Hindawi
Mathematical Problems in Engineering
Volume 2017, Article ID 6832575, 8 pages
https://doi.org/10.1155/2017/6832575

Research Article

The Controller Design of the Epilepsy Therapy Apparatus

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Received 27 June 2017; Revised 2 September 2017; Accepted 13 September 2017; Published 25 October 2017

Academic Editor: Quanxin Zhu

Deep brain stimulation (DBS) is one of the effective treatments of epilepsy. Based on the Lyapunov stability theory combined with the method of the periodically intermittent control and adaptive control, the abnormal synchronization neural network models with high frequency oscillation are achieved with multilag synchronization in this paper. Some simple criteria are derived for the multilag synchronization of the coupled neural networks with coupling delays. The adaptive periodically intermittent control which we have obtained can cut down control cost. The sufficient conditions of this paper for abnormal neural network multilag synchronization are less conservative and can be applied in a wider area. Finally, simulation results show the effectiveness of the proposed control strategy. The design of controllers and control strategy may provide a potential electrical stimulation therapy on neurological diseases caused by abnormal synchronization. And they can provide technical support for epilepsy treatment apparatus research and development.

1. Introduction

Epilepsy is one of common chronic diseases of the nervous system. The study shows that pathologically strong synchronization process may badly injure the brain function such as resting tremor in epilepsy and Parkinson's disease [1, 2]. Almost sixty million people survive with epilepsy in the world. The major treatment for epilepsy is drug therapy, surgery, and brain stimulation [3, 4]. Unfortunately, drug therapy and surgery have some defects, and it is a pity that there are still more than 20% of the epilepsy people that do not have control. Thus, it is urgent to study the mechanism of epilepsy, so that the abnormal epileptic discharge behavior can be controlled [5].

Recently, the development of complex network dynamics accelerates the research of epilepsy. It is reported that epileptic seizures, diffusion, and holding out are mostly due to the reciprocity of neuron network in the brain with functional connectivity with small world network characteristic [6–9]. If we can fully understand the functional connectivity of brain with epileptic seizures by complex network analysis methods, then, we will offer a new research approach for epileptic seizure prediction or controlling. It will greatly minimize risk or injury and improve the quality of life for many people with epilepsy. It is helpful to find an effective method via brain stimulation to prevent synchronization for treating such disease. In recent years, the brain stimulation is an effective substitution of medicaments and surgery. It is an emerging therapy for treating epilepsy.

Deep brain stimulation has received arising attention in treating epilepsy for the past few years. However, during current researches most of the stimulating modes are open-loop, which cannot emit stimulating pulses automatically according to the status of patients. Moreover, there are many disadvantages on feasibility and universality of nowadays' existing closed-loop electrical stimulators. As a result, it is of great significance for researches of epilepsy treatment with electrical stimulation to design a closed-loop system, which can detect epileptic seizures automatically and emit electrical stimuli accordingly [10–13]. Particularly, the research of synchronization of coupled neural networks is an important step for understanding brain science and designing coupled neural networks for practical use. Many control techniques, such as linear feedback control [14–16], adaptive feedback control [17, 18], stochastic control [19, 20], impulsive control [21, 22], pinning control [21–24], finite-time control [25–27], sliding mode control [28, 29], and intermittent control [30, 31], have been developed to drive the synchronization of networks.

Although closed-loop control is effective in epilepsy treatment, it is rare using neuron and neural network to achieve closed-loop control. Substituting model analysis for animal and human body is an effective way. In this paper, we combine computational neuron model with control theory and propose a closed-loop control strategy based on neuronal model, which achieved multilag exponential synchronization an abnormal neural network models. Based on the Lyapunov stability theory, combined with the method of the adaptive intermittent control, some simple criteria are derived for the multilag synchronization of the coupled neural networks with coupling delays. The adaptive periodically intermittent control which we have obtained can cut down control cost. At last, simulation results show the effectiveness of the proposed control strategy. The design of controllers and control strategy may provide a potential electrical stimulation therapy on neurological diseases caused by abnormal synchronization and provide technical support for epilepsy treatment apparatus research and development.

The rest of the paper is organized as follows. In Section 2, the model of abnormal synchronization neural networks is presented. And some assumptions and preliminaries are given. In Section 3, multilag exponential synchronization of abnormal neural networks via adaptive periodically intermittent control for the neural networks is designed, respectively. The simple and novel multilag exponential synchronization criterion is obtained. In Section 4, numerical examples of neural networks are given to demonstrate the effectiveness of the proposed controllers. Conclusions are given in Section 5.

2. Model and Preliminaries

The multilag synchronization method to design the main idea is as follows. First, choose the membrane potential of health neuron I from normal neural network as follows:

$$\dot{s}(t) = -\widetilde{C}s(t) + \widetilde{A}f(s(t)) + \widetilde{B}f(s(t-\tau)) + \widetilde{I}(t), \qquad (1)$$

where $s(t) = [s_1(t), s_2(t)]^{\mathrm{T}} \in \mathbb{R}^2$ represents the state vector of the health neuron I from undamaged neural networks at time t. $\widetilde{C} = \mathrm{diag}\{\widetilde{c}_1, \widetilde{c}_2\}$ with $\widetilde{c}_k > 0$, $k = 1, 2$, denotes the rate with which the cell k resets its potential to the resting state when isolated from other cells and inputs. $\widetilde{A} = (\widetilde{a}_{pq})_{2\times 2}$, $\widetilde{B} = (\widetilde{b}_{pq})_{2\times 2}$ represent the connection weight matrix and the delayed connection weight matrix, respectively. $\widetilde{a}_{pq}, \widetilde{b}_{pq}$ denote the strengths of connectivity between the cell p and q within the health neuron at time t and $t-\tau$, respectively. $\tau \geq 0$ is a transmission delay. $f(s(t)) = [\tanh(s_1(t)), \tanh(s_2(t))]^{\mathrm{T}}$ is activation function of healthy neural cell with the chaotic attractor. $\widetilde{I}(t) = [\widetilde{I}_1(t), \widetilde{I}_2(t)]^{\mathrm{T}} \in \mathbb{R}^2$ is an external input vector.

We move back T/N of the reference neuron I membrane voltage V_{refI} phase, where T is cycle of clusters neurons discharge. We can get reference neuron II membrane voltage V_{refII}. Then, membrane voltage difference $e_{\mathrm{refI}} = V_{\mathrm{refI}} - V_{\mathrm{refII}}$ of the reference neurons I and II is computed. The phase of e_{refI} is moved back T/N in turn $N-2$. The $N-1$ errors are seen as reference errors in the abnormal synchronization neural networks to multilag synchronization control. Using the Lyapunov stability theory combined with the method of the adaptive control and periodically intermittent control, a simple but robust adaptive intermittent controller is designed such that the abnormal neuron can realize the multilag synchronization and recover a health neuron, that is, the periodic orbit is synchronized into the chaos. Eventually, using the multilag synchronous controller, we can realize N neurons into the state of the given N. In medicine, the multilag synchronization can just play the role of accurate desynchronization of the abnormal synchronization neural cells to effectively prevent the occurrence of epilepsy [3].

Consider an abnormal synchronization neural networks model consisting of N abnormal identical nodes with high frequency oscillation, in which each node is a 2-dimensional nontrivial periodic orbit neural network; that is,

$$\begin{aligned}\dot{x}_i(t) &= -\overline{C}x_i(t) + \overline{A}f(x_i(t)) + \overline{B}f(x_i(t-\tau)) + \overline{I}(t) \\ &\quad + \sum_{j=1}^{N} g_{ij}\Gamma x_j(t) + u_i(t), \quad (i = 1, 2, \ldots, N),\end{aligned} \qquad (2)$$

where $x_i(t) = [x_{i1}(t), x_{i2}(t)]^{\mathrm{T}} \in \mathbb{R}^2$ represents the state vector of the ith abnormal synchronization neural network at time t. $\overline{C} = \mathrm{diag}\{\overline{c}_1, \overline{c}_2\}$ with $\overline{c}_k > 0$, $k = 1, 2$, denotes the rate with which the cell k resets its potential to the resting state when isolated from other cells and inputs. $\overline{A} = (a_{pq})_{2\times 2}$, $\overline{B} = (b_{pq})_{2\times 2}$ represent the connection weight matrix and the delayed connection weight matrix, respectively. $\overline{a}_{pq}, \overline{b}_{pq}$ denote the strengths of connectivity between the cell p and q within the ith node at time t and $t-\tau$, respectively. $\tau \geq 0$ is a transmission delay. $f(x_i(t)) = [\tanh(x_{i1}(t)), \tanh(x_{i2}(t))]^{\mathrm{T}}$ is activation function. $\overline{I}(t) = [\overline{I}_1(t), \overline{I}_2(t)]^{\mathrm{T}} \in \mathbb{R}^2$ is an external input vector. The constant matrix $G = (g_{ij})_{N\times N}$ represents the linear coupling configuration of the whole network, which satisfies $g_{ij} \geq 0$, for $i \neq j$, and $g_{ii} = -\sum_{j=1, j\neq i}^{N} g_{ij}$, $\overline{g}_{ii} = -\sum_{j=1, j\neq i}^{N} g_{ji}$, $i = 1, 2, \ldots, N$. $\Gamma = (\gamma_{ij})_{n\times n}$ is inner-coupling matrix between nodes. $u_i(t) = [u_{i1}(t), u_{i2}(t)]^{\mathrm{T}} \in \mathbb{R}^2$ is the input vector of node i, that is, the controller of epilepsy treatment apparatus to be designed.

Let $C([-\tau, 0], R^2)$ be the Banach space of continuous functions mapping the interval $[-\tau_m, 0]$ into R^2 with the norm $\|\phi\| = \sup_{-\tau_m \leq \theta \leq 0} \|\phi(\theta)\|$, where $\|\cdot\|$ is the Euclidean norm. The rigorous mathematical definition of multilag synchronization for the neural networks (2) is introduced as follows.

Definition 1. Let $x_i(t; t_0; \phi)$, $i = 1, 2, \ldots, N$, be a solution of abnormal neural networks (2), where $\phi = (\phi_1^{\mathrm{T}}, \phi_2^{\mathrm{T}}, \ldots, \phi_N^{\mathrm{T}})^{\mathrm{T}}$, $\phi_i = \phi_i(\theta) \in C([-\tau, 0], R^2)$ are initial conditions. If there exist constants > 0, $\lambda > 0$ and a nonempty subset $\Lambda \subseteq R^2$, such that ϕ_i take values in Λ and $x_i(t;t_0;\phi) \in R^2$ for all $t \geq t_0$ and

$$\begin{aligned}&\left\| x_i(t;t_0;\phi) - s\left(t - \frac{iT}{N}; t_0; s_0\right)\right\| \\ &\quad \leq \alpha e^{-\lambda t} \sup_{-\tau\leq\theta\leq 0} \|\phi(\theta) - s_0\|, \quad i = 1,2,\ldots,N,\end{aligned} \tag{3}$$

where $s(t-iT/N;t_0;s_0)$ is a healthy neural cell with the chaotic attractor, solution of an isolate node with $s_0 \in \mathbb{R}^2$, then the coupled neural networks (2) are said to realize multilag exponential synchronization.

Define the error vector by

$$e_i(t) = x_i(t) - s\left(t - \frac{iT}{N}\right), \quad i = 1,2,\ldots,N. \tag{4}$$

Then the error system can be described by

$$\begin{aligned}\dot{e}_i(t) &= -Ce_i(t) + AF(e_i(t)) + BF(e_i(t-\tau)) \\ &\quad + \sum_{j=1}^{N} g_{ij}\Gamma e_j(t) + u_i(t), \quad i = 1,2,\ldots,N,\end{aligned} \tag{5}$$

where $F(e_i(t)) = f(x_i(t)) - f(s(t - iT/N))$, $A = \overline{A} - \widetilde{A}$, $B = \overline{B} - \widetilde{B}$, and $C = \overline{C} - \widetilde{C}$.

Remark 2. The error vector we define is "$e_i(t) = x_i(t) - s(t - iT/N)$" instead of "$e_i(t) = x_i(t) - s(t)$" in [30]. Definition 1 in our letter is more general. When the term of "iT/N" in Definition 1 disappears, Definition 1 will degenerate into exponential synchronization presented in [30]. When "i" in the term of "iT/N" in the Definition 1 is a constant c, Definition 1 will degenerate into lag exponential synchronization presented in [17].

To achieve multilag synchronization of objective (3), we need the following lemmas.

Lemma 3 (see [32]). *For any vectors $x, y \in \mathbb{R}^n$ and positive definite matrix $Q \in \mathbb{R}^{n\times n}$, the following matrix inequality holds:*

$$2x^{\mathrm{T}}y \leq x^{\mathrm{T}}Qx + y^{\mathrm{T}}Q^{-1}y. \tag{6}$$

Lemma 4 (see [33]). *For any constant symmetric matrix $M \in R^{n\times n}$, $M > 0$, scalar $h > 0$, and vector function $\dot{x}(\cdot) \in C([-h,0],R^n)$ inequality (7) always holds.*

$$h\int_0^h x^{\mathrm{T}}(s)Mx(s)\,\mathrm{d}s \geq \left(\int_0^h x\,\mathrm{d}s\right)^{\mathrm{T}} M \left(\int_0^h x\,\mathrm{d}s\right). \tag{7}$$

3. Multilag Synchronization of Abnormal Neural Networks

In order to realize multilag synchronization of abnormal synchronization neural networks via adaptive periodically intermittent control, the controllers are added to nodes of the abnormal neural network. In system (5), we choose the adaptive periodically intermittent feedback controllers

$$u_i(t) = \begin{cases} -k_i(t)e_i(t), & t \in [\varpi T, \varpi T + h), \\ 0, & t \in [\varpi T + h, (\varpi + 1)T) \end{cases} \tag{8}$$

and the updating laws

$$\dot{k}_i(t) = \alpha_i \exp\{a_1 t\}\|e_i(t)\|^2, \tag{9}$$

where α_i ($i = 1,2,\ldots,N$) and a_1 are positive constants, $T > 0$ denotes the control period, $0 < h < T$, and $\varpi = 0,1,2,\ldots$. Thus error system (5) can be rewritten as

$$\begin{aligned}\dot{e}_i(t) &= -Ce_i(t) + AF(e_i(t)) + BF(e_i(t-\tau)) \\ &\quad + \sum_{j=1}^{N} g_{ij}\Gamma e_j(t) - k_i(t)e_i(t), \\ &\qquad t \in [\varpi T, \varpi T + h), \\ \dot{e}_i(t) &= -Ce_i(t) + AF(e_i(t)) + BF(e_i(t-\tau)) \\ &\quad + \sum_{j=1}^{N} g_{ij}\Gamma e_j(t), \quad t \in [\varpi T + h, (\varpi + 1)T).\end{aligned} \tag{10}$$

Denote $\gamma \neq 0$ as the minimum eigenvalue of the matrix $(\Gamma + \Gamma^{\mathrm{T}})/2$. Assume that $\|\Gamma\|_2 = \rho > 0$. Let $\widehat{G}^s = (G + G^{\mathrm{T}})/2$, where $\widehat{G}$ is a modified matrix of G via replacing the diagonal elements g_{ii} by $(\gamma/\rho)g_{ii}$; then $\widehat{G}^s$ is a symmetric irreducible matrix with nonnegative off-diagonal elements. Based on the adaptive periodically intermittent feedback controllers, the abnormal synchronization neural networks (2) multilag synchronization criterion is deduced as follows.

Theorem 5. *If there exist positive constants ε, a_1, a_2, q and α_i ($i = 1,2,\ldots,N$), such that*

$$\begin{aligned}&a_1 > \frac{b^2L^2}{q}, \\ &\left[q + \frac{a^2L^2}{2q} - \frac{1}{2}(a_2 - a_1)\right] I_N - C + \sigma\widehat{G}^s \leq 0, \\ &\varepsilon = \lambda - a_2\left(1 - \frac{h}{T}\right) > 0,\end{aligned} \tag{11}$$

where $\sigma = \|\Gamma\|_2$, $\lambda > 0$ is the smallest real root of the equation $a_1 - \lambda - (b^2L^2/q)\exp\{\lambda\tau\} = 0$, then the abnormal synchronization neural networks (2) multilag synchronization is under adaptive periodical intermittent controllers (8) and updating laws (9).

Proof. Construct the following Lyapunov-Krasovskii candidate function:

$$V(t) = \frac{1}{2}\sum_{i=1}^{N} e_i^{\mathrm{T}}(t)e_i(t) + \frac{1}{2}\sum_{i=1}^{N} \exp\{-a_1 t\}\frac{(k_i(t) - k)^2}{\alpha_i}. \tag{12}$$

And k is an undetermined sufficiently large positive constant. Then the derivative of $V(t)$ with respect to time t along the solutions of (10) can be calculated as follows.

When $\varpi T \le t < \varpi T + h$, for $\varpi = 0, 1, 2, \ldots$,

$$\begin{aligned}\dot{V}(t) &= \sum_{i=1}^{N} e_i^{\mathrm{T}}(t)\dot{e}_i(t) - \frac{1}{2}a_1\sum_{i=1}^{N}\exp\{-a_1 t\}\frac{(k_i(t)-k)^2}{\alpha_i} + \sum_{i=1}^{N}(k_i(t)-k)e_i^{\mathrm{T}}(t)e_i(t) \\ &= -\sum_{i=1}^{N} Ce_i^{\mathrm{T}}(t)e_i(t) + \sum_{i=1}^{N}e_i^{\mathrm{T}}(t)AF(e_i(t),t) + \sum_{i=1}^{N}e_i^{\mathrm{T}}(t)BF(e_i(t-\tau),t-\tau) \\ &\quad + \sum_{i=1}^{N}\sum_{j=1}^{N}g_{ij}e_i^{\mathrm{T}}(t)\Gamma e_j(t) - \sum_{i=1}^{N}k_i(t)e_i^{\mathrm{T}}(t)e_i(t) - \frac{1}{2}a_1\sum_{i=1}^{N}\exp\{-a_1 t\}\frac{(k_i(t)-k)^2}{\alpha_i} \\ &\quad + \sum_{i=1}^{N}(k_i(t)-k)e_i^{\mathrm{T}}(t)e_i(t).\end{aligned} \tag{13}$$

Because $F(e_i(t)) = \overline{f}(x_i(t)) - \widetilde{f}(s(t-iT/N))$, where $\overline{f}(x_i(t)) = \widetilde{f}(x_i(t)) = [\tanh(x_{i1}(t)), \tanh(x_{i2}(t))]^{\mathrm{T}}$, and $\tanh(x(t))$ is a bounded function, there exists positive constant L, such that $\|F(e_i(t))\| \le L$.

Let $Q = qI > 0$; we can get

$$\begin{aligned}\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)AF(e_i(t),t) &\le \frac{1}{2}\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)Qe_i(t) + \frac{1}{2}\sum_{i=1}^{N}F^{\mathrm{T}}(e_i(t))A^{\mathrm{T}}Q^{-1}AF(e_i(t)) \\ &\le \frac{1}{2}\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)Qe_i(t) + \frac{a^2L^2}{2}\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)Q^{-1}e_i(t) \\ &= \frac{q}{2}\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)e_i(t) + \frac{a^2L^2}{2q}\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)e_i(t).\end{aligned} \tag{14}$$

Similarly, we have

$$\begin{aligned}\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)BF(e_i(t-\tau),t-\tau) \le \frac{q}{2}\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)e_i(t) + \frac{b^2L^2}{2q}\sum_{i=1}^{N}e_i^{\mathrm{T}}(t-\tau)e_i(t-\tau).\end{aligned} \tag{15}$$

Substituting (13) and (14) into (12) gives

$$\begin{aligned}\dot{V}(t) &\le \sum_{i=1}^{N}\left(q + \frac{a^2L^2}{2q} - c_i\right)e_i^{\mathrm{T}}(t)e_i(t) + \sum_{i=1}^{N}\sum_{j=1,j\ne i}^{N}\rho g_{ij}\|e_i(t)\|_2\|e_j(t)\|_2 + \sum_{i=1}^{N}\gamma g_{ii}e_i^{\mathrm{T}}(t)e_i(t) \\ &\quad + \frac{b^2L^2}{2q}\sum_{i=1}^{N}e_i^{\mathrm{T}}(t-\tau)e_i(t-\tau) - \sum_{i=1}^{N}ke_i^{\mathrm{T}}(t)e_i(t) - \frac{1}{2} \cdot a_1\sum_{i=1}^{N}\exp\{-a_1 t\}\frac{(k_i(t)-k)^2}{\alpha_i} \le \overline{e}^{\mathrm{T}}(t) \\ &\quad \cdot \left[\left(q + \frac{a^2L^2}{2q} + \frac{1}{2}a_1\right)I_N - C + \sigma\widehat{G}^s - K\right]\overline{e}(t) - \frac{1}{2}a_1\overline{e}^{\mathrm{T}}(t)\overline{e}(t) + \frac{b^2L^2}{2q}\overline{e}_i^{\mathrm{T}}(t-\tau)\overline{e}_i(t-\tau) - \frac{1}{2} \\ &\quad \cdot a_1\sum_{i=1}^{N}\exp\{-a_1 t\}\frac{(k_i(t)-k)^2}{\alpha_i} + \frac{b^2L^2}{2q} \cdot \sum_{i=1}^{N}\exp\{-a_1(t-\tau)\}\frac{(k_i(t-\tau)-k)^2}{\alpha_i},\end{aligned} \tag{16}$$

where $\overline{e}^T(t) = (\|e_1(t)\|_2, \|e_2(t)\|_2, \ldots, \|e_N(t)\|_2)^{\mathrm{T}}$, $K = kI_N$. Because k is an undetermined sufficiently large positive constant, we can select k as

$$k > q + \frac{a^2L^2}{2q} - c + \frac{1}{2}a_1 + \sigma\lambda_{\max}\widehat{G}^s, \tag{17}$$

so we have

$$\dot{V}(t) \le -a_1V(t) + \frac{b^2L^2}{q}V(t-\tau). \tag{18}$$

Similarly, when $\varpi T + h \le t < (\varpi + 1)T$, using condition in the second inequality of (11), one has

$$\dot{V}(t) \le (a_2 - a_1)V(t) + \frac{b^2L^2}{q}V(t-\tau). \tag{19}$$

Because of the first inequality of (11), the equation $a_1 - \lambda - (b^2L^2/q)\exp\{\lambda\tau\} = 0$ has unique positive solution $\lambda > 0$, obviously. Take $\overline{V} = \sup_{-\tau\le s\le 0}V(s)$ and $U(t) = \exp\{\lambda t\}V(t)$, where $t \ge 0$. Let $\Omega(t) = U(t) - \beta\overline{V}$, where $\beta > 1$ is a constant. It is easy to see that

$$\Omega(t) < 0, \quad \forall t \in [-\tau, 0]. \tag{20}$$

Then, we want to prove that

$$\Omega(t) < 0, \quad \forall t \in [0, h]. \tag{21}$$

Otherwise, suppose $t_0 \in [0, h]$ to exist, such that

$$\Omega(t_0) = 0, \quad \dot{\Omega}(t_0) \ge 0, \tag{22}$$

$$\Omega(t) < 0, \quad -\tau \le t \le t_0. \tag{23}$$

Using (20), (22), and (23), we obtain

$$\begin{aligned}\dot{\Omega}(t_0) &= \lambda U(t_0) + \exp\{\lambda t_0\}\dot{V}(t_0)\\ &\le \lambda U(t_0) - a_1 \exp\{\lambda t_0\} V(t_0)\\ &\quad + \frac{b^2L^2}{q}\exp\{\lambda t_0\} V(t_0-\tau)\\ &\le (\lambda - a_1) U(t_0) + \frac{b^2L^2}{q}\exp\{\lambda\tau\} U(t_0-\tau)\\ &< (\lambda - a_1)\beta\overline{V} + \frac{b^2L^2}{q}\exp\{\lambda\tau\}\beta\overline{V}\\ &= \left(\lambda - a_1 + \frac{b^2L^2}{q}\exp\{\lambda\tau\}\right)\beta\overline{V} = 0.\end{aligned} \tag{24}$$

This contradicts the second inequality in (22), and so (21) holds.

Then, we prove that $\Delta(t) = U(t) - \beta\overline{V}\exp\{a_2(t-h)\} < 0$ for $t \in [h, T)$. Otherwise, there exists $t_1 \in [h, T)$, such that

$$\Delta(t_1) = 0, \quad \dot{\Delta}(t_1) \ge 0, \tag{25}$$

$$\Delta(t) < 0, \quad h \le t \le t_1. \tag{26}$$

For $\tau > 0$, if $h \le t_1 - \tau \le t_1$, it follows from (26) that

$$U(t_1-\tau) < \beta\overline{V}\exp\{a_2(t_1-h)\}, \tag{27}$$

and if $-\tau \le t_1 - \tau < h$, from (20) and (21), we have

$$U(t_1-\tau) < \beta\overline{V} \le \beta\overline{V}\exp\{a_2(t_1-h)\}. \tag{28}$$

Hence, for $\tau > 0$, we always have

$$U(t_1-\tau) < \beta\overline{V}\exp\{a_2(t_1-h)\}. \tag{29}$$

Then

$$\begin{aligned}&\dot{\Delta}(t_1)\\ &\quad= \lambda U(t_1) + \exp\{\lambda t_1\}\dot{V}(t_1)\\ &\qquad - a_2\beta\overline{V}\exp\{a_2(t_1-h)\}\\ &\quad\le \lambda U(t_1) + (a_2 - a_1)\exp\{\lambda t_1\} V(t_1)\\ &\qquad + \frac{b^2L^2}{q}\exp\{\lambda t_1\} V(t_1-\tau)\\ &\qquad - a_2\beta\overline{V}\exp\{a_2(t_1-h)\}\\ &\quad\le (\lambda + a_2 - a_1) U(t_1) + \frac{b^2L^2}{q}\exp\{\lambda\tau\} U(t_1-\tau)\\ &\qquad - a_2\beta\overline{V}\exp\{a_2(t_1-h)\}\\ &\quad< \left(\lambda - a_1 + \frac{b^2L^2}{q}\exp\{\lambda\tau\}\right)\beta\overline{V}\exp\{a_2(t_1-h)\}\\ &\quad= 0,\end{aligned} \tag{30}$$

which contradicts the second inequality in (25). Hence $\Delta(t) < 0$ holds; that is, for $t \in [h, T)$, we have

$$U(t) < \beta\overline{V}\exp\{a_2(t_1-h)\} \le \beta\overline{V}\exp\{a_2(T-h)\}, \tag{31}$$

On the other hand, it follows from (20) and (21) that for $t \in [-\tau, h)$

$$U(t) < \beta\overline{V} < \beta\overline{V}\exp\{a_2(T-h)\}. \tag{32}$$

so

$$U(t) < \beta\overline{V}\exp\{a_2(T-h)\}, \quad \forall t \in [-\tau, T). \tag{33}$$

Similarly, we can prove that, for $t \in [T, h+T)$,

$$U(t) < \beta\overline{V}\exp\{a_2(T-h)\}, \tag{34}$$

and, for $t \in [T+h, 2T)$,

$$U(t) < \beta\overline{V}\exp\{a_2(T-2h)\}. \tag{35}$$

By induction, we can derive the following estimation of $U(t)$ for any integer ϖ.

For $\varpi T \le t < \varpi T + h$, $\varpi = 0, 1, 2, \ldots$,

$$\begin{aligned}U(t) &< \beta\overline{V}\exp\{a_2\varpi(T-h)\}\\ &\le \beta\overline{V}\exp\left\{a_2\left(1-\frac{h}{T}\right)t\right\},\end{aligned} \tag{36}$$

and for $\varpi T + h \le t < (\varpi+1)T$, $\varpi = 0, 1, 2, \ldots$,

$$\begin{aligned}U(t) &< \beta\overline{V}\exp\{a_2[t-(\varpi+1)h]\}\\ &\le \beta\overline{V}\exp\left\{a_2\left(1-\frac{h}{T}\right)t\right\}.\end{aligned} \tag{37}$$

Let $\beta \to 1$, from the definition of $U(t)$, we obtain

$$\begin{aligned}V(t) &\le \overline{V}\exp\left\{-\left[\lambda - a_2\left(1-\frac{h}{T}\right)\right]t\right\}\\ &= \sup_{-\tau\le s\le 0} V(s)\exp\{-\varepsilon t\}, \quad t \ge 0.\end{aligned} \tag{38}$$

It follows from condition in the third inequality of (11) that the zero solution of the error dynamical system (10) is globally exponentially stable. So the abnormal synchronization neural networks (2) multilags synchronize under adaptive periodical intermittent controllers (8) and updating laws (9). This completes the proof of Theorem 5. □

4. Simulation

In this section, numerical examples of neural networks are given to demonstrate the effectiveness of the proposed controllers. First, choose the membrane potential of health neuron from normal neural network (1), where $\widetilde{I}(t) = (0, 0)^{\mathrm{T}}$ is external input vector and $\tau = 1.1$ is time delay. Taking $\widetilde{C} = \begin{pmatrix} 1.1 & 0 \\ 0 & 1 \end{pmatrix}$, $\widetilde{A} = \begin{pmatrix} 2.1 & -0.1 \\ -5.2 & 3.2 \end{pmatrix}$, $\widetilde{B} = \begin{pmatrix} -1.6 & -0.2 \\ -0.3 & -2.5 \end{pmatrix}$, then we have the dynamical behavior of model (1) with initial conditions

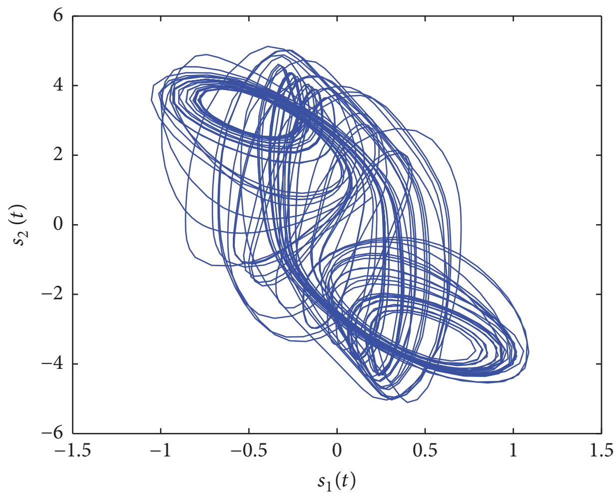

FIGURE 1: Chaotic trajectory of health neuron.

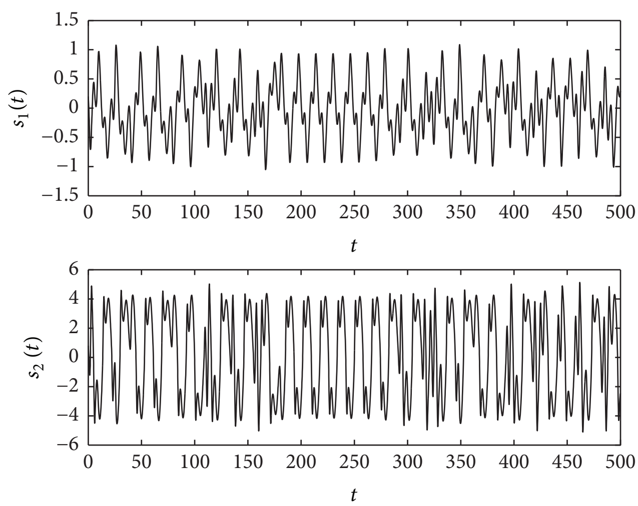

FIGURE 2: Time evolution of health neuron states $s(t)$.

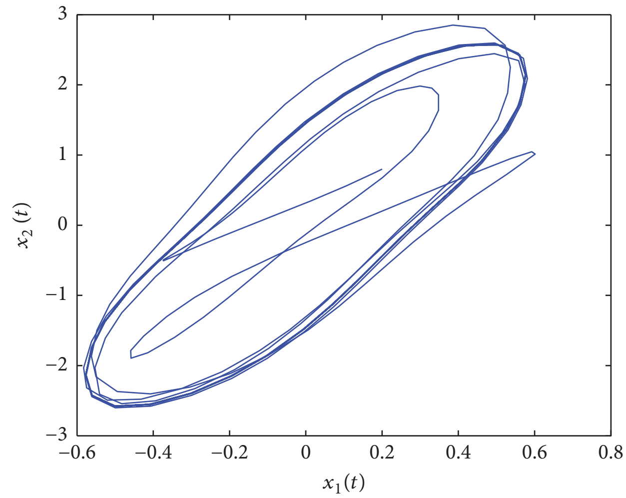

FIGURE 3: Periodic orbit of abnormal neuron.

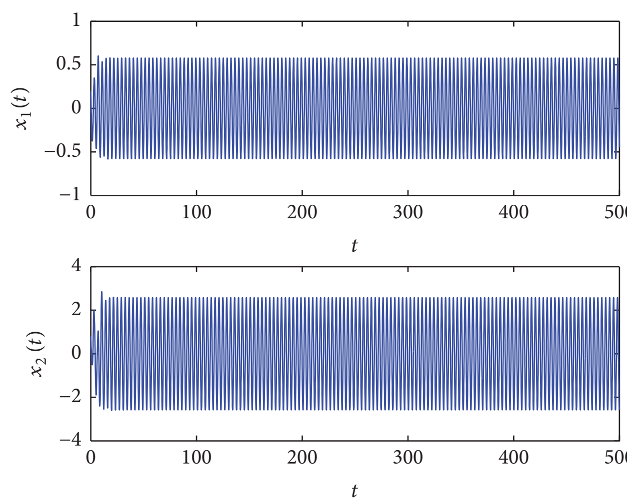

FIGURE 4: Time evolution of abnormal neuron states $x(t)$.

$x_1(t_0) = 0.8$, $x_2(t_0) = 0.2$, which can be seen in Figures 1 and 2. Obviously, this normal neuron I has a chaotic attractor.

Then, choose the membrane potential of abnormal synchronization neural networks from pathological neural network (2). Taking $\overline{C} = \begin{pmatrix} 2.1 & 0 \\ 0 & 1 \end{pmatrix}$, all other parameters are the same as health neuron I. Then we have the dynamical behavior of model (2) with initial conditions $x_1(t_0) = 0.8$, $x_2(t_0) = 0.2$, which can be seen in Figures 3 and 4. Evidently, this pathological neural network (2) has a nontrivial periodic orbit with high frequency oscillation. The abnormal neural network is in a state of an abnormal discharge.

In the simulation, the asymmetric coupling matrix as G is random and satisfied with the coupling condition. The parameters are set as $T = 1, h = 0.4$. According to Theorem 5, it is found that (11) is satisfied under the adaptive periodically intermittent controllers (8) and corresponding updating laws (9). The time series of the pathological neural network (2) by adaptive periodically intermittent controllers are numerically demonstrated as in Figure 5. Clearly, all errors are rapidly

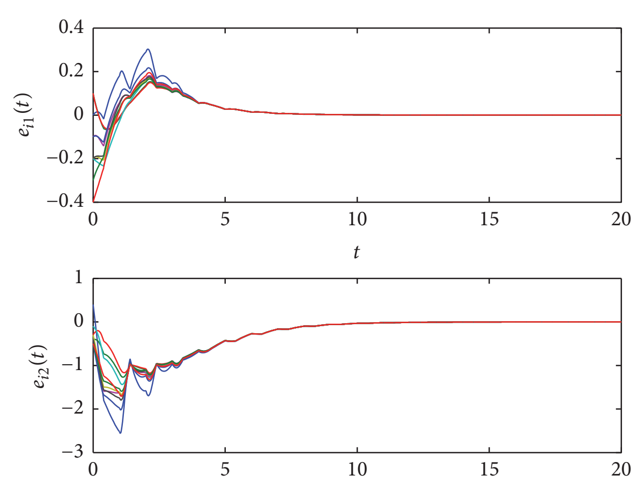

FIGURE 5: Errors e_{i1}, e_{i2} $(i = 1, 2, \ldots, 10)$ of network (2) under adaptive periodically intermittent feedback controllers (8) and updating laws (9).

achieving multilag synchronization. As synchronization of the pathological neural network, control strategies (8) reduce to common periodically intermittent controllers. For this simulation, the initial values of states $x_i(0)$ $(i = 0, 1, \ldots, 10)$ are random and $s(0) = 0$. It follows that a simple but robust adaptive intermittent controller is designed such that the abnormal neuron can have multilag synchronization and recover a healthy neuron. Eventually, using the multilag synchronous controller, we can realize 10 neurons into the state given, so as to realize multilag synchronization.

5. Conclusions

In this paper, we combine computational neuron model with control theory and propose a closed-loop control strategy based on neuronal model, which achieved multilag synchronization of abnormal synchronization neural network models. Based on the Lyapunov stability theory combined with the method of periodically intermittent control and the adaptive control, some simple criteria are derived for the multilag synchronization of the coupled neural networks with coupling delays. The adaptive periodically intermittent control which we have obtained can cut down control cost. The sufficient conditions of this paper for abnormal neural network multilag synchronization are less conservative. At last, simulation results show the effectiveness of the proposed control strategy. The multilag synchronization can just play the role of medicine to achieve the multilag exponential synchronization of the abnormal synchronization neural cells to effectively prevent the occurrence of epilepsy. Therefore, the design of controllers and control strategy may provide a potential electrical stimulation therapy on neurological diseases caused by abnormal synchronization and provide technical support for epilepsy treatment apparatus research and development.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

This work was partially supported by the National Science Foundation of China (Grant no. 61673100), Liaoning Province Doctor Starting Foundation (Grant no. 20170520283), the Natural Science Foundation of Liaoning Province (Grant no. 20170540769), Liaoning BaiQianWan Talents Program (Grant no. 2017076), the Fundamental Research Funds for the Central Universities (Grant no. N140504009), and Discipline Content Education Project of Shenyang Jianzhu University (Grant no. XKHY2-104).

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