tilde{t}_{9,j} \sim \tilde{t}_{16,j}$	(177, 20, 23.3)	(22.8, 25, 27.7)	(25.2, 35, 37.6)	(36.6, 45, 56.5)	(44.5, 55, 56)	(62.1, 65, 67.9)	(66.1, 75, 77.1)	(69.8, 85, 94.5)
$\tilde{t}_{17,j} \sim \tilde{t}_{24,j}$	(25.5, 30, 36.3)	(31.4, 35, 38.4)	(35.4, 40, 40.7)	(47.6, 50, 51.3)	(47.2, 60, 75.7)	(62.1, 70, 84.8)	(69.7, 80, 92.1)	(73.7, 90, 94.6)
$\tilde{t}_{25,j} \sim \tilde{t}_{32,j}$	(40, 40, 46.2)	(43.7, 45, 54.1)	(43.6, 50, 51.6)	(41.8, 55, 65.9)	(48, 65, 79.9)	(66.8, 75, 78.9)	(84.1, 85, 101.5)	(72.6, 95, 118.3)
$\tilde{t}_{33,j} \sim \tilde{t}_{40,j}$	(40.8, 50, 56.8)	(47.3, 55, 57.2)	(55.3, 60, 68.4)	(64.3, 65, 82.5)	(67.1, 70, 81.6)	(78.8, 80, 88.7)	(68.5, 90, 102.5)	(95.9, 100, 125.7)
$\tilde{t}_{41,j} \sim \tilde{t}_{48,j}$	(49.1, 60, 70.8)	(53.7, 65, 69)	(61.5, 70, 76.1)	(66.1, 75, 77.1)	(75.6, 80, 84.1)	(83.1, 85, 106.3)	(73.7, 95, 120.4)	(82.5, 105, 111.6)
$\tilde{t}_{49,j} \sim \tilde{t}_{56,j}$	(59, 70, 77.6)	(74.1, 75, 86.3)	(77.8, 80, 89.4)	(70.4, 85, 95.3)	(89.7, 90, 105.2)	(83.1, 95, 101.6)	(74.2, 100, 116)	(98.3, 110, 116.4)
$\tilde{t}_{57,j} \sim \tilde{t}_{64,j}$	(79.4, 80, 93.1)	(72.2, 85, 86.8)	(72.3, 90, 108.7)	(85.4, 95, 116.7)	(80.7, 100, 103)	(99.8, 105, 129.3)	(91.3, 110, 134.5)	(86.8, 115, 127.8)

TABLE 6: Comparison of average results after 10 operations by RGA, SA-GA, and TS.

Case	Inlets	Candidate pump stations	RGA			SA-GA			TS			Improvement on SA-GA		Improvement on TS	
			Searched solution of time $\bar{T}$	Searched solution of cost $\bar{Z}$	Running time (s)	Searched solution of time $\bar{T}$	Searched solution of cost $\bar{Z}$	Running time (s)	Searched solution of time $\bar{T}$	Searched solution of cost $\bar{Z}$	Running time (s)	Time $\bar{T}$ (%)	Cost $\bar{Z}$ (%)	Time $\bar{T}$ (%)	Cost $\bar{Z}$ (%)
1	64	4	3887.5	2496.08	173	3947.3	2532.65	162	3921.7	2518.14	187	1.54	1.47	0.88	0.88
2	64	6	2518.7	3630.75	368	2588.5	3703.82	355	2566.3	3686.85	376	2.77	2.01	1.89	1.55
3	64	8	1738.6	4841.74	676	1817.6	5022.02	645	1803.2	4918.36	744	4.54	3.72	3.71	1.58
4	64	10	1460.3	6104.17	1758	1583.4	6287.65	1702	1544.6	6267.56	1833	8.43	3.01	5.77	2.68
Min												1.54	1.47	0.88	0.88
Max												8.43	3.72	5.77	2.68
Mean												4.32	2.55	3.06	1.67

TABLE 7: Results after ten operations of Case 1 ($\lambda = 6.40$, $\beta = 0.2$) with stochastic and fuzzy features.

Test number	Inlets	Layout Pump stations	Fixed cost	Operation cost	Total cost $\bar{Z}$	Queue length of surface water		Sojourn time in all queues		Total time $\bar{T}$
						Min	Max	Min	Max	
1st	64	4	2415.36	85.01	2500.37	66.07	70.32	3426.5	3882.3	3882.3
2nd	64	4	2415.36	84.57	2499.93	66.03	70.35	3422.2	3885.4	3885.4
3rd	64	4	2415.36	79.03	2494.39	65.30	70.16	3347.1	3864.3	3864.3
4th	64	4	2415.36	77.35	2492.71	64.02	70.37	3216.9	3887.5	3887.5
5th	64	4	2415.36	78.77	2494.13	64.52	70.18	3267.6	3866.2	3866.2
6th	64	4	2415.36	82.61	2497.97	66.13	69.92	3433.2	3837.4	3837.4
7th	64	4	2415.36	80.43	2495.79	64.63	70.17	3278.7	3865.1	3865.1
8th	64	4	2415.36	79.36	2494.72	64.61	70.16	3276.5	3863.7	3863.7
9th	64	4	2415.36	81.65	2497.01	65.56	70.18	3374.2	3866.6	3866.6
10th	64	4	2415.36	78.42	2493.78	64.19	70.01	3234.9	3847.3	3847.3
Min	64	4	2415.36	77.35	2492.71	64.02	69.92	3216.9	3837.4	3837.4
Max	64	4	2415.36	85.01	2500.37	66.13	70.37	3433.2	3887.5	3887.5
Mean	64	4	2415.36	80.72	2496.08	65.10	70.18	3327.78	3866.6	3866.6

FIGURE 3: Iteration convergence feature of RGA in solving urban drainage system.

draining time is 3866.6 minutes, and time deviation is 0.76%; (4) maximum mean queue length of surface water to its inlet is 68.20 meters, minimum mean queue length is 67.10 meters and mean queue length is 67.64 meters, and length deviation is 0.83% and all length deviations in 64 inlets are in $[0.2\%, 1.5\%]$. All deviations are so small that total costs, total sojourn times, and queue lengths are considered to be unchanged in different trial of this test, which validates that RGA is reliable to solve this stochastic and fuzzy drainage system.

The iteration convergence graphs of total cost and total time at the 5th test, best close to the mean result, are observed and shown in Figures 3(a) and 3(b). After nearly 15 iterations, RGA searched both optimal solutions of total cost and total time. Other tests also demonstrate similar iteration convergence feature of RGA. Therefore, it is also validated that RGA is effective to solve this stochastic and fuzzy drainage system.

5. Discussions

Several parameters in this model are adjusted and their sensitivities are discussed to reveal patterns of urban drainage system under extreme rainstorms.

5.1. Quantity of Pump Stations. When quantity p of pump stations is adjusted, both objectives, total cost and total sojourn time, will change. When pump station quantity is 0 through 14, all feasible solutions of the drainage system are compared and shown in Figure 4.

When quantity of pump stations is less than two, none of available solutions can be found out. If pump stations are more than two, total cost increases significantly but total sojourn time of the rainfall decreases dramatically.

When pump stations are more than eight, the curve of total sojourn time declines a little but total cost keeps arising in the same slope, which reveals that a proposal to construct

FIGURE 4: Impact of pump station quantity on total cost and sojourn time.

eight more pump stations in this district is not a good idea considering trade-off between infrastructure investment and drainage capacity under extreme rainstorms.

5.2. Diameter of Drainage Pipes. If diameter of drainage pipe is adjusted, cost of drainage pipes and construction cost as well as its drainage capacity will change. When drainage pipe diameter is less than 600 mm, none of solutions is feasible because stormwater volume at collection points is far more than drainage capacity of drainage pipes. When drainage pipe diameter is more than 600 mm, feasible solutions of total sojourn time and total cost have been found out and shown in Figure 5.

With increase of pipe diameter r , total sojourn time $\bar{T}$ decreases and total cost $\bar{Z}$ climbs gradually until its average diameter is not more than 1600 mm. When pipe diameter r is more than 1600 mm, the optimal total sojourn time $\bar{T}$ remains the same but the optimal total cost $\bar{Z}$ increases still.

FIGURE 5: Impact of drainage pipe diameter on total cost and sojourn time.

FIGURE 6: Impact of precipitation intensity on total cost and total sojourn time.

This result reveals that enlarging drainage pipes will improve urban draining performance, but excessively huge drainage pipe in this district will not decrease stormwater drainage time but will increase drainage system investment.

5.3. Precipitation Intensity of Rainstorm. When precipitation intensity f_i is adjusted, namely, its Poisson distribution parameter τ varies in the model, both objectives are obtained after this model executed many times, and the results are compared in Figure 6.

With increase of precipitation intensity of rainstorm, the optimal total cost $\bar{Z}$ keeps growing and the optimal sojourn time $\bar{T}$ of stormwater is extended, which means that more investment in urban drainage system is demanded when it is designed to prevent frequent extreme rainstorm.

When precipitation intensity, the distribution parameter τ , is between $5.5 \text{ m}^3/\text{s}$ and $9.5 \text{ m}^3/\text{s}$, the optimal total sojourn time at drainage system increases significantly. When the parameter τ is less than $5.5 \text{ m}^3/\text{s}$, the drainage system is able to discharge all stormwater in time, so the curve of optimal total sojourn time is a little flat.

When the parameter τ is bigger than $9.5 \text{ m}^3/\text{s}$, for example, an extraordinary rainstorm, this drainage system cannot drain all stormwater in time. Because the total sojourn time

FIGURE 7: Confidence levels and feasible solutions.

is limited by the restriction value e_i , the total sojourn time $\bar{T}$ has to be subject to the subjective restrictions and it increases slowly along with the precipitant intensity.

It reveals that in a frequent extreme rainstorm district more money should be spent on its drainage system in order to prevent potential urban floods.

5.4. Confidence Levels. If confidence levels of both objective functions and constraint conditions are adjusted, feasible solutions change a lot. Their results are obtained after several executions of the model and shown in Figure 7.

If confidence level is 80%, feasible solutions are from the AB arc. If confidence level is 65%, more feasible solutions are available, and their solution domain is from the CD arc of the curve. When confidence level is higher than 95%, not a feasible solution is available after several trials. When confidence level is 95% or less, more and more feasible solutions can be found out and they all come from the solution domain.

This discussion reveals that confidence levels of objectives and constraint conditions dominate the solution space size to a large extent. When the confidence level is higher than a certain value, not a feasible solution is available, which means the urban drainage system sometimes cannot satisfy all requirements of draining capacities, sojourn times, restricted values, and confidence levels simultaneously. It is suggested that a proper confidence level should be determined based on actual requirements to prevent urban floods caused by extreme rainstorms.

6. Conclusion

Considering several characteristics in an urban drainage system such as randomness in extreme rainstorm, fuzziness in drainage capacity, and drainage system cost, this optimization model for urban drainage system layout is developed based on a two-stage single-counter queue method. Its two

objectives are total sojourn time of stormwater processed by the system and total cost including construction cost and operation cost. Restricted values which reflect draining priority in important areas are introduced to drain stormwater in important areas within limit time. This model can simulate a complex nonlinear problem under stochastic and fuzzy constraints, which represents actual operation of an urban drainage system.

Genetic algorithm is improved and redesigned by embedding with stochastic and fuzzy attributes to solve non-deterministic problems under hybrid constraints, and it is illustrated by a numerical example. It is verified that this model is flexible and effective in solving such a complex nonlinear problem.

The sensitivities of quantity of drainage pump stations, drainage pipe diameter, precipitant intensity of rainstorm, and confidence level on urban drainage system are discussed, respectively, and their results reveal the following: (1) a heavier precipitant intensity will increase total cost of urban drainage system construction and operation and total sojourn time of stormwater disposed by the system; (2) enlarging drainage pipes will increase drainage system cost and its drainage capacity, but total sojourn time of stormwater will decrease dramatically; (3) increasing quantity of pump stations will raise drainage system cost but shorten total sojourn time. But not an optimal solution is available when their quantity is less than a limit. So a minimum pump station quantity should be set in designing an urban drainage system; (4) a higher confidence level of drainage service time can significantly decrease feasible solutions. When confidence level is higher than a limit, not a feasible solution for the drainage system would be found out.

A future research work would be to improve the model features and extend its conditions. The model for drainage pipes will be developed with sophisticated connections and assorted pipes to increase accuracy in modeling an urban drainage system. The simulation algorithm will be upgraded with various distribution functions to represent stochastic and fuzzy characters of extreme rainstorms and drainage capacity.

Notations

Parameters

- i, j, k : A point representing a location of inlet, potential pump station, and selected pump station, respectively, where $i \in I$, $j \in J$, and $k \in J$
- I : Set of inlet points which collect stormwater runoff on ground
- J : Set of locations of potential pumping stations
- T : Total sojourn time that stormwater receives service from urban drainage system
- Z : Total cost including construction cost and operation cost of urban drainage system

- a_j : Cost of equipment related to pump station at location j (measured in drainage capacity)
- b_j : Cost of construction related to pump station at location j (measured in station)
- c_{ij} : Cost of wear and tear of drainage pipe from location i to j (measured in length)
- d_{ij} : Length of drainage pipe from locations i to j
- e_i : Restricted sojourn time for draining stormwater from location i
- f_i : Precipitation intensity at point i (measured in duration and area)
- h_r : Cost of installation of drainage pipe (diameter r) (measured in length)
- l_{ij}^r : Coefficient converting a drainage pipe coverage area o_r to its length d_{ij}
- o_r : Coverage area of a drainage pipe (diameter r) where groundwater is only collected and conveyed by this pipe
- t_d : Duration of rainstorm
- s_r : Water capacity of drainage pipe (diameter r) (measured in length)
- $\bar{t}_{ij}$: Sojourn time of stormwater conveyed from point i to point j
- λ_i : Demand rate of drainage pipe by stormwater at point i
- $\bar{\lambda}_j$: Arrival rate of stormwater at pump station j (measured in time)
- u_j : Service capacity of drainage pipe j
- p : Quantity of total pump stations in urban drainage system
- v_j : Flow velocity in drainage pipe j
- α, β : Confidence level of objectives and constraints.

Variables

- m_j : Indicating whether a pump station is located at point j or not: 1, yes; 0, no
- n_{ij} : Indicating whether stormwater from point i is disposed by pump station j or not: 1, yes; 0, no
- x_j : Draining service ability of pump station at point j .

Disclosure

Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of NSFC.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References

- [1] R.-S. Quan, "Rainstorm waterlogging risk assessment in central urban area of Shanghai based on multiple scenario simulation," *Natural Hazards*, vol. 73, no. 3, pp. 1569–1585, 2014.
- [2] S. Du, H. Gu, J. Wen, K. Chen, and A. Van Rompaey, "Detecting flood variations in shanghai over 1949–2009 with Mann-Kendall tests and a newspaper-based database," *Water*, vol. 7, no. 5, pp. 1808–1824, 2015.
- [3] A. S. Chandio, T. S. Lee, and M. S. Mirjat, "Simulation of horizontal and vertical drainage systems to combat waterlogging problems along the rohri canal in Khairpur District, Pakistan," *Journal of Irrigation and Drainage Engineering*, vol. 139, no. 9, pp. 710–717, 2013.
- [4] M.-S. Chen and H.-W. Yen, "A two-stage approach in solving the state probabilities of the multi-queue M/G/1 model," *International Journal of Systems Science*, vol. 47, no. 5, pp. 1230–1244, 2016.
- [5] J.-C. Ke and F.-M. Chang, "M[x]/(G1, G2)/1 retrial queue under Bernoulli vacation schedules with general repeated attempts and starting failures," *Applied Mathematical Modelling*, vol. 33, no. 7, pp. 3186–3196, 2009.
- [6] F. Dong, J. R. Deglise-Hawkinson, M. P. Van Oyen, and D. J. Singer, "Dynamic control of a closed two-stage queueing network for outfitting process in shipbuilding," *Computers & Operations Research*, vol. 72, pp. 1–11, 2016.
- [7] J. A. Filipe and M. A. M. Ferreira, "Infinite servers queue systems busy period—a practical case on logistics problems solving," *Applied Mathematical Sciences*, vol. 9, no. 25–28, pp. 1221–1228, 2015.
- [8] V. Hajipour, V. Khodakarami, and M. Tavana, "The redundancy queueing-location-allocation problem: a novel approach," *IEEE Transactions on Engineering Management*, vol. 61, no. 3, pp. 534–544, 2014.
- [9] M. Singer and P. Donoso, "Assessing an ambulance service with queueing theory," *Computers and Operations Research*, vol. 35, no. 8, pp. 2549–2560, 2008.
- [10] F. Silva and D. Serra, "Locating emergency services with different priorities: the priority queueing covering location problem," *Journal of the Operational Research Society*, vol. 59, no. 9, pp. 1229–1238, 2008.
- [11] M. Mohammadi, F. Jolai, and H. Rostami, "An M/M/c queue model for hub covering location problem," *Mathematical & Computer Modelling*, vol. 54, no. 11–12, pp. 2623–2638, 2011.
- [12] N. Vidyarthi and S. Jayaswal, "Efficient solution of a class of location-allocation problems with stochastic demand and congestion," *Computers and Operations Research*, vol. 48, pp. 20–30, 2014.
- [13] S. H. A. Rahmati, A. Ahmadi, M. Sharifi, and A. Chambari, "A multi-objective model for facility location-allocation problem with immobile servers within queueing framework," *Computers & Industrial Engineering*, vol. 74, pp. 1–10, 2014.
- [14] L. Fu, "A fuzzy queueing model for real-time, adaptive prediction of incident delay for ATMS/ATIS," *Transportation Planning and Technology*, vol. 27, no. 1, pp. 1–23, 2004.
- [15] H. Shavandi and H. Mahlooji, "A fuzzy queueing location model with a genetic algorithm for congested systems," *Applied Mathematics and Computation*, vol. 181, no. 1, pp. 440–456, 2006.
- [16] B. Zahiri, R. Tavakkoli-Moghaddam, M. Mohammadi, and P. Jula, "Multi-objective design of an organ transplant network under uncertainty," *Transportation Research Part E: Logistics and Transportation Review*, vol. 72, pp. 101–124, 2014.
- [17] S. H. R. Pasandideh and S. T. A. Niaki, "Genetic application in a facility location problem with random demand within queueing framework," *Journal of Intelligent Manufacturing*, vol. 23, no. 3, pp. 651–659, 2012.
- [18] B. Vahdani, R. Tavakkoli-Moghaddam, and F. Jolai, "Reliable design of a logistics network under uncertainty: a fuzzy possibilistic-queueing model," *Applied Mathematical Modelling*, vol. 37, no. 5, pp. 3254–3268, 2013.
- [19] W. Wang, W. Zhou, S. K. Fong et al., "Extreme rainfall and summer heat waves in Macau based on statistical theory of extreme values," *Climate Research*, vol. 66, no. 1, pp. 91–101, 2015.
- [20] M. E. Doyle, R. I. Saurral, and V. R. Barros, "Trends in the distributions of aggregated monthly precipitation over the La Plata Basin," *International Journal of Climatology*, vol. 32, no. 14, pp. 2149–2162, 2012.
- [21] V. A. Bavdekar, N. N. Nandola, and S. C. Patwardhan, "Maximum likelihood estimation of noise covariance matrices for state estimation of autonomous hybrid systems," *Computers & Chemical Engineering*, vol. 94, pp. 28–44, 2016.
- [22] D. Teguig, V. Le Nir, and B. Scheers, "Spectrum sensing method based on goodness of fit test using chi-square distribution," *Electronics Letters*, vol. 50, no. 9, pp. 713–715, 2014.
- [23] G. Mateu-Figueras, P. Puig, and A. Pewsey, "Goodness-of-fit tests for the skew-normal distribution when the parameters are estimated from the data," *Communications in Statistics—Theory and Methods*, vol. 36, no. 9, pp. 1735–1755, 2007.
- [24] J. Teng, N. Yasufuku, S. Zhang, and Y. He, "Modelling water content redistribution during evaporation from sandy soil in the presence of water table," *Computers and Geotechnics*, vol. 75, pp. 210–224, 2016.
- [25] F. Molinari, "A new criterion of choice between generalized triangular fuzzy numbers," *Fuzzy Sets and Systems*, vol. 296, pp. 51–69, 2016.
- [26] V. Chinnadurai and S. Muthukumar, "Solving the linear fractional programming problem in a fuzzy environment: numerical approach," *Applied Mathematical Modelling*, vol. 40, no. 11–12, pp. 6148–6164, 2016.
- [27] C. Carlsson and R. Fullér, "On possibilistic mean value and variance of fuzzy numbers," *Fuzzy Sets and Systems*, vol. 122, no. 2, pp. 315–326, 2001.
- [28] M. H. Korayem, A. K. Hoshier, and M. Nazarahari, "A hybrid co-evolutionary genetic algorithm for multiple nanoparticle assembly task path planning," *The International Journal of Advanced Manufacturing Technology*, vol. 87, no. 9, pp. 3527–3543, 2016.
- [29] H. Gopalakrishnan and D. Kusanovic, "Operational planning of combined heat and power plants through genetic algorithms for mixed 0-1 nonlinear programming," *Computers & Operations Research*, vol. 56, pp. 51–67, 2015.
- [30] A. Azadeh, M. Moghaddam, S. M. Asadzadeh, and A. Negahban, "An integrated fuzzy simulation-fuzzy data envelopment analysis algorithm for job-shop layout optimization: the case of injection process with ambiguous data," *European Journal of Operational Research*, vol. 214, no. 3, pp. 768–779, 2011.

- [31] P. Garambois, S. Besset, and L. Jézéquel, "Multi-objective structural robust optimization under stress criteria based on mixed plate super-elements and genetic algorithms," *Structural and Multidisciplinary Optimization*, vol. 53, no. 2, pp. 205–213, 2016.
- [32] M. Fazzolari, R. Alcalá, Y. Nojima, H. Ishibuchi, and F. Herrera, "A review of the application of multiobjective evolutionary fuzzy systems: current status and further directions," *IEEE Transactions on Fuzzy Systems*, vol. 21, no. 1, pp. 45–65, 2013.
- [33] N. Ghaffari-Nasab, S. Ghazanfar Ahari, and M. Ghazanfari, "A hybrid simulated annealing based heuristic for solving the location-routing problem with fuzzy demands," *Scientia Iranica*, vol. 20, no. 3, pp. 919–930, 2013.
- [34] M. H. F. Zarandi, A. Hemmati, S. Davari, and I. B. Turksen, "Capacitated location-routing problem with time windows under uncertainty," *Knowledge-Based Systems*, vol. 37, pp. 480–489, 2013.
- [35] G. Palubeckis, "Single row facility layout using multi-start simulated annealing," *Computers & Industrial Engineering*, vol. 103, pp. 1–16, 2017.
- [36] H. Ahonen, A. G. de Alvarenga, and A. R. Amaral, "Simulated annealing and tabu search approaches for the Corridor Allocation Problem," *European Journal of Operational Research*, vol. 232, no. 1, pp. 221–233, 2014.
- [37] S. Maskey, V. Guinot, and R. K. Price, "Treatment of precipitation uncertainty in rainfall-runoff modelling: a fuzzy set approach," *Advances in Water Resources*, vol. 27, no. 9, pp. 889–898, 2004.