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Research Article

City Sustainable Development Evaluation Based on Hesitant Multiplicative Fuzzy Information

Xiaorong He and Yingyu Wu

School of Economics and Management, Southeast University, Nanjing 211189, China

Correspondence should be addressed to Xiaorong He; shelley526@126.com

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Sustainable development evaluation is the basis of city sustainable development research, and effective evaluation is the foundation for guiding the formulation and implementation of sustainable development strategy. In this paper, we provided a new city sustainable development evaluation method called hesitant multiplicative fuzzy TODIM (HMF-TODIM). The main advantage of this method is that it can deal with the subjective preference information of the decision-makers. The comparison study of existing methods and HMF-TODIM is also carried out. Additionally, real case analysis is presented to show the validity and superiority of the proposed method. Research results in this paper can provide useful information for the construction of sustainable cities.

1. Introduction

Urbanization brings a series of ecological and environmental problems while bringing about economic and social benefits [1]. For example, urbanization leads to water quality deterioration, resource depletion, air pollution, traffic congestion, and so on. The implementation of sustainable development is the only way for the development of urbanization and the urgent need of the international situation [2]. At present, the study on the evaluation of urban sustainable development is a universal issue [3]. At present, there are many researches on the evaluation of urban sustainable development at home and abroad. Most of the research has been carried out from the aspects of structure, coordination, and continuity of the cities [4]. In addition, most of the research has been using the traditional methods, such as AHP and Delphi. These methods cannot effectively deal with the subjective preferences of decision makers, and most models are difficult to apply to practice. This paper presents a new city sustainable development evaluation method based on hesitant multiplicative fuzzy information.

Since Zadeh proposed the theory of fuzzy set [5], it has been extended to intuitionistic fuzzy sets [6–11], hesitant fuzzy sets [12–15], dual hesitant fuzzy set [16–18], and so on. And the above extended fuzzy set has been used in decision-making problems [19–23]. Recently, Xia et al. [24] presented the concept of intuitionistic multiplicative preference relation (IMPR). Xia and Xu [25] proposed the hesitant multiplicative preference relation (HMPR). These two kinds of preference relation are very useful in describing the preference relations of the decision-makers [26–28].

As a useful tool for the multicriteria decision-making (MCDM) problem, the TODIM method was first introduced by Gomes and Lima [29, 30] and later widely spread in many fields for solving MCDM problems [31–33]. The TODIM method is based on the Prospect Theory [31] and takes the DMs' psychological information and emotional preference into consideration. At present, the TODIM method has been extended to intuitionistic fuzzy [34, 35], interval-valued intuitionistic fuzzy [36], and hesitant fuzzy environment [37]. However, the TODIM method has not been studied with HMPR which is the focus of this paper.

The remainder of this paper is organized as follows. In Section 2, the concepts about hesitant fuzzy preference relation and hesitant multiplicative preference relation are presented; the aggregation operations for hesitant multiplicative values are also provided. In Section 3, we present a distance measure for the hesitant multiplicative elements. Section 4 proposes an extended TODIM method under hesitant multiplicative environment. In Section 5, an illustrative example about city sustainable development evaluation is shown to demonstrate the feasibility of the proposed method. Concluding remarks are presented in Section 6.

2. Preliminaries

2.1. Hesitant Fuzzy Preference Relation and Hesitant Multiplicative Preference Relation. Inspired by the HFS (hesitant fuzzy set) [12] and FPR (fuzzy preference relation) [38], Liao et al. [39] presented a new concept called HFPR (hesitant fuzzy preference relation).

Definition 1 (see [39]). In a HFPR, we have a set of fixed alternatives $A = (A_1, A_2, \ldots, A_n)$ and then derive a matrix $B = (h_{ij})_{n\times n} \in A \times A$, where $h_{ij} = \{h_{ij}^l \mid l = 1, 2, \ldots, l_{h_{ij}}\}$, through comparing every alternative with each other. And each h_{ij} is expressed with several hesitant fuzzy values so that DMs can provide their preferences for alternative h_i over alternative h_j in a hesitancy range. What is more, each h_{ij} should satisfy the following conditions:

$$\begin{aligned} h_{ij}^{\sigma(l)} + h_{ji}^{(l_{h_{ij}}-l+1)} &= 1, \\ h_{ii} &= \{0.5\}, \\ l_{h_{ij}} &= l_{h_{ji}}, \end{aligned} \tag{1}$$

where $h_{ij} = 0.5$ means there is no difference between alternative i and alternative j, $h_{ij} < 0.5$ means the alternative i is superior to alternative j, and the $h_{ij} > 0.5$ means the alternative j is superior to alternative i.

Based on the proposal of HFPR, an extended concept called HMPR (hesitant multiplicative preference relation), which utilizes a new information expressing method, is presented by Xia and Xu [25]. The only difference between the two concepts is that the HMPR uses Saaty's 1–9 scale while the HFPR uses the 0.1–0.9 scale. Firstly, we give the definitions of HMS (hesitant multiplicative set) and HME (hesitant multiplicative element) proposed by Zhang and Wu [40]. Then we introduce the concept of HMPR.

Definition 2 (see [40]). A HMS C on a fixed set X is defined as follows:

$$C = \{\langle x, h_C(x)\rangle \mid x \in X\}. \tag{2}$$

$h_C(x)$ is a HME of $x \in X$ and $h_C(x)$ satisfies the following conditions:

$$\begin{aligned} &\frac{1}{9} \le \xi \le 9, \\ &\forall x \in X, \\ &\forall \xi \in h_C(x). \end{aligned} \tag{3}$$

Definition 3 (see [25]). In a HMPR, we have a set of fixed alternatives $\widetilde{A} = (\widetilde{A_1}, \widetilde{A_2}, \ldots, \widetilde{A_n})$ and then derive a matrix $\widetilde{B} = (\widetilde{h_{ij}})_{n\times n} \in \widetilde{A} \times \widetilde{A}$, where $\widetilde{h_{ij}} = \{\widetilde{h_{ij}^l} \mid l = 1, 2, \ldots, \widetilde{l_{h_{ij}}}\}$, through comparing alternative i with alternative j. And each $\widetilde{h_{ij}}$ should satisfy the following conditions:

$$\begin{aligned} \widetilde{h_{ij}^{\sigma(l)}} \widetilde{h_{ji}^{(l_{h_{ij}}-l+1)}} &= 1, \\ \widetilde{h_{ii}} &= \{1\}, \\ l_{\widetilde{h_{ij}}} &= l_{\widetilde{h_{ji}}}. \end{aligned} \tag{4}$$

We define that $\widetilde{h_{ij}}$ is a HME where $\widetilde{h_{ij}} \in [1/9, 9]$ and $\widetilde{h_{ij}} = 1$ means the alternative i is equally preferred to alternative j; $1/9 \le \widetilde{h_{ij}} < 1$ means the alternative i is not preferred to alternative j; and $1 < \widetilde{h_{ij}} \le 9$ means the alternative i is preferred to alternative j. What is more, $\widetilde{h_{ij}} = \{1/9\}$ presents the alternative i is extremely not preferred to alternative j and contrarily $\widetilde{h_{ij}} = \{9\}$ presents the alternative i is extremely preferred to alternative j.

2.2. Aggregation Operations for Hesitant Multiplicative Values. Aggregation operator is an important research topic in decision-making problems [41–44]. Hesitant multiplicative information aggregation operators were first proposed by Xia and Xu [25]. Then Yu [45] made some changes and presented some extended aggregation operators. In this paper, we use the following operator [45] to aggregate hesitant multiplicative information. The operator is stated as follows:

$$\mathrm{HMFWA}(\rho_1, \rho_2, \ldots, \rho_n) = w_1\rho_1 \oplus w_2\rho_2 \oplus \cdots \oplus w_n\rho_n. \tag{5}$$

This operator is called HMFWA (hesitant multiplicative fuzzy weighted averaging) operator and $\rho = (\rho_1, \rho_2, \ldots, \rho_n)$ is a set of HMNs (hesitant multiplicative numbers). After that, Yu [45] presented a theorem with a condition that $w = (w_1, w_2, \ldots, w_n)$ is a set of weight vectors with respect to the HMNs $\rho = (\rho_1, \rho_2, \ldots, \rho_n)$, where $w_i \in [0, 1]$ and $\sum_{i=1}^n w_i = 1$. And η_i is the value in the HMN ρ_i. The theorem is provided below:

$$\begin{aligned} &\mathrm{HMFWA}(\rho_1, \rho_2, \ldots, \rho_n) \\ &\quad = \bigcup_{\eta_i \in \rho_i} \left\{ \frac{\prod_{i=1}^n (1 + 2\eta_i)^{w_i} - 1}{2} \right\}. \end{aligned} \tag{6}$$

3. Distance Measure for HMEs

In this section, we propose a new distance measure for integrating the HMEs in order to combine with the TODIM method. We would set a nonideal HME and derive a series of corresponding values through calculating the distance between the nonideal HME and each HME we picked. According to the definition of the distance measure, the higher the score is, the bigger the difference between two objects is. What is more, the bigger the difference between the ideal HME and the HME we picked is, the better the HME is. So we can realize the target of evaluating each HME via the distance measure and the effect of it. And the effect is similar to the compatibility degree between a pair of intuitionistic multiplicative values presented by Jiang et al. [46]. We concisely represent the distance measure presented by Xu and Xia [47], called hesitant normalized Hamming distance.

$$d_{hnh}(M,N)=\frac{1}{n}\sum_{i=1}^{n}\left[\frac{1}{l_{x_i}}\sum_{j=1}^{l_{x_i}}\left|h_M^{\sigma(j)}(x_i)-h_N^{\sigma(j)}(x_i)\right|\right]. \quad (7)$$

We need to calculate the distance between HMEs; however, this distance measure is for HFSs. In order to fit our method, we made a little change to this distance measure. And then we can calculate the distance between HMEs and present the measure as follows:

$$d(h_1,h_2)=\frac{1}{l_x}\sum_{j=1}^{l_x}\left|h_1^{\sigma(j)}(x)-h_2^{\sigma(j)}(x)\right|, \quad (8)$$

where h_1 and h_2 are two HMEs. All the values in each HME are in a monotone increasing order and $\sigma(j)$ is the jth largest value in the element. What is more, l_x represents the number of the values in the HME.

HMEs and HFEs have many similar characteristics. For instance, they both have a membership degree containing several possible valves to express DMs' preferences under hesitancy and uncertainty. The only difference is that HMEs are denoted by Saaty's 1–9 scale so the distance measure is also applicable to the calculation under hesitant multiplicative fuzzy environment.

4. Extended TODIM Method under Hesitant Multiplicative Environment

4.1. The Classical TODIM Method. Suppose there is a MCDM problem [19, 48–64]; the set of alternatives were represented as $A=(A_1,A_2,\ldots,A_m)$ as well as a set of criteria as shown as $C=(C_1,C_2,\ldots,C_n)$. Assume one of the criteria as the reference criterion and give the weight vectors denoted as $w=(w_1,w_2,\ldots,w_n)$ with respect to the set of criteria and the weight vectors satisfy $w_i\in[0,1]$ and $\sum_{i=1}^{n}w_i=1$. Then experts are asked to estimate each alternative. After obtaining the values of the alternatives, each value should be divided by the sum of all the values and then obtain a matrix where the values are normalized [65–73]. The matrix of the values is called the normalized alternatives' scores against criteria and the matrix is denoted as $A=[a_{mn}]$ in Table 1.

After assigning the proper values to the weights and its normalization, DMs have to select a reference criterion w_r which is the most vital compared with the other criteria. $w_{rc}=w_c/w_r$, w_c is the weight of criterion c.

Each alternative's performance should be transformed into the same dimension via using w_{rc}. And we use the following mathematical expression to calculate the dominance of alternative a_i over another alternative a_j.

$$\delta(a_i,a_j)=\sum_{c=1}^{m}\Phi_c(a_i,a_j), \quad \forall(i,j) \quad (9)$$

Table 1: Normalized alternatives' scores against criteria.

	C_1	C_2	$\cdots$	C_n
A_1	a_{11}	a_{12}	$\cdots$	a_{1n}
A_2	a_{21}	a_{22}	$\cdots$	a_{2n}
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
A_m	a_{m1}	a_{m2}	$\cdots$	a_{mn}

when

$$\Phi_c(a_i,a_j)=\begin{cases}\sqrt{\dfrac{w_{rc}(a_{ic}-a_{jc})}{\sum_{c=1}^{m}w_{rc}}} & a_{ic}-a_{jc}>0\\ 0 & a_{ic}-a_{jc}=0\\ \dfrac{-1}{\theta}\sqrt{\dfrac{(\sum_{c=1}^{m}w_{rc})(a_{jc}-a_{ic})}{w_{rc}}} & a_{ic}-a_{jc}<0.\end{cases} \quad (10)$$

θ is a parameter of the attenuation factor of the losses.

Then we need to calculate the global value of alternative a_i using the following expression:

$$\xi_i=\frac{\sum_{j=1}^{n}\delta(a_i,a_j)-\min_i\sum_{j=1}^{n}\delta(a_i,a_j)}{\max_i\sum_{j=1}^{n}\delta(a_i,a_j)-\min_i\sum_{j=1}^{n}\delta(a_i,a_j)}. \quad (11)$$

Finally, we can rank the alternatives varying from 1 to m according to the global value, respectively.

4.2. An Extended TODIM Method. We plan to present a new TODIM method synthesizing the aggregation operations, distance measures, and classical TODIM method mentioned above. Our innovation is to utilize the TODIM method's advantages such as considering the DMs' psychological information and emotional preference, to solve MCDM problems under hesitant multiplicative environment. Zhang and Wu [40] had solved a MCMD problem using some aggregation operations with interval-valued hesitant information. Nevertheless, they only relied on aggregation operations to deal with the alternatives' performances but not thinking about something psychological. So our method is necessary and valid for this black space.

Step 1. We provide a set of alternatives denoted as $A=(A_1,A_2,\ldots,A_m)$ and several criteria $C=(C_1,C_2,\ldots,C_n)$. Besides we set weight vectors $w=(w_1,w_2,\ldots,w_n)$ for the criteria with $w_i\in[0,1]$ and $\sum_{i=1}^{n}w_i=1$. What is more, we need a group of experts $E=(e_1,e_2,\ldots,e_k)$ to give their evaluation using hesitant multiplicative information. According to these, we obtain a HMPR called $B=(b_{ij})_{m\times m}$ constructed by

Table 2: The distance values between alternatives and the nonideal alternative.

	C_1	C_2	$\cdots$	C_n
A_1	$d_1^{(1)}$	$d_1^{(2)}$	$\cdots$	$d_1^{(n)}$
A_2	$d_2^{(1)}$	$d_2^{(2)}$	$\cdots$	$d_2^{(n)}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
A_m	$d_m^{(1)}$	$d_m^{(2)}$	$\cdots$	$d_m^{(n)}$

Saaty's 1–9 scale, where b_{ij} represents the dominance degree of alternative i over alternative j. Compared with the ordinary HMPR, there is a little difference in the following step which we use a set of criteria to replace the role of decision organization. In general, we obtain some HMPRs associated with each decision organization, respectively. However, in order to accomplish the new method combined with the TODIM method, we need experts to provide their preferences for each alternative with respect to the criteria. Furthermore due to the restriction of professions, experts are not permitted to evaluate certain alternative associated with a criterion if they know little about it. Note that experts are independent of each other. Then we can obtain a hesitant multiplicative preference relation called $R_n = ({r_{ij}}^{(n)})_{n\times n}$, where ${r_{ij}}^{(n)}$ denotes all the values given by experts comparing alternative i with alternative j with respect to the criterion n.

Step 2. We use (6) to aggregate every ${r_{ij}}^{(n)}$ $(j = 1, 2, \ldots, m)$ in the HMPR associated with each criterion, respectively, and then derive a series of $r_i^{(n)}$ for the alternative i under the criterion n.

Step 3. Set a nonideal element with the minimum hesitant multiplicative value (HMV) called A^*, where $A^* = \{1/9\}$. Then we calculate the distance between the nonideal alternative and the target element $r_i^{(n)}$ we calculated in the last step, via utilizing (8). It is obvious that the bigger the distance value is, the better the performance of the alternative is. According to this, we can obtain a new matrix called $D = ({d_i}^{(n)})_{n\times n}$ composed of the distance grades $d_i^{(n)}$ between the nonideal element and each alternative i associated with the criterion n, respectively. And this matrix was presented in Table 2.

In Table 2, $A = (A_1, A_2, \ldots, A_m)$ and $C = (C_1, C_2, \ldots, C_n)$ indicate the alternatives and criteria we selected, respectively.

Step 4. We utilize the TODIM method to deal with the matrix $D = ({d_i}^{(n)})_{n\times n}$. Through the process, we can obtain the global values of the alternatives and get the rank ordering for these alternatives. Specific processes are as follows:

(i) Calculate the dominance of alternative ${d_i}^{(n)}$ over another alternative ${d_j}^{(n)}$ corresponding to each criterion after determining the reference criterion w_r, via the expression

$$\delta\left(d_i, d_j\right) = \sum_{c=1}^{m} \Phi_c\left(d_i, d_j\right), \quad \forall (i, j) \tag{12}$$

when

$$\Phi_c\left(d_i, d_j\right) = \begin{cases} \sqrt{\dfrac{w_{rc}\left({d_i}^{(n)} - {d_j}^{(n)}\right)}{\sum_{c=1}^{m} w_{rc}}} & {d_i}^{(n)} - {d_j}^{(n)} > 0 \\ 0 & {d_i}^{(n)} - {d_j}^{(n)} = 0 \\ \dfrac{-1}{\theta}\sqrt{\dfrac{\left(\sum_{c=1}^{m} w_{rc}\right)\left({d_j}^{(n)} - {d_i}^{(n)}\right)}{w_{rc}}} & {d_i}^{(n)} - {d_j}^{(n)} < 0. \end{cases} \tag{13}$$

θ is a parameter to reflect the attenuation of the losses.

(ii) Calculate the global values of each alternative using the following formula:

$$\xi_i = \frac{\sum_{j=1}^{n} \delta\left(d_i, d_j\right) - \min_i \sum_{j=1}^{n} \delta\left(d_i, d_j\right)}{\max_i \sum_{j=1}^{n} \delta\left(d_i, d_j\right) - \min_i \sum_{j=1}^{n} \delta\left(d_i, d_j\right)}. \tag{14}$$

5. Case Studies

Sustainable development evaluation is the basis of urban sustainable development research, and correct evaluation is an important basis for guiding the formulation and implementation of sustainable development strategy. At the same time, due to the specificity and complexity of the city, the sustainable development of different cities is difficult to use the same indicators. Therefore, the sustainable development evaluation is both a hot and difficult point in the field of research.

Starting from the three aspects of economy (c_1), society (c_2), and resources and environment (c_3), the sustainable development level and ability of three cities $X = (x_1, x_2, x_3)$ in Zhejiang province, China, are evaluated.

We invite three experts $E = (e_1, e_2, e_3)$ to give their assessment with respective to the three cities associated with different criteria. What is more, each expert is not disturbed by each other.

First we obtain three matrices denoted by the HMPR for each criterion called $R = (R_1, R_2, R_3)$. The matrices R_1, R_2, R_3 are the HMPRs associated with the three cities under c_1 (economy), c_2 (society), and c_3 (resources and environment), respectively. The values in the HMPRs are the primary data provided by the experts and we present them in Tables 3–5.

Table 3: The hesitant multiplicative preference relation R_1.

	x_1	x_2	x_3
x_1	$\{1\}$	$\{3/2, 2\}$	$\{2, 3, 4\}$
x_2	$\{1/2, 2/3\}$	$\{1\}$	$\{3, 4\}$
x_3	$\{1/4, 1/3, 1/2\}$	$\{1/4, 1/3\}$	$\{1\}$

Table 4: The hesitant multiplicative preference relation R_2.

	x_1	x_2	x_3
x_1	$\{1\}$	$\{3/2, 3/4, 5/6\}$	$\{3/2, 2, 3\}$
x_2	$\{6/5, 4/3, 2\}$	$\{1\}$	$\{1/5, 1/3, 1/2\}$
x_3	$\{1/3, 1/2, 2/3\}$	$\{2, 3, 5\}$	$\{1\}$

Table 5: The hesitant multiplicative preference relation R_3.

	x_1	x_2	x_3
x_1	$\{1\}$	$\{2, 3\}$	$\{6, 7, 8\}$
x_2	$\{1/3, 1/2\}$	$\{1\}$	$\{2, 3, 4\}$
x_3	$\{1/8, 1/7, 1/6\}$	$\{1/4, 1/3, 1/2\}$	$\{1\}$

Table 6: The distance values between the three companies and the nonideal alternative.

	x_1	x_2	x_3
x_1	1.6490	1.1971	2.6062
x_2	1.2507	0.7470	1.0651
x_3	0.3944	1.5596	0.3282

Table 7: The dominance degrees of the three companies over each other.

	x_1	x_2	x_3
x_1	1	1.3689	1.5781
x_2	−4.8933	1	−0.1293
x_3	−5.3293	−2.8779	1

(i) Then we use the aggregation operations to deal with the primary data via (6) and derive nine results of each alternative with respect to the criteria denoted as $r_i^{(n)}$, where i means the alternative i and n means the criterion n.

$$\begin{aligned}
r_1^{(1)} &= \{1.4574, 1.6897, 1.8811, 1.6086, 1.8588, 2.0650\},\\
r_2^{(1)} &= \{1.2380, 1.3899, 1.3297, 1.4895\},\\
r_3^{(1)} &= \{0.4449, 0.4787, 0.4787, 0.5317, 0.5400, 0.5772\},\\
r_1^{(2)} &= \{1.3333, 1.4574, 1.6898, 1.0536, 1.1736, 1.3722, 1.0874, 1.2100, 1.4129\},\\
r_2^{(2)} &= \{0.7131, 0.7856, 0.8662, 0.7440, 0.8184, 0.9010, 0.8795, 0.9620, 1.0536\},\\
r_3^{(2)} &= \{0.9620, 1.1356, 1.4015, 1.0536, 1.2380, 1.5206, 1.1355, 1.3297, 1.6272\},\\
r_1^{(3)} &= \{2.3994, 2.5411, 2.6707, 2.7436, 2.99020, 3.0470\},\\
r_2^{(3)} &= \{0.9620, 1.1355, 1.2784, 1.0536, 1.2380, 1.3900\},\\
r_3^{(3)} &= \{0.3892, 0.4210, 0.4787, 0.3976, 0.4297, 0.4880, 0.4086, 0.4410, 0.500\}.
\end{aligned} \tag{15}$$

(ii) In general cases of group decision-making (GDM) problem, lots of researchers use a weighted aggregation operation to aggregate $r_i^{(n)}$ and obtain r_i which means the global value of alternative i over the other alternatives. Even Zhang and Wu [40] did the same work in a MCMD problem based on the interval-valued hesitant multiplicative preference relation. However we do not use this operation but lead a distance measure (8) into this step to calculate the distance between the alternatives we picked and a nonideal alternative $A^* = \{1/9\}$. So we can illustrate whether an alternative's value is bigger than another one through calculating their distance value $d_i^{(n)}$. And we obtain a matrix $D = \{d_i^{(n)}\}_{n\times n}$ composed of the distance values in Table 6.

(iii) The distance values of x_1 are bigger than those of x_2 and x_3 associated with c_1 and c_3. And the distance value of x_3 associated with c_2 ranks first. As we mentioned in Section 4.2, the bigger the distance value is, the better the alternative's performance is. Therefore the result we obtained in Table 7 means the performance of x_1 associated with c_1 and c_3 is best, while the performance of x_3 related to c_2 is better than the other two.

(iv) We use the TODIM method to deal with the matrix calculated above and gain the dominance degree of each alternative via (12). We present the dominance degree in Table 7. And then we can get the global valves of all the alternatives which are presented in Table 8.

From the result, it is obvious that x_1 is ranked first. We can roughly derive that x_1 possesses the biggest distance value with the nonideal alternative compared with the other two in Table 7. As we mentioned above, the bigger the distance value is, the better the alternative's performance is. Therefore, after calculating the distance values between the three companies and the nonideal alternative, the dominant position of x_1 is relatively manifest. And in Table 7, the superiority of x_1 is more obvious because only the dominance degree of x_1 is a positive number while the dominance degrees of the other two are negative numbers. And the definition of the dominance degree is the comparison of two alternatives' advantages and disadvantages. Therefore we can infer that x_1 is the best from the result of calculating the dominance degree.

5.1. Comparison between the New Method and an Aggregation Operation Method. In this subsection, we plan to do a comparison between our new method, HMF-TODIM, and an existing method based on aggregation operations proposed by Yu [45] which is provided as follows.

Table 8: The global values of the three companies and their rank ordering.

	Global value	Rank
x_1	1	1
x_2	0.2855	2
x_3	0	3

Table 9: The score values and the rank ordering concerning the three alternatives.

	Score value	Rank
x_1	1.7822	1
x_2	1.1136	2
x_3	0.6681	3

Step 1. Use (6) to aggregate every $r_{ij}^{(n)}$ $(j = 1, 2, \ldots, m)$ in the HMPRs associated with each criterion in Tables 3–5, respectively. And then derive a series of $r_i^{(n)}$ for the alternative i under the criterion n. This step is the same as the HMF-TODIM method.

Step 2. Through utilizing (6) again, we aggregate each $r_i^{(n)}$ to derive every alternative's global value r_i. The global value indicates the performance of each alternative related to the all criteria under the hesitant multiplicative environment.

Step 3. According to a score function defined by Xia [74], identify the sorting of the alternatives' values. And the score function is given as follows:

$$s(r) = \sqrt[1/\Delta h]{\prod_{\eta \in h} \eta}. \tag{16}$$

Noting that Δh is the number of the values of r, if $s(r_1) > s(r_2)$, then $r_1 > r_2$.

It is obvious to see that Step 1 of this method is the same as Step 2 of the HMF-TODIM method. Therefore we can obtain the same values to the set of $r_i^{(n)}$:

$$\begin{aligned}
r_1^{(1)} &= \{1.4574, 1.6897, 1.8811, 1.6086, 1.8588, 2.0650\}, \\
r_2^{(1)} &= \{1.2380, 1.3899, 1.3297, 1.4895\}, \\
r_3^{(1)} &= \{0.4449, 0.4787, 0.4787, 0.5317, 0.5400, 0.5772\}, \\
r_1^{(2)} &= \{1.3333, 1.4574, 1.6898, 1.0536, 1.1736, 1.3722, 1.0874, 1.2100, 1.4129\}, \\
r_2^{(2)} &= \{0.7131, 0.7856, 0.8662, 0.7440, 0.8184, 0.9010, 0.8795, 0.9620, 1.0536\}, \\
r_3^{(2)} &= \{0.9620, 1.1356, 1.4015, 1.0536, 1.2380, 1.5206, 1.1355, 1.3297, 1.6272\}, \\
r_1^{(3)} &= \{2.3994, 2.5411, 2.6707, 2.7436, 2.99020, 3.0470\}, \\
r_2^{(3)} &= \{0.9620, 1.1355, 1.2784, 1.0536, 1.2380, 1.3900\}, \\
r_3^{(3)} &= \{0.3892, 0.4210, 0.4787, 0.3976, 0.4297, 0.4880, 0.4086, 0.4410, 0.500\}.
\end{aligned} \tag{17}$$

Then we aggregate the set of $r_i^{(n)}$ to obtain the global values r_i via (6). Considering the enormous calculation process, we do not put the calculation result of Step 2 in our paper.

Finally, we identify the sorting of the three alternatives via the score function.

$s(r_1) = 1.7822$, $s(r_2) = 1.1136$, and $s(r_3) = 0.6681$. Since $s(r_1) > s(r_2) > s(r_3)$, then $r_1 > r_2 > r_3$, and we present the score values related to the alternatives in Table 9.

6. Conclusion

In this paper, we proposed a new TODIM method called HMF-TODIM method and applied it to the field of city sustainable development evaluation. The use of asymmetric information can solve more practical problems in the real world. Expressing the DMs' preferences with asymmetric information is more scientific and closer to the reality because the DMs' preference degrees may increase irregularly. For instance, the growth range from the extremely nonpreferred one to the equally preferred one may be smaller than that from the extremely preferred one to the equally preferred one. In addition, we also considered the hesitancy and uncertainty during decision-making process. A real case analysis is presented to show the validity and superiority of the proposed method. In the future study, we intend to apply the proposed methods to the field of energy [75–78] and supplier evaluation and selection [79–81].

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

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