Research Article

Study on the Elastoplastic Damage-Healing Coupled Constitutive Model of Mudstone

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Under the effect of high ground stress and water-rock chemical interaction, the fractures in the damaged mudstone wound undergo a self-healing process and recover the physical and mechanical properties, which has a significant impact on the wall-rock’s stability of high level radioactive waste repository and the migration of radioactive nuclide. According to the general thermodynamics and continuum damage mechanics, an internal variable describing mudstone healing properties is introduced and an elastoplastic damage-healing model reflecting mudstone deformation, damage, and self-healing evolution is put forward. This model is used to simulate the triaxial compression test of mudstone under different confining pressures, whose simulated results are compared with the test data. It is indicated that the model could embody the main mechanical properties of mudstone with the healing effect in an effective way, and the healing part of the model has a great influence on the simulated results.

1. Introduction

The reasonable disposal of nuclear waste relates to the sustainable development of society and economy and the survival of mankind. At present, the worldwide recognized nuclear waste disposal method is to bury nuclear waste deep underground for storage [1–4]. Among the many geologic bodies, mudstone is being paid extensive attention due to its characteristics of low permeability and good rheological and self-healing performance, which has been the main parent rock for nuclear waste disposal. Therefore, comprehending the mechanical behaviors of mudstone materials is beneficial to assessing, designing, and building the underground repository for nuclear waste.

The self-healing property of mudstone can improve the isolation performance and wall-rock’s stability of nuclear waste repository. Zhang et al. carried out a series of tests, such as porosity measurement, permeability determination, and AE monitoring, which validated the existence of damage self-healing process of mudstone [5, 6]. Bastiaens et al. pointed out that the main mechanism for mudstone damage self-healing effect was caused by the inflation, consolidation, and creep of mudstone materials in response to mechanics and water-power. Many domestic experts and scholars have also conducted a lot of researches on the damage self-healing effect of materials [7]. Jia et al. introduced the concept of healing stress and hydrochemistry healing factor to establish the self-healing model, which was used to describe the self-healing performance of fractured mudstone [8]. Based on the isotropic hypothesis, Miao et al. proposed a continuous healing mechanics model of crushed rock [9]. On the thermodynamic framework, Barbero built a continuous damage-healing constitutive model for the polymer composites [10]. Darabi et al. extended the concepts of the Kachanov effective construction and effective stress space and put forward microdamage-healing model based on continuum damage mechanics [11].

The current studies on mudstone self-healing performance mostly focus on the field and laboratory tests [12–15]. The mechanical analysis on mudstone self-healing performance is little. In this paper, an exploration is made to model
the continuous damage self-healing processes based on maximum energy dissipation principle, which is established by introducing the internal variable of mudstone self-healing performance.

2. Basic Framework of the Model

According to the generalized thermodynamics, an internal variable is introduced to the thermodynamic constitutive relation to describe the different mechanisms of material deformation, such as elastoplastic deformation and damage-healing evolution. The mudstone deformation is regarded as an isotropic process. The second law of thermodynamics can be expressed as

$$\sigma : \dot{\varepsilon} - \rho \psi \geq 0,$$  \hspace{1cm} (1)

where $\sigma$ and $\dot{\varepsilon}$ are the stress and strain tensors, $\rho$ is the mass density, $\psi$ is the Helmholtz free energy, and $\cdot$ is the derivative of variable with respect to time.

The Helmholtz free energy $\psi$ is set to the thermodynamic potential of mudstone and elastic strain $\varepsilon^e$, plastic strain $\varepsilon^p$, isotropic damage tensor $d$, and tensor $h$ describing the self-healing performance of materials are set to the internal variables, so the Helmholtz free energy is expressed as

$$\psi = \psi(\varepsilon^e, \varepsilon^p, d, h).$$ \hspace{1cm} (2)

It is assumed that the Helmholtz free energy can be divided into two parts:

$$\psi(\varepsilon^e, \varepsilon^p, d, h) = \frac{1}{2} \varepsilon^e E(d, h) \varepsilon^e + \phi(\varepsilon^p, d, h),$$ \hspace{1cm} (3)

where the first part of the formula is the strain energy, the second part is the dissipated energy, and $E(d, h)$ is the fourth-order damage-healing stiffness tensor.

In order to simplify the model, it is assumed that the evolution of damage and healing surface is isotropic. According to the study of [15], the dissipated energy is expressed as

$$\varphi(\varepsilon^p, d, h) = \varphi(\gamma, \eta, \omega)$$

$$= \frac{1}{2} a_0 \gamma^2 + a_1 \left(\eta - a_2 \varepsilon^{p/a_1}\right)$$

$$+ a_3 \left(a_4 \varepsilon^{p/a_1} - \omega\right),$$ \hspace{1cm} (4)

where $a_0$, $a_1$, $a_2$, and $a_3$ are material constant and $\gamma$, $\eta$, and $\omega$ are the isotropic hardening parameters which are designed as follows [16]:

$$\gamma = \int_0^t \sqrt{\frac{2}{3} \varepsilon^p : \dot{\varepsilon}^p} dt,$$

$$\eta = \int_0^t \sqrt{\frac{2}{3} \dot{d} : \dot{d}} dt,$$

$$\omega = \int_0^t \sqrt{\frac{2}{3} \dot{h} : \dot{h}} dt.$$ \hspace{1cm} (5)

According to formulas (1), (3), and (4), the thermodynamic driving forces associated with the corresponding internal variables are shown as follows:

$$\sigma = \frac{\partial \psi}{\partial \varepsilon^e} = E(d, h) : \dot{\varepsilon}^e,$$ \hspace{1cm} (6a)

$$\sigma^p = \sigma - \frac{\partial \psi}{\partial \varepsilon^p} = \sigma - a_0 \frac{\partial \psi}{\partial \varepsilon^p},$$ \hspace{1cm} (6b)

$$\sigma^d = -\frac{\partial \psi}{\partial d} = \frac{-1}{2} \varepsilon^p \frac{\partial E}{\partial \varepsilon^p} \varepsilon^p - a_1 \left(1 - \varepsilon^{p/a_1}\right) \frac{\partial \eta}{\partial d},$$ \hspace{1cm} (6c)

$$\sigma^h = \frac{\partial \psi}{\partial h} = \frac{1}{2} \varepsilon^p \frac{\partial E}{\partial \varepsilon^p} \varepsilon^p + a_3 \left(\varepsilon^{p/a_1} - 1\right) \frac{\partial \omega}{\partial h}.\hspace{1cm} (6d)$$

The mechanical dissipation potential of mudstone deformation is

$$\Pi = \sigma^p : \dot{\varepsilon}^p + \sigma^d : \dot{d} - \sigma^h : \dot{h} \geq 0.$$ \hspace{1cm} (7)

In addition, the other conditions need to be satisfied, and they are

Plastic dissipation energy $\Pi^p = \sigma^p : \dot{\varepsilon}^p \geq 0$ \hspace{1cm} (8a)

Damage dissipation potential $\Pi^d = \sigma^d : \dot{d} \geq 0$ \hspace{1cm} (8b)

Healing dissipation potential energy $\Pi^h = -\sigma^h : \dot{h} \leq 0.$ \hspace{1cm} (8c)

The maximum dissipation theory says that the true state of thermodynamic driving force is the state that makes the maximum of dissipation equation among all admissible states. It is a constrained optimization problem, which can be solved by Lagrange multiplier method. Now define the function

$$\Gamma = \Pi - \lambda^p \kappa^p (\sigma^p) - \lambda^d \kappa^d (\sigma^d) - \lambda^h \kappa^h (\sigma^h),$$ \hspace{1cm} (9)

where $\lambda^p$, $\lambda^d$, and $\lambda^h$ are Lagrange multipliers, related to plasticity, damage, and healing, respectively. $\kappa^p$, $\kappa^d$, and $\kappa^h$ are the dissipation potentials of plasticity, damage, and healing.

In order to make the maximum of $\Gamma$, the following condition needs to be satisfied:

$$\frac{\partial \Gamma}{\partial \sigma^p} = 0,$$

$$\frac{\partial \Gamma}{\partial \sigma^d} = 0,$$

$$\frac{\partial \Gamma}{\partial \sigma^h} = 0.$$ \hspace{1cm} (10)

The evolution equation of each internal variable can be expressed as

$$\dot{\varepsilon}^p = \lambda^p \frac{\partial \kappa^p}{\partial \sigma^p} ,$$ \hspace{1cm} (11)

$$\dot{d} = \lambda^d \frac{\partial \kappa^d}{\partial \sigma^d} ,$$

$$\dot{h} = \lambda^h \frac{\partial \kappa^h}{\partial \sigma^h}.$$
The process of mudstone plastic deformation, damage, and healing evolution is a coupling process, which needs to satisfy the consistency condition

\[ f^p = \frac{\partial f^p}{\partial \sigma} : \dot{\sigma} + \frac{\partial f^p}{\partial d} : \dot{d} + \frac{\partial f^p}{\partial h} \dot{h} = 0, \]

\[ f^d = \frac{\partial f^d}{\partial \sigma'} : \dot{\sigma'} + \frac{\partial f^d}{\partial d} : \dot{d} = 0, \]

\[ f^h = \frac{\partial f^h}{\partial \sigma''} : \dot{\sigma''} + \frac{\partial f^h}{\partial h} : \dot{h} = 0, \]  

(12)

where \( f^p, f^d, \) and \( f^h \) are yield function, damage, and healing evolution equations.

The derivative of (6a) with respect to time leads to the increment constitutive model of mudstone

\[ \dot{\sigma} = E (d, h) \dot{\varepsilon} + \dot{E} (d, h) \varepsilon' \]

\[ = E (d, h) (\dot{\varepsilon} - \dot{\varepsilon}') + (E'_d (d, h) : \dot{d} + E'_h (d, h) : \dot{h}) (\varepsilon - \varepsilon'), \]  

(13)

where \( E'_d \) and \( E'_h \) are the derivatives of \( E \) with respect to \( d \) and \( h \).

The plastic yield function, damage, and healing evolution equations are plugged into the consistency condition. Combined with the optimization problem of Lagrange multiplier, the Lagrange multipliers related to plasticity, damage, and healing can be solved, and then the mudstone constitutive model is obtained according to (13).

### 3. Mudstone Plastic Property

The plastic deformation can be described by yield function and plastic potential. Like many geomaterials, mudstone is a pressure-sensitive distant material, which reflects asymmetric response under the compression and tensile loadings. The yield function is expressed as

\[ f^p (\sigma, d, h) = q - \alpha_p P_c \left[ N \left( C_s + \frac{P}{P_s} \right) \right]^{1/2} = 0, \]  

(14)

where \( q = \sqrt{3J_2}, C_s \) is the cohesion coefficient, \( N \) is the curvature of failure surface, \( P_s \) is the uniaxial compressive strength of material, and \( \alpha_p \) is the function describing mudstone plastic hardening. The hardening function is obtained by introducing the healing variable \( h \):

\[ \alpha_p (\gamma, d, h) = (1 - b_1 \text{tr} d) (1 + b_2 \text{tr} h) \left[ \alpha_p^0 + (\alpha_p^m - \alpha_p^0) \frac{Y}{B + Y} \right], \]  

(15)

where \( b_1 \) and \( b_2 \) represent the influence of damage and healing to plasticity, whose value range is from 0 to 1. In order to simplify the calculation process, \( b_1 = b_2 = 1 \) is set. \( B \) is the parameter used to control plastic hardening rate. \( \alpha_p^0 \) and \( \alpha_p^m \) are the initial yield threshold and final plastic hardening value. The plastic flow rule is determined by plastic potential, and the mudstone plastic potential function proposed by Jia et al. [17] is modified to be

\[ g^p (\sigma_{ij}, \gamma, d, h) = q - \left( 1 - b_2 \text{tr} d \right) (1 + b_2 \text{tr} h) \cdot \left( \alpha_p^0 + (1 - \alpha_p^0) \frac{Y}{B + Y} \right), \]  

(16)

where \( \eta \) is the turning point between compression and dilatation.

### 4. Damage and Healing of Mudstone

When the loading mudstone reaches the conditions of damage nucleation and growth, some new microholes and microcracks begin to initiate and propagate. However, some microcracks may repair themselves. The damage can lead to some deterioration of mechanical property, such as strength reduction and deformation increase, while damage healing often gives rise to the strength recovery and compressibility reduction.

#### 4.1. Definition of Damage and Healing Variables

The damage and healing variables are defined based on the one-dimensional space. The cylinder sample under uniaxial tensile loading is shown in Figure 1.

The cross-sectional area \( A \) can be divided into three parts: undamaged area \( A_{ud} \), unhealed microcrack area \( A_{uh} \), and healed microcrack area \( A_h \), and

\[ A = A_{ud} + A_d, \]

\[ A_d = A_{uh} + A_h, \]  

(17)

where \( A_d \) is the damaged area.
Since the damaged mudstone cannot be healed to intact rock, unhealed microcrack area $A_{ah}$ is greater than 0, which means the damaged part of mudstone cannot heal completely under any circumstance. The carrying capacity of healing area $A_h$ does not always come back to that of the intact material, thus proposing a reduction coefficient to describe the effective healing area. In order to simplify the calculation, it is assumed in the paper that the mechanical property of healed areas is identical to that of intact material. That means when microdamage is fully repaired, its mechanical properties could recover completely. The effective bearing area is

$$A_e = A_{ud} + A_h.$$  

(18)

The one-dimensional internal variables of damage and healing are defined as

$$d = \frac{A_d}{A},$$  

$$h = \frac{A_h}{A_{ud}}.$$  

(19)

It is assumed that cracks and holes have no carrying capacity, and the main loads are carried on by the undamaged area and healed area. Thus the effective bearing area is also expressed as

$$A_e = (1 - d) (1 + h) A.$$  

(20)

The effective stress is

$$\sigma' = \frac{\sigma}{(1 - d) (1 + h)}.$$  

(21)

According to elastic mechanics theories, if one-dimensional strain $\varepsilon = \Delta L/L$ is extended to three-dimensional space, strain tensor $\varepsilon$ can be obtained. Analogously, the tensors $d$ and $h$ used to describe three-dimensional damage and healing also can be acquired with damage internal variable $d$ and healing internal variable $h$ extended to three-dimensional space. The formation and propagation of microcracks and holes have the prior direction, thus assuming that the damage and healing have the same principle direction. $d$ and $h$ can be divided into

$$d = \sum_{i=1}^{3} d_i n_i \otimes n_i,$$  

$$h = \sum_{i=1}^{3} h_i n_i \otimes n_i,$$  

(22)

where $n_i, d_i$, and $h_i$ are damage (healing) principle direction, damage principle value, and healing principle value and $\otimes$ is tensor product.

Based on the elastic energy equivalence principle, the relationship between $E(d, h)$ and elastic modulus $E$ of intact mudstone is

$$E(d, h) = \beta : E : \beta^T,$$  

(23)

where $\beta$ is a fourth-order transformational tensor, which is considered as function of $d$ and $h$.

4.2. Damage and Healing Evolution. The damage development of mudstone can be described by the damage criterion. It is generally acknowledged that weak tensile strength of geotechnical materials is one of the larger contributors to damage. Thus the damage criterion has to do with tensile stress or tensile strain. Given this, the following damage criterion is used to describe the damage development of mudstone in this paper:

$$f^d(\varepsilon, d) = (\varepsilon^+ : \varepsilon^-)^{1/2} - (r_0 + r_1 \text{tr} d) \leq 0,$$  

(24)

where $\varepsilon^+$ is the tensile part of strain and $r_0, r_1$ are material parameters.

$$\varepsilon^+ = \sum_{k=1}^{3} \varepsilon_k H(\varepsilon_k) \nu^k \otimes \nu^k,$$  

(25)

where $H(\varepsilon_k)$ is the Heaviside step function of the $k$th principal strain $\varepsilon_k$ and $\nu^k$ is the corresponding principal vector.

Like rock damage, the healing criterion can be used to describe the healing development process of rock. The healing of cracks is associated with compressive strain and thus the following healing criterion is adopted:

$$f^h(\sigma, h) = (\varepsilon^- : \varepsilon^-)^{1/2} - (r_2 + r_3 \text{tr} h) \leq 0,$$  

(26)

where $\varepsilon^-$ is the tensile part of strain and $r_2, r_3$ are material parameters:

$$\varepsilon^- = \varepsilon - \varepsilon^+.$$  

(27)

According to the nonassociated dissipation criterion, $f^d = \kappa^d$ is obtained. Similarly, $f^h = \kappa^h$ is also achieved, and thus the evolution equations of damage and healing are expressed as

$$\dot{d} = \lambda^d \frac{\partial f^d}{\partial \sigma},$$  

$$\dot{h} = \lambda^h \frac{\partial f^h}{\partial \sigma}.$$  

(28)

5. Model Validation

The parameters of the above model can be obtained by experiment. For example, the parameters associated with plastic yielding, $C_p$ and $N$, are acquired by drawing out failure curves on the $p-q$ plane with peak stresses in triaxial compression tests under different confining pressures. The elastic constants $E$ and $\nu$ can be determined by using the linear part of stress-strain curves before the initiation of damage and plastic deformation [10]. The controlling parameter of enhancement rate, $B$, is got by fitting the evolution of hardening function $\alpha_p$ with hardening variable $B$ according to (15). The initial yield threshold $\alpha_{p0}$ is determined through initial yield surface, while $\alpha_{pq}$ is acquired by fitting the peak strength. The turning point between compression and dilatation, $\eta$, is ascertained by the stress point in triaxial test where volumetric strain rate $\dot{\varepsilon}_v$ is equal to 0 [17].
In order to validate the rationality and validity of the model, the triaxial compression tests of mudstone samples are carried out, which are taken from deep rock roadway in Huainan Coalmine. Each sample was cut into cylinders 50 mm in diameter and 100 mm in height, and the surface of the samples appeared relatively smooth without apparent cracks. The samples were loaded by RMT-301, and the process of tests was as follow: load the samples to the prescribed confining pressure level (0, 10, and 20 MPa) at constant loading rate of 0.001 mm/s, then keep the confining pressure constant, and load axially the samples at loading rate of 0.001 mm/s until the complete stress-strain curve was available. Based on the above methods, the parameters of model are achieved as follows:

\[
E = 11.2 \text{ GPa},
\]

\[
\nu = 0.31,
\]

\[
a_0 = 2.5 \times 10^{-7},
\]

\[
P_c = 28.4 \text{ MPa},
\]

\[
C_s = 0.1,
\]

\[
N = 3.6,
\]

\[
B = 7 \times 10^{-3},
\]

\[
\alpha_p^0 = 0.29,
\]

\[
\alpha_p^m = 1,
\]

\[
\delta = 0.5,
\]

\[
a_1 = 0.021,
\]

\[
a_2 = -6.4 \times 10^4,
\]

\[
r_0 = 6.4 \times 10^{-5},
\]

\[
r_1 = 3.7 \times 10^{-3},
\]

\[
a_t = 0.011,
\]

\[
a_t = -0.15 \times 10^4,
\]

\[
r_2 = 9.3 \times 10^{-5},
\]

\[
r_3 = 4.1 \times 10^{-3}.
\]

The triaxial compression tests are simulated by crafting the finite element program based on the above model, which is denoted by Model I. In order to explore the influence of healing part on the simulated results, Mode II is obtained by taking out the healing part of Model I and then using it to simulate the triaxial compression tests again. The incremental constitutive equation of Mode II is shown as follows:

\[
\dot{\sigma} = E (d) (\dot{\epsilon} - \dot{\epsilon}^p) + E' (d) : \dot{d} : (\epsilon - \epsilon^p) .
\]

As indicated in Figures 2, 3, and 4, the comparisons between simulation values of Models I and II and experiment data under different confining pressures are carried out. It is found that the simulation values of Model I are exactly consistent with experiment data, especially in terms of lateral deformation, which reveals that it is feasible to describe the mechanical properties of deep mudstone with Model I proposed in this paper.

Although Model II without considering healing effect can also be used to simulate the triaxial compression tests of mudstone, the simulated results of Model II do not match well with experiment data compared with Model I. Adding the healing part to the model can improve the accuracy of the simulation.

6. Conclusion

(1) Based on the generalized thermodynamics, continuum damage mechanics, and specificity of mudstone,
the theoretical model proposed in this paper not only describes the elastic-plastic deformation and damage evolution of mudstone under different stress conditions but also reflects its healing properties.

(2) Comparison of calculation results with relevant test data shows that the presented elastoplastic damage-healing model can demonstrate mechanical properties of mudstone more effectively.

(3) The model proposed in this paper takes account of the healing properties of mudstone; at the same time, there are certain shortcomings as well: the model is put forward for the mudstone with fixed moisture content, not considering the influence of water migration and content variation on self-healing effect, and thus the model can not reflect the microscopic mechanism of self-healing property. Given this, further research will be conducted in a follow-up study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper, and the funding did not lead to any conflicts of interest regarding the publication of this paper.

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