Expression of Concern

Expression of Concern on “Reliability Analysis of Random Vibration Transmission Path Systems”

Mathematical Problems in Engineering

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Mathematical Problems in Engineering would like to express concern with the article titled “Reliability Analysis of Random Vibration Transmission Path Systems” [1], due to an error in the results and dual publication. The authors informed us that when the natural frequency of the system in Figure 2 was calculated, an error of parameter substitution occurred, and the deviation is about 5. This also affects the analysis shown in Figure 4. However, the authors were not reachable to provide details of this error and how this affects the conclusions.

The authors published another version of this work in July 2017 [2], shortly after the publication in *Mathematical Problems in Engineering*. The authors also did not discuss their related article from 2015 [3].

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Research Article

Reliability Analysis of Random Vibration Transmission Path Systems

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The vibration transmission path systems are generally composed of the vibration source, the vibration transfer path, and the vibration receiving structure. The transfer path is the medium of the vibration transmission. Moreover, the randomness of transfer path influences the transfer reliability greatly. In this paper, based on the matrix calculus, the generalized second moment technique, and the stochastic finite element theory, the effective approach for the transfer reliability of vibration transfer path systems was provided. The transfer reliability of vibration transfer path system with uncertain path parameters including path mass and path stiffness was analyzed theoretically and computed numerically, and the correlated mathematical expressions were derived. Thus, it provides the theoretical foundation for the dynamic design of vibration systems in practical project, so that most random path parameters can be considered to solve the random problems for vibration transfer path systems, which can avoid the system resonance failure.

1. Introduction

In the modern industry, the problem of vibration and noise has always been focused on widely by technical personnel. The suitable control method for the system vibration and noise is of great theoretical value and economic benefit. Generally speaking, the vibration system is composed of three parts, the vibration source, the vibration transfer path, and the vibration receiving structure. The transfer path is a specific medium, through which the vibration sources are transferred to the system receiving structure. In order to control the vibration and noise effectively, it is necessary to recognize and analyze the vibration transfer path accurately. Consequently, the prediction and dynamic design of the vibration transmission path system become especially important. At present, the analysis of the transfer path of the vibration system mainly concentrated in the experimental method and the energy transfer method [1–5]. Overall, the research on the prediction and the randomness of the parameters is limited.

It is generally known that the uncertain factors will appear inevitably in the vibration system. The random nature of the path parameters will change the system's vibration transmission characteristics and ultimately affect the output of the

system, which can bring out the system reliability problem. According to the actual working condition, the research on the reliability problems of vibration transmission path system can be made in two ways. On the one hand, it can be studied when the transfinite is considered as the measure indicator about the path transfer force or transfer rate. This kind of research is based on the system responses and depends on the random response analysis for the random systems. On the other hand, the study can be finished according to the structural failure coming from the resonance or potential resonance, which exists widely in the various structures with uncertainty factors. When the system is in a state of resonance, the dynamic stress is very large and the service life can be shortened greatly, which will influence the performance of mechanical equipment. In this paper, our works focus on the latter.

2. Stochastic Finite Element Method for System Eigensolutions Analysis

According to Taylor series expansion theory [6–9], if the vector $\mathbf{u}_{p \times 1}$ is the function of the vector $\mathbf{v}_{s \times 1}$, then the two-order Taylor expression of $\mathbf{u}$ at $\bar{\mathbf{v}} = E(\mathbf{v})$ is

$$\mathbf{u}(\mathbf{v}) = \mathbf{u}(\bar{\mathbf{v}}) + \left. \frac{\partial \mathbf{u}}{\partial \mathbf{v}^T} \right|_{\mathbf{v}=\bar{\mathbf{v}}} d\mathbf{v} + \frac{1}{2} \left. \frac{\partial^2 \mathbf{u}}{\partial^2 \mathbf{v}^T} \right|_{\mathbf{v}=\bar{\mathbf{v}}} (d\mathbf{v})^{[2]}, \quad (1)$$

where $d\mathbf{v}^{[2]} = d\mathbf{v} \otimes d\mathbf{v}$ is the second-order power. The symbol $\otimes$ represents Kronecker power, which is defined as $\mathbf{u}_{p \times 1} \otimes \mathbf{v}_{s \times 1} = (\mathbf{u}_i \mathbf{v})_{ps \times 1}$ and then the corresponding matrix derivatives are defined as $\partial \mathbf{u} / \partial \mathbf{v} = (\partial \mathbf{u} / \partial \mathbf{v}_i)_{ps \times 1}$.

Because the path parameters vector $\mathbf{b}_n$ of the vibration transmission path system is random, the eigenvalue $\omega_i^2(\mathbf{b}_n)$ ($i = 1, 2, \dots, n$) and characteristic vector $\mathbf{u}_i(\mathbf{b}_n)$ ($i = 1, 2, \dots, n$) of the system are also random and they also can be expanded into Taylor series.

The eigenvalue problem of the stochastic parameter system can be expressed as

$$\mathbf{K}_n(\mathbf{b}_n) \mathbf{u}_i(\mathbf{b}_n) = \omega_i^2(\mathbf{b}_n) \mathbf{M}_n(\mathbf{b}_n) \mathbf{u}_i(\mathbf{b}_n). \quad (2)$$

Both sides of (4) are expanded into two-order Taylor expression at the $\bar{\mathbf{b}}_n = E(\mathbf{b}_n)$. We can obtain the following equations through comparing the same order of $d\mathbf{b}_n$:

$$\mathbf{K}_n(\bar{\mathbf{b}}_n) \mathbf{u}_i(\bar{\mathbf{b}}_n) = \omega_i^2(\bar{\mathbf{b}}_n) \mathbf{M}_n(\bar{\mathbf{b}}_n) \mathbf{u}_i(\bar{\mathbf{b}}_n), \quad (3)$$

$$\begin{aligned} & \left(\left. \frac{\partial \mathbf{K}_n}{\partial \mathbf{b}_n^T} \right|_{\mathbf{b}_n=\bar{\mathbf{b}}_n} - \omega_i^2(\bar{\mathbf{b}}_n) \left. \frac{\partial \mathbf{M}_n}{\partial \mathbf{b}_n^T} \right|_{\mathbf{b}_n=\bar{\mathbf{b}}_n} \right. \\ & \left. - \frac{\partial \omega_i^2}{\partial \mathbf{b}_n^T} \right|_{\mathbf{b}_n=\bar{\mathbf{b}}_n} \mathbf{M}_n(\bar{\mathbf{b}}_n) \mathbf{u}_i(\bar{\mathbf{b}}_n) + (\mathbf{K}_n(\bar{\mathbf{b}}_n) \\ & - \omega_i^2(\bar{\mathbf{b}}_n) \mathbf{M}_n(\bar{\mathbf{b}}_n)) \left. \frac{\partial \mathbf{u}_i}{\partial \mathbf{b}_n^T} \right|_{\mathbf{b}_n=\bar{\mathbf{b}}_n} = 0. \end{aligned} \quad (4)$$

Equation (5) describes the problem with determined eigenvalue when the mean value of the structure parameters is taken. Thus, according to the conventional way, we can get the determined eigenvalue $\omega_i^2(\bar{\mathbf{b}}_n)$ and characteristic vector $\mathbf{u}_i(\bar{\mathbf{b}}_n)$, written as $\bar{\omega}_i^2$ and $\bar{\mathbf{u}}_i$ ($i = 1, 2, \dots, n$). The first-order sensitivities $(\partial \omega_i^2 / \partial \mathbf{b}_n^T)|_{\mathbf{b}_n=\bar{\mathbf{b}}_n}$ and $(\partial \mathbf{u}_i / \partial \mathbf{b}_n^T)|_{\mathbf{b}_n=\bar{\mathbf{b}}_n}$ can be obtained by (4), written as $\partial \bar{\omega}_i^2 / \partial \mathbf{b}_n^T$ and $\partial \bar{\mathbf{u}}_i / \partial \mathbf{b}_n^T$.

Since $\mathbf{K}_n$ and $\mathbf{M}_n$ both are real symmetric matrixes, (4) is multiplied by $\bar{\mathbf{u}}_i^T$ on the left; the following equation can be obtained:

$$\frac{\partial \bar{\omega}_i^2}{\partial \mathbf{b}_n^T} \bar{\mathbf{u}}_i^T \mathbf{M}_n \bar{\mathbf{u}}_i = \bar{\mathbf{u}}_i^T \left(\frac{\partial \bar{\mathbf{K}}_n}{\partial \mathbf{b}_n^T} - \bar{\omega}_i^2 \frac{\partial \bar{\mathbf{M}}_n}{\partial \mathbf{b}_n^T} \right) \bar{\mathbf{u}}_i. \quad (5)$$

By the regularization condition of the characteristic vector, the first-order sensitivity of the system eigenvalue to the mean of the random parameters can be obtained:

$$\frac{\partial \bar{\omega}_i}{\partial \mathbf{b}_n^T} = \frac{1}{2\bar{\omega}_i} \left[\bar{\mathbf{u}}_i^T \left(\frac{\partial \bar{\mathbf{K}}_n}{\partial \mathbf{b}_n^T} - \bar{\omega}_i^2 \frac{\partial \bar{\mathbf{M}}_n}{\partial \mathbf{b}_n^T} \right) \bar{\mathbf{u}}_i \right]. \quad (6)$$

According to the Taylor series expansion and the second moment method, the mean value and variance of the system eigenvalue can be written as

$$E(\omega_i) = \bar{\omega}_i + \frac{1}{2} \frac{\partial^2 \bar{\omega}_i}{\partial^2 \mathbf{b}_n} \text{Var}(\mathbf{b}_n), \quad (7)$$

$$\text{Var}(\omega_i) = \left(\frac{\partial \bar{\omega}_i}{\partial \mathbf{b}_n^T} \right)^{[2]} \text{Var}(\mathbf{b}_n), \quad (8)$$

where $\text{Var}(\mathbf{b}_n)$ is the variance matrix of the random parameters, including all of the variances and covariances.

By formula (8), the first-order sensitivity of the eigenvalue variances to the random parameter variances is

$$\frac{\partial \sigma_{\omega_i}}{\partial \text{Var}(\mathbf{b}_n)} = \frac{1}{2\sigma_{\omega_i}} \left(\frac{\partial \bar{\omega}_i}{\partial \mathbf{b}_n^T} \right)^{[2]}. \quad (9)$$

3. System Transmission Reliability Analysis

According to the interference theory of the reliability, the state function of the resonance problem for the vibration transmission path system can be described as

$$g_{ij}(p_j, \omega_i) = |p_j - \omega_i| \quad (i = 1, 2, \dots, n; j = 1, 2, \dots, m), \quad (10)$$

where p_j represents the excitation frequency and ω_i represents the natural frequency.

Based on the state function determined by the excitation frequency p_j and the natural frequency ω_i , the two states of the vibration transmission path system can be defined as

$$\begin{aligned} g_{ij}(p_j, \omega_i) &= |p_j - \omega_i| \leq \gamma \quad \text{invalid,} \\ g_{ij}(p_j, \omega_i) &= |p_j - \omega_i| > \gamma \quad \text{security,} \end{aligned} \quad (11)$$

where γ is a specific interval and is generally 5%–15% times of the natural frequency average.

Given $G_{ij} = p_j - \omega_i$, the mean value and variance of the function G_{ij} are, respectively,

$$\begin{aligned} \mu_{G_{ij}} &= E(G_{ij}) = \mu_{p_j} - \mu_{\omega_i}, \\ \sigma_{G_{ij}}^2 &= \text{Var}(G_{ij}) = \sigma_{p_j}^2 + \sigma_{\omega_i}^2. \end{aligned} \quad (12)$$

The quasi failure probability of the resonance for the vibration transmission path system is

$$P_f^{ij} = \int_{-\gamma_i \leq G_{ij} \leq \gamma_i} f_b(\mathbf{b}_n) d\mathbf{b}_n, \quad (13)$$

where $f_b(\mathbf{b}_n)$ is the joint probability density of the basic random parameter vector $\mathbf{b}_n$.

The random parameters of the vibration transmission path systems shown in Figures 1 and 2 are subject to the normal distribution, which brings convenience for the system reliability analysis. When the excitation frequency and natural frequency are also subject to normal distribution and independent, the system failure index is

$$\beta_1^{ij} = \frac{E(\gamma_i - G_{ij})}{\sqrt{\text{Var}(\gamma_i - G_{ij})}},$$

FIGURE 1: Vibration transfer path system model with vertical force excitation.

FIGURE 2: Vibration transfer path system model with vertical force excitation and moment excitation.

$$\beta_2^{ij} = \frac{E(-\gamma_i - G_{ij})}{\sqrt{\text{Var}(-\gamma_i - G_{ij})}}. \quad (14)$$

The system quasi failure probability is

$$\begin{aligned} P_f^{ij} &= \Phi(\beta_1^{ij}) - \Phi(\beta_2^{ij}) \\ &= \Phi\left(\frac{\gamma_i - \mu_{G_{ij}}}{\sigma_{G_{ij}}}\right) - \Phi\left(\frac{-\gamma_i - \mu_{G_{ij}}}{\sigma_{G_{ij}}}\right), \end{aligned} \quad (15)$$

where $\Phi(\cdot)$ is standard normal distribution function.

In the vibration transmission path system, when any one of the excitation frequencies is close to the natural frequency, the resonance will occur. The system is considered as the series system and the quasi failure probability of the whole system is

$$P_f = 1 - \prod_{i=1}^n \prod_{j=1}^m (1 - P_f^{ij}). \quad (16)$$

The transmission reliability is

$$R = 1 - P_f = \prod_{i=1}^n \prod_{j=1}^m (1 - P_f^{ij}). \quad (17)$$

4. Vibration Transmission Path System Model

In most vibration systems, there are several transfer paths between the vibration source and the receiving structure and the excitations are transmitted in the corresponding ways. For example, the car body vibration comes partly from the force and torque generated by the engine. In this system, the engine mount is transfer path. Based on the typical structure [10–12], we built the vibration transfer path system models with three parallel paths, as shown in Figures 1 and 2. The difference of two models is in the vibration sources. The system in Figure 1 is excited only by the vertical force and the system in Figure 2 is excited not only by the vertical force but also by the moment. Thus, the parameters that affect the path transmission characteristics include not only the quality, stiffness, and damping, but also the installation location of the transfer path. Figure 1 shows a 5-DoFs (degrees of freedom) system and Figure 2 shows a 7-DoFs system. Applying Lagrange equation, the vibration differential equation can be written as

$$\mathbf{M}_n \ddot{\mathbf{x}}_n + \mathbf{C}_n \dot{\mathbf{x}}_n + \mathbf{K}_n \mathbf{x}_n = \mathbf{F}_n(t), \quad (18)$$

where n represents the number of DoFs.

$$\mathbf{M}_5 = \text{diag}(m_s \ m_{p1} \ m_{p2} \ m_{p3} \ m_r),$$

$$\mathbf{M}_7 = \text{diag}(m_s \ I_s \ m_{p1} \ m_{p2} \ m_{p3} \ m_r \ I_r),$$

$$\mathbf{K}_5 = \begin{bmatrix} k_s + k_{sp1} + k_{sp2} + k_{sp3} & -k_{sp1} & -k_{sp2} & -k_{sp3} & 0 \\ -k_{sp1} & k_{sp1} + k_{rp1} & 0 & 0 & -k_{rp1} \\ -k_{sp2} & 0 & k_{sp2} + k_{rp2} & 0 & -k_{rp2} \\ -k_{sp3} & 0 & 0 & k_{sp3} + k_{rp3} & -k_{rp3} \\ 0 & -k_{rp1} & -k_{rp2} & -k_{rp3} & k_r + k_{rp1} + k_{rp2} + k_{rp3} \end{bmatrix},$$

$$\begin{aligned}
\mathbf{K}_7 = & \begin{bmatrix} \sum k_{sw} & -\sum k_{sw} \gamma_{sw} & -k_{sp1} & -k_{sp2} & -k_{sp3} & 0 & 0 \\ -\sum k_{sw} \gamma_{sw} & \sum k_{sw} \gamma_{sw}^2 & k_{sp1} \gamma_{sp1} & k_{sp2} \gamma_{sp2} & k_{sp3} \gamma_{sp3} & 0 & 0 \\ -k_{sp1} & k_{sp1} \gamma_{sp1} & k_{sp1} + k_{rp1} & 0 & 0 & -k_{rp1} & k_{rp1} \gamma_{rp1} \\ -k_{sp2} & k_{sp2} \gamma_{sp2} & 0 & k_{sp2} + k_{rp2} & 0 & -k_{rp2} & k_{rp2} \gamma_{rp2} \\ -k_{sp3} & k_{sp3} \gamma_{sp3} & 0 & 0 & k_{sp3} + k_{rp3} & -k_{rp3} & k_{rp3} \gamma_{rp3} \\ 0 & 0 & -k_{rp1} & -k_{rp2} & -k_{rp3} & \sum k_{rw} & -\sum k_{rw} \gamma_{rw} \\ 0 & 0 & k_{rp1} \gamma_{rp1} & k_{rp2} \gamma_{rp2} & k_{rp3} \gamma_{rp3} & -\sum k_{rw} \gamma_{rw} & \sum k_{rw} \gamma_{rw}^2 \end{bmatrix}, \\
\mathbf{C}_5 = & \begin{bmatrix} c_s + c_{sp1} + c_{sp2} + c_{sp3} & -c_{sp1} & -c_{sp2} & -c_{sp3} & 0 \\ -c_{sp1} & c_{sp1} + c_{rp1} & 0 & 0 & -c_{rp1} \\ -c_{sp2} & 0 & c_{sp2} + c_{rp2} & 0 & -c_{rp2} \\ -c_{sp3} & 0 & 0 & c_{sp3} + c_{rp3} & -c_{rp3} \\ 0 & -c_{rp1} & -c_{rp2} & -c_{rp3} & c_r + c_{rp1} + c_{rp2} + c_{rp3} \end{bmatrix}, \\
\mathbf{C}_7 = & \begin{bmatrix} \sum c_{sw} & -\sum c_{sw} \gamma_{sw} & -c_{sp1} & -c_{sp2} & -c_{sp3} & 0 & 0 \\ -\sum c_{sw} \gamma_{sw} & \sum c_{sw} \gamma_{sw}^2 & c_{sp1} \gamma_{sp1} & c_{sp2} \gamma_{sp2} & c_{sp3} \gamma_{sp3} & 0 & 0 \\ -c_{sp1} & c_{sp1} \gamma_{sp1} & c_{sp1} + c_{rp1} & 0 & 0 & -c_{rp1} & c_{rp1} \gamma_{rp1} \\ -c_{sp2} & c_{sp2} \gamma_{sp2} & 0 & c_{sp2} + c_{rp2} & 0 & -c_{rp2} & c_{rp2} \gamma_{rp2} \\ -c_{sp3} & c_{sp3} \gamma_{sp3} & 0 & 0 & c_{sp3} + c_{rp3} & -c_{rp3} & c_{rp3} \gamma_{rp3} \\ 0 & 0 & -c_{rp1} & -c_{rp2} & -c_{rp3} & \sum c_{rw} & -\sum c_{rw} \gamma_{rw} \\ 0 & 0 & c_{rp1} \gamma_{rp1} & c_{rp2} \gamma_{rp2} & c_{rp3} \gamma_{rp3} & -\sum c_{rw} \gamma_{rw} & \sum c_{rw} \gamma_{rw}^2 \end{bmatrix}.
\end{aligned} \tag{19}$$

Here, $w = 1, 2, p1, p2, p3$

$$\begin{aligned}
\mathbf{F}_5(t) &= \{F \ 0 \ 0 \ 0 \ 0\}^T, \\
\mathbf{x}_5(t) &= \{x_s \ x_{p1} \ x_{p2} \ x_{p3} \ x_r\}^T, \\
\mathbf{F}_7(t) &= \{F \ M \ 0 \ 0 \ 0 \ 0 \ 0\}^T, \\
\mathbf{x}_7(t) &= \{x_s \ \theta_s \ x_{p1} \ x_{p2} \ x_{p3} \ x_r \ \theta_r\}^T.
\end{aligned} \tag{20}$$

In the vibration systems, all the parameters in the random paths are represented by a vector $\mathbf{b}_n$. We assume the probability statistical properties of these random parameters are known here. According to the above models, the motion equation of the vibration transmission system with random path parameters can be expressed as

$$\begin{aligned}
\mathbf{M}_n(\mathbf{b}_n) \ddot{\mathbf{x}}_n(\mathbf{b}_n, t) + \mathbf{C}_n(\mathbf{b}_n) \dot{\mathbf{x}}_n(\mathbf{b}_n, t) \\
+ \mathbf{K}_n(\mathbf{b}_n) \mathbf{x}_n(\mathbf{b}_n, t) = \mathbf{F}_n(t),
\end{aligned} \tag{21}$$

where

$$\begin{aligned}
\mathbf{b}_5 &= (m_{p1} \ m_{p2} \ m_{p3} \ k_{sp1} \ k_{sp2} \ k_{sp3} \ k_{rp1} \ k_{rp2} \ k_{rp3} \ c_{sp1} \ c_{sp2} \ c_{sp3} \ c_{rp1} \ c_{rp2} \ c_{rp3})^T, \\
\mathbf{b}_7 &= (m_{p1} \ m_{p2} \ m_{p3} \ k_{sp1} \ k_{sp2} \ k_{sp3} \ k_{rp1} \ k_{rp2} \ k_{rp3} \ c_{sp1} \ c_{sp2} \ c_{sp3} \ c_{rp1} \ c_{rp2} \ c_{rp3} \ \gamma_{sp1} \ \gamma_{sp2} \ \gamma_{sp3} \ \gamma_{rp1} \ \gamma_{rp2} \ \gamma_{rp3})^T.
\end{aligned} \tag{22}$$

5. Numerical Examples

Example 1. Considering the lumped mass in the transfer paths as shown in Figure 1, the vibration transfer path system has 5 DoFs. The vibration source contains the following parameters: $m_s = 0.7724$ kg, $c_s = 0.5$ N·s/m, and $k_s = 100$ N/m; the receiver contains the following parameters: $m_r = 1.0556$ kg, $c_r = 1$ N·s/m, and $k_r = 1800$ N/m. The vibration source is excited by harmonic force and the excitation magnitude $F_0 = 10$ N. The random mass, damping, and stiffness of the transfer paths are all normal distribution with a variance equal to 0.05, and the mean values of these parameters are $m_{p1} = 0.9$ kg, $m_{p2} = 0.5$ kg, $m_{p3} = 0.6$ kg, $c_{sp1} = c_{rp1} = 1.0$ N·s/m, $c_{sp2} = c_{rp2} = 1.5$ N·s/m, $c_{sp3} = c_{rp3} = 0.5$ N·s/m, $k_{sp1} = k_{rp1} = 900$ N/m, $k_{sp2} = k_{rp2} = 450$ N/m, and $k_{sp3} = k_{rp3} = 600$ N/m, respectively.

The mean values of the natural frequencies are

$$\begin{aligned}\omega_1 &= 8.4552, \\ \omega_2 &= 42.9305, \\ \omega_3 &= 44.7214, \\ \omega_4 &= 47.7453, \\ \omega_5 &= 65.4045.\end{aligned}\quad (23)$$

According to formula (17), the transmission reliability of the vibration transfer path system is calculated. The curve changing with the excitation frequency is shown as follows. From Figure 3, we can know that the reliability is very low and close to 0 at every natural frequency point. The frequency regions should be avoided for the vibration system design. The large security area appears between the first and the second natural frequency, which should be used properly for the vibration system design.

Example 2. Because the system shown in Figure 2 is excited by the force and the moment together, it not only has the linear degree of freedom, but also has the rotational degree of freedom, which is a 7-DoFs vibration system. Thus, the vibration response is influenced not only by the physical characteristics parameters, such as mass, stiffness, and damping, but also by the geometric characteristics parameters, such as the location and shape. The corresponding parameter values are set: the source rotational inertia $I_{sz} = 2.5799 \times 10^{-3}$ kg·m² and the receiver rotational inertia $I_{rz} = 7.8722 \times 10^{-3}$ kg·m²; the source suspension positions $y_{s1} = -0.10$ m and $y_{s2} = 0.10$ m and the receiver suspension positions $y_{r1} = -0.15$ m and $y_{r2} = 0.15$ m; the excitation force is vertical to the source and the load position $y_f = 0.02$ m; the suspension position parameters of the transfer paths are $y_{sp1} = -0.09$ m, $y_{rp1} = -0.10$ m, $y_{sp2} = -0.03$ m, $y_{rp2} = -0.04$ m, $y_{sp3} = 0.10$ m, and $y_{rp3} = 0.09$ m. The other parameters are the same as the corresponding parameters shown in Example 1.

The mean values of natural frequencies are

$$\begin{aligned}\omega_1 &= 11.7566, \\ \omega_2 &= 19.0825,\end{aligned}$$

FIGURE 3: The transmission reliability with the exciting frequency.

$$\begin{aligned}\omega_3 &= 42.8390, \\ \omega_4 &= 46.8275, \\ \omega_5 &= 59.3125, \\ \omega_6 &= 65.1858, \\ \omega_7 &= 85.8403.\end{aligned}$$

(24)

The change trend of transmission reliability with the excitation frequency is shown in Figure 4. From Figure 4, we can obtain that the reliability is very low and close to 0 at every natural frequency point. The frequency regions should be avoided for the vibration system design. The large security area appears between the second and the third natural frequency and the reliability is close to 1.

6. Conclusions

It can be seen from the calculation results that the system has high reliability when the excitation frequency is far away from the natural frequency of the system. As the excitation frequency approaches the natural frequency, the reliability is gradually reduced until zero. In order to ensure the normal operation of the system, the related frequency area must be avoided in the actual project.

The examples illustrate that the dynamics analysis results of the system are ideal based on the theory of the transmission reliability proposed in this paper. The effective dynamic design and optimization can be carried out after determining the transmission reliability of the transfer path parameters. Particularly when the vibration and noise level do not meet the actual requirements, the engineers can quickly find out the key parameters and improve the design.

FIGURE 4: The transmission reliability with exciting frequency.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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