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Research Article

T-S Fuzzy Modelling and H_∞ Attitude Control for Hypersonic Gliding Vehicles

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This paper addresses the T-S fuzzy modelling and H_∞ attitude control in three channels for hypersonic gliding vehicles (HGVs). First, the control-oriented affine nonlinear model has been established which is transformed from the reentry dynamics. Then, based on Taylor's expansion approach and the fuzzy linearization approach, the homogeneous T-S local modelling technique for HGVs is proposed. Given the approximation accuracy and controller design complexity, appropriate fuzzy premise variables and operating points of interest are selected to construct the T-S homogeneous submodels. With so-called fuzzy blending, the original plant is transformed into the overall T-S fuzzy model with disturbance. By utilizing Lyapunov functional approach, a state feedback fuzzy controller has been designed based on relaxed linear matrix inequality (LMI) conditions to stable the original plants with a prescribed H_∞ performance of disturbance. Finally, numerical simulations are performed to demonstrate the effectiveness of the proposed H_∞ T-S fuzzy controller for the original attitude dynamics; the superiority of the designed T-S fuzzy controller compared with other local controllers based on the constructed fuzzy model is shown as well.

1. Introduction

Hypersonic gliding vehicles (HGVs) have been shown to endure a strongly nonlinear dynamic behaviour over the flight envelope; therefore, a control study for the natural nonlinearity is required. The reentry motion of HGVs presents the characteristics of strong nonlinearity, coupling, and uncertainty. In this period, hypersonic vehicles are extremely sensitive to changes in atmospheric conditions as well as physical and aerodynamic parameters. These characteristics emphasize that an effective nonlinear control approach represents an important and basic procedure. In fact, under these conditions, not only linear control approaches but also nonlinear control schemes face significant challenges. For example, the feedback linearization approach will be significantly restricted as the system models are not precisely known and the uncertain parameters are included in the models. Although the backstepping controllers guarantee global robustness of uncertain nonlinear systems, this approach can be effective only for systems with a serial form. In addition, appropriate Lyapunov functions and stability analysis have to be realized in each step, which is a drawback for high-order nonlinear systems. In the sliding mode approach, the chattering phenomenon derived from discontinuous switching control when the system state trajectory is converging along the selected sliding surface may excite the unmodelled dynamics, which is essential for system stability and steady state accuracy. Nevertheless, until now, several nonlinear control methodologies have been improved and successfully applied to the study of hypersonic vehicles [1–7].

On the other hand, because of its simple properties and easy implementation, fuzzy logic control has been well recognized as an effective methodology for complex nonlinear systems [8–13]. Control systems based on the T-S fuzzy model transfer nonlinear plants into fuzzy composition expressions with specific local linear models, which enables the T-S fuzzy system to consider the linear system theory. In fact, in contrast to other nonlinear control strategies, T-S fuzzy models have been shown to be universal function approximators in the sense that they can approximate any smooth nonlinear function to any degree of accuracy in any convex compact region [14–16]. Therefore, T-S fuzzy control has been suggested as an alternative approach to conventional techniques in many cases.

When a nonlinear dynamical model is given, it is more appropriate to construct the T-S fuzzy model by a derivation approach rather than an identification approach. Among the derivation approaches, the sector nonlinearity [17] approach usually needs too many rules because of miscellaneous nonlinear sections. Although the local approximation [18] approach reduces the number of model rules, the stability of the original nonlinear systems may not be guaranteed based on this T-S fuzzy controller [19].

Regardless of whether the above-mentioned modelling approaches are adopted, the T-S fuzzy model as an approximator can be divided into two categories: homogeneous fuzzy model and affine fuzzy model. The difference between the two models lies in whether the consequent part has a constant bias term in the local model. Since the difficulties of stability and stabilizability problems related to the nonconvex bilinear matrix inequality (BMI) cannot be efficiently solved by the convex linear matrix inequality (LMI) method, the affine fuzzy system is less advanced compared with the homogeneous fuzzy system [20–24]. Therefore, system analysis and controller design for homogeneous fuzzy systems would be less complex and conservative than that of affine fuzzy systems.

Up to date, numerous theoretical studies on the T-S fuzzy control for nonlinear systems and H_∞ control for the hypersonic vehicle have been conducted since the fuzzy controller was proposed in [25, 26]. Most of these existing investigations use the Lyapunov stability theory to obtain the LMI conditions with lesser conservatism and further obtain the fuzzy controller for the closed-loop feedback system [11, 27–30]. Intensive theoretical results have also resulted in numerous engineering applications. However, to the best of our knowledge, existing studies that introduce the T-S fuzzy control theory into the analysis of the hypersonic vehicle control system are still an open study [31–34].

Motivated by the previous discussion, the rest of this paper is organized as follows. In Section 2, the original nonlinear plant of the dynamics and the affine nonlinear model are presented. Then, in Section 3, a linearization technique based on Taylor's expansion method and fuzzy linearization method for this plant is analysed. Section 4 applies the proposed technique to the HGV affine model. In Section 5, a simulation is presented to show the feasibility and effectiveness of the proposed fuzzy model and fuzzy controller. Finally, Section 6 draws some conclusions.

2. Reentry Dynamics

The original nonlinear reentry motion equations of the hypersonic vehicle can be expressed as follows [35]:

$$\begin{aligned}
\dot{\alpha} &= q - \tan\beta\left(p\cos\alpha + r\sin\alpha\right) - \frac{\cos\mu}{\cos\beta}\left(\dot{\gamma} - \dot{\varphi}\cos\chi\right. \\
&\quad \left. - \left(\dot{\lambda} + \omega_e\right)\cos\varphi\sin\chi\right) + \frac{\cos\mu}{\cos\beta}\left(\dot{\chi}\cos\gamma\right. \\
&\quad - \dot{\varphi}\cos\chi\sin\gamma + \left(\dot{\lambda} + \omega_e\right) \\
&\quad \left.\cdot\left(\cos\varphi\cos\chi\sin\gamma - \sin\varphi\cos\gamma\right)\right), \\
\dot{\beta} &= p\sin\alpha - r\cos\alpha \\
&\quad + \sin\mu\left(\dot{\gamma} - \dot{\varphi}\cos\chi - \left(\dot{\lambda} + \omega_e\right)\cos\varphi\sin\chi\right) + \cos\mu \\
&\quad \cdot\left(\dot{\chi}\cos\gamma - \dot{\varphi}\cos\chi\sin\gamma + \left(\dot{\lambda} + \omega_e\right)\right. \\
&\quad \left.\cdot\left(\cos\varphi\cos\chi\sin\gamma - \sin\varphi\cos\gamma\right)\right), \\
\dot{\mu} &= -q\sin\beta - \cos\beta\left(p\cos\alpha + r\sin\alpha\right) + \dot{\alpha}\sin\beta - \dot{\chi} \\
&\quad \cdot\sin\gamma - \dot{\varphi}\sin\chi\cos\gamma + \left(\dot{\lambda} + \omega_e\right)\left(\cos\varphi\cos\chi\cos\gamma\right. \\
&\quad \left. + \sin\varphi\sin\gamma\right), \\
\dot{p} &= I^p_{pq}pq + I^p_{qr}qr + g^p_n n + g^p_l l, \\
\dot{q} &= I^q_{pp}p^2 + I^q_{rr}r^2 + I^q_{pr}pr + g^q_m m, \\
\dot{r} &= I^r_{qr}qr + I^r_{pq}pq + g^r_l l + g^r_n n.
\end{aligned} \tag{1}$$

Here, α, β, μ denote the angle of attack, sideslip angle, and bank angle, respectively; p, q, r denote the bank angle rate of rotation, angle of attack rate, and sideslip angle rate of rotation, respectively. R, φ, λ and V, γ, χ are variables related to the vehicle location and velocity in the particle kinematic. ω_e denotes the earth rotation angular rate, and $I^{\square}_{\square}$ and $g^{\square}_{\square}$ are the constant coefficients with respect to the moment of inertia of the vehicle.

Unlike the air-breathing hypersonic vehicle dynamics in the longitude plane [36] and the GHV (generic hypersonic vehicle) reentry dynamics [37], the reentry dynamics of HGV not only contain the dynamics of six attitude variables in three channels but also couple the trajectory variable $[\gamma, \chi]$ and the earth rotation angular rate ω_e, which makes the HGV exhibit a strong coupling and complex nonlinearity.

Since one of the key purposes of this study is to establish the fuzzy model of the reentry attitude dynamics of HRVs, the differential equations in (1) have a general form; therefore, it is necessary to recast (1) to a control-oriented form. In the previous studies, the transformation process for motion equations (1) to the control-oriented model is often omitted or a further model is directly provided. In contrast, this study explores this operation in detail.

Compared with the motions of variables $[\alpha, \beta, \mu, p, q, r]^T$ in reentry dynamics equation (1), only a slight effect on system dynamics is shown when sections of ω_e are considered; instead of that obvious effects will act on the motions of trajectory variables in reentry trajectory equations for researches of trajectory optimization and guidance. Therefore, the sections of the earth rotation angular rate ω_e will be omitted in the next discussion.

In fact, when $\omega_e = 0$ is applied in (1), the following truth will be discovered based on the trajectory equations with respect to $[\dot{\lambda}, \dot{\varphi}, \dot{\gamma}, \dot{\chi}]$ in [35].

$$\begin{aligned}
&\left(\dot{\gamma} - \dot{\varphi}\cos\chi - \left(\dot{\lambda} + \omega_e\right)\cos\varphi\sin\chi\right) = \frac{F_\gamma}{MV},\\
&\dot{\chi}\cos\gamma - \dot{\varphi}\cos\chi\sin\gamma + \left(\dot{\lambda} + \omega_e\right)\\
&\quad\cdot\left(\cos\varphi\cos\chi\sin\gamma - \sin\varphi\cos\gamma\right) = \frac{F_\chi}{MV},\\
&-\dot{\chi}\sin\gamma - \dot{\varphi}\sin\chi\cos\gamma + \left(\dot{\lambda} + \omega_e\right)\\
&\quad\cdot\left(\cos\varphi\cos\chi\cos\gamma + \sin\varphi\sin\gamma\right) = -\frac{F_\chi}{MV}\tan\gamma.
\end{aligned} \tag{2}$$

Here, F_V, F_γ, and F_χ denote the vehicle aerodynamic force decomposed in the velocity coordinate system, which are defined as

$$\begin{aligned}
F_\gamma &= -Y\sin\mu + L\cos\mu - Mg\cos\gamma,\\
F_\chi &= -Y\cos\mu - L\sin\mu.
\end{aligned} \tag{3}$$

By substituting (2), reentry dynamics (1) can be represented as

$$\begin{aligned}
\dot{\alpha} &= q - \tan\beta\left(p\cos\alpha + r\sin\alpha\right) - \frac{\cos\mu}{\cos\beta}\frac{F_\gamma}{MV}\\
&\quad + \frac{\cos\mu}{\cos\beta}\frac{F_\chi}{MV},\\
\dot{\beta} &= p\sin\alpha - r\cos\alpha + \sin\mu\frac{F_\gamma}{MV} + \cos\mu\frac{F_\chi}{MV},\\
\dot{\mu} &= -q\sin\beta - \cos\beta\left(p\cos\alpha + r\sin\alpha\right) + \dot{\alpha}\sin\beta\\
&\quad - \frac{F_\chi}{MV}\tan\gamma,\\
\dot{p} &= I^p_{pq}pq + I^p_{qr}qr + g^p_n n + g^p_l l,\\
\dot{q} &= I^q_{pp}p^2 + I^q_{rr}r^2 + I^q_{pr}pr + g^q_m m,\\
\dot{r} &= I^r_{qr}qr + I^r_{pq}pq + g^r_l l + g^r_n n.
\end{aligned} \tag{4}$$

Here, the force and force moment are expressed as follows:

$$\begin{aligned}
Y &= C_Y QS,\\
L &= C_L QS,\\
l &= C_l QSb,\\
m &= C_m QSc,\\
n &= C_n QSb,\\
C_Y &= C_{Y,\beta}\beta + C_{Y,\delta_e}\delta_e + C_{Y,\delta_a}\delta_a + C_{Y,\delta_r}\delta_r\\
&= f_Y\left(\alpha, \beta, \delta_e, \delta_a, \delta_r\right),\\
C_L &= C_{L,\alpha} + C_{L,\delta_e}\delta_e + C_{L,\delta_a}\delta_a = f_L\left(\alpha, \beta, \delta_e, \delta_a, \delta_r\right),\\
C_l &= C_{l_\beta}\beta + C_{l,\delta_e}\delta_e + C_{l,\delta_a}\delta_a + C_{l,\delta_r}\delta_r\\
&= f_l\left(\alpha, \beta, \delta_e, \delta_a, \delta_r\right),\\
C_m &= C_{m_\alpha} + C_{m,\delta_e}\delta_e + C_{m,\delta_a}\delta_a + C_{m,\delta_r}\delta_r\\
&= f_m\left(\alpha, \beta, \delta_e, \delta_a, \delta_r\right),\\
C_n &= C_{n_\beta}\beta + C_{n,\delta_e}\delta_e + C_{n,\delta_a}\delta_a + C_{n,\delta_r}\delta_r\\
&= f_n\left(\alpha, \beta, \delta_e, \delta_a, \delta_r\right).
\end{aligned} \tag{5}$$

Substituting (5) into (4) and denoting $\Omega = [\alpha, \beta, \mu]^T$ and $\omega = [p, q, r]^T$, nonlinear reentry plant (4) is inferred as the following nonlinear affine model:

$$\begin{aligned}
\dot{\Omega} &= f_s + g_{s1}\omega + g_{s2}\delta,\\
\dot{\omega} &= f_f + g_f M_c = f_f + g_f \cdot g_{f\delta} \cdot \delta,
\end{aligned} \tag{6}$$

where

$$f_s = \frac{1}{MV}\begin{bmatrix} \dfrac{-C_{L,\alpha}QS + Mg\cos\gamma\cos\mu}{\cos\beta} \\ -C_{Y,\beta}\beta QS - Mg\cos\gamma\sin\mu \\ C_{L,\alpha}QS\left(\tan\beta + \cos\mu\right) + C_{Y,\beta}\beta QS\cos\beta - Mg\cos\gamma\cos\mu\tan\beta \end{bmatrix},$$

$$g_{s1} = \begin{bmatrix} -\tan\beta\cos\alpha & 1 & -\tan\beta\sin\alpha \\ \sin\alpha & 0 & -\cos\alpha \\ -\sec\beta\cos\alpha & 0 & -\sec\beta\sin\alpha \end{bmatrix},$$

$$g_{s2} = \frac{QS}{MV}\begin{bmatrix} \dfrac{C_{L,\delta_e}}{\cos\beta} & \dfrac{C_{L,\delta_a}}{\cos\beta} & \dfrac{C_{L,\delta_r}}{\cos\beta} \\ C_{Y,\delta_e} & C_{Y,\delta_a} & C_{Y,\delta_r} \\ C_{L,\delta_e}\left(\tan\beta + \cos\mu\right) + C_{Y,\delta_e}\cos\beta & C_{L,\delta_a}\left(\tan\beta + \cos\mu\right) + C_{Y,\delta_a}\cos\beta & C_{L,\delta_r}\left(\tan\beta + \cos\mu\right) + C_{Y,\delta_r}\cos\beta \end{bmatrix},$$

$$f_f = \begin{bmatrix} I_{pq}^p pq + I_{qr}^p qr + g_l^p l_A \\ I_{pp}^q p^2 + I_{rr}^q r^2 + I_{pr}^q pr + g_m^q m_A \\ I_{qr}^r qr + I_{pq}^r pq + g_l^r n_A \end{bmatrix},$$

$$g_f = \begin{bmatrix} g_l^p & 0 & g_n^p \\ 0 & g_m^q & 0 \\ g_l^r & 0 & g_n^r \end{bmatrix},$$

$$\delta = \begin{bmatrix} \delta_e \\ \delta_a \\ \delta_r \end{bmatrix},$$

$$M_c = g_{f\delta}\delta,$$

$$g_{f\delta} = QS \begin{bmatrix} bC_{l,\delta_e} & bC_{l,\delta_a} & bC_{l,\delta_r} \\ g_{m,\delta_e} & g_{m,\delta_a} & g_{m,\delta_r} \\ bC_{n,\delta_e} + X_{cg}C_{Y,\delta_e} & bC_{n,\delta_a} + X_{cg}C_{Y,\delta_a} & bC_{n,\delta_r} + X_{cg}C_{Y,\delta_r} \end{bmatrix}. \tag{7}$$

In particular, as the aerodynamic force generated by the rudder is far lesser than the force moment generated by the vehicle body, $g_{s2}\delta$ has a negligible effect on variables $\Omega = [\alpha, \beta, \mu]^T$ in (6). Therefore, recast equation (6) yields

$$\begin{bmatrix} \dot{\Omega} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} g_{s1}\omega \\ f_{f1} \end{bmatrix} + \begin{bmatrix} 0 \\ g_f \end{bmatrix} M_c + \begin{bmatrix} f_s \\ f_{f2} \end{bmatrix}. \tag{8}$$

Let $x(t) = [\Omega^T, \omega^T]^T = [\alpha, \beta, \mu, p, q, r]^T$, $\left[\begin{smallmatrix} g_{s1}\omega \\ f_{f1} \end{smallmatrix}\right] = f(x)$, $\left[\begin{smallmatrix} 0_{3\times 3} \\ g_f \end{smallmatrix}\right] = g(x)$, $(M_c) = u(t)$, and $\left[\begin{smallmatrix} f_s \\ f_{f2} \end{smallmatrix}\right] = \Delta(x)$,

Finally, the nonlinear affine model can be rewritten as

$$\dot{x}(t) = f(x) + Gu(t) + \Delta(x), \tag{9}$$

in which

$$f(x) = \begin{bmatrix} -p\tan\beta\cos\alpha + q - r\tan\beta\sin\alpha \\ p\sin\alpha - r\cos\alpha \\ -p\sec\beta\cos\alpha - r\sec\beta\sin\alpha \\ I_{pq}^p pq + I_{qr}^p qr \\ I_{pp}^q p^2 + I_{rr}^q r^2 + I_{pr}^q pr \\ I_{qr}^r qr + I_{pq}^r pq \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \end{bmatrix},$$

$$g(x) = \begin{bmatrix} & \mathbf{0}_{3\times 3} & \\ g_l^p & 0 & g_n^p \\ 0 & g_m^q & 0 \\ g_l^r & 0 & g_n^r \end{bmatrix}, \tag{10}$$

$$\Delta(x) = \begin{bmatrix} \dfrac{-C_{L_\alpha}QS + Mg\cos\gamma\cos\mu}{MV\cos\beta} \\ \dfrac{-C_{Y_\beta}\beta QS - Mg\cos\gamma\sin\mu}{MV} \\ \dfrac{C_{L_\alpha}QS(\tan\beta + \cos\mu) + C_{Y_\beta}\beta QS\cos\beta - Mg\cos\gamma\cos\mu\tan\beta}{MV} \\ g_l^p l_A + g_n^p n_A \\ g_m^q m_A \\ g_l^r l_A + g_n^r n_A \end{bmatrix}.$$

$\Delta(x)$ is considered as the uncertain term related to the vehicle aerodynamic parameters.

$u(t)$ is the control variable with respect to the control surface deflections $\delta = [\delta_e, \delta_a, \delta_r]^T$.

3. Fuzzy Linearization Approaches

3.1. Problem Formulation. As can be seen in (9), the reentry dynamics of the HGV presents distinguished features of multivariable coupling and high-order nonlinearity, which cause difficulties for direct decoupling and controlling.

To solve this problem, the homogeneous fuzzy T-S models are aimed at being derived from truth dynamic model (9). In the following discussion, the uncertain term $\Delta(x)$ will be neglected as it can be solved by the controller design with the H_∞ specification:

Represent model (9) as

$$\dot{x}(t) = f(x) + Gu(t) = F(x,u). \tag{11}$$

Here, $g(x) = G \in R^{n\times m}$ and $F(x,u) = [F_1, F_2, \ldots, F_n]^T$.

Expanding $F(x,u)$ by means of a Taylor series in the neighbourhood of the operating point of interest (x^j, u^j) yields

$$\begin{aligned} F(x,u) &\approx F\left(x^j, u^j\right) + \left[\frac{\partial F}{\partial x}\right]_{(x^j,u^j)} \left(x - x^j\right) \\ &\quad + \left[\frac{\partial F}{\partial x}\right]_{(x^j,u^j)} \left(u - u^j\right) \\ &= f\left(x^j\right) + Gu^j + A_j\left(x - x^j\right) + G\left(u - u^j\right) \\ &= A_j x + B_j u + \left(f\left(x^j\right) - A_j x^j\right), \end{aligned} \tag{12}$$

where

$$A_j = \left[\frac{\partial F}{\partial x}\right]_{(x^j,u^j)} = \begin{bmatrix} \dfrac{\partial f_1}{\partial x_1} & \dfrac{\partial f_1}{\partial x_2} & \cdots & \dfrac{\partial f_1}{\partial x_n} \\ \dfrac{\partial f_2}{\partial x_1} & \dfrac{\partial f_2}{\partial x_2} & \cdots & \dfrac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & & \vdots \\ \dfrac{\partial f_n}{\partial x_1} & \dfrac{\partial f_n}{\partial x_2} & \cdots & \dfrac{\partial f_n}{\partial x_n} \end{bmatrix}_{(x^j)} \tag{13}$$

is the Jacobi matrix of $\mathbf{f}(x)$, $B_j = [\partial F/\partial u]_{(x^j,u^j)} = G$, and $\mu_j = f(x^j) - A_j x^j$.

Note that the bias term μ_j is not necessarily zero; only when $\mu_j = 0$, the homogeneous models $\dot{x} = F(x,u) = A_j x + B_j u$ could be derived from the truth model based on Taylor's expansion method.

In a traditional case, Taylor's expansion method can also be used to linearize the local nonlinear sectors for multivariable decoupling. However, the same problem of a nonzero μ_j exists as well.

For example, the nonlinear term $I^p_{pq}pq$ in (9) expanded Taylor series around the nonzero operating point (p_0, q_0) yields

$$\begin{aligned} pq &= p_0 q_0 + \left[\frac{\partial(pq)}{\partial p}\right]_{(p_0,q_0)} (p - p_0) \\ &\quad + \left[\frac{\partial(pq)}{\partial q}\right]_{(p_0,q_0)} (q - q_0) = q_0 p + p_0 q - p_0 q_0. \end{aligned} \tag{14}$$

Under normal circumstances, $p_0 q_0 \neq 0$, this implies that a linear local model could not be constructed in the neighbourhood of (p_0, q_0).

In (12), the possible nonzero term μ_j transforms the nonlinear system into a local affine subsystem at the operating point, which will increase the difficulty to a certain extent for a closed-loop controller design of the nonlinear system.

3.2. Jacobi Matrix Linearization. In this case, another linearization method has to be utilized to deconstruct the original system into homogeneous models in the vicinity of the nonzero operating points of interest.

That is, find constant matrices A and B such that, in the neighbourhood of nonequilibrium operating points, $\mathbf{x}^0$ yields

$$f(x) + g(x)u \approx Ax + Bu. \tag{15}$$

And point $\mathbf{x}^0$ satisfies

$$f\left(x^0\right) + g\left(x^0\right)u \approx Ax^0 + Bu. \tag{16}$$

From formula (15), it can be deducted that

$$B = g(x). \tag{17}$$

Subtracting formula (16) from formula (15) gives

$$\left[f(x) - f\left(x^0\right)\right] + \left[g(x) - g\left(x^0\right)\right]u = A\left(x - x^0\right). \tag{18}$$

For the arbitrary of u in (18),

$$g(x) = g\left(x^0\right). \tag{19}$$

Finally,

$$B = g\left(x^0\right). \tag{20}$$

Denote $A_j = [a_1 \ \ a_2 \ \ \cdots \ \ a_n]^T$ as the ith subsystem matrix linearized around the operating point $\mathbf{x}^j$, where a_i^T is the row vector of the subsystem matrix A_j. From the approach in [38], the vector of subsystem matrix A_j yields

$$a_i = \nabla f_i\left(x^j\right) + \frac{f_i\left(x^j\right) - \left(x^j\right)^T \nabla f_i\left(x^j\right)}{\left\|x^j\right\|^2} x^j, \qquad i = 1, 2, \ldots, n, \ \ j = 1, 2, \ldots, r. \tag{21}$$

Here, $f_i(x^j)$ is the value of the nonlinear function $f_i(x)$ on the point x^j, and the column vector $\nabla f_i(x^j)$ is the gradient of $f_i(x)$ evaluated at point x^j.

Thus, an integrated matrix form result can be deduced as

$$A_j = [a_1 \ a_2 \ \cdots \ a_n]^T = J\left(f\left(x^j\right)\right) + \left(x^j \cdot Z_j\right)^T. \quad (22)$$

Here, $Z_j = (1/\|x^j\|^2)([f(x^j)]^T - (x^j)^T[J(f(x^j))]^T)$, $i = 1,2,\ldots,n$, $j = 1,2,\ldots,r$, and the Jacobi matrix of $\mathbf{f}(\mathbf{x}) = [f_1, f_2, \ldots, f_n]^T$ is

$$J(f) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix}. \quad (23)$$

In summary, the local homogeneous models that approximate the original plant at every operating point of interest are obtained. It is not difficult to learn that the subsystem control matrices obtained based on Taylor's expansion method at equilibrium points are the same as those obtained based on the Jacobi matrix linearization method at other operating points.

4. T-S Modelling and Control for HGVs

4.1. T-S Fuzzy Model. The ith rules of the continuous T-S fuzzy models are as follows.

Model Rule i. If $z_1(t)$ is M_{i1} and $z_p(t)$ is M_{ip},
then

$$\dot{x} = A_i x + B_i u \quad i = 1,2,\ldots,r. \quad (24)$$

The fuzzy controller is designed by sharing the same fuzzy sets with the fuzzy model in the premise parts via the PDC (parallel distributed compensation).

Control Rule i. If $z_1(t)$ is M_{i1} and $z_p(t)$ is M_{ip},
then

$$u(t) = F_i x(t) \quad i = 1,2,\ldots,r. \quad (25)$$

Thus, given a pair of $(x(t), u(t))$, the final state space equation of the fuzzy systems can be presented according to the so-called fuzzy blending, which is inferred as follows:

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))\left[A_i x(t) + B_i u(t)\right], \quad (26)$$

$$u(t) = \sum_{i=1}^{r} h_i(z(t)) F_i x(t). \quad (27)$$

Substituting (26) into (27), the closed-loop T-S fuzzy control system can be represented by

$$\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t)) h_j(z(t))\left(A_i + B_i F_j\right)x(t). \quad (28)$$

Here, $\mathbf{z}(t) = [z_1(t) \ z_2(t) \ \cdots \ z_p(t)]$ is the premise variable vector, M_{ij} is the fuzzy set, $x(t) \in R^n$ is the state vector, $u(t)$ is the input vector, $\omega_i(z(t)) = \prod_{j=1}^{p} M_{ij}(z_j(t))$, $h_i(z(t)) = \omega_i(z(t))/\sum_{i=1}^{r}\omega_i(z(t))$, and $\sum_{i=1}^{r} h_i(z(t)) = 1$. This is a natural nonlinear system because the membership functions of the premise variables are generally nonlinear.

4.2. T-S Model for HGVs. According to the analysis above, in this section, the investigation focuses on the homogeneous T-S fuzzy modelling for reentry dynamics plants (9).

For a high-dimensional control system, the requirement of constructing its exact fuzzy model will lead to the introduction of excessive premise variables so that problems of "curse of dimension" and "rule explosion" can be encountered. Although the exact fuzzy model possesses high approximation with the original plant, the complex formulation is unrealistic for application to hypersonic vehicles. In addition to the possible application problem, more fuzzy subsystems, especially for the high-order dynamic system, will inevitably increase the complexity and conservativeness of controller design, thus resulting in more problems for system synthesis.

In view of this, the local fuzzy modelling for HGVs considered in this section will circumvent the aforementioned problems to ensure modelling accuracy as possible by appropriate fuzzy rules and premise variable selection.

As can been seen from the nonlinear equations (9), the state of bank angle rate of rotation p and sideslip angle rate of rotation r plays an important role in the severe nonlinearity and response divergence of the original dynamics. On the other hand, as in the aerospace reentry engineering, the angle of attack α is an important system parameter in longitudinal plane to measure the quality of the gliding flight. To sum up, the angle of attack α and the bank angle rate of rotation p are chosen as the fuzzy premise variables.

The membership functions of the chosen premise variables are represented in Figure 1.

Thus, dynamics plant (6) can be represented by the following T-S fuzzy model.

Rule i. If α is about α_k^i rad/s and p is about p_l^i rad/s, where $k, l = 1,2,3$,
then,

$$\dot{x} = A_i x + B_i u + D_i \omega \quad i = 1,2,\ldots,9. \quad (29)$$

Here, $\alpha_k^i \in \{0, \alpha_{\text{mid}}, \alpha_m\}$ $p_l^i \in \{-p_m, 0, p_m\}$, and α_m and p_m are the positive maximum of rotation of α and p, and $\alpha_{\text{mid}} = 0.5\alpha_m$. The control matrix $g(x) = G$ in dynamics plants (9) is constant; thus, at every operating point $B_i = G$, $i = 1,2,\ldots,9$, is widely suitable. The subsystem matrices A_i and B_i are calculated from (22) and (20); the results are shown in Appendix. D_i is disturbance matrix, and ω is the disturbance.

From (26), the blending nonlinear T-S fuzzy model for original model (9) can be constructed as

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))\left[A_i x(t) + B_i u(t) + D_i \omega\right]. \quad (30)$$

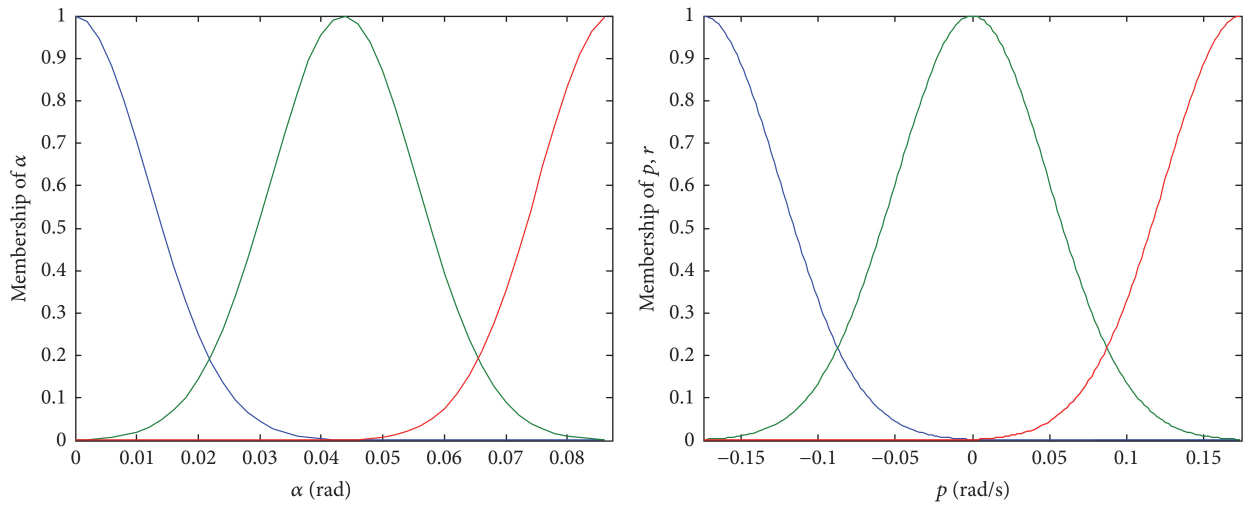


FIGURE 1: Membership functions of angle of attack α and angular rate of rotation p.

4.3. H_∞ Controller Design. When the T-S fuzzy model for HGVs has been established, a question which naturally arises is that whether a fuzzy model based controller can stabilize the original plant rather than only the T-S fuzzy model, which leads to the system stability control problem.

Before proceeding, we present the following lemma.

Lemma 1. *The parameterised linear matrix inequalities*

$$\sum_{i,j=1}^{r} \mu_i \mu_j M_{ij} < 0 \tag{31}$$

are fulfilled, if the following condition holds:

$$\begin{cases} M_{ii} < 0 & i = 1, \dots, r \\ \dfrac{1}{r-1} M_{ii} + \dfrac{1}{2}\left(M_{ij} + M_{ji}\right) < 0 & 1 \le i \ne j \le r. \end{cases} \tag{32}$$

Theorem 2. *If there exist matrices $Z > 0$, M_i, $i = 1, 2, \dots, r_s$, satisfy the following LMIs:*

$$\Theta_{ii} < 0 \quad i = 1, \dots, r, \tag{33}$$

$$\frac{1}{r-1}\Theta_{ii} + \frac{1}{2}\left(\Theta_{ij} + \Theta_{ji}\right) < 0 \quad 1 \le i \ne j \le r, \tag{34}$$

$$\Theta_{ij} = \begin{bmatrix} A_i Z + Z A_i^T + B_i M_j + M_j^T B_i^T & D & Z C^T \\ D^T & -\rho^2 I & 0 \\ CZ & 0 & -I \end{bmatrix}. \tag{35}$$

Then, T-S fuzzy system (30) is quadratically stabilizable via fuzzy control (27) for a prescribed performance index $\rho > 0$, where $F_i = M_i Z^{-1}$.

Proof. For system (30), define the following Lyapunov function:

$$V = x^T P x. \tag{36}$$

Then

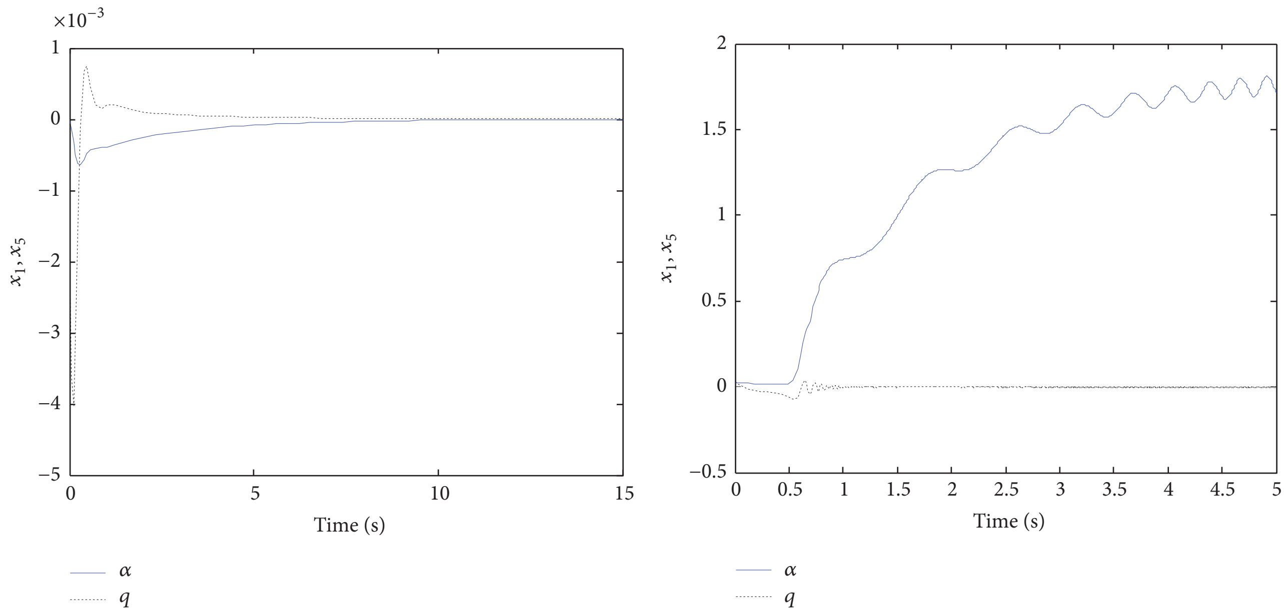
$$\begin{aligned} \dot{V} &= x^T P (\mathscr{A} x + \mathscr{B}\omega) + * \\ &= \left\{x^T P \mathscr{A} x + x^T P \mathscr{B} \omega\right\} + * \\ z^T z &= x^T \mathscr{C}^T \mathscr{C} x. \end{aligned} \tag{37}$$

We can obtain

$$\begin{aligned} &\dot{V} + z^T z - \rho^2 \omega^T \omega \\ &\quad = x^T (P\mathscr{A} + {}^*) x + x^T P \mathscr{B} \omega + \omega^T \mathscr{B}^T P x \\ &\qquad + x^T \mathscr{C}^T \mathscr{C} x + \omega^T \left\{-\rho^2 I\right\} \omega \\ &\quad = x^T \left\{(P\mathscr{A} + {}^*) + \mathscr{C}^T \mathscr{C}\right\} x + x^T \left\{P\mathscr{B} + \mathscr{C}^T \mathscr{D}\right\} \omega \\ &\qquad + \omega^T \left\{\left(P\mathscr{B} + \mathscr{C}^T \mathscr{D}\right)^T\right\} x \\ &\qquad + \omega^T \left\{\mathscr{D}^T \mathscr{D} - \rho^2 I\right\} \omega \\ &\quad = \begin{bmatrix} x^T & \omega^T \end{bmatrix} \begin{bmatrix} (P\mathscr{A} + {}^*) + \mathscr{C}^T \mathscr{C} & P\mathscr{B} \\ (P\mathscr{B})^T & -\rho^2 I \end{bmatrix} \begin{bmatrix} x \\ \omega \end{bmatrix}, \end{aligned} \tag{38}$$

as

$$\begin{aligned} &\begin{bmatrix} (P\mathscr{A} + {}^*) + \mathscr{C}^T \mathscr{C} & P\mathscr{B} \\ (P\mathscr{B})^T & -\rho^2 I \end{bmatrix} \\ &\quad = \begin{bmatrix} P\mathscr{A} + {}^* & P\mathscr{B} \\ (P\mathscr{B})^T & -\rho^2 I \end{bmatrix} - \begin{bmatrix} \mathscr{C}^T \\ 0 \end{bmatrix} (-I)^{-1} \begin{bmatrix} \mathscr{C} & 0 \end{bmatrix} < 0. \end{aligned} \tag{39}$$



(a) Subfigure 1

(b) Subfigure 2

FIGURE 2: Response of the pitch channel in system (9) under the controller I.

If and only if

$$\begin{bmatrix} P\mathscr{A} + * & P\mathscr{B} & \mathscr{C}^T \\ (P\mathscr{B})^T & -\rho^2 I & 0 \\ \mathscr{C} & 0 & -I \end{bmatrix} < 0,$$
$$\sum_{i=1}^{r_s}\sum_{j=1}^{r_s} h_i h_j \begin{bmatrix} P\left(A_i + B_i F_j\right) + * & PD_i & \mathscr{C}^T \\ \left(PD_i\right)^T & -\rho^2 I & 0 \\ C & 0 & -I \end{bmatrix} < 0. \tag{40}$$

Pre- and postmultiplying by $\mathrm{diag}(P^{-1}, I, I)$, we obtain

$$\sum_{i=1}^{r_s}\sum_{j=1}^{r_s} h_i h_j \begin{bmatrix} \left(A_i + B_i F_j\right) P^{-1} + * & D_i & P^{-1}C^T \\ D_i^T & -\rho^2 I & 0 \\ CP^{-1} & 0 & -I \end{bmatrix} < 0. \tag{41}$$

Let $Z = P^{-1}$ and $F_i = M_i Z^{-1}$

Then the above equation is equivalent to

$$\sum_{i=1}^{r_s}\sum_{j=1}^{r_s} h_i h_j \begin{bmatrix} \left(A_i Z + B_i M_j\right) + * & D_i & ZC^T \\ D_i^T & -\rho^2 I & 0 \\ CZ & 0 & -I \end{bmatrix} < 0. \tag{42}$$

From Lemma 1, we know that LMI conditions (33) and (34) are sufficient conditions to guarantee

$$\dot{V} + z^T z - \rho^2 \omega^T \omega < 0. \tag{43}$$

Let $\omega = 0$; we have $\dot{V} < \rho^2 \omega^T \omega - z^T z \leq 0$, so the closed-loop system is asymptotically stable with $\omega = 0$.

Furthermore, under zeros initial conditions, we have

$$\int_0^{+\infty} \left(z^T z - \rho^2 \omega^T \omega\right) d\tau < -\int_0^{+\infty} \dot{V}\, d\tau$$
$$= V(0) - V(+\infty) \leq V(0) = x(0)^T P x(0) = 0. \tag{44}$$

That is,

$$\int_0^{+\infty} z^T z\, d\tau < \rho^2 \int_0^{+\infty} \omega^T \omega\, d\tau. \tag{45}$$

Therefore, the H_∞ performance is achieved with a prescribed level ρ. The proof is completed. □

Theorem 2 gives the relaxed LMI condition for the H_∞ stability of the fuzzy system (30) and obtains the global T-S fuzzy controller by solving LMI. The fuzzy controller can guarantee the stability of the fuzzy system but not necessarily guarantee the stability of the original system. In the following, we will verify that the T-S fuzzy controller designed by Theorem 2 is a H_∞ stable controller of the original closed-loop system.

5. Simulations

In this section, the obtained hypersonic T-S fuzzy model is compared with the original nonlinear plant at the single operating points and the state region.

To illustrate the effectiveness of the designed controller based on T-S fuzzy model (30) for HGVs, the original nonlinear model (not the T-S fuzzy model) will be used to test the H_∞ stability of the control system.

Without loss of generality, the control performance for system (6) with the following three controllers will be

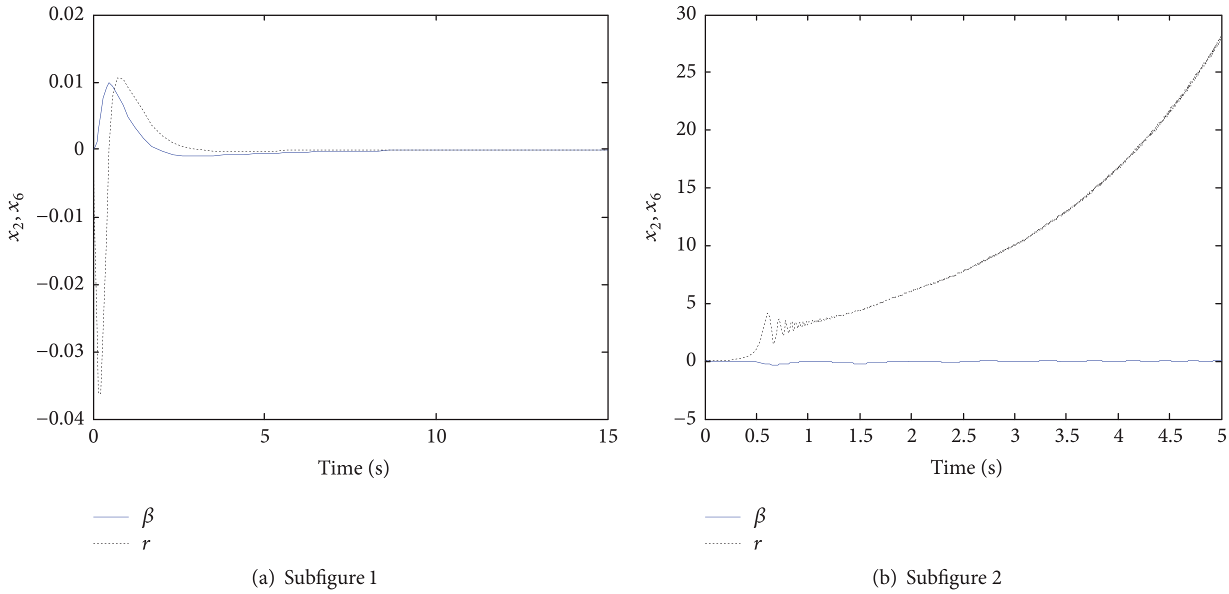


FIGURE 3: Response of the yaw channel in system (9) under the controller I.

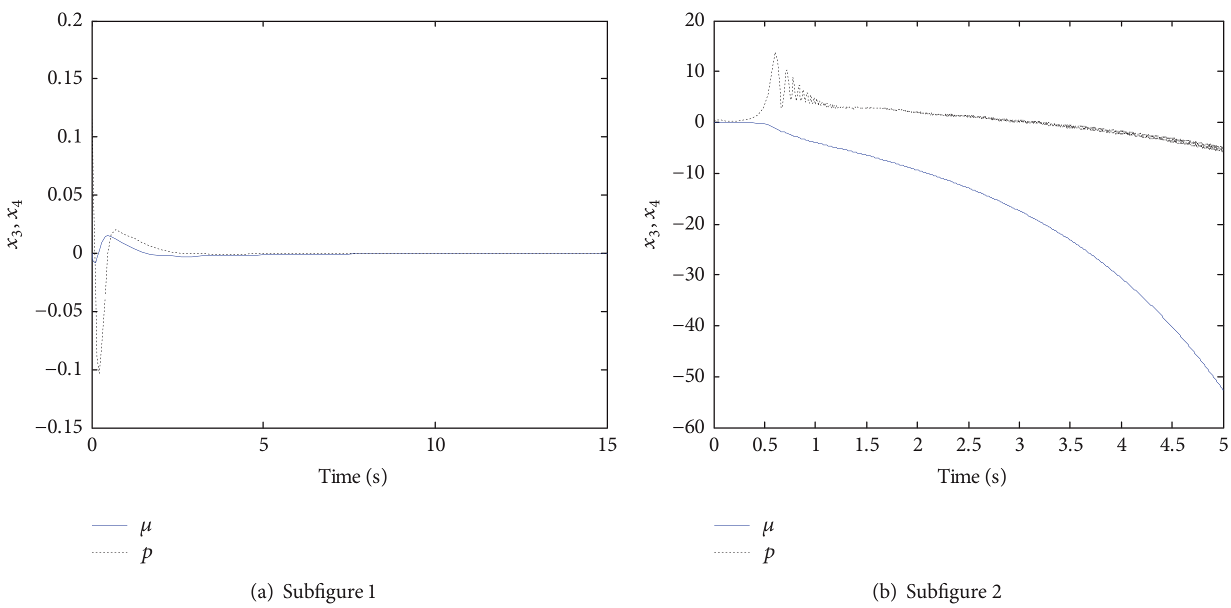


FIGURE 4: Response of the rolling channel in system (9) under the controller I.

depicted and compared at the operating point and its neighbourhood:

Controller I. The local linear controller designed with the pole-placement method.

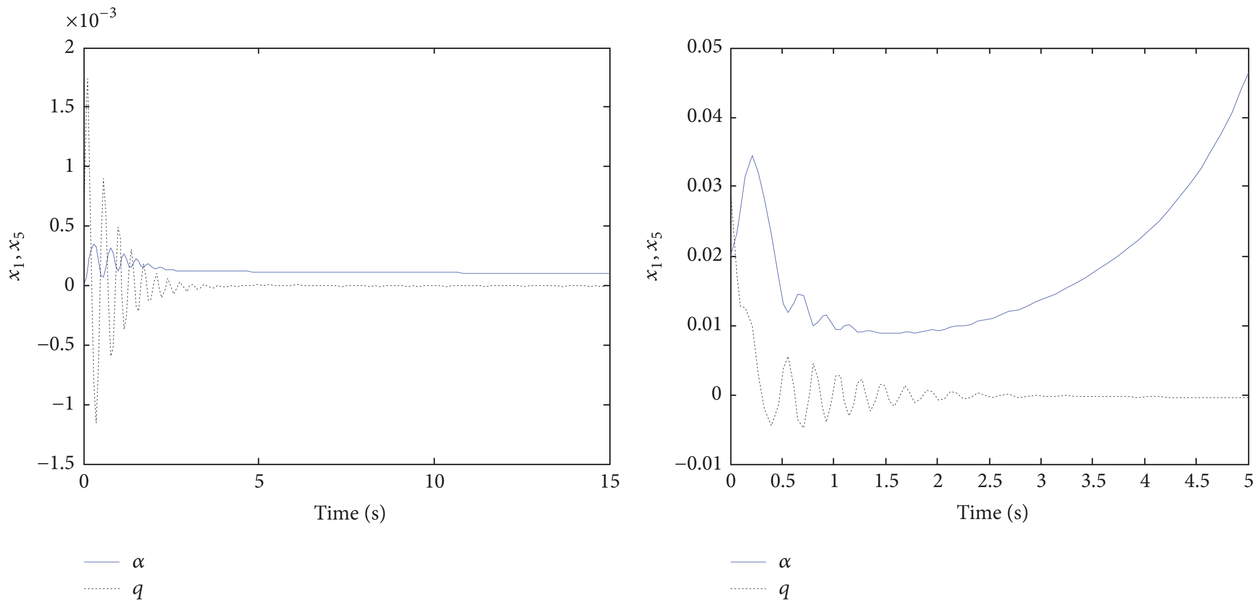
Controller II. The local linear T-S fuzzy controller designed with Theorem 2.

Controller III. The nonlinear T-S fuzzy controller designed with Theorem 2.

Disturbance matrix $D_i = D = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}^T$, and the disturbance is only added to controller III, which is shown as follows:

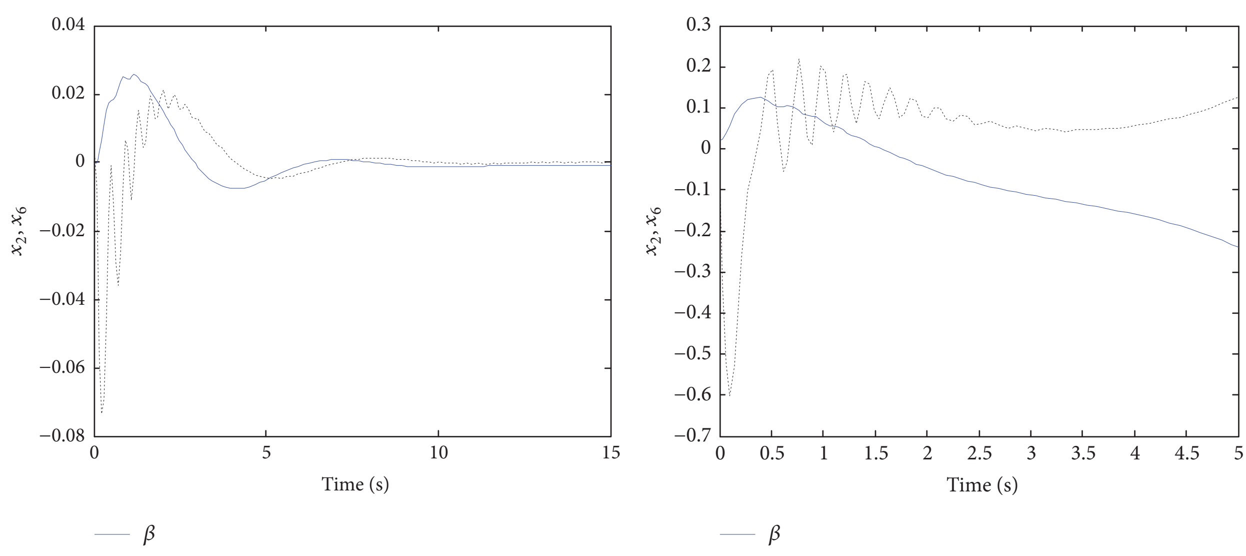
$$\omega(t) = \begin{cases} \text{rand} - 0.5 & 0.5 \le t \le 1 \\ 0 & \text{others.} \end{cases} \tag{46}$$

For ease of presentation, we chose the operation point as $[0, 0, 0, \pi/18, 0, 0]$ and its neighbourhood point as $[0.02, 0.02, 0.02, \pi/18, 0.03, 0.02]$. The responses of HGV



(a) Subfigure 1 (b) Subfigure 2

FIGURE 5: Response of the pitch channel in system (9) under the controller II.



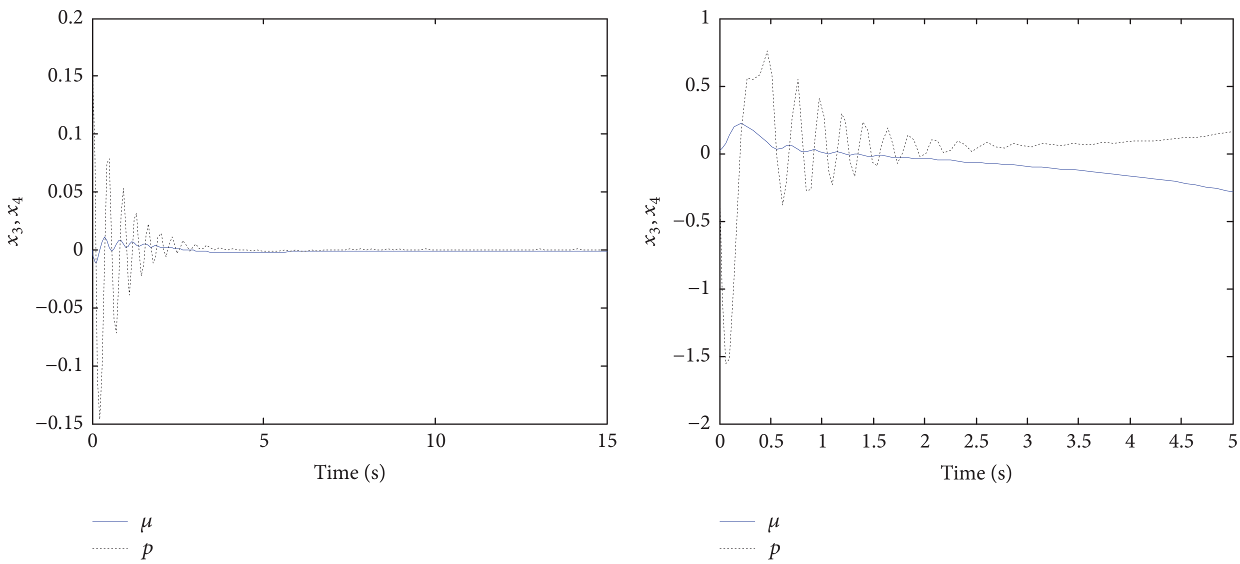
(a) Subfigure 1 (b) Subfigure 2

FIGURE 6: Response of the yaw channel in system (9) under the controller II.

control system (9) in three control channels have been drawn in Figures 2–10.

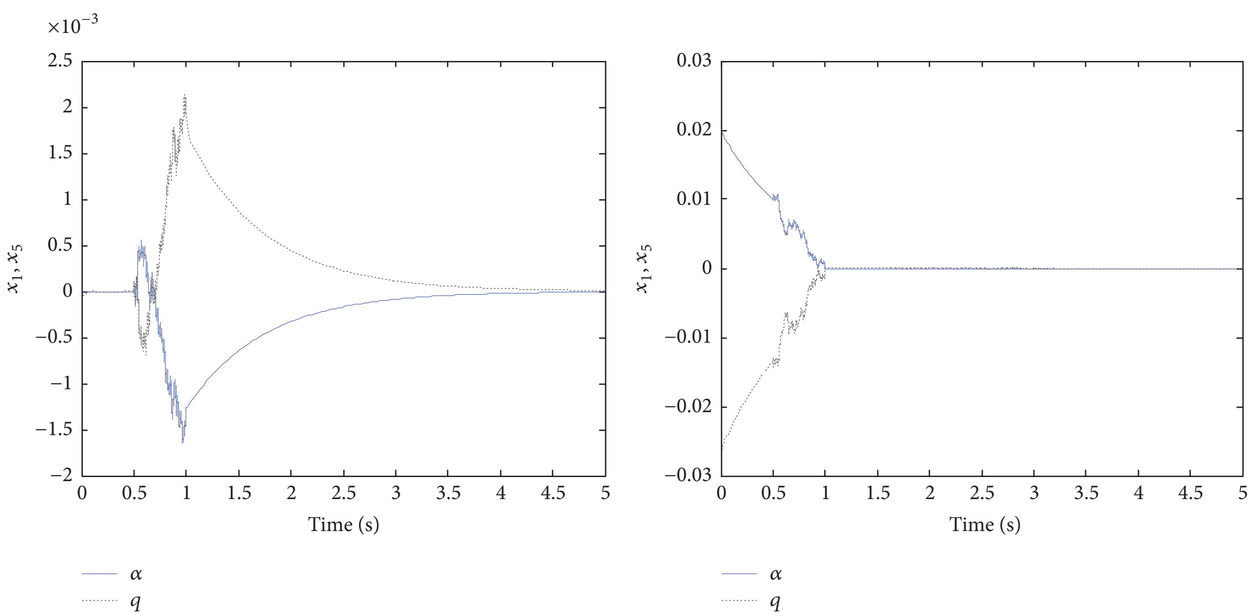
Figures 2–10 show the responses of system (9) on the operation point $[0, 0, 0, \pi/18, 0, 0]$ (Subfigure 1) and its neighbourhood point $[0.02, 0.02, 0.02, \pi/18, 0.03, 0.02]$ (Subfigure 2) under the local linear controllers and designed T-S controller (27).

Obviously, although the linear controller I and linear controller II can stabilize truth plant (9) at a single operating point, which are, respectively, depicted in Subfigure 1 from Figures 2–4 and Subfigure 1 from Figures 5–7, they all diverge in the neighbourhood of this operating point, which are depicted in Subfigure 2 from Figures 2–7. However, the T-S fuzzy controller designed based on T-S fuzzy model (30)



(a) Subfigure 1 (b) Subfigure 2

Figure 7: Response of the rolling channel in system (9) under the controller II.



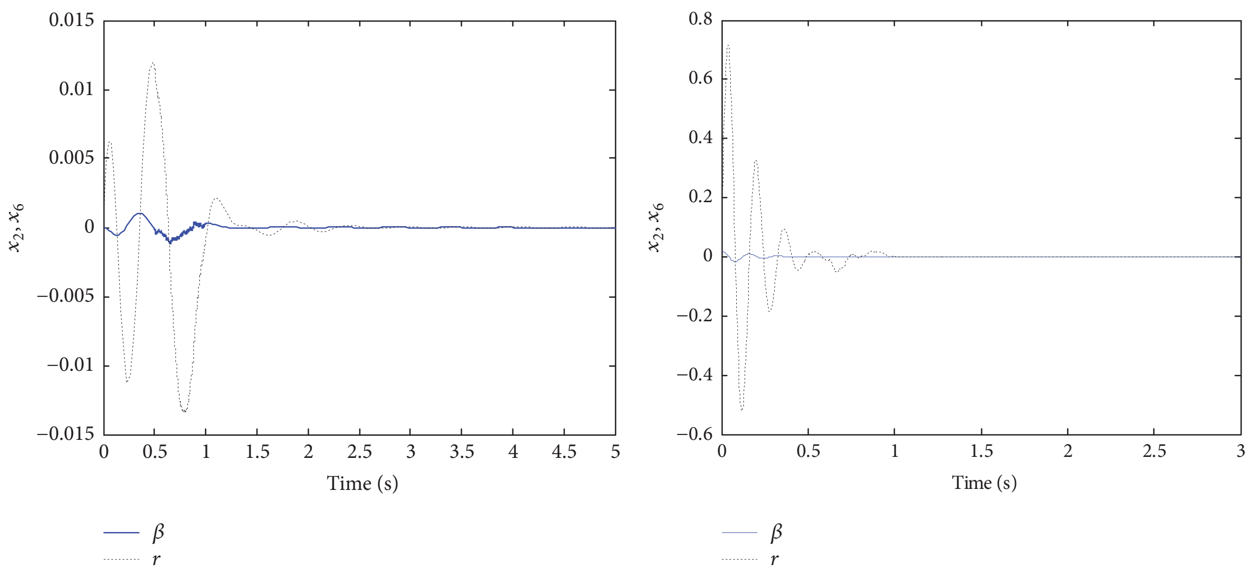
(a) Subfigure 1 (b) Subfigure 2

Figure 8: Response of the pitch channel in system (9) under the controller III.

and Theorem 2 can stabilize the truth plant (9) at both points with external disturbance depicted, respectively, in Subfigure 1 and Subfigure 2 from Figures 8–10, which demonstrates the effectiveness of the T-S fuzzy model and T-S fuzzy H_∞ controller design for the HGV control system.

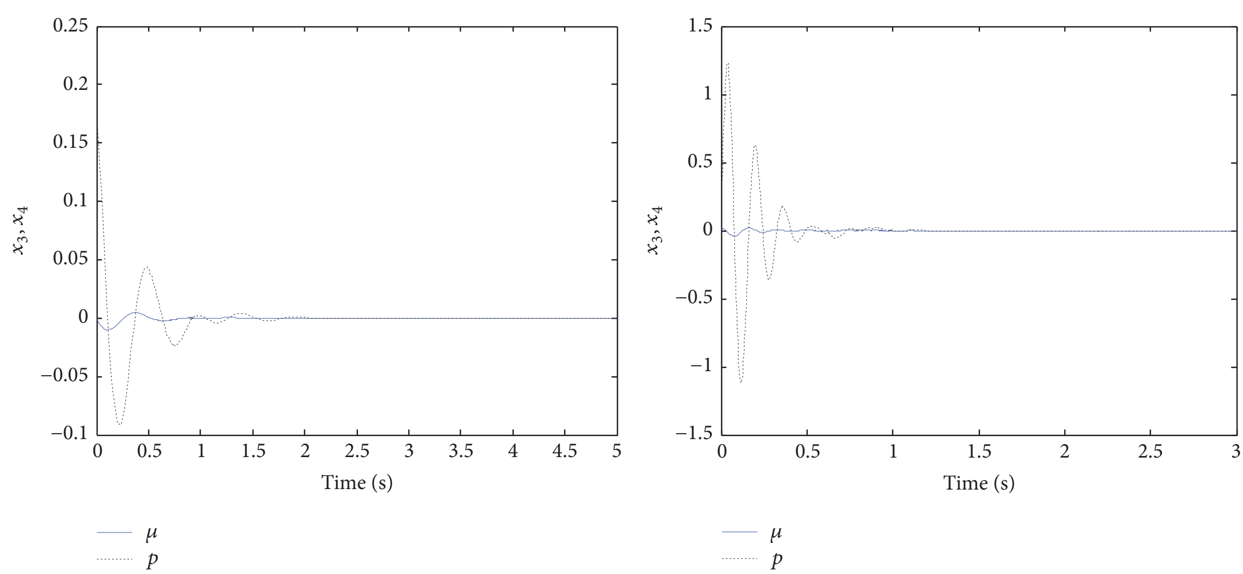
6. Conclusions

The problems of fuzzy modelling and H_∞ fuzzy control for the HGV reentry dynamics system have been investigated in this paper. The affine nonlinear system is established by reasonable assumptions, and then, the T-S fuzzy model of the HGV reentry dynamics system has been constructed based on Taylor's expansion and the fuzzy linearization approaches. A T-S fuzzy H_∞ controller has been designed for the HGV original plant based on the designed T-S fuzzy model by using the LMI technology. Moreover, numerical simulations have been carried out to demonstrate the effectiveness of the proposed design scheme.



(a) Subfigure 1 (b) Subfigure 2

Figure 9: Response of the yaw channel in system (9) under the controller III.



(a) Subfigure 1 (b) Subfigure 2

Figure 10: Response of the rolling channel in system (9) under the controller III.

Appendix

$$A_1 = \begin{bmatrix} 0 & 0.17453 & 0 & 0 & 1 & 0 \\ -0.17453 & 0 & 0 & 0 & 0 & -1 \\ 0.17453 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1253.5 & 0 \\ 0 & 0 & 0 & 0.011223 & 0 & -0.17222 \\ 0 & 0 & 0 & 0 & 332.21 & 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 0.17453 & 0 & 0 & 1 & 0 \\ -0.17453 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -451.69 & 0 \\ 0 & 0 & 0 & 0.04586 & 0 & -0.16099 \\ 0 & 0 & 0 & 0 & -119.48 & 0 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 0 & 0.17453 & 0 & 0 & 1 & 0 \\ -0.17453 & 0 & 0 & 0 & 0 & -1 \\ -0.17453 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2156.9 & 0 \\ 0 & 0 & 0 & 0.17222 & 0 & 0.011223 \\ 0 & 0 & 0 & 0 & -571.17 & 0 \end{bmatrix},$$

$$A_4 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0.17453 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1705.2 & 0 \\ 0 & 0 & 0 & -0.16099 & 0 & -0.04586 \\ 0 & 0 & 0 & 0 & 451.69 & 0 \end{bmatrix},$$

$$A_5 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$A_6 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ -0.17453 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1705.2 & 0 & 0 \\ 0 & 0 & 0 & 0.16099 & 0 & 0.04586 \\ 0 & 0 & 0 & 0 & -451.69 & 0 \end{bmatrix},$$

$$A_7 = \begin{bmatrix} 0 & -0.17453 & 0 & 0 & 1 & 0 \\ 0.17453 & 0 & 0 & 0 & 0 & -1 \\ 0.17453 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 2156.9 & 0 & 0 \\ 0 & 0 & 0 & -0.17222 & 0 & -0.011223 \\ 0 & 0 & 0 & 0 & 571.17 & 0 \end{bmatrix},$$

$$A_8 = \begin{bmatrix} 0 & -0.17453 & 0 & 0 & 1 & 0 \\ 0.17453 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 451.69 & 0 & 0 \\ 0 & 0 & 0 & -0.04586 & 0 & -0.16099 \\ 0 & 0 & 0 & 0 & 119.48 & 0 \end{bmatrix},$$

$$A_9 = \begin{bmatrix} 0 & -0.17453 & 0 & 0 & 1 & 0 \\ 0.17453 & 0 & 0 & 0 & 0 & -1 \\ -0.17453 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1253.5 & 0 \\ 0 & 0 & 0 & -0.011223 & 0 & 0.17222 \\ 0 & 0 & 0 & 0 & -332.21 & 0 \end{bmatrix},$$

$$B_i = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 31.983 & 0 & 8.4714 \\ 0 & 0.0002 & 0 \\ 8.4714 & 0 & 2.244 \end{bmatrix}, \quad i = 1, 2, \ldots, 9. \tag{A.1}$$

Conflicts of Interest

The authors declare that there are no conflicts of interest and confirm that the funding of the paper did not lead to any conflicts of interest regarding the publication of this paper.

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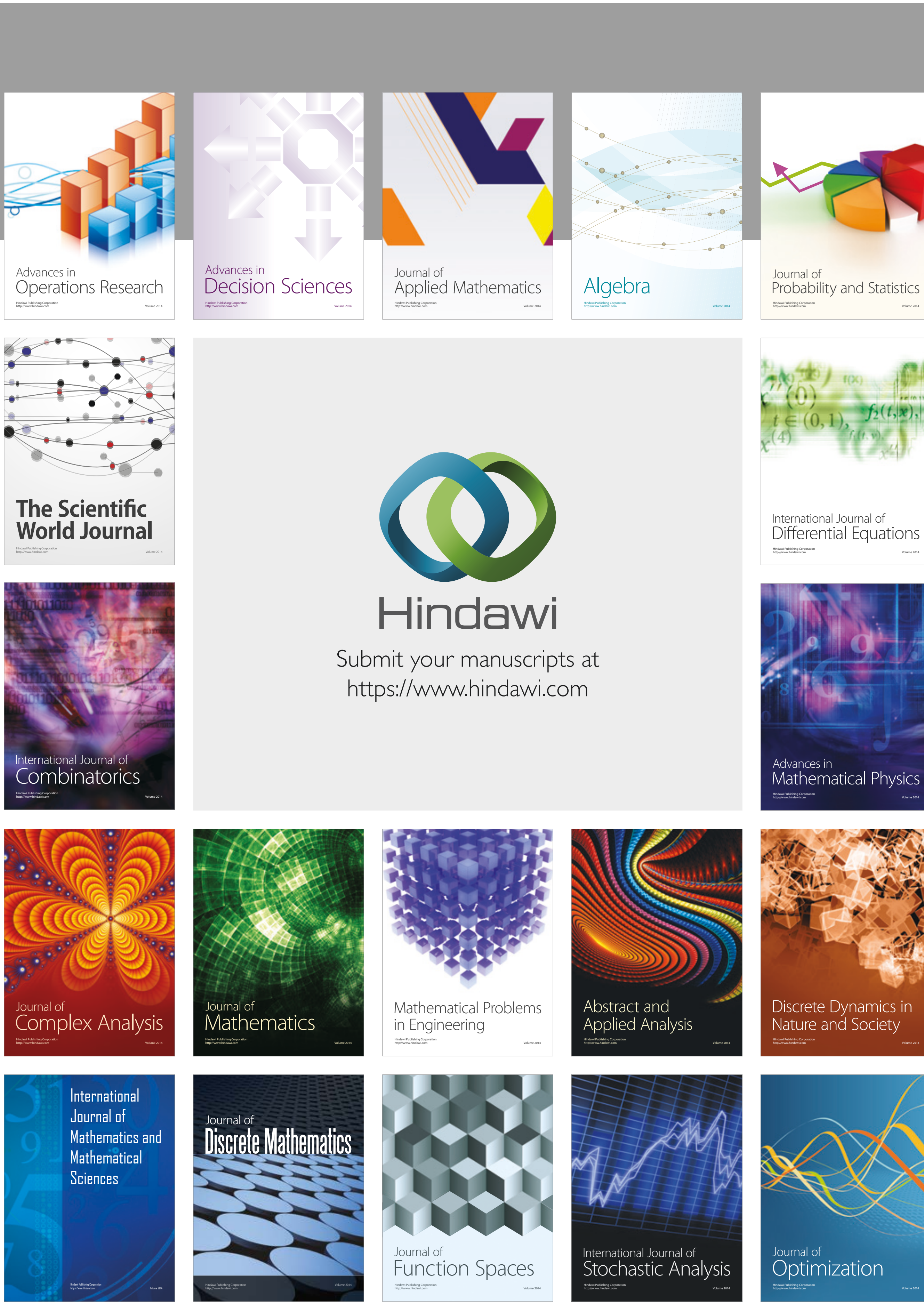
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