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Research Article

Dynamic Stochastic Multiattribute Decision-Making That Considers Stochastic Variable Variance Characteristics under Time-Sequence Contingency Environments

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This paper presents a dynamic stochastic decision-making method that considers the characteristics of stochastic variable variances under time-sequence contingency environments for solving stochastic decision-making problems with information from different periods and of indeterminate attribute weights. First, time-sequence weights are obtained using the technique for order preference by similarity to ideal solution (TOPSIS), corresponding with the idea of "stressing the present rather than the past." After determining the time degree and fully considering the characteristics of normally distributed stochastic variable variances, the attribute weight is determined based on vertical projection distance. Decision-making information is then assembled from two dimensions of time-sequence and attributes, based on the two categories of weighted arithmetic averaging operators of normally distributed stochastic variables, resulting in comprehensive dynamic decision-making from single solution dimensions and a priority sequence of solutions per the order relation criteria of normally distributed stochastic variables. Finally, the validity and practicability of the methods proposed in this paper are verified using an example numerical analysis.

1. Introduction

In modern socioeconomic decision-making systems, people often face decision-making problems with a variety of information conditions, such as stochastic information [1–12], 2-dimension uncertain linguistic information [13, 14], intuitionistic fuzzy information [15–19], intuitionistic trapezoidal fuzzy information [20, 21], triangular fuzzy [22], single-valued neutrosophic numbers [23], trapezoidal fuzzy numbers [24], and neutrosophic hesitant fuzzy information [25]. One of them, which have different time-sequence phases and multiple attribute indexes of normally distributed stochastic variables, like street traffic flow, shopping centre popularity at different times, or the customer waiting times, is referred to as dynamic stochastic multiattribute decision-making problems.

Compared with dynamic stochastic decision-making, which comprehensively considers multiple time-sequences, stochastic multi-attribute decision-making under a single static time-sequence environment has received more widespread attention from scholars. These research findings mainly expand on information aggregation operators [1], stochastic dominance [2, 3], stochastic multiattribute analysis [4–6], set pair connection number analysis [7], prospect stochastic dominance [8, 9], bivariate expectation in decision-making [10], probability weighted means [3], and possibility degree interval-valued numbers [11, 12], of attribute weights in an unknown state—further expanding the research boundary of stochastic multiattribute decision-making. It is difficult for most decision-making results to be comprehensive and rationally optimised based on single time-sequence nodes. Therefore, past and current decision-making information should also be considered when conducting stochastic decision-making. Appropriately determining the weight of different time-sequences is key to successful dynamic stochastic decision-making. Current

scholars have mainly developed the following objective methods based on sample information to establish time-sequence weights: discrete time-sequences [12], normal distribution [15], time-sequence information entropy [16], difference geometric progression [26], and exponential distribution [27]. Moreover, scholars such as Park et al. [17], Li et al. [24], Cao et al. [28], and Liu et al. [29] et al. have conducted weight allocation on different time-sequences by relying on expert experience and subjective preference. These methods provide a reference for solving for time-sequence weights in dynamic stochastic problems, but they derive weights based on an agent's preference in each period, which inevitably results in randomness in the time-sequence weight distribution. For instance, methods based on objective information often tend to emphasise time-sequences with more information in an historical period; at these times, sequence weight increases [12, 15, 16]. However, methods that merely rely on the subjective perception of information at different times, owing to diversity in subjective perception, greatly impact the stability of decision-making results [17, 24, 28, 29]. In real life, people pay more attention to the most current information when making judgments of the present. Therefore, this paper focuses on allocating time-sequence weights in accordance with the closeness of historical information to the latest information.

Dynamic stochastic multiattribute decision-making problems possess a time dimension and an attribute dimension; therefore, determining attribute weights is a prerequisite for assembling the attribute information required for the final decision-making result. Relevant scholars have developed a variety of methods for successfully determining attribute weight. For instance, Chen and Li have obtained attribute weights [22] by solving the grey relation function of attribute information per the grey correlation model. Scholars, such as Yue [18], Cao et al. [19], and Zhang et al. [30] et al., have designed linear optimisation models of attribute weight using TOPSIS-based Euclidean distance. Xu and Wan [31] have calculated attribute weights based on an uncertainty-ordered weighted averaging operator and an attribute information ordering method that relates to attribute weight size and attribute evaluation value size and order. This idea provides a basis for attribute weight calculation in normally distributed stochastic decision-making problems; however, due to its assumed conditions and the characteristic requirements of normally distributed stochastic variables, this method's applications are limited. For instance, grey system theory and TOPSIS are based on Euclidean distance. Since Euclidean distance does not consider the distance relationship between a decision-making solution and the positive and negative ideal solutions, the weight calculation can easily lead to a "reversed order" phenomenon, in which the attribute information ordering method assigns weights per the attribute information size criteria, but attribute information is the decision-making information after inverting the normally distributed stochastic variables into common interval numbers, and thus the attribute information of the original normally distributed stochastic variables is lost completely. Therefore, attribute weight information that considers normally distributed stochastic variable characteristics is expanded upon in this paper.

Based on these analyses, this study proposes a dynamic stochastic multiattribute decision-making method that considers random variable variance characteristics in time-sequence contingencies with respect to time-sequence and attribute. The remainder of this paper is organised as follows: Section 2 reviews random decision-making and related algorithms in which attribute values are normally distributed stochastic variables. Section 3 provides a time-sequence weight calculation model using the ideal time-sequence solution by introducing the concept of time degree. The validity and applicability of the proposed dynamic stochastic decision-making method are examined in Section 4 using a numerical example. Conclusions and directions for future work are discussed in Section 5.

2. Background

Here, we offer a concise overview of the essentials of normally distributed numbers and stochastic variables and the operation rules thereof.

Definition 1 (see [32]). A stochastic (random) variable, usually written as X, is a variable whose possible values are numerical outcomes of a random phenomenon. There are two types of stochastic variables: discrete and continuous.

Definition 2 (see [12]). Suppose that the probability density of a continuous type of stochastic variable X is $f(x) = (1/\sqrt{2\pi}\sigma)e^{-(x-\mu)^2/2\sigma^2}$, where parameters μ and σ^2 are the expectation and variance of the stochastic variables, respectively. X is a normal distribution, denoted by $X \sim N(\mu, \sigma^2)$, with a cumulative probability function of $F(x) = (1/\sqrt{2\pi}\sigma)\int_{-\infty}^{x} e^{-(t-\mu)^2/2\sigma^2}dt$. $\{\mu, \sigma\}$ is the normal distribution numbers of stochastic variable X, denoted by $x = \{\mu, \sigma\}$, and Θ is the set of all normal distribution numbers.

Lahdelma et al. (2006) [33] proposed that the stochastic parameters σ and μ depend on the basic discrete sample $\{Z^l,\ l = 1, 2, \ldots, L\}$, where Z is criteria values and l is sample size. The stochastic parameter μ is estimated by the sample mean

$$\mu \cong \overline{Z} = \frac{\sum_{l=1}^{L} Z^l}{L} \tag{1}$$

and the stochastic parameter σ is the square root of the sample variance

$$\sigma \cong \sqrt{\frac{\sum_{l=1}^{L}\left(Z^l - \overline{Z}\right)^2}{(L-1)}}. \tag{2}$$

Definition 3 (see [11]). For two arbitrary normal distribution numbers $x_1 = \{\mu_1, \sigma_1\}$ and $x_2 = \{\mu_2, \sigma_2\}$,

(1) $x_1 \oplus x_2 = \{\mu_1 + \mu_2, \sqrt{\sigma_1^2 + \sigma_2^2}\}$;

(2) $\lambda x_1 = \{\lambda\mu_1, \lambda\sigma_1\}$;

(3) $\lambda(x_1 \oplus x_2) = \lambda x_1 \oplus \lambda x_2$.

Definition 4 (see [11, 12]). For an arbitrary normal distribution number set $x_j = \{\mu_j, \sigma_j\}$, a normally distributed number weighted arithmetic average (NDNWAA) operator of dimension n is a mapping NDNWAA: $\Theta^n \to \Theta$,

$$\begin{aligned} &\mathrm{NDNWAA}_w(x_1, x_2, \ldots, x_n) \\ &\quad = w_1 x_1 \oplus w_2 x_2 \oplus \cdots \oplus w_n x_n \\ &\quad = \left\{ \sum_{j=1}^{n} w_j \mu_j, \sqrt{\sum_{j=1}^{n} w_j^2 \sigma_j^2} \right\}, \end{aligned} \tag{3}$$

where $w = (w_1, w_2, \ldots, w_n)^T$ is the weight vector of the stochastic variable X_j $(j = 1, 2, \ldots, n)$ attribute, $w_j \in [0, 1]$, $\sum w_j = 1$.

Definition 5 (see [12]). Suppose that $x(t) = (x(t_1), x(t_2), \ldots, x(t_p))$ is a group of normally distributed numbers under p different moments t_k $(k = 1, 2, \ldots, p)$; a dynamic normal distribution number weighted arithmetic average (DNDNWAA) operator of dimension p is a mapping NDNWAA: $\Theta^p \to \Theta$

$$\begin{aligned} &\mathrm{DNDNWAA}_{w(t)}\left(x(t_1), x(t_2), \ldots, x(t_p)\right) \\ &\quad = w(t_1) x(t_1) \oplus w(t_2) x(t_2) \oplus \cdots \oplus w(t_p) x(t_p) \\ &\quad = \left\{ \sum_{k=1}^{p} w(t_k) \mu(t_k), \sqrt{\sum_{k=1}^{p} w(t_k)^2 \sigma(t_k)^2} \right\}, \end{aligned} \tag{4}$$

where $w(t) = (w(t_1), w(t_2), \ldots, w(t_p))^T$ is the weight of time-sequence t_k $(k = 1, 2, \ldots, p)$, $w(t_k) \in [0, 1]$, $\sum_{k=1}^{p} w(t_k) = 1$.

3. The Dynamic Stochastic Decision-Making Method

3.1. Problem Description. Suppose that there exists a dynamic stochastic decision-making problem, for which $S = \{S_1, S_2, \ldots, S_m\}$ is a discrete and independent alternative solution set, $C = \{C_1, C_2, \ldots, C_n\}$ is the attribute set to which the solution is subject, $w = (w_1, w_2, \ldots, w_n)^T$ is the weight vector, and $w(t) = (w(t_1), w(t_2), \ldots, w(t_p))^T$ is the time-weight vector. The value of attribute C_j, to which solution S_i is subject at moment t_k, is denoted as $X_{ij}(t_k)$, which is subject to a normal distribution, denoted by $X_{ij}(t_k) \sim N(\mu_{ij}(t_k), \sigma_{ij}(t_k)^2)$ and its corresponding normal distribution number $x_{ij}(t_k) = N(\mu_{ij}(t_k), \sigma_{ij}(t_k))$, forming a primitive decision-making matrix $D_{X(t_k)} = N(\mu_{ij}(t_k), \sigma_{ij}(t_k)^2)_{m\times n}$ based on p moments of decision-making information. Dynamic stochastic decision-making problems consist of multiple dimensions, such as solution, attribute, and time. Determining attribute and time-sequence weights is key to reducing dimensionality and solving dynamic stochastic problems.

3.2. Determination of Time-Sequence Weight. In dynamic stochastic multiattribute decision-making problems, time-sequence weight vector $w(t_k) = (w(t_1), w(t_2), \ldots, w(t_p))^T$ is a prerequisite for assembling time-sequence information and obtaining a decision-making result. Contrary to previous similar research, for example, Sun and Xu [12] and Tan and Chen [16], which are merely based on the absolute quantity of time-sequence information or completely rely on subjective arbitrary judgments, this paper obtains a time-sequence weight vector using TOPSIS and by introducing time-degree criteria in accord with the findings of Yager [34] and Zhang and Zhu [35].

Definition 6 (see [35]). Denote $\lambda = \sum_{k=1}^{p}((p-k)/(p-1))w(t_k)$, where $0 \le \lambda \le 1$ and λ is the time degree of the time-sequence weight vector $w(t_k) = (w(t_1), w(t_2), \ldots, w(t_p))^T$. Time degree describes the degree of attention the decision maker attaches to decision-making information from different time-sequence phases. When λ decreases, the decision maker attaches more importance to recent information; when λ increases, the decision maker attaches more importance to older information.

Based on this analysis and guided by the principle of "stress the present rather than the past," when $\lambda = 0$, then $w(t) = (0, 0, \ldots, 1)^T$, indicating that the decision maker attaches full importance to current information, and $w(t)^+ = (0, 0, \ldots, 1)^T$ can be denoted by the positive ideal time-weight vector; when $\lambda = 1$, then $w(t) = (1, 0, \ldots, 0)^T$, indicating that the decision maker attaches full importance to past information, and $w(t)^- = (1, 0, \ldots, 0)^T$ can be denoted by the negative ideal time-weight vector.

Per the definition of Euclidean distance, the distance between time-weight vectors $w^1(t_k) = (w^1(t_1), w^1(t_2), \ldots, w^1(t_p))^T$ and $w^2(t_k) = (w^2(t_1), w^2(t_2), \ldots, w^2(t_p))^T$ can be denoted by

$$d\left(w^1(t_k), w^2(t_k)\right) = \sqrt{\sum_{k=1}^{p} \left|w^1(t_k) - w^2(t_k)\right|^2}. \tag{5}$$

Thus, the distances of a time-weight vector $w(t_k) = (w(t_1), w(t_2), \ldots, w(t_p))^T$ from the positive and negative ideal time-weight vectors are, respectively,

$$\begin{aligned} d\left(w(t_k), w(t_k)^+\right) &= \sqrt{\sum_{k=1}^{p-1} w(t_k)^2 + \left(1 - w(t_p)\right)^2}, \\ d\left(w(t_k), w(t_k)^-\right) &= \sqrt{\left(1 - w(t_1)\right)^2 + \sum_{k=2}^{p} w(t_k)^2}. \end{aligned} \tag{6}$$

The closeness of time-weight vector $w(t_k)$ to the ideal time-weight vector can be obtained by

$$C = \frac{d\left(w(t_k), w(t_k)^-\right)}{d\left(w(t_k), w(t_k)^+\right) + d\left(w(t_k), w(t_k)^-\right)}. \tag{7}$$

The greater the closeness degree C is, the more the attention is attached to current information. To maximise the closeness of a time-sequence weight vector, the following model is optimised based on nonlinear programming under the given time-degree conditions

$$\begin{aligned}
\max \quad & C = \frac{\sqrt{\left(1 - w\left(t_1\right)\right)^2 + \sum_{k=2}^{p} w\left(t_k\right)^2}}{\sqrt{\sum_{k=1}^{p-1} w\left(t_k\right)^2 + \left(1 - w\left(t_p\right)\right)^2} + \sqrt{\left(1 - w\left(t_1\right)\right)^2 + \sum_{k=2}^{p} w\left(t_k\right)^2}}, \\
\text{s.t.} \quad & \lambda = \sum_{k=1}^{p} \frac{p-k}{p-1} w\left(t_k\right), \\
& \sum_{k=1}^{p} w\left(t_k\right) = 1, \\
& w\left(t_k\right) \in [0,1], \ k = 1, 2, \ldots, p,
\end{aligned} \tag{8}$$

solved via Lingo software, from which time-sequence weight vector $w(t_k) = (w(t_1), w(t_2), \ldots, w(t_p))^T$ is acquired.

Upon determining the time-sequence weights, the decision-making information matrix $D_{X(t_k)} = N(\mu_{ij}(t_k), \sigma_{ij}(t_k)^2)_{m\times n}$ of different time-sequences is assembled with the $\text{DNDNWAA}_{w(t)}$ operator from Formula (4) to form a comprehensive decision-making information matrix $D_X = N(\mu_{ij}, \sigma_{ij}{}^2)_{m\times n}$ that consists of attribute and solution dimensions. The normally distributed stochastic variable that corresponds to attribute C_j, subject to solution S_i, is $X_{ij} \sim N(\mu_{ij}, \sigma_{ij}{}^2)$. Its corresponding normal distribution number is $x_{ij} = N(\mu_{ij}, \sigma_{ij})$, where $\mu_{ij} = \sum_{k=1}^{p} w(t_k)\mu_{ij}(t_k)$, $\sigma_{ij} = \sqrt{\sum_{k=1}^{p} w(t_k)^2 \sigma_{ij}(t_k)^2}$.

3.3. Determination of Attribute Weight. Attribute weight determination in normally distributed stochastic decision-making seldom stems from the characteristics of the attribute itself, and, from the characteristics of normally distributed stochastic variables, the superiority or inferiority of a variable attribute is closely related to its variance size—the smaller the variance, the better the attribute [36]. With the previous analysis, the variance characteristics of normally distributed stochastic variables can be used to determine attribute weights. Hence, this study obtains attribute weight by establishing an attribute-weight-solving optimised model based on stochastic variable variance characteristics guided by the shortest vertical projection distance.

Vertical projection distance refers to the distance between the planes that take the connections crossing the positive and negative ideal solution points as a normal vector [37]. Vertical projection distance can prevent a reversed order solution, which may occur in traditional Euclidean distance methods, and allows the characteristics of stochastic variable variances to be highlighted when calculating attribute weight in stochastic decision-making problems. Vertical projection distance has been elaborated upon in the literature [37]; in this study, it mainly provides an optimised model for determining attribute weight.

Like the comprehensive decision-making information matrix $D_X = N(\mu_{ij}, \sigma_{ij}{}^2)_{m\times n}$, assembled based on time-sequences, owing to the difference in data dimensionality of different attributes, nondimensionalisation is carried out for comprehensive decision-making information, to avoid the influence of dimensionality differences on the decision-making result. Considering the normal distribution number $x_{ij} = N(\mu_{ij}, \sigma_{ij})$ to which each attribute corresponds, the nondimensionalisation formulas for benefit-oriented and cost-oriented indexes and for index variance are, respectively [11, 12],

$$\tilde{\mu}_{ij} = \frac{\mu_{ij}}{\max_i\left(\mu_{ij}\right)}, \tag{9}$$

$$\tilde{\mu}_{ij}\left(t_k\right) = \frac{\min_i\left(\mu_{ij}\right)}{\mu_{ij}}, \tag{10}$$

$$\tilde{\sigma}_{ij} = \frac{\sigma_{ij}}{\max_i\left(\mu_{ij}\right)}. \tag{11}$$

A normal distribution number after nondimensionalisation is denoted by $\tilde{x}_{ij} = N(\tilde{\mu}_{ij}, \tilde{\sigma}_{ij})$ and the decision-making information matrix is denoted by $\widetilde{D}_X = N(\tilde{\mu}_{ij}, \tilde{\sigma}_{ij}{}^2)_{m\times n}$.

Attribute weights are determined after conducting nondimensionalisation of the comprehensive decision-making information using vertical projection distance. The specific steps of this calculation are as follows.

Suppose attribute-weight vector $w = (w_1, w_2, \ldots, w_n)$ and denote the root extraction (i.e., standard deviation) matrix of the standardised stochastic variable based on attribute weighting as

$$R = \left(r_{ij}\right)_{m\times n} = \left(\sqrt{\tilde{\sigma}_{ij}{}^2}\right)_{m\times n} \times \left(w_j\right)_{n\times 1}. \tag{12}$$

Shift the origin coordinates of matrix R to the ideal variance point to obtain ideal variance $S^+ = (S_j^+ \mid j = 1, 2, \ldots, n)$. The shifted matrix is denoted by $Z = (z_{ij})_{m\times n}$.

Per the characteristics of stochastic variable variances, $S_j^+ = \min_{1\le i\le m}\{\tilde{\sigma}_{ij}{}^2\}$ can be obtained; then

$$z_{ij} = \sqrt{\tilde{\sigma}_{ij}{}^2} - S_j^+ = \left(\sqrt{\tilde{\sigma}_{ij}{}^2} - \sqrt{\tilde{\sigma}_j^{+2}}\right) w_j, \quad i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n. \tag{13}$$

To determine the negative ideal variance of matrix Z as $H^- = (H_j^- \mid j = 1, 2, \ldots, m)$, the ideal variance is $(0, 0, \ldots, 0)$, where

$$H_j^- = z_{\eta j} = \sqrt{\tilde{\sigma}_{ij}{}^2} w_j. \tag{14}$$

Set $|z_{\eta j}| \ge |z_{ij}|$ or $|\sqrt{\tilde{\sigma}_{\eta j}{}^2}| \ge |\sqrt{\tilde{\sigma}_{ij}{}^2} - \sqrt{\tilde{\sigma}_j^{+2}}|$, where $1 \le \eta \le m$.

Vertical projection distance V_i of each variance and the ideal variance, because the distance between the positive and negative ideal variances is constant for each solution, may be determined by

$$V_i = \sum_{j=1}^{n} H_j^- \times z_{ij}. \tag{15}$$

The variances in normally distributed stochastic variables V_i can be thought of as the degree of closeness between the variance and ideal variance of all attributes in different solutions—smaller values of V_i indicate that the variance and ideal variance of attributes in a solution are closer. To achieve more optimised stochastic variables, variances should be as small as possible. Therefore, these problems can be inverted to the following optimised ones:

$$\begin{aligned} \min \quad & \sum_{i=1}^{m} V_i = \sum_{i=1}^{m}\sum_{j=1}^{n} H_j^- \times z_{ij} \\ & = \sum_{i=1}^{m}\sum_{j=1}^{n} \sqrt{\tilde{\sigma}_{\eta j}{}^2}\left(\sqrt{\tilde{\sigma}_{ij}{}^2} - \sqrt{\tilde{\sigma}_j^{+2}}\right) w_j^2 \\ \text{s.t.} \quad & \sum_{j=1}^{n} w_j = 1, \\ & w_j > 0,\ \ j = 1, 2, \ldots, n. \end{aligned} \tag{16}$$

Attribute weight $w = (w_1, w_2, \ldots, w_n)^T$ can be solved via Lingo software.

3.4. Decision-Making Based on Order Relation. Upon obtaining attribute weight, comprehensive decision-making information is assembled after nondimensionalisation from attribute dimensions, per the NDNWAA$_w$ operator in Formula (3), forming a decision-making information matrix $\widetilde{D}_{X_i} = N(\tilde{\mu}_i, \tilde{\sigma}_i{}^2)_{m\times 1}$ constituted by a single solution dimension. Because of the characteristics of expectation and variance attributes in normally distributed stochastic variables, the greater expectation $\tilde{\mu}_i$ is, the smaller $\tilde{\sigma}_i{}^2$ is and the greater corresponding stochastic variable X_i is [36]. Normally, $\tilde{\sigma}_i{}^2 > 0$; if $\tilde{\sigma}_i{}^2 = 0$, then the normally distributed stochastic variable is degraded into a real number. Therefore, the order relation criteria between any two arbitrary normally distributed stochastic variables $X_1 \sim N(\tilde{\mu}_1, \tilde{\sigma}_1{}^2)$ and $X_2 \sim N(\tilde{\mu}_2, \tilde{\sigma}_2{}^2)$ can be defined as follows:

If $\tilde{\mu}_1/\tilde{\sigma}_1{}^1 > \tilde{\mu}_2/\tilde{\sigma}_2{}^2$, then $X_1 > X_2$.

The order relation criteria of normally distributed stochastic variables state that the superiority-inferiority degree value ($X_i = X_1, X_2, \ldots, X_m$) of related normally distributed stochastic variables can be obtained to determine the priority sequence of the solution; that is, the greater X_i is, the more optimised corresponding ith solution is.

3.5. Summary of the New Dynamic Stochastic Decision-Making Method. In brief, the calculation steps for dynamic stochastic multiattribute decision-making that considers stochastic variable characteristics under time-sequence contingencies are as follows.

Step 1. Obtain p moments of time-sequence weight vector $w(t_k) = (w(t_1), w(t_2), \ldots, w(t_p))^T$ using Formula (8) after obtaining p moments of dynamic stochastic decision-making information matrix $D_{X(t_k)} = N(\mu_{ij}(t_k), \sigma_{ij}(t_k)^2)_{m\times n}$.

Step 2. Assemble p moments of dynamic stochastic decision-making information with the time-sequence weights and Formula (4) to form a comprehensive decision-making information matrix $D_X = N(\mu_{ij}, \sigma_{ij}{}^2)_{m\times n}$ comprising attributes and solutions.

Step 3. Conduct nondimensionalisation for comprehensive decision-making information D_X, using Formulas (9) through (11) to form a decision-making information matrix $\widetilde{D}_X = N(\tilde{\mu}_{ij}, \tilde{\sigma}_{ij}{}^2)_{m\times n}$ after nondimensionalisation, and obtain attribute-weight vector $w = (w_1, w_2, \ldots, w_n)^T$ as in Formula (16).

Step 4. Further assemble decision-making information matrix $\widetilde{D}_X$ from attributes, utilising attribute weights and Formula (3) to form a comprehensive decision-making information matrix $\widetilde{D}_{X_i} = N(\tilde{\mu}_i, \tilde{\sigma}_i{}^2)_{m\times 1}$ that is only constituted of a single solution dimension.

Step 5. Finally determine the order ($X_i = X_1, X_2, \ldots, X_m$) of each alternative solution using the order relation criteria of normally distributed stochastic variables and further determine the priority sequence of alternative solutions based on this order.

4. Illustrative Cases of Study

There is a major emerging technical project seeking technical partners. Five enterprises, denoted by $S_i = \{S_1, S_2, S_3, S_4, S_5\}$, are options for potential partners or solutions. The attribute parameters for evaluating each enterprise refer to the stochastic variables subject to stochastic normal distribution: the R&D cycle of the enterprise's emerging technology (C_1), the market share of its emerging technology products (C_2), the

life of its emerging technology products (C_3), the conversion rate of its emerging technology achievements (C_4), and the attribute set $C_j = \{C_1, C_2, C_3, C_4\}$, corresponding to attribute weights $w = (w_1, w_2, w_3, w_4)$. In addition, the investor of the emerging technical project has investigated related attribute information in four historical periods for the previously mentioned enterprises and synthesised historical attribute information for reference. Suppose that the time-sequence set of different historical periods is $t_k = \{t_1, t_2, t_3, t_4\}$.

To obtain decision-making information for each evaluation attribute that obeys or approximately obeys the law of normal distribution, we first collect sample data from completed emerging technical project developments of each enterprise as the basic data sample $\{Z_{ij}^l(t_k)\}$. Basic data were provided by each candidate enterprise for attribute set $C_j = \{C_1, C_2, C_3, C_4\}$ in four different periods $\{t_1, t_2, t_3, t_4\}$. Then, the collected sample data were transformed; thus the expectation $\mu_{ij}(t_k)$ and variance $\sigma_{ij}(t_k)^2$ of each attribute index can be calculated by the sample data in accordance with Formulas (1) and (2). Finally, the basic data samples were converted into the original evaluation attribute information data shown in Tables 1–4.

Based on the original evaluation attribute information, per Step 1, where the time degree parameter $\lambda = 0.4$, the time-sequence weights of the different time-sequences were obtained:

$$\begin{aligned} w(t_k) &= (w(t_1), w(t_2), w(t_3), w(t_4)) \\ &= (0.255, 0.218, 0, 0.527). \end{aligned} \tag{17}$$

The four historical time-sequences were assembled as in Step 2 and synthesised into comprehensive decision-making attribute information shown in Table 5.

Nondimensionalisation was conducted on the comprehensive original evaluation attributes in Table 5 with the procedure in Step 3, as shown in Table 6.

Attribute weights were obtained as in Step 3:

$$w = (w_1, w_2, w_3, w_4) = (0.421, 0.103, 0.177, 0.299). \tag{18}$$

Comprehensive decision-making information was assembled from attribute information, based on Step 4, forming comprehensive normal distribution numbers constituted by target single dimensions:

$$\begin{aligned} \widetilde{D}_{X_1} &= N(0.88, 0.033^2), \\ \widetilde{D}_{X_2} &= N(0.934, 0.032^2), \\ \widetilde{D}_{X_3} &= N(0.862, 0.031^2), \\ \widetilde{D}_{X_4} &= N(0.843, 0.032^2), \\ \widetilde{D}_{X_5} &= N(0.997, 0.034^2). \end{aligned} \tag{19}$$

The value of each alternative cooperative enterprise was determined based on Step 5:

$$\begin{aligned} X &= (X_1, X_2, X_3, X_4, X_5) \\ &= (26.86, 29.03, 28.18, 26.22, 29.3). \end{aligned} \tag{20}$$

The priority sequence of the five alternative enterprises, $S_5 > S_2 > S_3 > S_1 > S_4$, was obtained by arranging each enterprise's value from largest to smallest and the optimal solution was determined to be S_5.

Further, the decision-making solution sequence was also calculated using the possibility degree method described in the literature [11, 12].

First, the comprehensive normal distribution number $\widetilde{D}_{X_i}$ $(i = 1, 2, 3, 4, 5)$ is converted to an interval number:

$$\begin{aligned} \overline{D}_{X_1} &= [0.782, 0.979], \\ \overline{D}_{X_2} &= [0.838, 1.031], \\ \overline{D}_{X_3} &= [0.771, 0.954], \\ \overline{D}_{X_4} &= [0.747, 0.94], \\ \overline{D}_{X_5} &= [0.895, 1.099]. \end{aligned} \tag{21}$$

Then, the possibility matrices of the two compared schemes are calculated:

$$P = \begin{bmatrix} 0.5 & 0.361 & 0.547 & 0.595 & 0.209 \\ 0.639 & 0.5 & 0.691 & 0.735 & 0.343 \\ 0.453 & 0.309 & 0.5 & 0.55 & 0.153 \\ 0.405 & 0.265 & 0.45 & 0.5 & 0.114 \\ 0.791 & 0.657 & 0.847 & 0.886 & 0.5 \end{bmatrix}. \tag{22}$$

Finally, the following ranking value was acquired, based on the fuzzy compensation judgment matrix in the possibility degree method:

$$\begin{aligned} X &= (X_1, X_2, X_3, X_4, X_5) \\ &= (0.286, 0.32, 0.273, 0.261, 0.359), \end{aligned} \tag{23}$$

so that the priority order obtained from the ranking value is $S_5 > S_2 > S_1 > S_3 > S_4$, where S_5 is the optimal investment scheme.

Obviously, the result based on the possibility degree method described in the literature [11, 12] was consistent with the result of Section 4, indicating the rationality of the method proposed in this study. Compared with the possibility degree method, the method proposed herein requires fewer nonlinear conversions during calculation and model construction, and its calculation is simple and easy to implement. In addition, compared with similar normally distributed stochastic decision-making methods, this study considers the characteristics of normally distributed stochastic variable variances while determining attribute weight; thus, the final ordering result of the method proposed in this paper is more comprehensive.

5. Conclusions

This paper describes a dynamic stochastic decision-making method that fully considers the characteristics of normally distributed stochastic variable variances with time-sequence

Table 1: Original evaluation attribute information at t_1.

	C_1	C_2	C_3	C_4
S_1	$N(716, 75^2)$	$N(11.8, 1.8^2)$	$N(8.5, 1.5^2)$	$N(34.3, 2.2^2)$
S_2	$N(899, 89^2)$	$N(9.5, 1.7^2)$	$N(9.3, 1.8^2)$	$N(47.4, 4^2)$
S_3	$N(653, 74^2)$	$N(12.5, 1.1^2)$	$N(7.4, 1.2^2)$	$N(51, 4.1^2)$
S_4	$N(816, 84^2)$	$N(10.9, 1.9^2)$	$N(10.1, 2.4^2)$	$N(38, 3.9^2)$
S_5	$N(786, 74^2)$	$N(12, 2^2)$	$N(11, 1.4^2)$	$N(44, 4.1^2)$

Table 2: Original evaluation attribute information at t_2.

	C_1	C_2	C_3	C_4
S_1	$N(671, 66^2)$	$N(12.4, 2.1^2)$	$N(6.5, 0.9^2)$	$N(45, 3.2^2)$
S_2	$N(790, 87^2)$	$N(11.5, 1.9^2)$	$N(7.6, 1.3^2)$	$N(51, 5^2)$
S_3	$N(816, 78^2)$	$N(10.5, 1.2^2)$	$N(11.4, 1.7^2)$	$N(60, 4.9^2)$
S_4	$N(718, 77^2)$	$N(9.9, 1.2^2)$	$N(8.1, 1.4^2)$	$N(43, 3.2^2)$
S_5	$N(765, 74^2)$	$N(13, 1.8^2)$	$N(7.9, 1.6^2)$	$N(62, 5.4^2)$

Table 3: Original evaluation attribute information at t_3.

	C_1	C_2	C_3	C_4
S_1	$N(790, 80^2)$	$N(9.7, 1.3^2)$	$N(10.6, 1.2^2)$	$N(58.8, 4.9^2)$
S_2	$N(611, 61^2)$	$N(10.1, 1.2^2)$	$N(9.8, 1.7^2)$	$N(60.3, 4.1^2)$
S_3	$N(530, 47^2)$	$N(12.1, 2.1^2)$	$N(12.1, 2.4^2)$	$N(37.3, 2.8^2)$
S_4	$N(799, 71^2)$	$N(8.9, 0.8^2)$	$N(11.1, 2.1^2)$	$N(31.3, 2.8^2)$
S_5	$N(854, 78^2)$	$N(11.9, 1.4^2)$	$N(12, 1.7^2)$	$N(53, 4.2^2)$

Table 4: Original evaluation attribute information at t_4.

	C_1	C_2	C_3	C_4
S_1	$N(806, 81^2)$	$N(10.8, 1^2)$	$N(11.7, 1.6^2)$	$N(66, 5.1^2)$
S_2	$N(744, 71^2)$	$N(11.1, 1.1^2)$	$N(12.6, 1.3^2)$	$N(67, 5.1^2)$
S_3	$N(614, 65^2)$	$N(12.6, 1.9^2)$	$N(11.8, 1.4^2)$	$N(54,4.8^2)$
S_4	$N(709, 69^2)$	$N(9.9, 1.2^2)$	$N(10.6, 1.7^2)$	$N(56, 3.7^2)$
S_5	$N(883, 80^2)$	$N(12.7, 2.2^2)$	$N(11.3, 1.4^2)$	$N(77, 5.2^2)$

Table 5: Comprehensive decision-making attribute information.

	C_1	C_2	C_3	C_4
S_1	$N(753.6, 48.9^2)$	$N(11.4, 0.8^2)$	$N(9.8, 0.9^2)$	$N(53.3, 2.8^2)$
S_2	$N(793.6, 47.7^2)$	$N(10.8, 0.8^2)$	$N(10.7, 0.9^2)$	$N(58.5, 3.1^2)$
S_3	$N(668, 32.6^2)$	$N(12.1, 1.1^2)$	$N(10.6, 0.8^2)$	$N(54.5,2.9^2)$
S_4	$N(738.3, 45.4^2)$	$N(10.2, 0.8^2)$	$N(9.9, 1.1^2)$	$N(48.6, 2.3^2)$
S_5	$N(832.5, 48.9^2)$	$N(12.6, 1.3^2)$	$N(10.5, 0.9^2)$	$N(65.3, 3.2^2)$

Table 6: Nondimensionalised comprehensive decision-making attribute information.

	C_1	C_2	C_3	C_4
S_1	$N(0.905, 0.059^2)$	$N(0.906, 0.066^2)$	$N(0.914, 0.089^2)$	$N(0.817, 0.043^2)$
S_2	$N(0.953, 0.057^2)$	$N(0.856, 0.066^2)$	$N(1, 0.082^2)$	$N(0.896, 0.047^2)$
S_3	$N(0.802, 0.051^2)$	$N(0.963, 0.085^2)$	$N(0.993, 0.083^2)$	$N(0.835,0.045^2)$
S_4	$N(0.887, 0.055^2)$	$N(0.807, 0.067^2)$	$N(0.931, 0.106^2)$	$N(0.744, 0.035^2)$
S_5	$N(1, 0.059^2)$	$N(1, 0.105^2)$	$N(0.983, 0.083^2)$	$N(1, 0.048^2)$

contingencies. It first obtains time-sequence weights by combining time-degree theory and TOPSIS. It calculates attribute weights based on the characteristics of normally distributed stochastic variable variances and vertical projection distance. Decision-making information is then assembled from the attribute and time-sequence weights via related operators, to obtain the stochastic normally distributed comprehensive decision-making matrix constituted by target single dimensions, finally providing the priority sequence of alternative solutions using order relation criteria. The primary contributions of this paper are as follows:

(1) In time-sequence weight calculations, this method fully considers decision-makers' preference to "stress the present rather than the past," which more accurately models real-world decision-making processes.

(2) The characteristics of normally distributed stochastic variable variances are highlighted in attribute-weight calculations and solution priority sequence ordering, making the final ordering result more comprehensive and reasonable.

(3) The method has a clear theoretical backing, requires only simple calculation, and is easy to implement. It provides a new, broadly applicable way of thinking of dynamic stochastic decision-making problems with time-sequence contingencies.

Our future study will extend the proposed method for multiattribute decision-making problems with intuitionistic fuzzy numbers and two-dimensional linguistic information.

Additional Points

Highlights. Stochastic decision-making is based on variance and time-sequence characteristics. A time-sequence weight calculation in line with TOPSIS and time degree is introduced. A new vertical projection distance-based attribute-weight model is proposed. The final decision-making result in accordance with the order relation criteria is given.

Competing Interests

The authors declare that they have no competing interests.

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