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Research Article

Adaptive PID and Model Reference Adaptive Control Switch Controller for Nonlinear Hydraulic Actuator

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Nonlinear systems are modeled as piecewise linear systems at multiple operating points, where the operating points are modeled as switches between constituent linearized systems. In this paper, adaptive piecewise linear switch controller is proposed for improving the response time and tracking performance of the hydraulic actuator control system, which is essentially piecewise linear. The controller composed of PID and Model Reference Adaptive Control (MRAC) adaptively chooses the proportion of these two components and makes the designed system have faster response time at the transient phase and better tracking performance, simultaneously. Then, their stability and tracking performance are analyzed and evaluated by the hydraulic actuator control system, the hydraulic actuator is controlled by the electrohydraulic system, and its model is built, which has piecewise linear characteristic. Then the controller results are compared between PID and MRAC and the switch controller designed in this paper is applied to the hydraulic actuator; it is obvious that adaptive switch controller has better effects both on response time and on tracking performance.

1. Introduction

Fast response time and tracking performance are the two primary conditions in oil and gas production control system. Improving the efficiency of hydraulic actuator system is the most important part during the oil and gas production. As the control type of production, electrohydraulic system has been studied and many achievements have been investigated by researchers [1–3].

As Figure 1 shows, multiphase flow and hydrate in the pipeline distributed unevenly during the oil and gas production, which produces different resistance characteristics and makes the hydraulic actuator exhibit nonlinear behaviors in the opening/closing process.

In order to deal with the control problem of this kind of nonlinear system, many techniques have been used. For example, in [4], the authors study fuzzy control of nonlinear systems with unmodeled dynamics and input saturation using small-gain approach. And adaptive control for stochastic switched nonlinear triangular system is investigated in [5], different from the approaches investigated in [4, 5]. The approach is to linearize the system by using standard linear estimation techniques at some operating points [6, 7], within a neighborhood which can accurately describe the nonlinear system behavior, and a wealth of linearization-based design techniques can be applied to achieve the control objective [7]. So the nonlinear system consists of a set of linear time-invariant subsystems [8, 9].

In practical systems, the piecewise linear system is an efficient method to approximate nonlinear system. For example, the control for the linear parameter-varying systems with piecewise constant parameters, the signal transmission system, and the fluid power electrohydraulic actuator system have been extensively investigated; see examples including [10–13].

Some efforts have been done toward developing control strategies for such systems [14–24]. There are mainly two control strategies: PID control and Model Reference Adaptive Control (MRAC).

Some researches have been studied for the piecewise linear system by using PID control strategy [14–16]. A new algorithm of PID controller online tuning has been presented

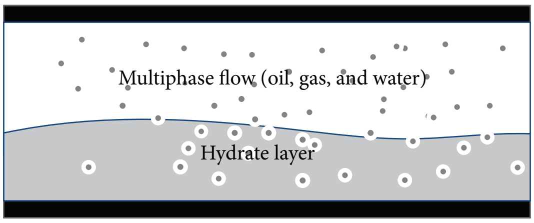

Figure 1: Multiphase flow and hydrate distribution in pipeline.

in [14], which uses a model of the controlled process in the shape of piecewise linear that is linearized continuously and resulting in linearized model. Dolezel et al. proposed algorithm for PID controller tuning using pole assignment technique and piecewise linear network [15]; the tuning technique generally provides stable control response, and its quality of performance depends on accuracy of the used piecewise linear neural model of the system and on the polynomial. PID controller has been spread widely in various applications till these days due to its simple structure, and different control objectives can be got by adjusting PID parameters, which has been described in [16]; the PID controller has been used very effectively for vibration reduction of piecewise linear system.

Different from PID controller, adaptive control strategy can guarantee the stability and tracking properties by designing proper adaptive laws. Many researches about adaptive control for piecewise linear system have been studied in recent years [17–21]. Linear time-invariant (LTI) is chosen as the reference model in adaptive control [17], which is conventional in adaptive control [20, 21, 25]. Sang and Tao put forward the adaptive state tracking control in [7] achieving closed-loop stability and small tracking error.

From these studies, PID controller has obvious advantages at the transient phase and is widely used; it has simple structure and we can tune the parameters to obtain the control objective by different algorithms [26, 27]. However, when used alone, it can give poor performance when the PID loop gains must be reduced so that the control system does not overshoot, oscillate, or hunt about the control setpoint value. They also have difficulties in the presence of nonlinearity. Adaptive control is more complex; however, it has better tracking performance when we identify the plant's model. Comparison of robustness between adaptive control and PID control has been researched by Shibata and Mitsukawa in [28].

It is worth mentioning that fast response time and tracking performance are the two primary requirements in electrohydraulic actuator control problem, especially for the subsea electrohydraulic actuator control systems; the control of such systems may become difficult and complicated due to two facts:

(1) The nonlinear electrohydraulic system exhibits piecewise linear characteristics. And the piecewise linear system consists of a set of linear subsystems; "switching points" exist between different linear subsystems.

(2) For the electrohydraulic piecewise linear system, it is needed to design a control strategy to reduce vibration and get less response time at the transient phase and get better tracking performance in steady-state.

Motivated by the above discussion, in this paper, we aim to combine the PID method and MRAC technique together and establish a unified framework to get fast transient tracking at the transient phase and better tracking performance for the piecewise linear control system. It should be noted that the piecewise linear system, which consists of a set of linear subsystems, and piecewise adaptive switch controller which consists of PID and Model Reference Adaptive Control (MRAC) are developed to control the system. First, piecewise linear reference model is given; PID controller and MRAC controller are designed for every linear subsystem; both PID controller and model reference adaptive laws are established based on the state tracking error. Then, the switch controller is designed for the piecewise linear system. Second, by constructing Lyapunov function, a sufficient condition is presented to ensure the asymptotic stability of the state tracking control. Finally, the switch controller is applied to the electrohydraulic actuator system.

The main contributions of this paper can be highlighted as follows: (i) adaptive switch controller is established for the piecewise linear system that tackles the tracking time at the transient phase and gets better tracking performance in steady-state; (ii) the estimated state function is designed at the PID control stage, and the "switch point" is determined based on the norm of the error between the estimated state and the piecewise linear plant state; (iii) four possible cases with different mode switches are studied for evaluating the tracking performance; (iv) the model of the electrohydraulic actuator system is built and the switch controller is used for the system.

The rest of this paper is organized as follows. Section 2 briefly introduces the problem under consideration. In Section 3, adaptive switch controller integrated with PID and MRAC adaptively chooses the proportion of these two components based on the norm of the error between the estimated state and the piecewise linear plant state; estimated state function is designed at the PID control stage; then the stability properties of the adaptive designs are studied. Model of hydraulic actuator is studied in Section 4 and PID, MRAC, and switch have been used for this piecewise model to prove the switch controller's advantages. At last, concluding remarks are in Section 5.

2. Problem Statement

In this paper, we consider the piecewise linear system described by the following state-space equation:

$$\dot{x}(t) = A_i x(t) + b_i u(t), \quad x(t) \in S_i, \; i \in \mathscr{M}, \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}$ is the scalar input, $\{S_i\}_{i\in\mathscr{M}}$ denotes a partition of the state space into a number of closed (possibly unbounded) polyhedral subspaces, $\mathscr{M} = \{1, \ldots, l\}$ is the index set of subspaces such that $\bigcup_{i=1}^{l} S_i = S$ with $S \in \mathbb{R}^n$, and $S_j \cap S_k = \emptyset$ for all $j \neq k$. A_i and b_i are supposed to be unknown.

The problem is to find a controller $u(t)$ to ensure that the state variables of the plant track asymptotically the reference states, say $x_m(t)$.

Here, we assume that the piecewise linear reference model is given by

$$\dot{x}_m(t) = A_{mi}x(t) + b_{mi}r(t), \quad (2)$$

where A_{mi}, $i \in \mathcal{M}$, are stable and $r(t) \in R$ is a bounded reference input signal, when (A_{mi}, b_{mi}) is active, as indicated by $x(t) \in S_i$.

Remark 1. The control systems with such piecewise components have become an important area of research. As discussed in Section 1, many results have been investigated. For the nonlinear hydraulic actuator [29, 30] in subsea oil and gas production, fast response time and better tracking performance are pursued.

In this paper, the control objective is to develop a switch controller such that the closed-loop system is stable and $x(t)$ asymptotically tracks $x_m(t)$, which has faster response time at the transient phase and better tracking performance.

3. Adaptive Switch Controller Design

In this section, a switch controller is proposed for the piecewise linear system (1) to achieve closed-loop stability and state tracking. The controller consists of PID and MRAC control stage.

To simplify our discussion, we make the following assumptions about the piecewise linear system.

Assumption 2. In this note, assume that all the plant's state variables can be observed.

Assumption 3. For every linear subsystem, assume that the PID and MRAC controller will switch at the switching point, and the switching point is decided by the norm of the error between the state output and the state estimated during PID control stage, which will be studied in the following.

The bounded $x_m(t)$ is generated by the time-invariant parameter set (A_{mi}, b_{mi}). For the stability of piecewise linear system (2), exponential stability of the homogeneous system $\dot{z}(t) = A_m(t)z(t)$ is necessary and sufficient for stability of system (2) [7, 31, 32].

As studied in [7], let $T_0 = \min_{k \in \mathbb{Z}^+}\{t_k - t_{k-1}\}$, where $\mathbb{Z}^+$ stands for all possible integers, and let P_{mi}, $Q_{mi} \in \mathbb{R}^{n\times n}$ be symmetric, positive definite satisfying

$$A_{mi}^T P_{mi} + P_{mi}A_{mi} = -Q_{mi}, \quad i \in \mathcal{M}. \quad (3)$$

And the following lemma ensures exponential stability of homogeneous system and thus the stability of (2), which has been proofed by Sang and Tao in [7].

Lemma 4. *The system $\dot{z}(t) = A_m(t)z(t)$ is exponentially stable with decay rate $\sigma \in (0, 1/2\alpha)$ if T_0 satisfies*

$$T_0 \geq \frac{\alpha}{1-2\sigma\alpha}\ln\left(1+\mu\Delta A_m\right), \quad \mu = \frac{a_m^2}{\lambda_m\beta}\max_{i\in\mathcal{M}}\left\|P_{mi}\right\|, \quad (4)$$

where $\Delta A_m = \max_{i,j\in\mathcal{M}}\|A_{mi} - A_{mj}\|$ and $a_{mi}, \lambda_{mi} > 0$ such that $\|e^{A_{mi}t}\| \leq a_{mi}e^{-\lambda_{mi}t}$, with $a_m = \max_{i\in\mathcal{M}} a_{mi}$, $\lambda_m = \min_{i\in\mathcal{M}}\lambda_{mi}$, $\alpha = \max_{i\in\mathcal{M}}\lambda_{max}[P_{mi}]$, and $\beta = \min_{i\in\mathcal{M}}\lambda_{min}[P_{mi}]$, where λ_{min} and λ_{max} denote the minimum and maximum eigenvalues of a matrix.

3.1. Adaptive Controller Structure. In this section, state feedback adaptive controller is proposed for the piecewise linear system (1) to make its closed-loop system stable and state tracking. And the following controller is applied:

$$u(t) = F(t)x(t) + K(t)r(t), \quad (5)$$

where $K \in \mathbb{R}$ is the gain scalar and $F \in \mathbb{R}^{1\times n}$ is the gain matrix.

As shown in Figure 2 the ith region of the system adaptive control input is

$$u(t) = F_i x(t) + K_i r(t), \quad i \in \mathcal{M}, \quad (6)$$

with (1) and (6), and we get the closed-loop system

$$\begin{aligned}\dot{x}(t) &= A_i x(t) + b_i F_i x(t) + b_i K_i r(t), \\ &\qquad x(t) \in S_i, \quad i \in \mathcal{M},\end{aligned} \quad (7)$$

where the nominal parameters F_{0i}, K_{0i} exist, such that

$$\begin{aligned}A_{mi} &= A_i + b_i F_{0i}, \\ b_{mi} &= b_i K_{0i}.\end{aligned} \quad (8)$$

The state tracking error is

$$\begin{aligned}e(t) &= x_m(t) - x(t), \\ \dot{e}(t) &= \dot{x}_m(t) - \dot{x}(t).\end{aligned} \quad (9)$$

In view of (7)–(9), we have

$$\begin{aligned}\dot{e}(t) &= \dot{x}_m(t) - \dot{x}(t) \\ &= A_{mi}e(t) + \left(A_{mi} - A_i - b_i F_i\right)x(t) \\ &\quad + \left(b_{mi} - b_i K_i\right)r(t) \\ &= A_{mi}e + b_{mi}K_{0i}^{-1}\widetilde{F}_i x(t) + b_{mi}K_{0i}^{-1}\widetilde{K}_i r(t),\end{aligned} \quad (10)$$

where $\widetilde{F}_i = F_{0i} - F$ and $\widetilde{K}_i = K_{0i} - K$.

3.2. Adaptive Laws. For updating the parameters F_i, K_i, we need to develop adaptive laws based on $e(t)$, with the

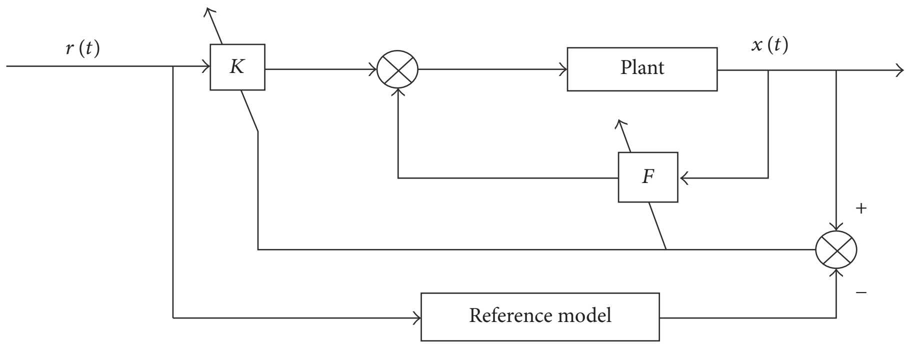

Figure 2: MRAC controller structure.

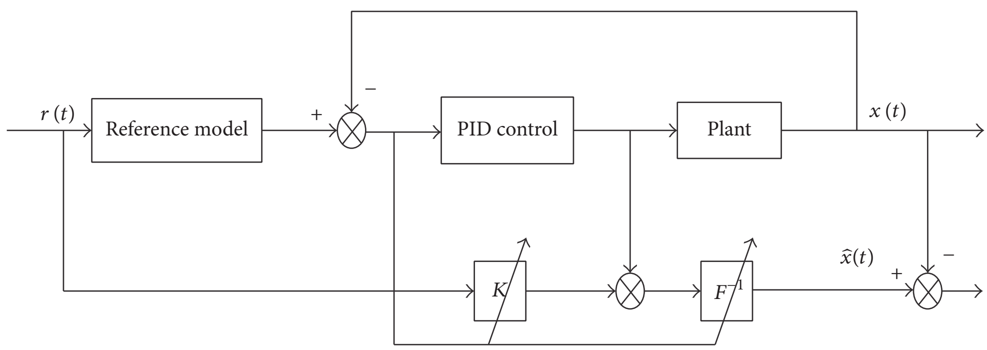

Figure 3: PID controller structure.

knowledge of lower and upper bounds of the parameters in F_{0i}, K_{0i}; the parameter projection adaptive laws

$$\dot{F}_i = \begin{cases} R_{i1} b_{mi}^T P_{mi} e_i x_i^T, & F_i(t) \in \Xi_F, \\ 0, & \text{otherwise}, \end{cases}$$
$$\dot{K}_i = \begin{cases} R_{i2} b_{mi}^T P_{mi} e_i r, & K_i(t) \in \Xi_K, \\ 0, & \text{otherwise}, \end{cases} \tag{11}$$

where Ξ_F and Ξ_K are the known bound regions of $F_i(t)$ and $K_i(t)$ and $R_{i1}, R_{i2} \in \mathbb{R}$, $i \in \mathscr{M}$, and satisfy

$$\begin{gathered} R_{i1} = P_{Fi} K_{0i}^{-T}, \\ R_{i2} = P_{Ki} K_{0i}^{-T}, \\ i \in \mathscr{M}, \end{gathered} \tag{12}$$

with P_{Fi}, P_{Ki} being symmetric positive definite matrix.

Remark 5. The parameter projection adaptive laws for piecewise linear state tracking case in [7] are based on the knowledge of known sign K_{0i}; in this paper, the parameter projection laws need not have the knowledge of sign K_{0i}.

Remark 6. Without the knowledge of K_{0i}, it is difficult to get the parameters R_{i1} and R_{i2}; however, as P_{Fi} and P_{Ki} are defined matrices, in this paper, R_{i1} and R_{i2} are defined matrices.

3.3. PID Controller Design. PID controller has been widely researched; its basic structure is shown in Figure 3. For system (1), the control objective is to make system (1) asymptotic tracking $x_m(t)$ with less time.

Continuous-time PID controller itself is defined by several different algorithms. The common version of its algorithm defined

$$u_{\text{PID}}(t) = K_p^i e(t) + K_i^i \int_0^t e(\tau)\, d\tau + K_d^i \dot{e}(t), \tag{13}$$

with

$$e(t) = x_m(t) - x(t), \tag{14}$$

which is the state tracking error, where $x_m(t) = (x_{m1}, x_{m2}, \ldots, x_{mn})^T$, $x(t) = (x_1, x_2, \ldots, x_n)^T$, and $e(t) = (e_1, e_2, \ldots, e_n)^T$. The control objective is to guarantee that the plant state $x(t)$ tracks the reference trajectory $x_m(t)$; that is, $\lim_{t\to\infty} e(t) = 0$.

The control variable consists of three parts: proportional one, integral one, and derivative one. K_p^i is the proportional gain, K_i^i is the integral time, and K_d^i is the derivative time for the ith linear subsystem, which can be tuned to achieve the asymptotic tracking of $x_m(t)$ by $x(t)$ and takes less time.

3.4. Switch Controller Design. In this section, a new controller structure is proposed for the piecewise linear system (1) to achieve the asymptotic tracking of $x_m(t)$ by $x(t)$ with less time.

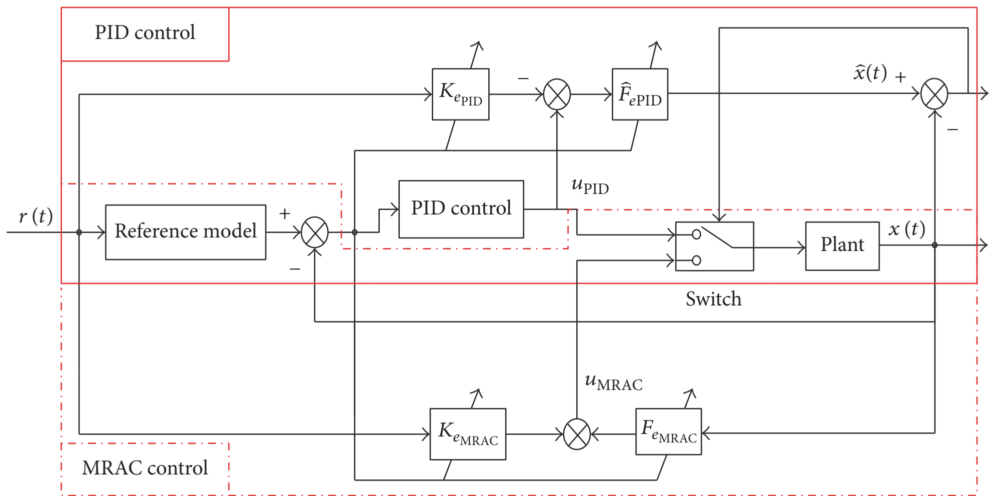

FIGURE 4: Switch controller structure.

Notes. $K_{e\mathrm{PID}}$, $F_{e\mathrm{PID}}$, $K_{e\mathrm{MRAC}}$, and $F_{e\mathrm{MRAC}}$ are all updated based on the projection laws (11); the differences are that $K_{e\mathrm{PID}}$ and $F_{e\mathrm{PID}}$ are updated at the PID control stage and $K_{e\mathrm{MRAC}}$ and $F_{e\mathrm{MRAC}}$ are updated at the MRAC control stage.

We design the switching controller consisting of 2 parts: PID and MRAC, as shown in Figure 4. At the first stage, PID controller is used to guarantee the error convergence to a certain extent. At the same time, adaptive laws $K_{e\mathrm{PID}}$ and $F_{e\mathrm{PID}}$ (11) are updated by the error between the state output and the reference, with the plant controlled by the PID; we have the equation

$$u_{\mathrm{PID}} = K^{i}_{e\mathrm{PID}} r(t) + F^{i}_{e\mathrm{PID}} \widehat{x}(t), \tag{15}$$

where $\widehat{x}(t)$ is the estimated states.

In the first stage, PID control laws are used to guarantee boundedness of asymptotic tracking, and MRAC laws are used to achieve asymptotic tracking of $x_m(t)$ by $x(t)$, with the assumption that $\varepsilon > 0$ exist; we set the switching "condition"

$$\|\widehat{x}(t) - x(t)\|_2 \le \varepsilon. \tag{16}$$

Remark 7. As described in Figure 4, $\widehat{x}(t)$ is the estimated states to evaluate the MRAC controller during PID control stage. Under the condition $\|\widehat{x}(t) - x(t)\|_2 > \varepsilon$, K and F are updated by the error $e(t) = x_m(t) - x(t)$ controlled by PID controller. When K and F make the estimated state $\widehat{x}(t)$ get the condition (16), MRAC controller is used to control the plant because it has better tracking performance and smaller error.

With (15), there are two conditions during the process of getting $\widehat{x}(t)$; the first is $n = 1$ and the other is $n > 1$. If $n = 1$, from the equation

$$u_{\mathrm{PID}} = K^{i}_{e\mathrm{PID}} r(t) + F^{i}_{e\mathrm{PID}} \widehat{x}(t), \tag{17}$$

we can get

$$\widehat{x}(t) = \left(F^{i}_{e\mathrm{PID}}\right)^{-1} \left(u_{\mathrm{PID}} - K^{i}_{e\mathrm{PID}} r(t)\right). \tag{18}$$

Under the system with $n > 1$, however, $\widehat{x}(t)$ cannot be got only by solving (17), as the inverse matrix of F does not exist in many systems, and (15) can be written

$$F^{i}_{e\mathrm{PID}} \widehat{x}(t) = u_{\mathrm{PID}} - K^{i}_{e\mathrm{PID}} r(t). \tag{19}$$

Then we use the following equation to describe the vector of output discrepancies

$$\xi = u_{\mathrm{PID}} - K^{i}_{e\mathrm{PID}} r(t) - F^{i}_{e\mathrm{PID}} \widehat{x}(t). \tag{20}$$

Hence, the loss function can be written as

$$\begin{aligned} L(\widehat{x}, S) = \|\xi\|_2^2 &= \left(u_{\mathrm{PID}} - K^{i}_{e\mathrm{PID}} r(t) - F^{i}_{e\mathrm{PID}} \widehat{x}(t)\right)^T \\ &\cdot \left(u_{\mathrm{PID}} - K^{i}_{e\mathrm{PID}} r(t) - F^{i}_{e\mathrm{PID}} \widehat{x}(t)\right), \end{aligned} \tag{21}$$

where $L(\widehat{x}, S) = \xi^2$ denotes the squared error or loss of $\widehat{x}(t)$ on $(\widehat{x}, u_{\mathrm{PID}} - K^{i}_{e\mathrm{PID}} r(t))$ and $L(\widehat{x}, S)$ denotes collective loss of a function on the training set S.

We can seek the optimal $\widehat{x}(t)$ by taking the derivatives of the loss with respect to the parameters $\widehat{x}(t)$ which satisfies the following equation:

$$\begin{aligned} \frac{\partial L(\widehat{x}, S)}{\partial \widehat{x}} &= -2\left(F^{i}_{e\mathrm{PID}}\right)^T \left(u_{\mathrm{PID}} - K^{i}_{e\mathrm{PID}} r(t)\right) \\ &\quad + 2\left(F^{i}_{e\mathrm{PID}}\right)^T F^{i}_{e\mathrm{PID}} \widehat{x}. \end{aligned} \tag{22}$$

With the inverse of $(F^{i}_{e\mathrm{PID}})^T F^{i}_{e\mathrm{PID}}$ existing, we can get

$$\widehat{x} = \left(\left(F^{i}_{e\mathrm{PID}}\right)^T F^{i}_{e\mathrm{PID}}\right)^{-1} \left(F^{i}_{e\mathrm{PID}}\right)^T \left(u_{\mathrm{PID}} - K^{i}_{e\mathrm{PID}} r(t)\right). \tag{23}$$

Then, we can get that $\widehat{F}^{i}_{e\mathrm{PID}}$ in Figure 4 is

$$\widehat{F}^{i}_{e\mathrm{PID}} = \begin{cases} F^{i}_{e\mathrm{PID}}, & n = 1, \\ \left(\left(F^{i}_{e\mathrm{PID}}\right)^T F^{i}_{e\mathrm{PID}}\right)^{-1} \left(F^{i}_{e\mathrm{PID}}\right)^T, & n > 1. \end{cases} \tag{24}$$

For the piecewise linear system (1), which consists of l regions linear subsystems, a new adaptive switch controller structure is proposed to achieve closed-loop stability (signal boundedness) and state tracking.

$$u_S = \begin{cases} u_{S1} = \begin{cases} \text{PID stage: } \begin{cases} u_{\text{PID}} = K_p^1 e(t) + K_i^1 \int_0^t e(t)\, d(t) + K_d^1 \dot{e}(t) \\ u_{\text{PID}} = K_{e\text{PID}}^i r(t) + F_{e\text{PID}}^i \hat{x}(t) \\ \|\hat{x}(t) - x(t)\|_2 \geq \varepsilon \end{cases} \\ \text{MRAC control: } u_{\text{MRAC}} = F_{e\text{MRAC}}^1 x(t) + K_{e\text{MRAC}}^1 r(t) \end{cases} \\ \vdots \\ u_{Si} = \begin{cases} \text{PID stage: } \begin{cases} u_{\text{PID}} = K_p^i e(t) + K_i^i \int_0^t e(t)\, d(t) + K_d^i \dot{e}(t) \\ u_{\text{PID}} = K_{e\text{PID}}^i r(t) + F_{e\text{PID}}^i \hat{x}(t) \\ \|\hat{x}(t) - x(t)\|_2 \geq \varepsilon \end{cases} \\ \text{MRAC control: } u_{\text{MRAC}} = F_{e\text{MRAC}}^i x(t) + K_{e\text{MRAC}}^i r(t) \end{cases} \\ \vdots \\ u_{Sl} = \begin{cases} \text{PID stage: } \begin{cases} u_{\text{PID}} = K_p^l e(t) + K_i^l \int_0^t e(t)\, d(t) + K_d^l \dot{e}(t) \\ u_{\text{PID}} = K_{e\text{PID}}^l r(t) + F_{e\text{PID}}^l \hat{x}(t) \\ \|\hat{x}(t) - x(t)\|_2 \geq \varepsilon \end{cases} \\ \text{MRAC control: } u_{\text{MRAC}} = F_{e\text{MRAC}}^l x(t) + K_{e\text{MRAC}}^l r(t). \end{cases} \end{cases} \tag{25}$$

Remark 8. At the PID control stage, the estimated state $\hat{x}(t)$ is got by the controller u_{PID}, and the controller switches at the condition $\|\hat{x}(t) - x(t)\|_2 \leq \varepsilon$.

Remark 9. This novel developed control strategy combines the advantages of the PID controller and MRAC controller, the two major approaches for piecewise linear system, and leads to a flexible controller, allowing us to exploit, maximally, the benefits of two control algorithms. The MRAC technique is designed based on the model; however, as the plant model is not known and the MRAC controller cannot get good tracking performance at this stage, PID controller is used to guarantee the closed-loop system's tracking response time and stability at this stage. The proposed controller is composed of two components; one component is the PID controller, which has faster tracking response time at the transient phase, and the other is the MRAC, which has better tracking performance when the plant's model is identified by the adaptive laws. In other words, we will design a switch controller, which is integrated with PID and MRAC controller, adaptively chooses the proportion of these two components, and makes the designed system have faster tracking response time and better tracking performance, simultaneously. Therefore, we need to design the integrated controller and develop a state feedback control law $u(t)$ so that the closed-loop system is stable and $x(t)$ asymptotically tracks $x_m(t)$, where $x_m(t)$ is the reference model.

3.5. Stability and Tracking Properties. For system (1), every linear subsystem has two phases as shown in Figure 4, PID control stage and MRAC control stage. The parameters of PID controllers are designed by Ziegler-Nichols Method, which can guarantee the stability of the system. So we mainly study the stability and tracking properties controlled by MRAC controller.

Let $\{t_k\}_{k=1}^{\infty}$ denote the time instants at which (1) switches mode. With the definitions of a_m, λ_m, α, β, μ, ΔA_m in Lemma 4, we have the following stability and tracking properties.

Theorem 10. *Consider the system consisting of the piecewise linear system (1), the adaptive switch controller (6), and adaptive laws (11). If*

$$T_0 \geq T_d = \alpha(1+k)\ln(1+\mu\Delta A_m), \quad k > 0, \tag{26}$$

the transient tracking error performance is given by

$$\|e\|_2^2 \leq f\frac{T}{T_0} + \frac{g}{\beta} + f, \tag{27}$$

where $f = \mu\Delta A_m g$ *and* $g = \max\{c(1+\mu\Delta A_m)(1+k)/k, V(t_0)\}$.

Proof. For the PID controller, it has been much used in the control loops of industrial process. Parameters K_p, T_i, and T_d can be tuned to make the system stable and take less time for $x(t)$ asymptotic tracks $x_m(t)$.

For the MRAC controller, due to the fact that $T_d = \alpha(1+k)\ln(1+\mu\Delta A_m)$, it follows from Lemma 4 that $T_0 \geq T_d$ ensures the stability of (2). For analyzing the stability and tracking properties about closed-loop system consisting of piecewise linear system (1), there, build the piecewise Lyapunov function

$$V = e^T P_{mi} e + \widetilde{F}_i^T P_{Fi} \widetilde{F}_i + \widetilde{K}_i^T P_{Ki} \widetilde{K}_i, \tag{28}$$

where P_{Fi}, P_{Ki} are symmetric positive definite matrices.

The derivative of V_i by applying the properties of matrix eigenvalues, along (8)–(12), is

$$\begin{aligned}\dot{V} &= \dot{e}^T P_{mi} e + e^T P_{mi} \dot{e} + tr\left(\dot{\widetilde{F}}_i^T P_{Fi} \widetilde{F}_i + \widetilde{F}_i^T P_{Fi} \dot{\widetilde{F}}_i\right) \\ &\quad + tr\left(\dot{\widetilde{K}}_i^T P_{Fi} \widetilde{K}_i + \widetilde{K}_i^T P_{Fi} \dot{\widetilde{K}}_i\right) \\ &= \dot{e}^T A_{mi}^T P_{mi} e + e^T P_{mi} A_{mi} e \\ &\quad + 2tr\left(\dot{\widetilde{F}}_i^T P_{Fi} \widetilde{F}_i + x e^T P_{mi} b_{mi} K_{0i}^{-1} \widetilde{F}_i\right) \\ &\quad + 2tr\left(\dot{\widetilde{K}}_i^T P_{Ki} \widetilde{K}_i + r e^T P_{mi} b_{mi} K_{0i}^{-1} \widetilde{K}_i\right).\end{aligned} \tag{29}$$

With (11) and (12), we can get that

$$\dot{V} = \dot{e}^T \left(A_{mi}^T P_{mi} + P_{mi} A_{mi}\right) e = -\dot{e}^T Q_{mi} e; \tag{30}$$

here, without loss of generality, we consider the case $Q_{mi} = I_n$ for $i \in \mathscr{M}$. With (12), the parameter projection adaptive laws, with the parameter estimates F_i, K_i, are bounded; thus there exists a $c > 0$, defined as

$$c = \max_{i \in \mathscr{M}} \left(\widetilde{F}_i^T P_{Fi} \widetilde{F}_i + \widetilde{K}_i^T P_{Ki} \widetilde{K}_i\right), \tag{31}$$

such that

$$\dot{V} \leq -\frac{V}{(1+\kappa)\alpha} - \frac{\kappa V - (1+\kappa) c}{(1+\kappa)\alpha}, \quad \kappa > 0. \tag{32}$$

As described in Lemma 4, $\alpha = \max_{i \in \mathscr{M}} \lambda_{\max}[P_{mi}]$, where $\lambda_{\max}$ denote the maximum eigenvalues of a matrix. That is, for $V > (1+\kappa)c/\kappa$, V decays faster than exponential stability at the rate $-1/(1+\kappa)\alpha$ and is nonincreasing otherwise.

When a mode switch occurs at $t = t_k$, we have

$$V(t_k) \leq (1 + \mu \Delta A_m) V(t_k^-). \tag{33}$$

Remark 11. The inequality provides $V(t)$ at $t = t_k$, which has been studied by Sang and Tao in [7].

Then, the switching condition $T_0 \geq T_d$ ensures that

$$V(t_k) \leq \begin{cases} c(1 + \mu \Delta A_m) \dfrac{k+1}{k}, & V(t_k^-) \leq c \dfrac{1+k}{k}, \\ V(t_{k-1}), & V(t_k^-) > c \dfrac{1+k}{k}. \end{cases} \tag{34}$$

thus $V(t) \leq g \doteq \max\{c(1 + \mu \Delta A_m)(1+k)/k, V(t_0)\}$.

For evaluating the tracking performance, there are four possible cases depending on the integration interval $[t, t+T]$.

(i) $T \leq T_0, t_{k-1} \leq t \leq t+T < t_k$. There is no mode switch over $[t, t+T]$, since V is nonincreasing; we have

$$\begin{aligned}\|e\|_2^2 &= \int_t^{t+T} |e(\tau)|^2 \, d\tau \leq (V(t) - V(t+T)) \leq V(t) \\ &= g.\end{aligned} \tag{35}$$

(ii) $T \leq T_0$, $t < t_k \leq t+T$. There is one and only one switch over $[t, t+T]$. We have

$$\begin{aligned}\|e\|_2^2 &\leq (V(t) - V(t+T)) + e^T(t_k) \Delta P_{m(k)} e(t_k) \\ &\leq g + \mu \Delta A_m g.\end{aligned} \tag{36}$$

(iii) $T > T_0$, $t < t_k$, $t+T < t_{k+N}$. N is less than or equal to T/T_0, which means that there are at most N mode switches.

$$\begin{aligned}\|e\|_2^2 &\leq (V(t) - V(t+T)) \\ &\quad + \sum_{j=0}^{N-1} e^T(t_{k+j}) \Delta P_{m(k)}^{k+j} e(t_{k+j}) \\ &\leq g + \mu \Delta A_m g \frac{T}{T_0}.\end{aligned} \tag{37}$$

(iv) $T > T_0$, $t < t_k$, $t+T \geq t_{k+N}$. There are at most $N+1$ mode switches

$$\|e\|_2^2 \leq g + \mu \Delta A_m g \frac{T}{T_0} + \mu \Delta A_m g. \tag{38}$$

Define $f = \mu \Delta A_m g$. Concluding from (i) to (iv), the tracking error is given by

$$\|e\|_2^2 \leq f \frac{T}{T_0} + \frac{g}{\beta} + f. \tag{39}$$

Proof is completed. □

4. Application to Hydraulic Actuator Control System

Simulations are performed to demonstrate the system stability and tracking performance with proposed adaptive control schemes applied to the piecewise linear system model of the longitudinal dynamics of the hydraulic control valves [29, 30].

To demonstrate the above technique, the hydraulic actuator control system is used.

4.1. System Description. In this work, we consider the electrohydraulic actuator system studied in [29, 30], which consists of surface hydraulic supply unit and subsea parts: servo valve, hydraulic actuator, and control unit. In the subsea oil/gas production system, the multiphase flow and hydrate distribution can be detected by using the detector.

As shown in Figure 5, the plant input u is the control voltage based on the reference input, the actuator displacement, and the fluid distribution in the pipe line.

The efficiency of production system is determined by hydraulic actuator's response time, which possesses piecewise

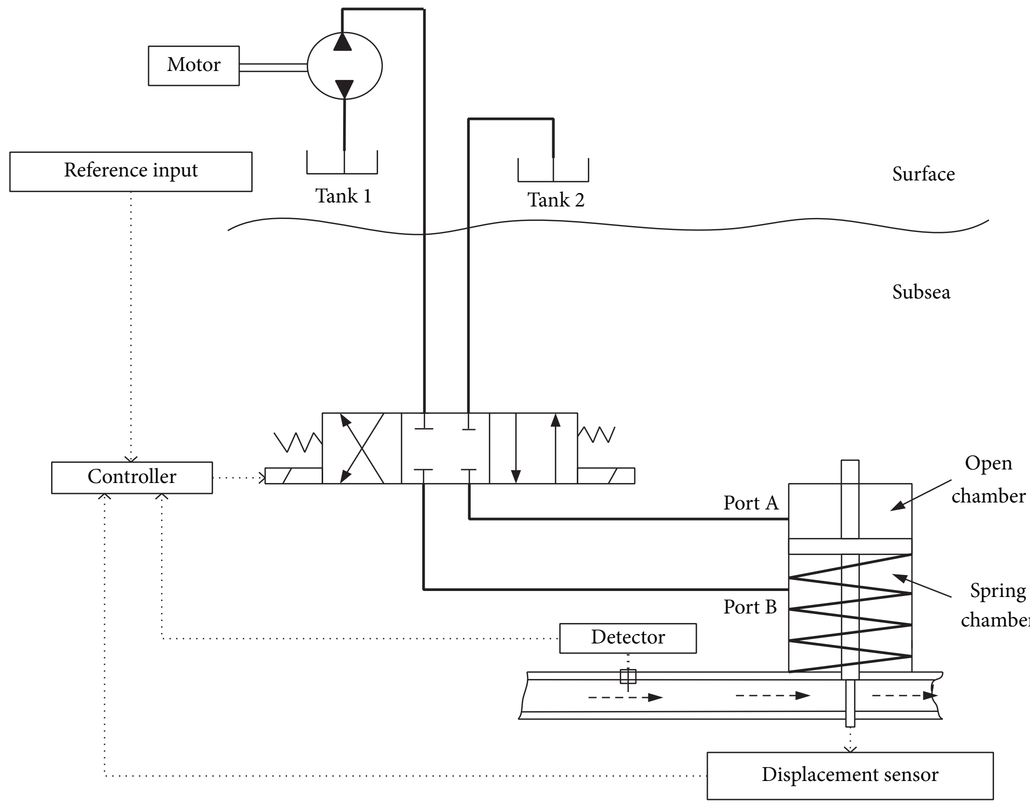

Figure 5: The electrohydraulic actuator control system.

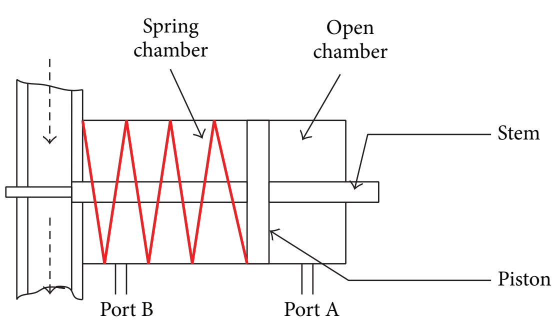

Figure 6: The hydraulic actuator's structure diagram.

linear characteristics in opening/closing process. The structure diagram of the actuator is shown in Figure 6.

The equations of actuator system are given as follows.

$$p(S-S_1)+\rho gHS_1 = m\frac{d^2y}{dt^2}+\left(\rho gH+\frac{128\mu l_p Q}{\pi d_p^4}\right)(S-S_2) + k(L_0+y)+f_i(y), \quad (40)$$

$$Q=(S-S_1)\frac{dy}{dt}. \quad (41)$$

Table 1: Variable description.

Variable	Variable description
p	Supply pressure of open chamber
y	Piston displacement
S	Piston area
S_2	Valve stem area
ρ	Sea water density
k	Spring elastic coefficient
d_p	Inner diameter of guide hole
l_p	Length of guide hole
H	Working water depth
L_0	Spring precompression
S_1	Piston rod area
m	Mass of piston
μ	Control fluid viscosity
g	Gravity acceleration
$f_i(y)$	Resistance in operating process
γ_i	Resistance coefficient

With the different flow in the pipe line, piecewise linear resistance characteristics exist during the hydraulic actuator in opening/closing process.

$$f_i(y)=\gamma_i * y, \quad (42)$$

where the variable description is shown in Table 1.

Figure 5 shows the electrohydraulic actuator control system, which consists of power unit (Tank 1, Tank 2), electrohydraulic proportional control valve, hydraulic actuator, and the control unit. As shown in (41) and (42), $f_i(y)$ changes with y in practical operation, which is piecewise linear. Let $\rho g H(A-A_2)+kL_0 \doteq \rho g H A_1$; for the piecewise linear model, the system can be further expressed by the following integer program:

$$\begin{aligned}\dot{x}_1 &= x_2,\\ \dot{x}_2 &= \frac{1}{m}(S-S_1)\,p - \frac{1}{m}\left(kx_1 \right.\\ &\quad \left. + \gamma_i \frac{128\mu l_p}{\pi d_p^4}(S-S_2)(S-S_1)\,x_2 - k_i x_1\right),\end{aligned} \tag{43}$$

where $x_1 = y$, $x_2 = dy/dt$, and the piecewise linear function

$$\begin{gathered} f(y) = \gamma_i * y,\\ \gamma_i = \begin{cases} a, & x_1 \in \Omega_1,\\ b, & x_1 \in \Omega_2,\end{cases}\end{gathered} \tag{44}$$

where Ω_1 and Ω_2 are the fluid element distribution in the pipeline, which is determined by the detector shown in Figure 6; in this paper, Ω_1 is the region of $x_1 \le 0.1$; Ω_2 is the region of $x_1 > 0.1$. For the servo valve, there exist $p = K_u \bullet p_{\text{in}}$ and $K_u = k_{\text{pv}} \bullet u$ (where k_{pv} represents the function of electrohydraulic proportional control valve and p_{in} is the input pressure of the electrohydraulic proportional control valve). Then the system can be written

$$\begin{aligned}\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} &= \begin{bmatrix} 0 & 1\\ -\dfrac{1}{m}(k+\gamma_i) & -\dfrac{1}{m}\left(\gamma_i \dfrac{128\mu l_p}{\pi d_p^4}(S-S_2)(S-S_1)\right)\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}\\ &\quad + \begin{bmatrix} 0\\ \dfrac{1}{m}(S-S_1)\,k_{\text{pv}} \bullet p_{\text{in}}\end{bmatrix} u.\end{aligned} \tag{45}$$

For simplicity of presentation, we choose $k_{\text{pv}} = 1$, $p_{\text{in}} = 1$, with the elements being the state variables to control the piston displacement, with (44) and (45) state equation of hydraulic actuator with parameters in Table 2.

4.2. Switch Controller Synthesis. For the electrohydraulic actuator control system, we first construct the reference model.

$$\begin{bmatrix}\dot{x}_{m1}\\ \dot{x}_{m2}\end{bmatrix} = \begin{bmatrix}A_{m11} & A_{m12}\\ A_{m21} & A_{m22}\end{bmatrix}\begin{bmatrix}x_{m1}(t)\\ x_{m2}(t)\end{bmatrix} + \begin{bmatrix}b_{m11}\\ b_{m21}\end{bmatrix} r(t), \tag{46}$$

where $x_{m1} = \hat{y}$, $x_{m2} = d\hat{y}/dt$ are the reference trajectory of x_1 and x_2. $r(t)$ is the bounded continuous reference input signal, while the initial conditions are set to $x_m = [0 \;\; 0]^T$. With assumption 1 fulfilled, let

Table 2: Parameters of the hydraulic actuator in simulation.

k (kN/m)	221
S (mm^2)	10
S_2 (mm^2)	4
μ (mm^2/s)	20.4
γ_2	−223
l_p	40
S_1 (mm^2)	2
d_p (mm)	10
γ_1	−229
m (kg)	1

$$\begin{aligned} A_{m1} &= \begin{bmatrix}0 & 1\\ -10 & -5\end{bmatrix},\\ b_{m1} &= \begin{bmatrix}0\\ 2\end{bmatrix},\\ A_{m2} &= \begin{bmatrix}0 & 1\\ -5 & -5\end{bmatrix},\\ b_{m1} &= \begin{bmatrix}0\\ 2\end{bmatrix}.\end{aligned} \tag{47}$$

There are two stages for every region of the nonlinear system; the first one is PID controller which plays a role in the process of x_{mi} track to x_{pi}; the second is the MRAC controller used.

At the PID control stage, we have

$$u = k_p^i e(t) + k_i^i \int_0^t e(\tau)\,d\tau + k_d^i \dot{e}(t), \quad i = 1, 2, \tag{48}$$

where $e(t) = x_m(t) - x(t)$. And the PID controller parameters $k_p^i = [k_{p1}^i \;\; k_{p2}^i]$, $k_i^i = [k_{i1}^i \;\; k_{i2}^i]$, and $k_d^i = [k_{d1}^i \;\; k_{d2}^i]$ are determined based on Ziegler-Nichols' methods in practice operation. In this example, we control the plant by PI controller with the following parameters:

$$\begin{aligned} k_p^1 &= [20 \;\; 1],\\ k_p^2 &= [15 \;\; 3],\\ k_i^1 &= [10 \;\; 1],\\ k_i^2 &= [2 \;\; 2].\end{aligned} \tag{49}$$

For the model reference adaptive laws, the parameters

$$\begin{aligned} R_{11} &= R_{21} = 1,\\ R_{12} &= R_{22} = 1,\end{aligned}$$

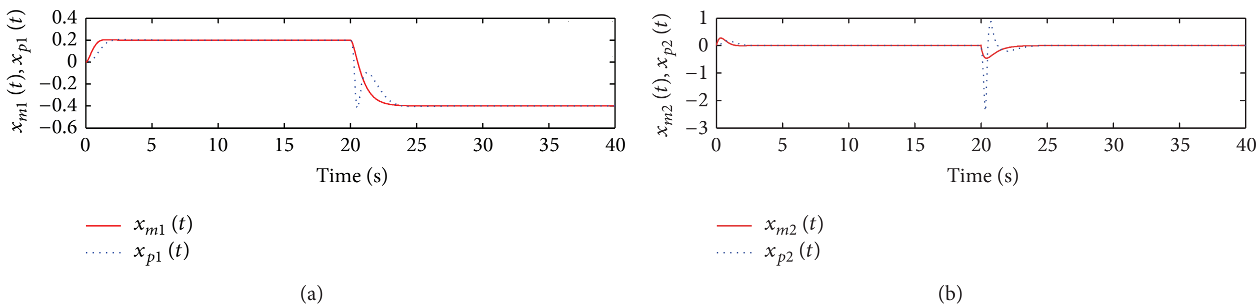

FIGURE 7: MRAC controller for piecewise linear system.

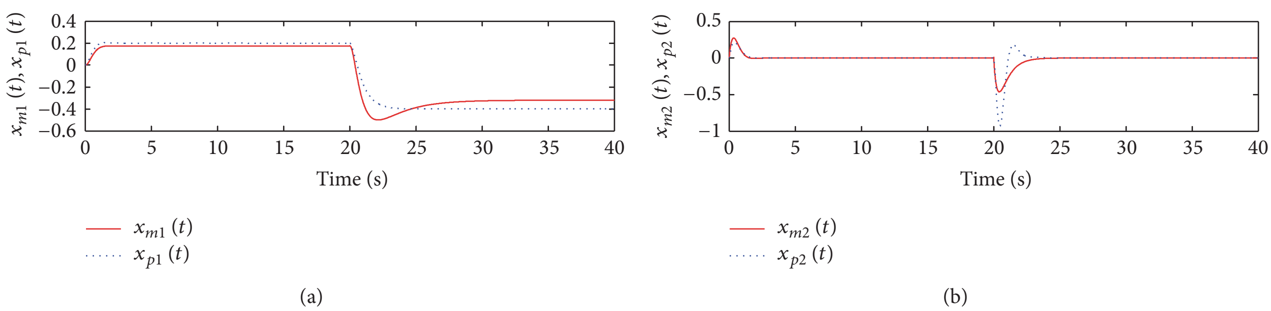

FIGURE 8: PID controller for piecewise linear system.

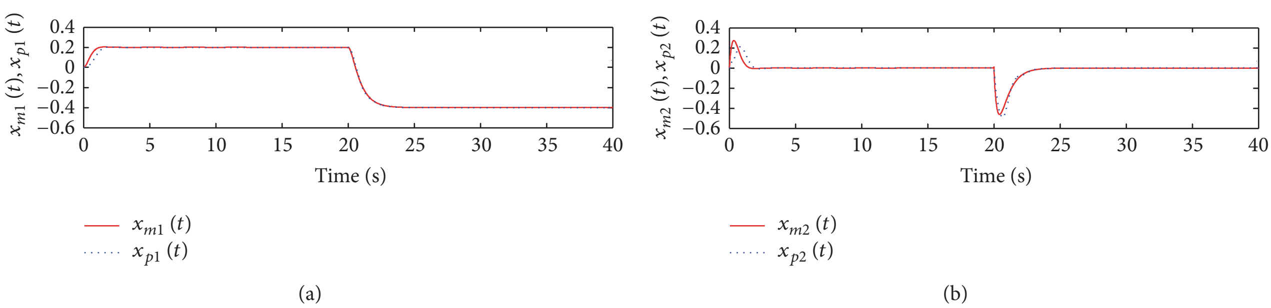

FIGURE 9: Switch controller for piecewise linear system.

$$P_{m1} = \begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix},$$
$$P_{m2} = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}. \tag{50}$$

$\varepsilon = 0.01$ is the switch point from PID controller to MRAC controller.

Notes. Figures 7–9 show the two plant states' tracking performance. Figures 7(a), 8(a), and 9(a) show x_{p1} track to x_{m1}. Figures 7(b), 8(b), and 9(b) show x_{p2} track to x_{m2}.

Notes. Figures 11–13 show the plant states tracking performance at the transient phase. Figures 11(a), 12(a), and 13(a) show x_{p1} track to x_{m1}. Figures 11(b), 12(b), and 13(b) show x_{p2} track to x_{m2}.

Case 1. Simulations are performed for the reference input signal $r(t) = \{1, t \le 20; -1, t > 20\}$; we can get the state tracking performance by only using MRAC controller in Figure 7, PID controller in Figure 8, and switch controller in Figure 9. Both the switch controller and MRAC controller can get stabile, but the MRAC controller needs longer time to make state x track x_m; PID controller needs less time for x to track x_m, but x cannot be equal to x_m in Figure 8, which is dangerous in practical operation because the actuator displacement cannot be controlled exactly. With comparison of the state tracking errors by using switch controller, MRAC, and PID in Figure 10, there is large deviation at the beginning of tracking stage if we use MRAC; especially, error will increase and will be larger by using MRAC controller when the plant's input changes.

The mean square errors contrasts are shown in Tables 3 and 4. There, we set the sample time as 0.01 s in simulation; Table 3 shows the mean square errors of Figures 7–9, and

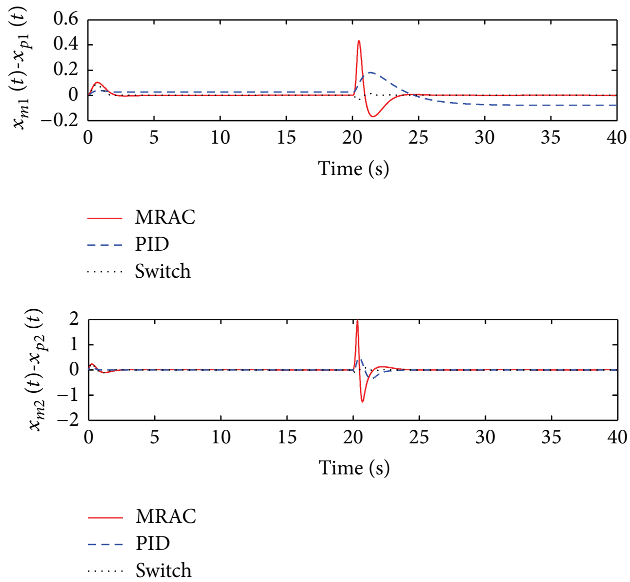

FIGURE 10: Errors controlled by switch controller, MRAC controller, and PID controller.

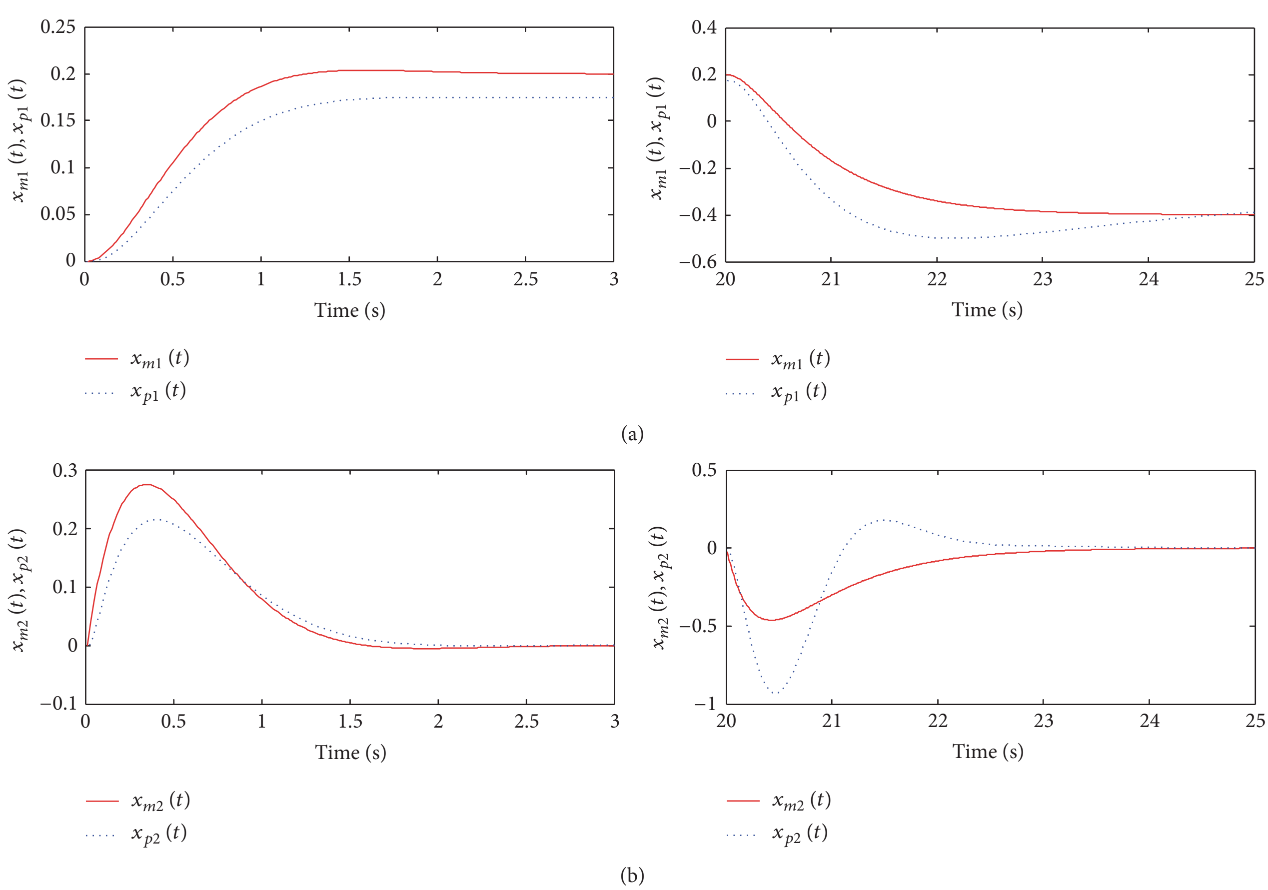

FIGURE 11: PID controller for piecewise linear system at the transient phase.

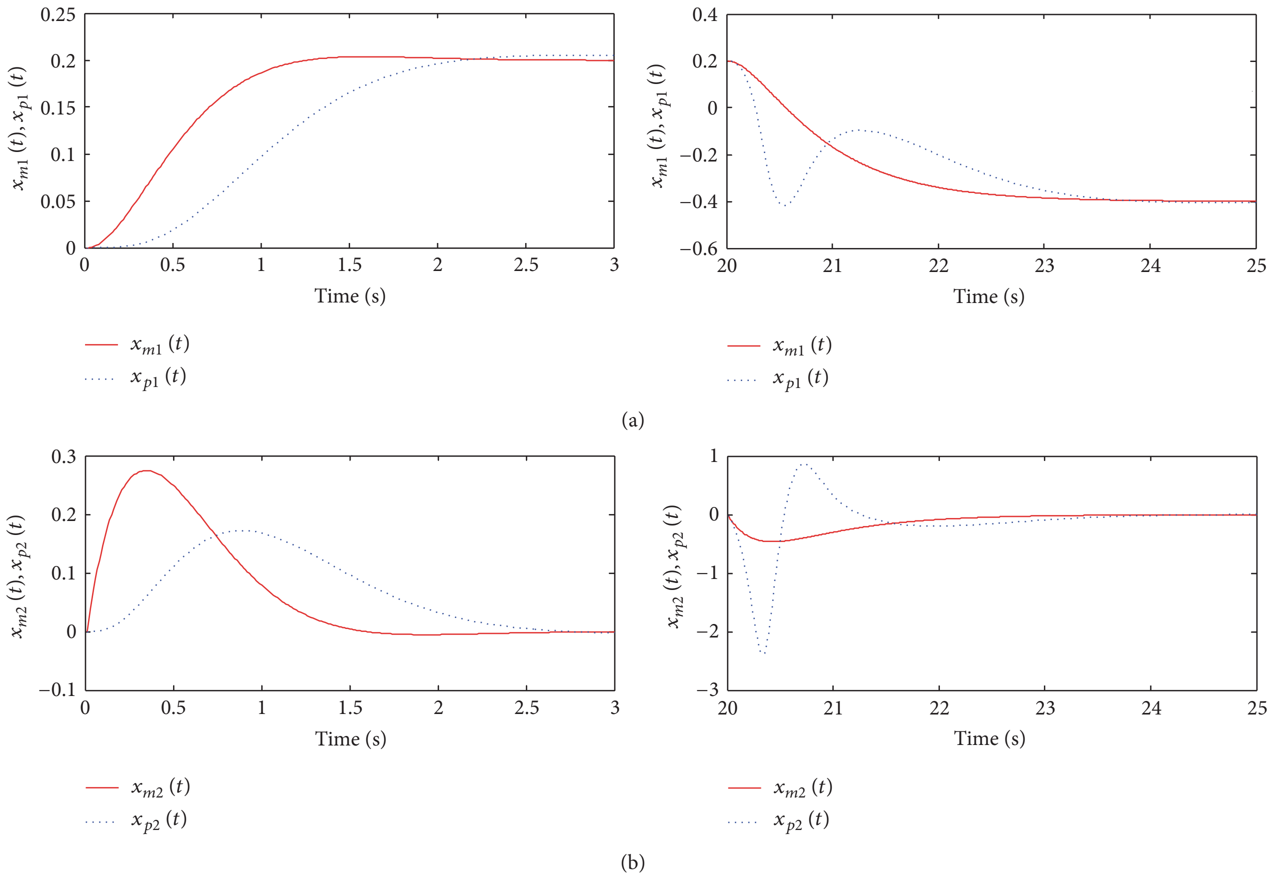

FIGURE 12: MRAC controller for piecewise linear system at the transient phase.

TABLE 3: Mean square error contrast.

	PID	MRAC	Switch
$f_{\mathrm{MSE}}(x_{m1} - x_{p1})$	0.616	0.0493	0.0015
$f_{\mathrm{MSE}}(x_{m2} - x_{p2})$	0.0655	0.1721	0.0266

$f_{\mathrm{MSE}}(x_{m1}(t) - x_{p1}(t))$ is the mean square error of x_{p1} to track x_{m1}, $f_{\mathrm{MSE}}(x_{m2}(t) - x_{p2}(t))$ is the mean square error of x_{p2} to track x_{m2}. From Table 3, we can get that the switch controller has better performance than PID controller and MRAC controller. Table 4 shows the system performance at transient phase controlled by PID, MRAC, and the switch controller. With Table 4, PID technique has smaller mean square errors than MRAC and switch techniques at the transient phase; at the same time, switch controller has smaller mean square errors when the input changes at $t = 20$ s.

Case 2. Simulations are performed for the reference input signal $r(t) = 10$. From Figures 14 and 15, we can get that x needs less time to track x_m. For the piecewise linear model, it takes about 1 second for x_{p1} to track x_{m1} and about 1.25 seconds for x_{p2} to track x_{m2} by using MRAC controller and more than that by using the switch controller, which takes about 0.5 seconds for x_{p1} to track x_{m1} and about 0.6 seconds for x_{p2} to track x_{m2}. We can get that the switch controller has better performance for the piecewise linear system than using MRAC controller. And the mean square errors are smaller, controlled by switch controller, shown in Table 5.

5. Conclusion

In this paper, the adaptive switch controller consisting of PID and MRAC controllers for piecewise linear systems is studied. The controller adaptively chooses the proportion of these two components and makes the designed system have faster response time and better tracking performance, simultaneously. The integrated controller and a state feedback control law $u(t)$ have been designed so that the closed-loop system is stable and $x(t)$ asymptotically tracks $x_m(t)$. Hydraulic actuator model was built and its piecewise linear characteristics were used to demonstrate the asymptotic tracking of $x_m(t)$ by piston displacement. Unlike conventional MRAC and PID controllers, the adaptive switch controller combines advantages of PID and MRAC; it makes the piecewise linear control have faster tracking response time and better performance, especially in oil production; it needs less time to control the actuator displacement and has less hydraulic shock.

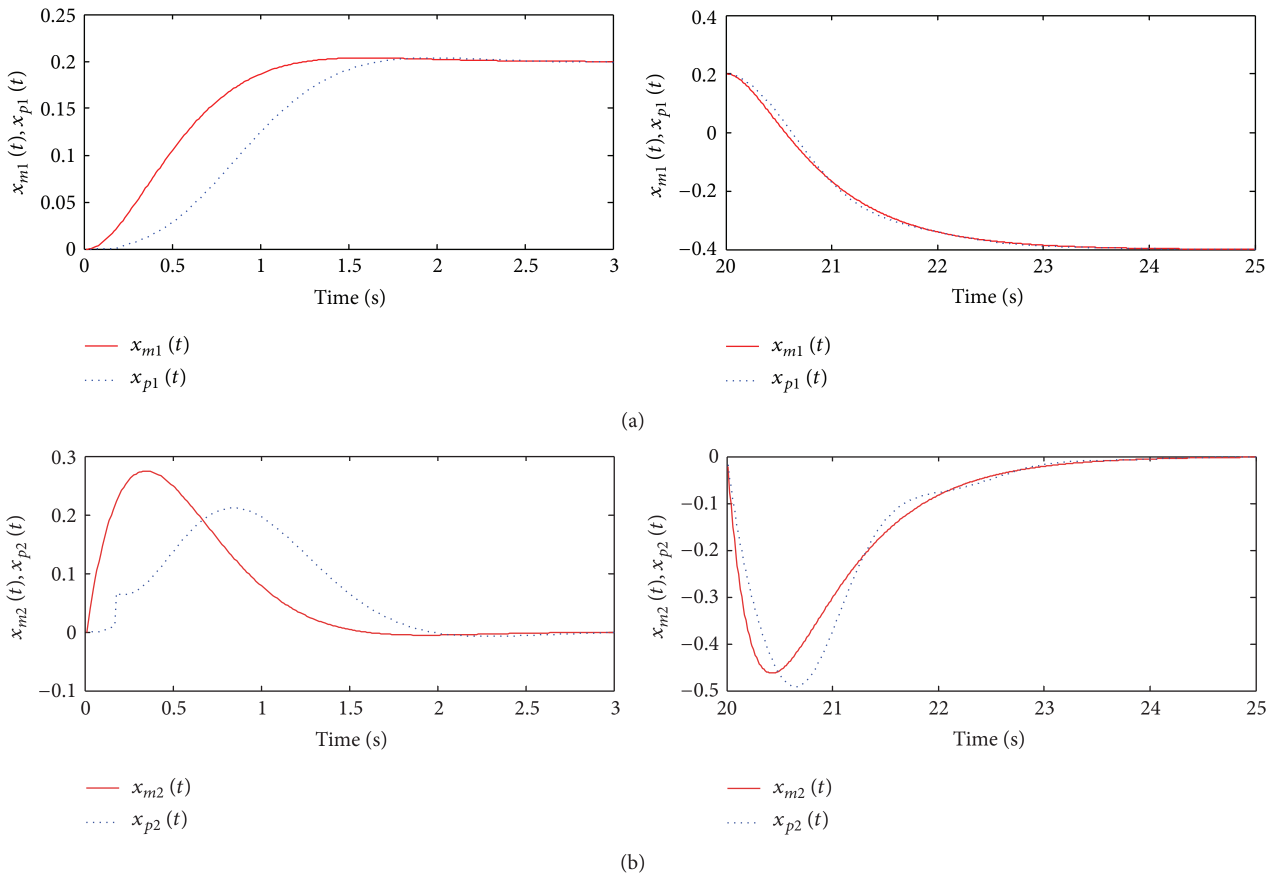

FIGURE 13: Switch controller for piecewise linear system at the transient phase.

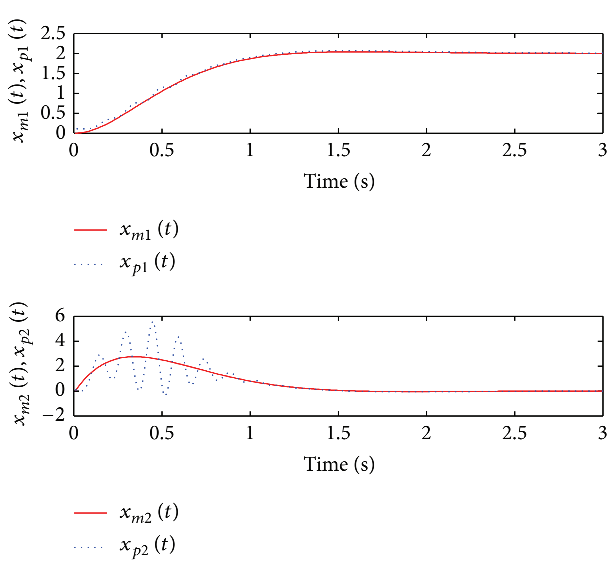

FIGURE 14: MRAC controller's response.

TABLE 4: Mean square error contrast at the transient phase.

	PID		MRAC		Switch	
Input signal	$r(t) = 1$	$r(t) = -1$	$r(t) = 1$	$r(t) = -1$	$r(t) = 1$	$r(t) = -1$
$f_{\mathrm{MSE}}(x_{m1} - x_{p1})$	0.0288	0.1101	0.0527	0.1333	0.0400	0.1040
$f_{\mathrm{MSE}}(x_{m2} - x_{p2})$	0.0277	0.0518	0.0946	0.4813	0.0850	0.0361

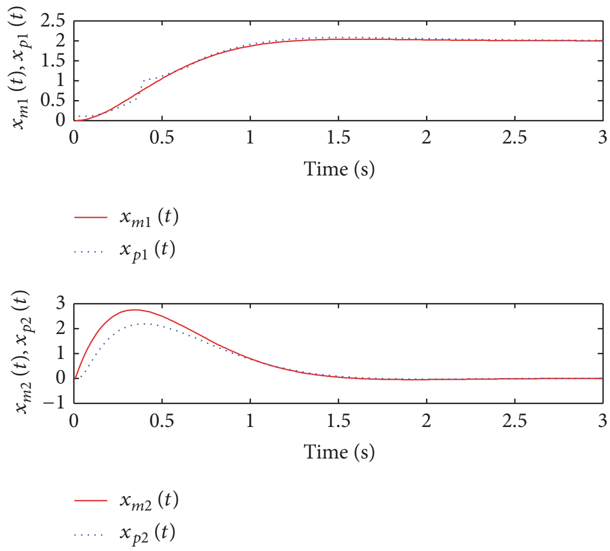

Figure 15: Switch controller's response time.

Table 5: Mean square error contrast.

	MRAC	SWITCH
$f_{\mathrm{MSE}}(x_{m1} - x_{p1})$	0.0499	0.0387
$f_{\mathrm{MSE}}(x_{m2} - x_{p2})$	0.7186	0.2753

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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