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Research Article

Disordered Stabilization of Stochastic Delay Systems: The Disorder-Dependent Approach

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In this paper, a general stabilization problem of stochastic delay systems is realized by a disordered controller and studied by exploiting the disorder-dependent approach. Different from the traditional results, the stabilizing controller here experiences a disorder between control gains and system states. Firstly, the above disorder is described by the robust method, whose probability distribution is embodied by a Markov process with two modes. Based on this description, a kind of disordered controller having special uncertainties and depending on a Markov process is proposed. Then, by exploiting a disorder-dependent Lyapunov functional, two respective conditions for the existence of such a disordered controller are provided with LMIs. Moreover, the presented results are further extended to a general case that the corresponding transition rate matrix of the disordered controller has uncertainties. Finally, a numerical example is exploited to demonstrate the effectiveness and superiority of the proposed methods.

1. Introduction

It is very known that time delay is commonly encountered in variously practical dynamical systems, such as chemical systems, heating systems, biological systems, networked control systems (NCSs), and telecommunication and economic systems. Due to the presence of time delay in such practical systems, many negative effects, for example, oscillation, instability, and poor performance, could be caused. Motivated by these facts, various research topics of time-delayed systems have been considered. By investigating the existing results, it is found that they are mainly classified into two classes: delay-dependent and delay-independent ones. Because of delay-dependent results making use of the information on the length of delays, they are less conservative than delay-independent ones, especially when time delay is small. During the past decades, many important results on all kinds of delay systems have emerged, such as stability analysis [1–9], stabilization [10–18], dissipativity analysis [19] and dissipative and passive control [20–22], output control [23, 24], H_∞ control and filtering [25–34], state estimation [35–37], synchronization [38–40], slide control [41, 42], and positivity analysis [43]. It is said that the more information about time delay is used, the less conservative results will be obtained. In order to achieve this aim, some methods or techniques applied to improved Lyapunov functionals are proposed and used, such as slack variable method [1, 5, 11], Jensen inequality algorithm [2, 4, 12], delay decomposition technique [3, 15, 26], and stochastic approach [13, 14, 17, 27], where the other results or methods can be found in the existing large references.

By investigating the most results on the system synthesis in the literature, it is seen that there were few references to consider the disordering problem. The motivation of disordering problem usually comes from the data transmitted through the shared communication networks. It is a phenomenon that the transmitted data arriving at the destination is usually out of order [44] and usually complicates its analysis and synthesis. During the past years, very few results were considered on this issue. Some new interesting and challenging problems could also been introduced. In [45], some LMI conditions were presented by exploiting a packet disordering compensation method. Based on transforming the underlying system into a discrete-time system with multistep delays, the stability and H_∞ control problems of NCSs with packet disordering were considered in [46, 47], while some less conservative results were given in [48]. Recently, a kind of packet reordering method based on the average dwell-time method was proposed in [49]. By investigating such references, it will be seen that the considered problems and studied methods between these references and this paper are quite different. Firstly, the considered systems between these references and this paper are different. The originally considered systems of such references are ones without any time delay, while there is time delay in our considered systems. Secondly, the places of disordering happening are different. In the above references, the disordering only exists in system states transmitted through networks, while the disordering to be considered in this paper takes place between system states and control gains. A suitable model to describe such problems correctly should be established firstly. Thirdly, but not the last, even if a suitable model is presented, how to make the existence conditions of the desired controller with solvable forms is also necessary studied. To our best knowledge, very few results are available to design a disordered controller for delay systems. All the facts motivate the current research.

In this paper, the general stabilization for a class of stochastic delay systems closed a disordered controller is studied by a disorder-dependent approach. The main contributions of this paper are summarized as follows: (1) a kind of stabilizing controller experiencing a disorder between control gains and system states is proposed. Not only is the disorder described by the robust method but also its probability distribution is expressed by a Markov process with two modes; (2) based on the established model of disordered controller, two different sufficient conditions for such a controller are presented with LMI forms, where a disorder-dependent approach in terms of depending on the Markov process is exploited; it was also shown by a numerical example that the conservatism of above conditions is not constant and should be considered on the concrete situations; (3) because of all the results being LMIs, they are further extended to another general case that the TRM describing the disorder has uncertainties.

Notation. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. $\mathbb{R}^{m\times n}$ is the set of all $m \times n$ real matrices. $\mathscr{E}[\cdot]$ means the mathematical expectation of $[\cdot]$. $\|\cdot\|$ refers to the Euclidean vector norm or spectral matrix norm. In symmetric block matrices, we use "$*$" as an ellipsis for the terms induced by symmetry, diag$\{\cdots\}$ for a block-diagonal matrix, and $(M)^\star \triangleq M + M^T$.

2. Problem Formulation

Consider a kind of stochastic delay systems described as

$$\begin{aligned} dx(t) &= [Ax(t) + A_\tau x(t-\tau) + Bu(t)]\,dt \\ &\quad + [Cx(t) + C_\tau x(t-\tau) + Du(t)]\,d\omega(t) \\ x(t) &= \phi(t),\ t \in [-\tau, 0], \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the system state, $u(t) \in \mathbb{R}^m$ is the control input, and $\omega(t)$ is a one-dimensional Brownian motion or Wiener process. Matrices A, A_τ, B, C, C_τ, and D are known matrices of compatible dimension. Time delay τ satisfies $\tau \geq 0$. $\phi(t)$ is a continuous function and defined from $[-\tau, 0]$ to $\mathbb{R}^n$.

As we know, the traditional state feedback controllers for delay systems are commonly described as follows:

$$u_1(t) = \overline{K}x(t), \tag{2}$$

$$u_2(t) = \overline{K}_\tau x(t-\tau), \tag{3}$$

$$u_3(t) = Kx(t) + K_\tau x(t-\tau), \tag{4}$$

where $\overline{K}$, $\overline{K}_\tau$, K, and K_τ are control gains to be determined. It is said that controller (4) compared to (2) and (3) is more general and has some advantages in terms of being less conservatism. The main reason is both delay and nondelay states are taken into account. However, it is said that the action of controller (4) needs an assumption that the control gains and theirs related states should be available in a right sequence. Unfortunately, due to some practice constraints, this assumption may be very hard satisfied. In this paper, a kind of controller experiencing disordering phenomenon is proposed and described by

$$u(t) = \begin{cases} Kx(t) + K_\tau x(t-\tau), & \text{no disordering} \\ K_\tau x(t) + Kx(t-\tau), & \text{disordering occurring.} \end{cases} \tag{5}$$

It is rewritten to be

$$\begin{aligned} u(t) &= \left(\overline{K} + \Delta\overline{K}(\eta_t)\right)x(t) \\ &\quad + \left(\overline{K}_\tau + \Delta\overline{K}_\tau(\eta_t)\right)x(t-\tau), \end{aligned} \tag{6}$$

where $\overline{K} = (1/2)(K + K_\tau)$ and $\overline{K}_\tau = (1/2)(K_\tau - K)$. Particularly, the process $\{\eta_t,\ t \geq 0\}$ introduced here is a Markov process having two modes and assumed to take values in a finite set $\mathbb{S} \triangleq \{1, 2\}$. Its transition rate matrix (TRM) $\Pi \triangleq (\pi_{ij}) \in \mathbb{R}^{2\times 2}$ is given by

$$\Pr(\eta_{t+\Delta t} = j \mid \eta_t = i) = \begin{cases} \pi_{ij}\Delta t + o(\Delta t), & i \neq j \\ 1 + \pi_{ii}\Delta t + o(\Delta t), & i = j, \end{cases} \tag{7}$$

where $\Delta t > 0$, $\pi_{ij} \geq 0$, if $i \neq j$, and $\pi_{ii} = -\sum_{j\neq i}\pi_{ij}$. Here, the gain fluctuations are selected to be

$$\begin{aligned} \Delta\overline{K}(\eta_t) &= \begin{cases} \dfrac{1}{2}(K - K_\tau), & \text{if } \eta_t = 1 \text{ or no disordering} \\ \dfrac{1}{2}(K_\tau - K), & \text{if } \eta_t = 2 \text{ or disordering occurring} \end{cases} \\ \Delta\overline{K}_\tau(\eta_t) &= \begin{cases} \dfrac{1}{2}(K + K_\tau), & \text{if } \eta_t = 1 \text{ or no disordering} \\ \dfrac{1}{2}(3K - K_\tau), & \text{if } \eta_t = 2 \text{ or disordering occurring.} \end{cases} \end{aligned} \tag{8}$$

Applying controller (6) to system (1), we get

$$\begin{aligned} dx(t) = {} & \left[\left(\overline{A} + B\Delta\overline{K}(\eta_t)\right)x(t) \right. \\ & \left. + \left(\overline{A}_\tau + B\Delta\overline{K}_\tau(\eta_t)\right)x(t-\tau)\right]dt \\ & + \left[\left(\overline{C} + D\Delta\overline{K}(\eta_t)\right)x(t) \right. \\ & \left. + \left(\overline{C}_\tau + D\Delta\overline{K}_\tau(\eta_t)\right)x(t-\tau)\right]d\omega(t) \\ & x(t) = \phi(t),\ \eta_t = \eta_0,\ \forall t \in [-\tau, 0], \end{aligned} \tag{9}$$

where

$$\begin{aligned} \overline{A} &= A + B\overline{K}, \\ \overline{A}_\tau &= A_\tau + B\overline{K}_\tau \\ \overline{C} &= C + D\overline{K}, \\ \overline{C}_\tau &= C_\tau + D\overline{K}_\tau. \end{aligned} \tag{10}$$

In this paper, the gain fluctuations $\Delta\overline{K}(\eta_t)$ and $\Delta\overline{K}_\tau(\eta_t)$ with forms (8) satisfy

$$\begin{aligned} \Delta\overline{K}^T(\eta_t)\Delta\overline{K}(\eta_t) &\le W, \\ \Delta\overline{K}_\tau^T(\eta_t)\Delta\overline{K}_\tau(\eta_t) &\le W_\tau(\eta_t), \end{aligned} \tag{11}$$

where W and $W_\tau(\eta_t)$ are positive-definite matrix to be determined.

Remark 1. Different from the traditional stabilization methods of delay systems that no disorder occurs in controllers, controller (5) has a disordering phenomenon between control gains and system states. Based on the robust method, the controller with disorder is transformed into a controller with special uncertainties. Moreover, the probabilities of nondisorder and disorder happening are described by a Markov process with two operation modes. It is seen that the proposed model (6) with conditions (8) and (11) is fundamental in the disorder-dependent approach.

Remark 2. It is said that disordered controller (5) not only is important in theory but also has the practical significance. From [44], it is known that disordering is usually encountered in practice especially in NCSs. This phenomenon makes the system analysis and synthesis very complicated. In order to deal with this practical problem, some methods have been proposed in [45–48]. Though such results are useful in studying this problem, more generally practical problem about disordering could be proposed. By investigating these references, it is seen that there is no delay in the originally considered systems. More importantly, the place of disorder occurring only exists in system states transmitted through networks. From [14, 50], it is seen that not only system state but also operation mode of controller could be transmitted through networks and suffer the effect of network. Based on these facts, it is said that the proposed model for controller having a disorder between control gain and system state is seen to be a meaningful extension of the existing results and will play important roles in the developments of theory and applications.

Definition 3. System (9) is said to be stochastically stable, if there exists a constant $M(x_0, \eta_0)$ such that

$$\mathscr{E}\left\{\int_0^\infty \|x(t)\|^2 dt \mid x_0, \eta_0\right\} \le M(x_0, \eta_0). \tag{12}$$

Lemma 4. *Let A, D, L, W, and F be real matrices of appropriate dimensions such that $W > 0$ and $F^TF \le I$. Then, for any $\varepsilon > 0$ such that $W - \varepsilon DD^T > 0$,*

$$\begin{aligned} &(A + DFL)^T W^{-1}(A + DFL) \\ &\quad \le A^T\left(W - \varepsilon DD^T\right)^{-1} A + \varepsilon^{-1} L^T L. \end{aligned} \tag{13}$$

3. Main Results

Theorem 5. *Given stochastic delay system (1), there is disordered controller (5) such that the resulting closed-loop system (9) is stochastically stable, if given positive scalars $\varepsilon_i > 0$, there exist matrices $X_i > 0$, $\overline{Q}_i > 0$, $\overline{Q} > 0$, $\overline{W} > 0$, $\overline{W}_{\tau i} > 0$, G, Y, and Y_τ, such that the following LMIs hold for all $i \in \mathbb{S}$:*

$$\begin{bmatrix} \Theta_{i1} & \Theta_{i2} & \Lambda_{i2} & \Lambda_{i3} & X_i & \tau^{1/2}X_i & \bar{\varepsilon}_i X_i & \Phi_i \\ * & \Theta_{i3} & 0 & \Lambda_{i3} & 0 & 0 & 0 & 0 \\ * & * & \Theta_{i4} & \Lambda_{i4} & 0 & 0 & 0 & 0 \\ * & * & * & \Theta_{i5} & 0 & 0 & 0 & 0 \\ * & * & * & * & \Theta_{i6} & 0 & 0 & 0 \\ * & * & * & * & * & \Theta_{i7} & 0 & 0 \\ * & * & * & * & * & * & \Theta_{i8} & 0 \\ * & * & * & * & * & * & * & \Gamma_i \end{bmatrix} < 0 \tag{14}$$

$$\sum_{j=1}^{2} \pi_{ij}\overline{Q}_j \le \overline{Q} \tag{15}$$

$$\begin{bmatrix} -\overline{W}_{\tau i} & \overline{Y}_i^T \\ * & -I \end{bmatrix} \le 0 \tag{16}$$

$$\begin{bmatrix} -\overline{W} & Y^T - Y_\tau^T \\ * & -I \end{bmatrix} \le 0, \tag{17}$$

where

$$\begin{aligned} \Theta_{i1} &= (\Lambda_{i1})^\star + \pi_{ii}X_i + 2BB^T, \\ \Theta_{i2} &= \Lambda_{i1} + X_i - G^T, \\ \Theta_{i3} &= (-G)^\star, \end{aligned}$$

$$\begin{aligned}
\Theta_{i4} &= -\overline{Q}_i + \overline{\varepsilon}_i^2 \overline{W}_{\tau i}, \\
\Theta_{i5} &= -\left(X_i - \varepsilon_i \widetilde{D}\widetilde{D}^T\right), \\
\Theta_{i6} &= \Theta_{i3} + \overline{Q}_i, \\
\Theta_{i7} &= \Theta_{i3} + \overline{Q}, \\
\Theta_{i8} &= \Theta_{i3} + \overline{W}, \\
\Lambda_{i1} &= AG + B\left(Y + Y_\tau\right), \\
\Lambda_{i2} &= A_\tau G + B\left(Y_\tau - Y\right), \\
\Lambda_{i3} &= G^T C^T + \left(Y + Y_\tau\right)^T D^T, \\
\Lambda_{i4} &= G^T C_\tau^T + \left(Y_\tau - Y\right)^T D^T, \\
\Phi_1 &= \sqrt{\pi_{12}} X_1, \\
\Phi_2 &= \sqrt{\pi_{21}} X_2, \\
\Gamma_1 &= -X_2, \\
\Gamma_2 &= -X_1, \\
\overline{\varepsilon}_i &= \left(1 + \varepsilon_i^{-1}\right)^{1/2}, \\
\overline{Y}_1 &= Y + Y_\tau, \\
\overline{Y}_2 &= 3Y - Y_\tau.
\end{aligned} \tag{18}$$

Then, the gains of disordered controller with form (5) or (6) are obtained by

$$\begin{aligned}
K &= 2YG^{-1}, \\
K_\tau &= 2Y_\tau G^{-1}.
\end{aligned} \tag{19}$$

Proof. Choose a stochastic Lyapunov functional for the closed-loop system (9) as

$$\begin{aligned}
V\left(x_t, \eta_t\right) &= x^T(t) P\left(\eta_t\right) x(t) \\
&\quad + \int_{t-\tau}^{t} x^T(s) Q\left(\eta_t\right) x(s)\, ds \\
&\quad + \int_{-\tau}^{0} \int_{t+\theta}^{t} x^T(s) Q x(s)\, ds.
\end{aligned} \tag{20}$$

Let $\mathscr{L}$ be the weak infinitesimal generator of random process $\{x_t, \eta_t\}$ for each $\eta_t = i \in \mathbb{S}$; it is defined as

$$\begin{aligned}
&\mathscr{L}V\left(x_t, t, i\right) \\
&= \lim_{\Delta t \to 0^+} \frac{1}{\Delta t} \left\{\mathscr{E}\left[V\left(x_{t+\Delta t}, \eta_{t+\Delta t}, t + \Delta t\right) \mid x_t, \eta_t = i\right]\right. \\
&\quad \left. - V\left(x_t, i, t\right)\right\}.
\end{aligned} \tag{21}$$

Then, based on condition (11) and Lemma 4, it is obtained that

$$\begin{aligned}
&\mathscr{L}V\left(x_t, t, i\right) = x^T(t)\left(P_i\overline{A}\right)^* x(t) + x^T(t) \\
&\quad \cdot \sum_{j=1}^{2} \pi_{ij} P_j x(t) + 2x^T(t) P_i \overline{A}_\tau x(t-\tau) + 2x^T(t) \\
&\quad \cdot P_i B \Delta\overline{K}_i x(t) + 2x^T(t) P_i B \Delta\overline{K}_{\tau i} x(t-\tau) \\
&\quad + \left[\left(\overline{C} + D\Delta\overline{K}_i\right) x(t) + \left(\overline{C}_\tau + D\Delta\overline{K}_{\tau i}\right) x(t-\tau)\right]^T \\
&\quad \times P_i \left[\left(\overline{C} + D\Delta\overline{K}_i\right) x(t) + \left(\overline{C}_\tau + D\Delta\overline{K}_{\tau i}\right) x(t-\tau)\right] \\
&\quad + x^T(t) Q_i x(t) - x^T(t-\tau) Q_i x(t-\tau) \\
&\quad + \int_{t-\tau}^{t} x^T(s) \sum_{j=1}^{N} \pi_{ij} Q_j x(s)\, ds + \tau x^T(t) Q x(t) \\
&\quad - \int_{t-\tau}^{t} x^T(s) Q x(s)\, ds = x^T(t) \\
&\quad \cdot \left[\left(P_i\overline{A}\right)^* + \sum_{j=1}^{2} \pi_{ij} P_j + Q_i + \tau Q\right] x(t) + 2x^T(t) \\
&\quad \cdot P_i \overline{A}_\tau x(t-\tau) + 2x^T(t) P_i B \Delta\overline{K}_i x(t) + 2x^T(t) \\
&\quad \cdot P_i B \Delta\overline{K}_{\tau i} x(t-\tau) \\
&\quad + \left[\left(\overline{C} + D\Delta\overline{K}_i\right) x(t) + \left(\overline{C}_\tau + D\Delta\overline{K}_{\tau i}\right) x(t-\tau)\right]^T \\
&\quad \times P_i \left[\left(\overline{C} + D\Delta\overline{K}_i\right) x(t) + \left(\overline{C}_\tau + D\Delta\overline{K}_{\tau i}\right) x(t-\tau)\right] \\
&\quad - x^T(t-\tau) Q_i x(t-\tau) \\
&\quad + \int_{t-\tau}^{t} x^T(s) \left(\sum_{j=1}^{N} \pi_{ij} Q_j - Q\right) x(s)\, ds = \xi^T(t) \\
&\quad \cdot \left[\left(\widetilde{C} + \widetilde{D}\Delta\widetilde{K}_i\right)^T P_i \left(\widetilde{C} + \widetilde{D}\Delta\widetilde{K}_i\right) + \Omega_i\right] \xi(t) \\
&\quad + 2x^T(t) P_i B \Delta\overline{K}_i x(t) + 2x^T(t) P_i B \Delta\overline{K}_{\tau i} x(t-\tau) \\
&\quad + \int_{t-\tau}^{t} x^T(s) \left(\sum_{j=1}^{2} \pi_{ij} Q_j - Q\right) x(s)\, ds \le \xi^T(t) \\
&\quad \cdot \left[\widetilde{C}^T \left(P_i^{-1} - \varepsilon_i \widetilde{D}\widetilde{D}^T\right)^{-1} \widetilde{C} + \varepsilon_i^{-1} \widetilde{W} + \Omega_i\right] \xi(t) \\
&\quad + \xi^T(t) \begin{bmatrix} 2P_i B B^T P_i + W & 0 \\ * & W_{\tau i} \end{bmatrix} \xi(t) \\
&\quad + \int_{t-\tau}^{t} x^T(s) \left(\sum_{j=1}^{2} \pi_{ij} Q_j - Q\right) x(s)\, ds < 0,
\end{aligned} \tag{22}$$

where

$$\xi(t)=\begin{bmatrix} x(t) \\ x(t-\tau) \end{bmatrix},$$
$$\Omega_i=\begin{bmatrix} \Omega_{i1} & P_i\overline{A}_\tau \\ * & -Q_i \end{bmatrix},$$
$$\widetilde{C}=\begin{bmatrix} \overline{C} & \overline{C}_\tau \end{bmatrix},$$
$$\widetilde{D}=\begin{bmatrix} D & D \end{bmatrix}, \tag{23}$$
$$\Delta\widetilde{K}_i=\begin{bmatrix} \Delta\overline{K}_i & 0 \\ * & \Delta\overline{K}_{\tau i} \end{bmatrix},$$
$$\widetilde{W}=\begin{bmatrix} W & 0 \\ * & W_{\tau i} \end{bmatrix},$$
$$\Omega_{i1}=\left(P_i\overline{A}\right)^*+\sum_{j=1}^{N}\pi_{ij}P_j+Q_i+\tau Q.$$

From condition (11), it is implied by

$$\widetilde{C}^T\left(P_i^{-1}-\varepsilon_i\widetilde{D}\widetilde{D}^T\right)^{-1}\widetilde{C}+\widetilde{\Omega}_i<0, \tag{24}$$

which is equivalent to

$$\begin{bmatrix} \widetilde{\Omega}_i & \widetilde{C}^T \\ * & -\left(P_i^{-1}-\varepsilon_i\widetilde{D}\widetilde{D}^T\right) \end{bmatrix}<0, \tag{25}$$

$$\sum_{j=1}^{2}\pi_{ij}Q_j\leq Q, \tag{26}$$

where

$$\widetilde{\Omega}_i=\begin{bmatrix} \widetilde{\Omega}_{i1} & P_i\overline{A}_\tau \\ * & -Q_i+\overline{\varepsilon}_i^2W_{\tau i} \end{bmatrix},$$
$$\widetilde{\Omega}_{i1}=\left(P_i\overline{A}\right)^*+\sum_{j=1}^{2}\pi_{ij}P_j+Q_i+\tau Q+2P_iBB^TP_i +\overline{\varepsilon}_i^2W. \tag{27}$$

From (14), one concludes that G is nonsingular. Then, it is known that condition (25) is equivalent to

$$\begin{bmatrix} \widehat{\Omega}_{i1} & \overline{A}_\tau G & X_i\overline{C}^T \\ * & -\overline{Q}_i+\overline{\varepsilon}_i^2\overline{W}_{\tau i} & G^T\overline{C}_\tau^T \\ * & * & -\left(X_i-\varepsilon_i\widetilde{D}\widetilde{D}^T\right) \end{bmatrix}<0, \tag{28}$$

where

$$\widehat{\Omega}_{i1}=\left(\overline{A}X_i\right)^*+\sum_{j=1}^{2}\pi_{ij}X_iP_jX_i+X_iQ_iX_i+\tau X_iQX_i +2BB^T+\overline{\varepsilon}_i^2X_iWX_i,$$
$$X_i=P_i^{-1}, \tag{29}$$
$$\overline{Q}_i=G^TQ_iG,$$
$$\overline{W}=G^TWG,$$
$$\overline{W}_{\tau i}=G^TW_{\tau i}G,$$

which is obtained by pre- and postmultiplying both sides of (25) with $\operatorname{diag}\{X_i, G^T, I\}$ and its transpose, respectively. Moreover, it is further implied by

$$\begin{bmatrix} \check{\Omega}_{i1} & \check{\Omega}_{i2} & \overline{A}_\tau G & G^T\overline{C}^T \\ * & \Theta_{i3} & 0 & G^T\overline{C}^T \\ * & * & -\overline{Q}_i+\overline{\varepsilon}_i^2\overline{W}_{\tau i} & G^T\overline{C}_\tau^T \\ * & * & * & -\left(X_i-\varepsilon_i\widetilde{D}\widetilde{D}^T\right) \end{bmatrix}<0, \tag{30}$$

where

$$\check{\Omega}_{i1}=\left(\overline{A}G\right)^*+\sum_{j=1}^{2}\pi_{ij}X_iP_jX_i+X_iQ_iX_i+\tau X_iQX_i +2BB^T+\overline{\varepsilon}_i^2X_iWX_i, \tag{31}$$
$$\check{\Omega}_{i2}=\overline{A}G+X_i-G^T,$$
$$\Theta_{i3}=(-G)^*,$$

via pre- and postmultiplying it with the following matrix

$$\begin{bmatrix} I & \overline{A} & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & \overline{C} & 0 & I \end{bmatrix} \tag{32}$$

and its transpose. Taking into the representation (19), condition (30) is equivalent to

$$\begin{bmatrix} \Theta_{i1} & \Theta_{i2} & \Lambda_{i2} & \Lambda_{i3} & X_i & \tau^{1/2}X_i & \overline{\varepsilon}_iX_i & \Phi_i \\ * & \Theta_{i3} & 0 & \Lambda_{i3} & 0 & 0 & 0 & 0 \\ * & * & \Theta_{i4} & \Lambda_{i4} & 0 & 0 & 0 & 0 \\ * & * & * & \Theta_{i5} & 0 & 0 & 0 & 0 \\ * & * & * & * & -Q_i^{-1} & 0 & 0 & 0 \\ * & * & * & * & * & -Q^{-1} & 0 & 0 \\ * & * & * & * & * & * & -W^{-1} & 0 \\ * & * & * & * & * & * & * & \Gamma_i \end{bmatrix}<0. \tag{33}$$

As for $-Q_i^{-1}$, based on representation $\overline{Q}_i = G^T Q_i G$, it is known that

$$-Q_i^{-1} = -G\left(G^T Q_i G\right)^{-1} G^T \le \Theta_{i3} + \overline{Q}_i, \tag{34}$$

while $-Q^{-1}$ and $-W^{-1}$ can be done similarly with representation $\overline{Q} = G^T QG$ and $\overline{W} = G^T WG$. Based on (34), we have (14) implying (33). As for condition (26), based on the above representations, it is claimed that it is equivalent to (15). Moreover, from the proof of this theorem, it is seen that condition (11) is important, which is equivalent to

$$\begin{bmatrix} -W & \left(\Delta\overline{K}(\eta_t)\right)^T \\ * & -I \end{bmatrix} \le 0, \tag{35}$$

or

$$\begin{bmatrix} -W_{\tau i} & \left(\Delta\overline{K}_\tau(\eta_t)\right)^T \\ * & -I \end{bmatrix} \le 0. \tag{36}$$

Because of $\overline{W} = G^T WG$ and considering representation (19), it is obvious that either of them is equivalent to (16) and (17). This completes the proof. □

Remark 6. Based on the disordered controller (6), a stochastic Lyapunov functional depending on system mode η_t is exploited. It is different from the traditional methods in which a single or common Lyapunov functional is used. Because of the Markov process coming from the disorder, the adopted Lyapunov functional is said to be disorder-dependent, which could reduce the conservatism of common or disorder-independent Lyapunov functional. It is worth mentioning that the introduced stochastic Lyapunov functional (20) is not unique, which could be replaced by others similarly. In addition, the existing improved techniques such as slack variable method, Jensen inequality approach, and delay decomposition technique may be used in the process of controller design.

Theorem 7. *Given stochastic delay system (1), there is disordered controller (5) such that the resulting closed-loop system (9) is stochastically stable, if given positive scalars $\beta_i > 0$, there exist matrices $X_i > 0$, $\overline{Q}_i > 0$, $\overline{Q} > 0$, $\overline{W} > 0$, $\overline{W}_{\tau i} > 0$, G, Y, and Y_τ satisfying conditions (15), (16), (17), and*

$$\begin{bmatrix} \Theta_{i1} & \Theta_{i2} & \Lambda_{i2} & \overline{\Lambda}_{i3} & X_i & \tau^{1/2}X_i & \overline{\beta}_i X_i & \Phi_i \\ * & \Theta_{i3} & 0 & \overline{\Lambda}_{i3} & 0 & 0 & 0 & 0 \\ * & * & \overline{\Theta}_{i4} & \overline{\Lambda}_{i4} & 0 & 0 & 0 & 0 \\ * & * & * & -X_i & 0 & 0 & 0 & 0 \\ * & * & * & * & \Theta_{i6} & 0 & 0 & 0 \\ * & * & * & * & * & \Theta_{i7} & 0 & 0 \\ * & * & * & * & * & * & \Theta_{i8} & 0 \\ * & * & * & * & * & * & * & \Gamma_i \end{bmatrix} < 0, \tag{37}$$

$$\begin{bmatrix} -\beta_i I & D^T \\ * & -X_i \end{bmatrix} \le 0, \tag{38}$$

where

$$\begin{aligned} \overline{\Theta}_{i4} &= -\overline{Q}_i + \overline{\beta}_i^2 \overline{W}_{\tau i}, \\ \overline{\Lambda}_{i3} &= \sqrt{2}\Lambda_{i3}, \\ \overline{\Lambda}_{i4} &= \sqrt{2}\Lambda_{i4}, \\ \overline{\beta}_i &= \left(1 + 4\beta_i\right)^{1/2}. \end{aligned} \tag{39}$$

The other symbols are given in Theorem 5. Then, the gains of controller (5) are computed by (19).

Proof. Choosing the same Lyapunov functional (20), similar to the computation of (22), it is obtained that

$$\begin{aligned} \mathscr{L}V(x_t, t, i) &= x^T(t)\left[\left(P_i\overline{A}\right)^* + \sum_{j=1}^{2}\pi_{ij}P_j + Q_i + \tau Q\right] \\ &\cdot x(t) + 2x^T(t)P_i\overline{A}_\tau x(t-\tau) + 2x^T(t) \\ &\cdot P_i B\Delta\overline{K}_i x(t) + 2x^T(t)P_i B\Delta\overline{K}_{\tau i}x(t-\tau) + \xi^T(t) \\ &\cdot \left(\widetilde{C} + \widetilde{D}\Delta\widetilde{K}_i\right)^T P_i\left(\widetilde{C} + \widetilde{D}\Delta\widetilde{K}_i\right)\xi^T(t) - x^T(t-\tau) \\ &\cdot Q_i x(t-\tau) + \int_{t-\tau}^{t} x^T(s)\left(\sum_{j=1}^{N}\pi_{ij}Q_j - Q\right)x(s)\,ds \\ &= \xi^T(t)\left[\Omega_i + \widetilde{C}^T P_i\widetilde{C} + 2\widetilde{C}^T P_i\widetilde{D}\Delta\widetilde{K}_i\right. \\ &\left. + \Delta\widetilde{K}_i^T\widetilde{D}^T P_i\widetilde{D}\Delta\widetilde{K}_i\right]\xi(t) + 2x^T(t)P_i B\Delta\overline{K}_i x(t) \\ &+ 2x^T(t)P_i B\Delta\overline{K}_{\tau i}x(t-\tau) \\ &+ \int_{t-\tau}^{t} x^T(s)\left(\sum_{j=1}^{2}\pi_{ij}Q_j - Q\right)x(s)\,ds \le \xi^T(t) \\ &\cdot \left[\Omega_i + \operatorname{diag}\left\{2P_iBB^TP_i + W, W_{\tau i}\right\} + \widetilde{C}^T P_i\widetilde{C}\right. \\ &\left. + 2\widetilde{C}^T P_i\widetilde{D}\Delta\widetilde{K}_i + \Delta\widetilde{K}_i^T\widetilde{D}^T P_i\widetilde{D}\Delta\widetilde{K}_i\right]\xi(t) \\ &+ \int_{t-\tau}^{t} x^T(s)\left(\sum_{j=1}^{2}\pi_{ij}Q_j - Q\right)x(s)\,ds < 0 \end{aligned} \tag{40}$$

which is guaranteed by

$$\begin{aligned} &\Omega_i + \operatorname{diag}\left\{2P_iBB^TP_i + W, W_{\tau i}\right\} + \widetilde{C}^T P_i\widetilde{C} \\ &\quad + 2\widetilde{C}^T P_i\widetilde{D}\Delta\widetilde{K}_i + \Delta\widetilde{K}_i^T\widetilde{D}^T P_i\widetilde{D}\Delta\widetilde{K}_i < 0, \end{aligned} \tag{41}$$

and condition (26). From the representations of $\widetilde{C}$, $\widetilde{D}$, and $\Delta\widetilde{K}_i$ and the following condition

$$-\beta_i I + D^T P_i D \le 0, \tag{42}$$

condition (41) is guaranteed by

$$\begin{aligned}
&\Omega_i + \operatorname{diag}\left\{2P_iBB^TP_i + W, W_{\tau i}\right\} + 2\widetilde{C}^TP_i\widetilde{C} \\
&\quad + 2\Delta\widetilde{K}_i^T\widetilde{D}^TP_i\widetilde{D}\Delta\widetilde{K}_i \le \Omega_i \\
&\quad + \operatorname{diag}\left\{2P_iBB^TP_i + W, W_{\tau i}\right\} + 2\widetilde{C}^TP_i\widetilde{C} \\
&\quad + 2\Delta\widetilde{K}_i^T\widehat{I}^TD^TP_iD\widehat{I}\Delta\widetilde{K}_i \le \Omega_i \\
&\quad + \operatorname{diag}\left\{2P_iBB^TP_i + W, W_{\tau i}\right\} + 2\widetilde{C}^TP_i\widetilde{C} \\
&\quad + 2\beta_i\Delta\widetilde{K}_i^T\widehat{I}^T\widehat{I}\Delta\widetilde{K}_i \le \Omega_i \\
&\quad + \operatorname{diag}\left\{2P_iBB^TP_i + \overline{\beta}_i^2W, \overline{\beta}_i^2W_{\tau i}\right\} + 2\widetilde{C}^TP_i\widetilde{C} < 0,
\end{aligned} \tag{43}$$

where $\widehat{I} = [I \;\; I]$ and $\overline{\beta}_i = (1 + 4\beta_i)^{1/2}$. Similar to the proof of (28) implying (25), it is obtained that condition (43) is guaranteed by

$$\begin{bmatrix} \breve{\Omega}_{i1} & \overline{A}_\tau G & \sqrt{2}X_i\overline{C}^T \\ * & -\overline{Q}_i + \overline{\beta}_i^2\overline{W}_{\tau i} & \sqrt{2}G^T\overline{C}_\tau^T \\ * & * & -X_i \end{bmatrix} < 0, \tag{44}$$

where

$$\begin{aligned}
\breve{\Omega}_{i1} &= \left(\overline{A}X_i\right)^* + \sum_{j=1}^{2}\pi_{ij}X_iP_jX_i + X_iQ_iX_i + \tau X_iQX_i \\
&\quad + 2BB^T + \overline{\beta}_i^2X_iWX_i.
\end{aligned} \tag{45}$$

The next process is similar to the process of (28) guaranteed by condition (14), which is omitted here. As for condition (42), it is known that it is equivalent to (38). Similar to the proof of Theorem 5, conditions (11) and (34) are also needed. This completes the proof. □

Remark 8. From the forms of Theorems 5 and 7, one cannot conclude which one is less conservative. However, in the following numerical example, it is seen that sometimes the former theorem is less conservative, while sometimes the latter one has less conservatism. Thus, it is concluded that it is impossible to claim that either of them is less conservative and should be considered in the concrete situations.

From the established result, it is seen that TRM Π related to disorder is assumed to be exact. However, due to many practical situations in practice, it is impossible or high cost to get it exactly. Instead, it has admissible uncertainty $\Delta\widetilde{\Pi} \triangleq (\Delta\widetilde{\pi}_{ij})$ and is described as

$$\begin{gathered}
\Pi = \widetilde{\Pi} + \Delta\widetilde{\Pi} \\
\text{with } \left|\Delta\widetilde{\pi}_{ij}\right| \le \epsilon_{ij}, \quad \epsilon_{ij} \ge 0, \;\; j \ne i.
\end{gathered} \tag{46}$$

In (46), TRM $\widetilde{\Pi} \triangleq (\widetilde{\pi}_{ij})$ is the known constant estimation of Π with $\widetilde{\pi}_{ij}$ satisfying (7). It is assumed that $\Delta\widetilde{\pi}_{ij}$, $j \ne i$, takes any value in $[-\epsilon_{ij}, \epsilon_{ij}]$. Then, it is concluded that $|\Delta\widetilde{\pi}_{ii}| \le -\epsilon_{ii}$, where $\epsilon_{ii} \triangleq -\epsilon_{ij}$, $\alpha_{ij} \triangleq \widetilde{\pi}_{ij} - \epsilon_{ij}$, and $\alpha_{ii} \triangleq -\alpha_{ij}$, $\forall j \ne i$.

Theorem 9. *Given stochastic delay system (1), there is disordered controller (5) such that the resulting closed-loop system (9) is stochastically stable over uncertainty (46), if given positive scalars $\varepsilon_i > 0$, if there exist matrices $X_i > 0$, $\overline{Q}_i > 0$, $\overline{Q} > 0$, $\overline{W}_i > 0$, $\overline{W}_{\tau i} > 0$, $T_i > 0$, $S_i > 0$, $M_i = M_i^T$, $V_i = V_i^T$, Y, and Y_τ satisfying conditions (16), (17), and*

$$\begin{bmatrix}
\overline{\Theta}_{i1} & \Theta_{i2} & \Lambda_{i2} & \Lambda_{i3} & X_i & \tau^{1/2}X_i & \overline{\varepsilon}_iX_i & \overline{\Phi}_i & M_i \\
* & \Theta_{i3} & 0 & \Lambda_{i3} & 0 & 0 & 0 & 0 & 0 \\
* & * & \Theta_{i4} & \Lambda_{i4} & 0 & 0 & 0 & 0 & 0 \\
* & * & * & \Theta_{i5} & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & \Theta_{i6} & 0 & 0 & 0 & 0 \\
* & * & * & * & * & \Theta_{i7} & 0 & 0 & 0 \\
* & * & * & * & * & * & \Theta_{i8} & 0 & 0 \\
* & * & * & * & * & * & * & \Gamma_i & 0 \\
* & * & * & * & * & * & * & * & -T_i
\end{bmatrix} < 0, \tag{47}$$

$$\begin{bmatrix} \Theta_{i9} & V_i \\ * & -S_i \end{bmatrix} < 0, \tag{48}$$

$$\begin{bmatrix} -X_i - M_i & X_i \\ * & -X_j \end{bmatrix} \le 0, \quad j \ne i \tag{49}$$

$$\overline{Q}_j - \overline{Q}_i - V_i < 0, \quad j \ne i, \tag{50}$$

where

$$\begin{aligned}
\overline{\Theta}_{i1} &= \left(\Lambda_{i1}\right)^* + \alpha_{ii}X_i + 2BB^T - \epsilon_{ii}M_i + 0.25\epsilon_{ii}^2T_i, \\
\overline{\Phi}_1 &= \sqrt{\alpha_{12}}X_1, \\
\overline{\Phi}_2 &= \sqrt{\alpha_{21}}X_2, \\
\Theta_{i9} &= -\overline{Q} - \epsilon_{ii}V_i + 0.25\epsilon_{ii}^2S_i + \sum_{j=1}^{2}\alpha_{ij}\overline{Q}_j.
\end{aligned} \tag{51}$$

The other symbols are given in Theorem 5. Then, the gains of controller (5) can be get by (19).

Proof. From the proof of Theorem 5, it is seen that only some terms are related to uncertainty (46), which are referred to be (26) and (30), respectively. As for $\breve{\Omega}_{i1}$ in (30), it is equivalent to

$$\begin{aligned}
&\left(\overline{A}G\right)^* + X_iQ_iX_i + \tau X_iQX_i + 2BB^T + \overline{\varepsilon}_i^2X_iWX_i \\
&\quad + \sum_{j=1}^{2}\left(\widetilde{\pi}_{ij} + \Delta\widetilde{\pi}_{ij}\right)X_iP_jX_i - \sum_{j=1}^{2}\epsilon_{ij}X_iP_jX_i \\
&\quad + \sum_{j=1}^{2}\epsilon_{ij}X_iP_jX_i - \sum_{j=1}^{2}\left(\Delta\widetilde{\pi}_{ij} + \epsilon_{ij}\right)M_i
\end{aligned}$$

$$\begin{aligned} &= \left(\overline{A}G\right)^{*} + X_iQ_iX_i + \tau X_iQX_i + 2BB^T + \bar{\varepsilon}_i^2 X_iWX_i \\ &\quad + \sum_{j=1}^{2}\alpha_{ij}X_iP_jX_i - \left(\Delta\tilde{\pi}_{ii} + \epsilon_{ii}\right)M_i \\ &\quad + \sum_{j\neq i}\left(\Delta\tilde{\pi}_{ij} + \epsilon_{ij}\right)\left(X_iP_jX_i - X_i - M_i\right), \end{aligned} \tag{52}$$

where $M_i = M_i^T$, which is guaranteed by

$$\begin{aligned} &\left(\overline{A}G\right)^{*} + X_iQ_iX_i + \tau X_iQX_i + 2BB^T + \bar{\varepsilon}_i^2 X_iWX_i \\ &\quad + \sum_{j=1}^{2}\alpha_{ij}X_iP_jX_i - \left(\Delta\tilde{\pi}_{ii} + \epsilon_{ii}\right)M_i < 0, \end{aligned} \tag{53}$$

$$\sum_{j\neq i}\left(\Delta\tilde{\pi}_{ij} + \epsilon_{ij}\right)\left(X_iP_jX_i - X_i - M_i\right) \leq 0. \tag{54}$$

For any $T_i > 0$, it is concluded that

$$-\Delta\tilde{\pi}_{ii}M_i \leq 0.25\epsilon_{ii}^2T_i + M_iT_i^{-1}M_i. \tag{55}$$

Based on conditions (54) and (55), (52) is implied by

$$\begin{aligned} &\left(\overline{A}G\right)^{*} + X_iQ_iX_i + \tau X_iQX_i + 2BB^T + \bar{\varepsilon}_i^2 X_iWX_i \\ &\quad + \sum_{j=1}^{2}\alpha_{ij}X_iP_jX_i - \epsilon_{ii}M_i + 0.25\epsilon_{ii}^2T_i \\ &\quad + M_iT_i^{-1}M_i < 0, \\ &X_iP_jX_i - X_i - M_i \leq 0, \end{aligned} \tag{56}$$

where the latter one is equivalent to (49). Then, based on the proof of Theorem 5, it is concluded that conditions (47) and (49) imply (30) with Π replaced by (46), where $\overline{Q}_i = G^TQ_iG$, $\overline{Q} = G^TQG$, and $\overline{W} = G^TWG$. As for condition (15), similar to the method of (52), it is equivalent to

$$\begin{aligned} &-\overline{Q} - \epsilon_{ii}V_i + \sum_{j=1}^{2}\alpha_{ij}\overline{Q}_j - \Delta\tilde{\pi}_{ii}V_i \\ &\quad + \sum_{j\neq i}\left(\Delta\tilde{\pi}_{ij} + \epsilon_{ij}\right)\left(\overline{Q}_j - \overline{Q}_i - V_i\right) \leq 0 \end{aligned} \tag{57}$$

with $V_i = V_i^T$. It is further guaranteed by

$$\begin{aligned} &-\overline{Q} - \epsilon_{ii}V_i + 0.25\epsilon_{ii}^2S_i + \sum_{j=1}^{2}\alpha_{ij}\overline{Q}_j + V_iS_i^{-1}V_i < 0, \\ &\sum_{j\neq i}\left(\Delta\tilde{\pi}_{ij} + \epsilon_{ij}\right)\left(\overline{Q}_j - \overline{Q}_i - V_i\right) \leq 0, \end{aligned} \tag{58}$$

which are implied by (48) and (50). This completes the proof. □

Theorem 10. *Given stochastic delay system (1), there is disordered controller (5) such that the resulting closed-loop system (9) is stochastically stable over uncertainty (46), if given positive scalars $\beta_i > 0$, there exist matrices $X_i > 0$, $\overline{Q}_i > 0$, $\overline{Q} > 0$, $\overline{W}_i > 0$, $\overline{W}_{\tau i} > 0$, $T_i > 0$, $S_i > 0$, $M_i = M_i^T$, $V_i = V_i^T$, Y, and Y_τ satisfying conditions (16), (17), (38), (48), (49), (50), and*

$$\begin{bmatrix} \overline{\Theta}_{i1} & \Theta_{i2} & \Lambda_{i2} & \overline{\Lambda}_{i3} & X_i & \tau^{1/2}X_i & \overline{\beta}_iX_i & \overline{\Phi}_i & M_i \\ * & \Theta_{i3} & 0 & \overline{\Lambda}_{i3} & 0 & 0 & 0 & 0 & 0 \\ * & * & \overline{\Theta}_{i4} & \overline{\Lambda}_{i4} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -X_i & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Theta_{i6} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Theta_{i7} & 0 & 0 & 0 \\ * & * & * & * & * & * & \Theta_{i8} & 0 & 0 \\ * & * & * & * & * & * & * & \Gamma_i & 0 \\ * & * & * & * & * & * & * & * & -T_i \end{bmatrix} < 0, \tag{59}$$

where symbols are given in Theorems 5–9. Then the gain of controller (5) can be get by (19).

Proof. Similar to the proof of Theorem 9, its proof could be obtained and is omitted. This completes the proof. □

4. Numerical Examples

Example 1. Consider a stochastic delay system (1) with parameters as follows:

$$\begin{aligned} A &= \begin{bmatrix} -2 & -0.6 \\ 0.5 & -1 \end{bmatrix}, \\ A_\tau &= \begin{bmatrix} -0.4 & -0.1 \\ 0 & -0.5 \end{bmatrix}, \\ B &= \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \\ C &= \begin{bmatrix} -0.4 & 0 \\ 0.2 & -0.1 \end{bmatrix}, \\ C_\tau &= \begin{bmatrix} -0.3 & 0 \\ 0.2 & -0.1 \end{bmatrix}, \\ D &= \begin{bmatrix} 0 \\ 0.2 \end{bmatrix}. \end{aligned} \tag{60}$$

Firstly, by the traditional methods such as [10–12, 15, 17] where no disorder between control gains and system states happens, one can design controller (4) by exploiting a disorder-independent Lyapunov functional $V(x_t) = x^T(t)Px(t) + \int_{t-\tau}^{t} x^T(s)Qx(s)ds$. Here, it should be pointed out that the above selected Lyapunov functional is without

Table 1: The correlation between $\tau_{\max}$ and ε_2 with given different ε_1.

$\varepsilon_1 = 2$ & $\varepsilon_2 =$	0.221	0.3	0.5	1	2	5
$\tau_{\max}$	0.002	0.760	1.559	2.092	2.312	2.425
$\varepsilon_1 = 11$ & $\varepsilon_2 =$	0.221	0.3	0.5	1	2	5
$\tau_{\max}$	0.005	0.950	1.759	2.306	2.547	2.677
$\varepsilon_1 = 20$ & $\varepsilon_2 =$	0.221	0.3	0.5	1	2	5
$\tau_{\max}$	0.002	0.760	1.559	2.092	2.312	2.425

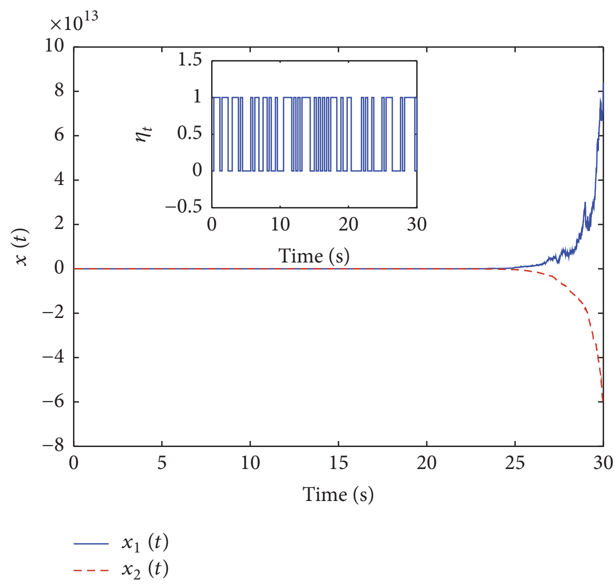


Figure 1: The curves of the resulting system via controller experiencing disorder.

loss of generality. Its form given here is determined by the one exploited in (20). In other words, in order to make some comparisons in this example, they should be coincident. On the other hand, based on the existing methods where some improved Lyapunov functionals or some techniques are used to deal with delay systems, less conservative results could be obtained. Thus, if either of the Lyapunov functionals referred to cases (4) and (5), respectively, is selected to be an improved form or some techniques are exploited, the other one should be done similarly. Based on these illustrations, it is said that the comparisons between controllers (4) and (5) based on similar Lyapunov functionals and similar techniques are without loss of generality. Then, the corresponding gains of controller (4) are computed as

$$K = \begin{bmatrix} -0.2600 & 0.0923 \end{bmatrix}, \quad K_\tau = \begin{bmatrix} -2.0000 & 1.0000 \end{bmatrix}, \tag{61}$$

where delay is $\tau = 0.3$. When the above desired controller experiences a disorder between control gains and system states, without loss of generality, the TRM of such a disorder is described as

$$\Pi = \begin{bmatrix} -0.4 & 0.4 \\ 0.6 & -0.6 \end{bmatrix}. \tag{62}$$

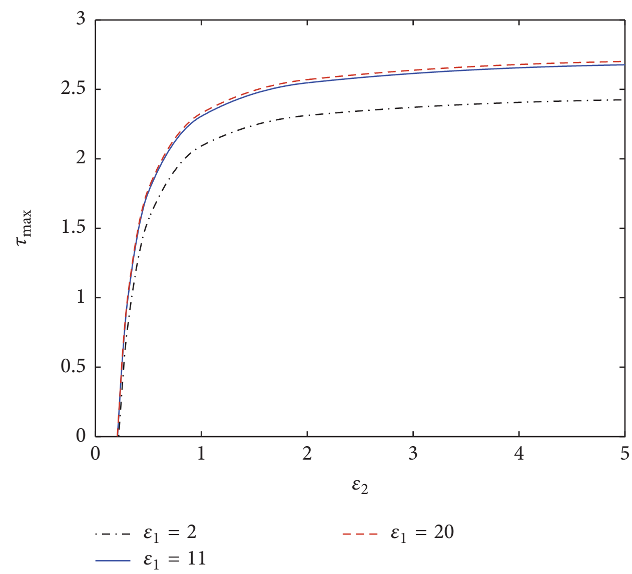


Figure 2: The simulation of correlation between $\tau_{\max}$ and ε_2.

Its simulation is given in the smaller subgraph of Figure 1. Under the initial condition $x_0 = \begin{bmatrix} 1 & -1 \end{bmatrix}^T$, the state response of the resulting closed-loop system is presented in the larger subgraph of Figure 1, which is obviously stable. It is claimed that such a disorder plays a negative effect and could make the resulting closed-loop system unstable. Thus, it is necessary and important to consider such a disorder problem. Under the same TRM and letting $\varepsilon_1 = 11$ and $\varepsilon_2 = 2$, we could have the gains of controller (5) with disorders (8) obtained by Theorem 5 and given as

$$K = \begin{bmatrix} -0.0396 & -0.0289 \end{bmatrix}, \quad K_\tau = \begin{bmatrix} -0.1443 & -0.0454 \end{bmatrix}. \tag{63}$$

However, when the parameters of Theorem 7 are selected to be $\beta_1 = 11$ and $\beta_2 = 2$, respectively, it is known that there is no solution to controller (5). In this case, it is said that Theorem 5 is less conservative. However, when the above parameters are chosen to be $\varepsilon_1 = 0.2$ ($\beta_2 = 0.2$) and $\varepsilon_2 = 0.1$ ($\beta_1 = 0.1$), it is concluded that there is no solution to Theorem 5, while the gains of controller (5) can be gotten by Theorem 7 and given as

$$K = \begin{bmatrix} -0.0083 & -0.0128 \end{bmatrix}, \quad K_\tau = \begin{bmatrix} -0.0650 & -0.0258 \end{bmatrix}. \tag{64}$$

As for this case, it is obtained that Theorem 5 has less conservatism. Thus, it is claimed that which one of such two theorems is less conservative is not constant and should be considered in the concrete situations. In order to further demonstrate this conclusion, more additional comparisons are done in Tables 1 and 2, where $\tau_{\max}$ denotes the allowable upper bound of τ. Moreover, the simulations of such comparisons given in the above tables are further demonstrated in Figures 2 and 3, which are used to show the statements about the conservatism of Theorems 5 and 7 vividly. For a given parameter

TABLE 2: The correlation between τ_{max} and β_2 with given different β_1.

$\beta_1 = 0.2$ & $\beta_2 =$	0.001	0.005	0.01	0.05	0.1	0.3	0.5	0.749
τ_{max}	1.498	1.493	1.488	1.444	1.385	1.061	0.614	0.001
$\beta_1 = 0.3$ & $\beta_2 =$	0.001	0.005	0.01	0.05	0.1	0.3	0.5	0.694
τ_{max}	1.196	1.192	1.188	1.149	1.099	0.844	0.452	0.001
$\beta_1 = 0.5$ & $\beta_2 =$	0.001	0.005	0.01	0.05	0.1	0.3	0.5	0.527
τ_{max}	0.578	0.575	0.571	0.539	0.497	0.305	0.044	0.0001

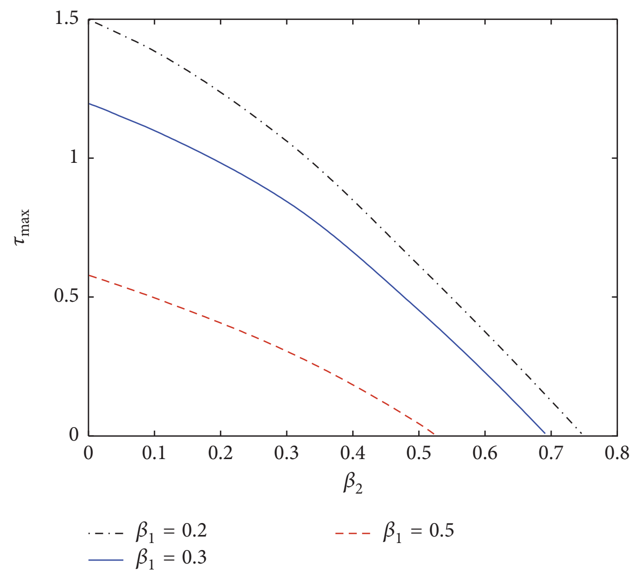


FIGURE 3: The simulation of correlation between τ_{max} and β_2.

ε_1, it is known from Figure 2 that larger ε_2 will result in less conservative results in terms of larger τ_{max}. In addition, it is also seen that smaller ε_1 will lead to less conservative results with larger τ_{max}. To the contrary, from Figure 3, it is found that, for a given parameter β_1, larger β_2 will make the results more conservative in terms of smaller τ_{max}. Moreover, there is an inverse phenomenon about the correlation between τ_{max} and β_1. From these simulations, it is claimed that the effects of parameters ε_i and β_i, $i = 1, 2$, are different, which are contrary. More importantly, based on the curves of such figures in addition to considering the correlation between the upper and lower bounds, it is known that there is a cross section of Figures 2 and 3. In other words, it is concluded that sometimes Theorem 5 is less conservative, while sometimes Theorem 7 is less conservative. Based on these facts, it is said that the conservatism of such theorems is not deterministic, and their applications should be considered in the concrete situations. In addition, even there is uncertainty (46) in Π, such as

$$\widetilde{\Pi} = \begin{bmatrix} -0.4 & 0.4 \\ 0.6 & -0.6 \end{bmatrix} \tag{65}$$

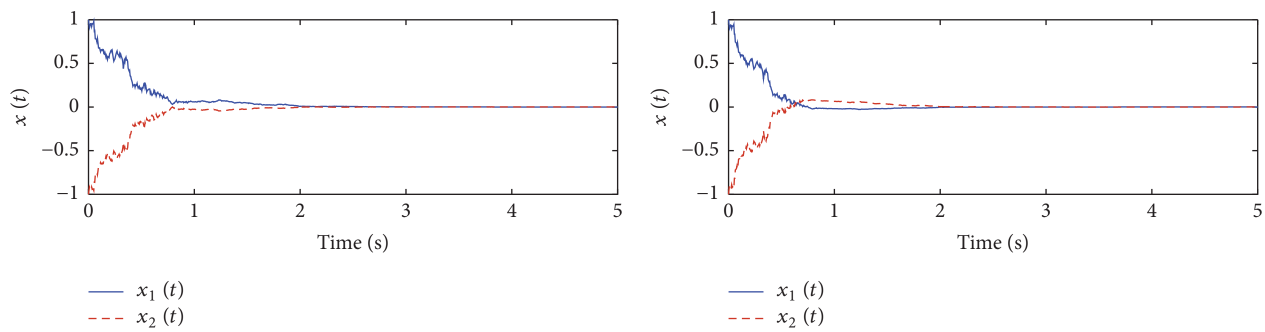
with $\Delta\widetilde{\Pi} = (\Delta\widetilde{\pi}_{ij})$ satisfying $\Delta\widetilde{\pi}_{ij} \leq \epsilon_{ij} = 0.3\widetilde{\pi}_{ij}$, $\forall i, j \in \mathbb{S}$, and $i \neq j$; we can also design effective stabilizing controllers with form (5). Firstly, based on Theorem 9 with $\varepsilon_1 = 11$ and $\varepsilon_2 = 2$, we have the gains of disordered controller (5) computed as

$$\begin{aligned} K &= [-0.0065 \quad -0.0027], \\ K_\tau &= [-0.0180 \quad -0.0053]. \end{aligned} \tag{66}$$

On the other hand, based on Theorem 7 with $\beta_1 = 0.2$ and $\beta_2 = 0.1$, the corresponding gains are given by

$$\begin{aligned} K &= [-0.0030 \quad -0.0045], \\ K_\tau &= [-0.0268 \quad -0.0101]. \end{aligned} \tag{67}$$

Under the same initial condition, after applying the above desired controllers, respectively, we have the state response of the resulting closed-loop systems illustrated in Figure 4. There, the upper subgraph is simulation of the resulting systems obtained by Theorem 9, while the under one is gotten by Theorem 10. It is found that all the states of the resulting systems are stable. Based on these simulations, it is seen that both the desired controllers are useful; even TRM Π experiences uncertainties.



(a) (b)

FIGURE 4: The state responses of the resulting system with uncertain TRM.

5. Conclusions

In this paper, the general stabilization for stochastic delay systems closed by a disordered controller has been studied by a disorder-dependent approach. Firstly, a kind of disordered controller whose control gains and system states experience a disorder has been proposed. Based on the robust method and exploiting a Markov process, the above controller is transformed to be a controller having special uncertainties and depending on a Markov process with two modes. Several sufficient LMI conditions for the desired controller are obtained by using the disorder-dependent Lyapunov functional. In addition, more applications about the TRM of the described disorder having uncertainties have been considered too. Finally, a numerical example has been used to demonstrate the effectiveness of the proposed methods.

Disclosure

A preliminary version of this work first appeared at the 29th Chinese Control and Decision Conference, Chongqing, China.

Competing Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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