Research Article

Design of Initial Value Control for Modified Lorenz-Stenflo System

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For the sake of complexity, unpredictability, and exceeding sensitivity to initial conditions in the chaotic systems, there were many studies for information encryption of chaotic systems in recent years. Enhancing the security in information encryption of chaotic systems, an initial value control circuit for chaotic systems is proposed in this paper. By way of changing the initial value, we can change the behavior of chaotic systems and also change the key of information encryption. An analog circuit is implemented to verify the initial value control circuit design.

1. Introduction

Chaotic systems provide a rich mechanism for signal design and generation, with potential applications to communications and signal processing. Because chaotic signals are typically broadband, noise-like, and difficult to predict, they can be used in various contexts for masking information bearing waveforms. They can also be used as modulating waveforms in spread spectrum systems. This can be useful in many practical circumstances like securing communication channels [1–3], masking signals [4–6], and spreading data sequence or for generating random signals [7]. Four-dimensional Lorenz-Stenflo system is a famous four-dimensional chaotic system; there were many recent works on it [8–11].

The cryptographic system is composed of plaintext, ciphertext, encryption algorithm, key, and decryption algorithm. Dynamic update of the key can reduce its probability of being correctly guessed and improve the security of cryptographic system. In this paper, we introduce an initial value control circuit design for the modified four-dimensional Lorenz-Stenflo system. Furthermore, an analog circuit of the chaotic system is implemented to verify the initial value control circuit design. By way of changing the initial value, we can change the behavior of chaotic systems and also change the key of information encryption.

This paper is organized as follows. In Section 2, nonlinear dynamic characteristics of the modified four-dimensional Lorenz-Stenflo system are analyzed. In Section 3, an analog circuit of the modified chaotic system is implemented to verify the initial value control circuit design. Finally, some concluding remarks are given in Section 4.

2. Dynamic Analysis of a New Chaotic System

2.1. Chaotic Equations and Phase Portraits. The Lorenz-Stenflo system is described as [12, 13]

$$\begin{aligned}\dot{x}_1 &= ax_2 - ax_1 + bx_4, \\ \dot{x}_2 &= cx_1 - x_1x_3 - x_2, \\ \dot{x}_3 &= x_1x_2 - dx_3, \\ \dot{x}_4 &= -x_1 - ax_4,\end{aligned}\tag{1}$$

where $x = [x_1, x_2, x_3, x_4]^T$, a is Prandtl number, b is rotation number, c is Rayleigh number, and d is geometric parameter. Because four-dimensional Lorenz-Stenflo system is a famous four-dimensional chaotic system, we modify four-dimensional Lorenz-Stenflo system as a study example. We

FIGURE 1: Two-dimensional phase portraits of system equations (2).

modify system equations (1) where new system equations (2) still keep chaotic characteristics [14].

$$\begin{aligned}
 \dot{x}_1 &= ax_2 - ax_1, \\
 \dot{x}_2 &= cx_1 - x_1x_3, \\
 \dot{x}_3 &= x_1x_2 - dx_3 + bx_4, \\
 \dot{x}_4 &= -x_1 - ax_4.
 \end{aligned} \tag{2}$$

The two-dimensional phase portraits of chaotic system equations (2) are shown in Figure 1, in which symbol “*” denotes equilibrium points, with EP_1 , EP_2 , and EP_3 , with the parameters $a = 3.7$, $b = 1.5$, $c = 25$, $d = 0.7$, and $x_1(0) = x_2(0) = x_3(0) = x_4(0) = 1$.

2.2. Symmetry and Invariance. New chaotic system equations (2) are invariance under the coordinate transform $(x_1, x_2, x_3, x_4) \rightarrow (-x_1, -x_2, x_3, -x_4)$; chaotic system equations (2) are rotation symmetrical about the x_3 axis.

2.3. Dissipativity and Existence of Attractor. The state space of chaotic system equations (2) is four-dimensional. The vector

field on the right-hand sides of chaotic system equations (2) is defined by

$$\Phi(x) = \begin{bmatrix} \Phi_1(x) \\ \Phi_2(x) \\ \Phi_3(x) \\ \Phi_4(x) \end{bmatrix} = \begin{bmatrix} ax_2 - ax_1 \\ cx_1 - x_1x_3 \\ x_1x_2 - dx_3 + bx_4 \\ -x_1 - ax_4 \end{bmatrix}. \tag{3}$$

The divergence of the vector field Φ is calculated as

$$\begin{aligned}
 \nabla \Phi(x) &= \frac{\partial \Phi_1(x)}{\partial x_1} + \frac{\partial \Phi_2(x)}{\partial x_2} + \frac{\partial \Phi_3(x)}{\partial x_3} + \frac{\partial \Phi_4(x)}{\partial x_4} \\
 &= -2a - d.
 \end{aligned} \tag{4}$$

A necessary and sufficient condition for system equations (2) to be dissipative is that the divergence of the vector field Φ is negative. In (4), it is immediate that system equations (2) are dissipative if and only if $a, d \in \mathbb{R}^+$ with an exponential rate:

$$\frac{d\Phi}{dt} = e^{-(2a+d)}. \tag{5}$$

Thus, in dynamical system equations (2), a volume element Φ_0 is apparently contracted by the flow into a volume element $\Phi_0 e^{-(2a+d)t}$ in time domain. This means that each volume containing the trajectories of this dynamical system shrinks to zero as time approached infinity

FIGURE 2: Circuit of chaotic system (2).

at an exponential rate $\nabla\Phi$. So, all the orbits of chaotic system equations (2) will be eventually confined to a special subset that has zero volume, and the asymptotic motion of system equations (2) will settle onto an attractor of the system. Then, the existence of an attractor is proved.

2.4. Equilibrium Points Analysis. The equilibrium points of system equations (2) can be found by solving the following algebraic equations simultaneously:

$$\begin{aligned} ax_2 - ax_1 &= 0, \\ cx_1 - x_1x_3 &= 0, \end{aligned}$$

FIGURE 3: Two-dimensional phase portraits of circuit in Figure 2 (default initial value voltage $x_1(0) = V_{c1} = 2$ V, without initial value control).

$$\begin{aligned} x_1 x_2 - dx_3 + bx_4 &= 0, \\ -x_1 - ax_4 &= 0. \end{aligned}$$

(6)

Chaotic system equations (2) have three equilibrium points, respectively, represented as

$$\begin{aligned} &EP_1(0, 0, 0, 0), \\ &EP_2\left(\frac{b + \sqrt{4a^2cd + b^2}}{2a}, \frac{b + \sqrt{4a^2cd + b^2}}{2a}, c, \right. \\ &\quad \left. - \frac{b + \sqrt{4a^2cd + b^2}}{2a^2}\right), \\ &EP_3\left(\frac{b - \sqrt{4a^2cd + b^2}}{2a}, \frac{b - \sqrt{4a^2cd + b^2}}{2a}, c, \right. \\ &\quad \left. - \frac{b - \sqrt{4a^2cd + b^2}}{2a^2}\right). \end{aligned} \quad (7)$$

When the parameters are $a = 3.7$, $b = 1.5$, $c = 25$, and $d = 0.7$, chaotic system equations (2) have three equilibrium points,

given by $EP_1(0, 0, 0)$, $EP_2(4.39091, 4.39091, 25, -1.18673)$, and $EP_3(-3.98551, -3.98551, 25, 1.07716)$.

By linearizing chaotic system equations (2), the Jacobian matrix is obtained as

$$J = \begin{bmatrix} -a & a & 0 & 0 \\ c - x_3 & 0 & -x_1 & 0 \\ x_2 & x_1 & -d & b \\ -1 & 0 & 0 & -a \end{bmatrix}. \quad (8)$$

The Jacobian matrix for chaotic system equations (2) at equilibrium point $EP_1(0, 0, 0, 0)$ is obtained as

$$J(EP_1) = \begin{bmatrix} -a & a & 0 & 0 \\ c & 0 & 0 & 0 \\ 0 & 0 & -d & b \\ -1 & 0 & 0 & -a \end{bmatrix} \Big|_{x=EP_1}$$

FIGURE 4: Initial value control circuit of chaotic system equations (2).

$$= \begin{bmatrix} -a & a & 0 & 0 \\ c & 0 & 0 & 0 \\ 0 & 0 & -d & b \\ -1 & 0 & 0 & -a \end{bmatrix}$$

(9)

which has the eigenvalues

$$\lambda_1^{(EP_1)} = -0.7,$$

$$\lambda_2^{(EP_1)} = -3.7,$$

$$\lambda_3^{(EP_1)} = -11.6440,$$

$$\lambda_4^{(EP_1)} = 7.9440.$$

(10)

The Jacobian matrix for chaotic system equations (2) at equilibrium point $EP_2(4.39091, 4.39091, 25, -1.18673)$ is obtained as

$$J(EP_2) = \begin{bmatrix} -a & a & 0 & 0 \\ c - x_3 & 0 & -x_1 & 0 \\ x_2 & x_1 & -d & b \\ -1 & 0 & 0 & -a \end{bmatrix} \bigg|_{x=EP_2}$$

(11)

$$= \begin{bmatrix} -a & a & 0 & 0 \\ c - 25 & 0 & -4.39091 & 0 \\ 4.39091 & 4.39091 & -d & b \\ -1 & 0 & 0 & -a \end{bmatrix}$$

FIGURE 5: Two-dimensional phase portrait of Figure 4 with control switch turn-on time 3 sec (initial value voltage $x_1(0) = V_{c1} = 0.5 \text{ V} * 3 \text{ sec} = 1.5 \text{ V}$, with initial value control).

which has the eigenvalues

$$\begin{aligned}\lambda_1^{(EP_2)} &= 0.4161 + 5.1585i, \\ \lambda_2^{(EP_2)} &= 0.4161 - 5.1585i, \\ \lambda_3^{(EP_2)} &= -5.5368, \\ \lambda_4^{(EP_2)} &= -3.3954.\end{aligned}\tag{12}$$

The Jacobian matrix for chaotic system equations (2) at equilibrium point $EP_3(-3.98551, -3.98551, 25, 1.07716)$ is obtained as

$$J(EP_3) = \begin{bmatrix} -a & a & 0 & 0 \\ c - x_3 & 0 & -x_1 & 0 \\ x_2 & x_1 & -d & b \\ -1 & 0 & 0 & -a \end{bmatrix}_{x=EP_3}$$

$$= \begin{bmatrix} -a & a & 0 & 0 \\ c - 25 & 0 & 3.98551 & 0 \\ -3.98551 & -3.98551 & -d & b \\ -1 & 0 & 0 & -a \end{bmatrix}\tag{13}$$

which has the eigenvalues

$$\begin{aligned}\lambda_1^{(EP_3)} &= 0.4488 + 4.7403i, \\ \lambda_2^{(EP_3)} &= 0.4488 - 4.7403i, \\ \lambda_3^{(EP_3)} &= -4.7828, \\ \lambda_4^{(EP_3)} &= -4.2148.\end{aligned}\tag{14}$$

Since the linearization matrices $J(EP_1)$, $J(EP_2)$, and $J(EP_3)$ have at least one eigenvalue with positive real parts, according to Lyapunov stability method [15], then the equilibrium points EP_1 , EP_2 , and EP_3 are unstable, and this implies

FIGURE 6: Two-dimensional phase portrait of Figure 4 with control switch turn-on time 4 sec (initial value voltage $x_1(0) = V_{c1} = 0.5 \text{ V} * 4 \text{ sec} = 2 \text{ V}$, with initial value control).

chaos in the dissipative chaotic system equation (2). So, the trajectories of chaotic system equation (2) diverge from the three equilibrium points and orbit into the strange attractor of chaotic system Eq. (2).

3. Initial Value Control Circuit for Chaotic Systems

The four-dimensional modified Lorenz-Stenflo system can be modeled by an electronic circuit [16–19]. To implement system equations (2), a circuit is constructed in Figure 2; the governing integral equation of the circuit can be written as (15) and some simulation results are showed in Figure 3. A initial value control circuit design for system equations (2) is designed in Figure 4; the two-dimensional phase portraits of chaotic system (2) with initial value control circuit are shown in Figures 5 and 6. From Figures 5 and 6, we can observe the results, when the control switch turn-on time is different, the capacitor is charged to different voltage level. For example, if the control switch turn-on time = 3 sec, the capacitor is charged to $V_{c1} = 0.5 \text{ V} * 3 \text{ sec} = 1.5 \text{ V}$. By way of changing the charged voltage level in capacitor, we can change the initial

value in system equations (2) and the behavior of system equations (2) also is modified. The real circuit of the modified four-dimensional Lorenz-Stenflo system is constructed on a breadboard shown in Figure 7. The experimental setup is shown in Figure 8 and the experimental results are shown in Figure 9.

$$\begin{aligned}
 \dot{x}_1 &= \frac{1}{R_2 C_1} x_2 - \frac{1}{R_1 C_1} x_1, \\
 \dot{x}_2 &= \frac{1}{R_3 C_2} x_1 - \frac{1}{R_4 C_2} x_1 x_3, \\
 \dot{x}_3 &= \frac{1}{R_7 C_3} x_1 x_2 - \frac{1}{R_8 C_3} x_3 + \frac{1}{R_6 C_3} x_4, \\
 \dot{x}_4 &= -\frac{1}{R_9 C_4} x_1 - \frac{1}{R_{10} C_4} x_4.
 \end{aligned} \tag{15}$$

4. Conclusions

In summary, this paper introduces an initial value control circuit design and nonlinear dynamic analysis of the modified

FIGURE 7: Realization circuit of the modified four-dimensional Lorenz-Stenflo system.

FIGURE 8: Experimental setup for realization of the modified four-dimensional Lorenz-Stenflo system.

FIGURE 9: Realization circuit results for voltage signals outputs with different control switch turn-on time.

four-dimensional Lorenz-Stenflo system is presented. Furthermore, an analog circuit of the chaotic system is implemented to verify the initial value control circuit design. We design a charge circuit embedded in modified four-dimensional Lorenz-Stenflo analog circuit and use analog circuit to implement the concept of changing initial values for chaotic systems in mathematics. Because there are operational amplifier (OPA) integrators in the modified four-dimensional Lorenz-Stenflo analog circuit, we only need Metal-Oxide-Semiconductor Field-Effect Transistor (MOS-FET) to implement the charge circuit; it is easy to realize. By way of changing the initial value, we can change the behavior of chaotic system and also change the key of information encryption. Next step we will apply the initial value control circuit of chaotic systems for information encryption application.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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