Research Article

The Snap Tension Analysis of Taut-Slack Mooring Line with tanh Method

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The continuous model is introduced, and the nonlinear partial differential equations of taut-slack mooring line system are transferred to nonlinear algebraic equations through tanh method, and four solitary solutions are obtained further. At the same time, to express the results clearly, the curve surfaces of strains, displacements, and tension are plotted. The results show that there are four different solutions in the system. With the pretension increasing, the tension changes from one solitary solution to snap tension, and when the pretension is increased further, the curve converts to continuous line, until straight line, which is corresponding to the taut mooring line. In the process of increasing of pretension, the mooring line transfers from slack to taut, accompanied with tension skipping, which is reduced by the system parameters, and different combination of parameters may introduce different tension in line, and the uncertainty may cause the breakage of mooring system. The results have an agreement with experiment, which shows that the calculating method in this paper may be believable and feasible. This work may provide reference for design of mooring system.

1. Introduction

With the development of exploitation of deep-water resources, TLP and SPAR platforms are designed and used in some areas and are considered as the excellent platforms in deep water, in which mooring line is one of the most important parts of platforms.

Mooring lines are flexible structures and have unidirectional stiffness; that is, they can only bear tension and cannot bear compressive loading. When the platform moves with large amplitude, the mooring line will transfer from taut to slack, accompanied with snap tension. The research showed that the snap tension was usually several times to dozen times more than the mean tension [1], which may be the real reason causing breakage of mooring line.

Many works have been done by the scholars to reveal the mechanism of breakage of mooring line. Niedzwecki and Thampi [2] investigated the snap load behavior of marine cable systems in regular seas. Huang and Vassalos studied the impulse loads under alternative taut-slack conditions by using the model of bilinear string [3, 4]. They neglected the effect of floating structure, the cable was simplified into the

model subjected to periodic sinusoidal excitation at the top end of cable, and the snap tension of the mooring cable was calculated [5]. The results show that the effect of tension magnification is due to the shock. Experiments show that, under sinusoidal excitation and quasi-static condition, the result calculated by the quasi-static method is near the experiment result. But under the impact condition, the impact tension is much larger than the result calculated by the quasi-static method [6]. The equivalent force model is used to analyze the effect of mooring systems on the horizontal motions [7]. Zhang et al. studied the analytical solution of taut-slack mooring line with tanh method and obtained snap tension curves in four cases [8]. Until now, most of the solutions to the equations are numerical simulation, and the analytical solution is not fully developed.

At the same time, different methods solving the nonlinear evolution equations have been developed. The tanh method, developed for years, is one of the most direct and effective algebraic methods for finding exact solutions of nonlinear diffusion equations. Recently, much work has been concentrated on the various extension and applications of the method [9]. Huibin and Kelin [10] introduced a

power series in $\tanh$ as a possible solution and substituted this expansion directly into a higher-order KdV equation. Wazwaz [11] employed the $\tanh$ method for traveling wave solutions of nonlinear equations and the work was extended to equations that do not have $\tanh$ polynomial solutions. Abdou and Soliman [12] presented and implemented the solutions of nonlinear physical equations for constructing multiple traveling wave in a computer algebraic system by means of computerized symbolic computation and a modified extended $\tanh$ -function method. Zheng et al. [13] presented the generalized extended $\tanh$ -function method for constructing the exact solutions of nonlinear partial differential equations (NPDEs) in a unified way making use of a new generalized analysis. As a result, the solitary wave solutions and other new and more general solutions were obtained. Based on the symbolic computation, Zhi and Zhang [14] combined the $\tanh$ -function method with the symmetry method to construct new type of solutions of the nonlinear evolution equations for the first time. With the combined method, some new types of solutions of the coupled $(2 + 1)$ -dimensional nonlinear system of Schrodinger equations were obtained.

In this paper, to reveal the mechanism of snap tension in mooring line while transforming from taut to slack or from slack to taut, the continuous model is introduced and $\tanh$ method is used to resolve the nonlinear equations of taut-slack mooring line. Four types of analytical solutions of taut-slack mooring line are obtained, and the displacement and tension curve surfaces in different cases are presented, and the effects of pretension will be discussed in Section 4. The snap tension is obtained, which may be the reason causing the breakage of mooring line.

2. The $\tanh$ Method

Consider a given evolution, say in two variables [15],

$$H(u, u_t, u_x, u_{xx}, \dots) = 0, \quad (1)$$

where $u = u(x, t)$ and H is a polynomial about u and its derivatives.

The fact that the solutions of many nonlinear equations can be expressed as a finite series of $\tanh$ function motivates us to seek for the solutions of (1) in the form

$$u(x, t) = u(\xi) = \sum_{i=0}^m a_i T^i = \sum_{i=0}^m a_i \tanh(\xi)^i, \quad (2)$$

where $\xi = \kappa(x - ct) + \xi_0$. Notice that the highest order of $\mathbf{d}^p u / \mathbf{d}\xi^p$ is

$$\begin{aligned} o\left(\frac{\mathbf{d}^p u}{\mathbf{d}\xi^p}\right) &= m + p, \quad p = 1, 2, 3, \dots, \\ o\left(u^q \frac{\mathbf{d}^p u}{\mathbf{d}\xi^p}\right) &= (q + 1)m + p, \quad (3) \\ q &= 1, 2, 3, \dots; \quad p = 1, 2, 3, \dots \end{aligned}$$

So m can be obtained by balancing the derivative term of the highest order with the nonlinear term in (1). $k, c, a_0, \dots, a_m$

are parameters to be determined. Substituting (2) into (1) will yield a set of algebraic equations for $k, c, a_0, \dots, a_m$ because all coefficients of $\tanh(\xi)^i$ have to vanish. Using these relations, $k, c, a_0, \dots, a_m$ can be obtained. Therefore, the traveling solitary wave solutions are obtained. Usually m is a positive integer; however, once in a while, the value of m is a negative or a fraction. In these cases, we can introduce a transformation $u(\xi) = v(\xi)^{\text{sign}(m)/d}$, where d is the denominator of m , and transform (1) into another equation for v , whose balancing number will be a positive integer. Then it can be dealt with by the above method. Therefore the final solution u may be a rational or radical function about $\tanh(\xi)$.

3. Equations of Motion

The model of mooring line is shown in Figure 1. The horizontal uniform mooring line is considered, and the shear deformation is neglected. The left end of the line is fixed, and the right end is attached to the buoy; here the condition is simplified by assuming harmonic excitation $A_b \sin \omega t$. e_1 and e_2 are the unit vector in tangential and normal direction, respectively, and e_3 is the binormal unit vector. χ^o and χ^f represent the static and dynamic configuration, and $U(U_1, U_2, U_3)$ is the displacement relative to the static configuration.

Only the in-plane motions of mooring line are considered, and the equations are as follows [16]:

$$\begin{aligned} \rho A U_{1,tt} &= [(P + EA\varepsilon)(1 + U_{1,s} - \psi U_2)]_{,s} \\ &\quad - \psi(P + EA\varepsilon)(U_{2,s} + \psi U_1) \\ &\quad - (\rho - \rho_w) Agl_t + F_1 \end{aligned} \quad (4)$$

$$\begin{aligned} \rho A U_{2,tt} &= [(P + EA\varepsilon)(U_{2,s} + \psi U_1)]_{,s} \\ &\quad + \psi(P + EA\varepsilon)(1 + U_{1,s} - \psi U_2) \\ &\quad - (\rho - \rho_w) Agl_n + F_2, \end{aligned} \quad (5)$$

where ε is the dynamic strain, and the expression is as follows:

$$\varepsilon = U_{1,s} - \psi U_2 + \frac{1}{2} [(U_{1,s} - \psi U_2)^2 + (U_{2,s} + \psi U_1)^2]. \quad (6)$$

EA represents the cable section modulus, ρA represents the cable mass/length, and F_1, F_2 are components of an arbitrary external excitation, including the hydrodynamic loading and exciting from the buoy attached at the end, mainly the end exciting. In other words, the transforming from taut to slack of mooring line is mainly caused by the large amplitude motion of attached buoy. The hydrodynamic loading is calculated with Morison's equation as follows:

$$F_1 = C_{at} \rho_w \frac{\pi d^2}{4} U_{1,tt} + C_{dt} \frac{\rho_w d}{2} |U_{1,t} - v_t| (U_{1,t} - v_t) \quad (7)$$

$$F_2 = C_{an} \rho_w \frac{\pi d^2}{4} U_{2,tt} + C_{dn} \frac{\rho_w d}{2} |U_{2,t} - v_n| (U_{2,t} - v_n).$$

C_{an} and C_{at} are the added mass coefficients in tangential and normal direction, respectively, C_{dt} and C_{dn} are fluid drag

FIGURE 1: The model of mooring line.

coefficients in tangential and normal direction, respectively, v is the velocity of fluid particle, and the subscripts t and n denote the projection in tangential and normal directions, respectively. U_1 and U_2 are the displacements in tangential and normal directions, subscript s denotes partial differentiation with respect to arc lengths, and t denotes partial differentiation with respect to time. Moreover, $P(s)$ and $\psi(s)$ are the equilibrium tension and curvature distributions describing a catenary as given by

$$P = \sqrt{P_0^2 + (\rho A g s)^2}, \quad (8)$$

$$\psi = \frac{\rho A g P_0}{(P_0^2 + (\rho A g s)^2)},$$

where P_0 denotes the horizontal component of the equilibrium cable tension.

Now considering the effects of curvature and tension in line, substitute (6) and (8) into (4) and (5), and the equations can be reduced as

$$\begin{aligned} & g \left(\frac{3}{2} \lambda^3 s^2 - \frac{1}{2} \lambda^7 s^6 - \lambda \right) U_1 + \left(g \lambda^2 s + 2g \lambda^4 s^3 \right. \\ & \quad \left. + 2 \frac{E}{\rho} \lambda^3 s \right) U_2 + g \lambda s U_{1,s} + 2 \frac{E}{\rho} (\lambda^6 s^3 - \lambda^4 s) U_1^2 \\ & \quad + \left(g \lambda^2 s^2 + \frac{E}{\rho} \lambda^3 s^2 + g \lambda^4 s^4 - \frac{E}{\rho} \lambda - 2g \right) U_{2,s} + 6 \\ & \quad \cdot \frac{E}{\rho} (\lambda^6 s^3 - \lambda^4 s) U_2^2 + \frac{E}{\rho} (\lambda^3 s^2 - \lambda) U_2 U_{2,s} U_{2,ss} \\ & \quad + \frac{E}{\rho} \left(2\lambda^6 s^2 - 3\lambda^8 s^4 + 2\lambda^{10} s^6 - \frac{1}{2} \lambda^{12} s^8 - \frac{1}{2} \lambda^4 \right) U_1^3 \\ & \quad + 3 \frac{E}{\rho} (\lambda^5 s - 2\lambda^7 s^3 + \lambda^9 s^5) U_2^3 + \left(\frac{E}{\rho} + \frac{P_0}{\rho A} \right. \\ & \quad \left. + \frac{1}{2} g \lambda s^2 \right) U_{1,ss} - \left(1 + C_{at} \rho_w \frac{\pi d^2}{4 \rho A} \right) U_{1,tt} + C_{dt} \end{aligned}$$

$$\begin{aligned} & \cdot \frac{\rho_w d}{2 \rho A} |U_{1,t} - v_t| (U_{1,t} - v_t) + g \lambda s + 3 \frac{E}{\rho} U_{1,s} U_{1,ss} \\ & \quad + \frac{E}{\rho} U_{2,s} U_{2,ss} + \frac{3}{2} \frac{E}{\rho} U_{1,s}^2 U_{1,ss} + 4 \frac{E}{\rho} (\lambda^2 - 2\lambda^4 s^2 \\ & \quad + \lambda^6 s^4) U_2 U_{2,s} + \frac{E}{\rho} (\lambda^3 s^2 - \lambda) U + 2 \frac{E}{\rho} (3\lambda^5 s^2 \\ & \quad - 3\lambda^7 s^4 + \lambda^9 s^6 - \lambda^3) U_2^2 U_{2,s} + 2 \frac{E}{\rho} (\lambda^6 s^3 - \lambda^4 s) \\ & \quad \cdot U_1^2 U_{1,s} + \frac{E}{\rho} \lambda^3 s U_{2,s}^2 U_{2,ss} + \frac{E}{\rho} \left(\frac{1}{2} \lambda^6 s^4 + \frac{1}{2} \lambda^2 \right. \\ & \quad \left. - \lambda^4 s^2 \right) U_{1,s} U_1 + \frac{E}{\rho} \left(\frac{1}{2} \lambda^2 - \lambda^4 s^2 + \frac{1}{2} \lambda^6 s^4 \right) U_1^2 U_{1,ss} \\ & \quad = 0 \\ & \quad g (1 - \lambda^4 s^4 - \lambda^2 s^2) + g (-\lambda^2 s - 2\lambda^4 s^3) U_1 + g \lambda s U_{2,s} \\ & \quad + \frac{5}{2} \frac{E}{\rho} (\lambda - \lambda^3 s^2) U_{1,s} - \left(1 + C_{an} \rho_w \frac{\pi d^2}{4 \rho A} \right) U_{2,tt} \\ & \quad + C_{dn} \frac{\rho_w d}{2 \rho A} |U_{2,t} - v_n| (U_{2,t} - v_n) + \left(2 \frac{E}{\rho} \lambda^4 s^2 \right. \\ & \quad \left. - \frac{E}{\rho} \lambda^6 s^4 - g \lambda - \frac{1}{2} g \lambda^7 s^6 + \frac{3}{2} g \lambda^3 s^2 - \frac{E}{\rho} \lambda^2 \right) U_2 + 3 \\ & \quad \cdot \frac{E}{\rho} (-\lambda^5 s + 2\lambda^7 s^3 - \lambda^9 s^5) U_1^3 + \frac{E}{\rho} \left(2\lambda^6 s^2 - 3\lambda^8 s^4 \right. \\ & \quad \left. + 2\lambda^{10} s^6 - \frac{1}{2} \lambda^{12} s^8 - \frac{1}{2} \lambda^4 \right) U_2^3 + \left(2g - \frac{E}{\rho} \lambda^3 s^2 \right. \\ & \quad \left. - g \lambda^4 s^4 - g \lambda^2 s^2 + \frac{E}{\rho} \lambda \right) U_{1,s} + \frac{1}{2} \frac{E}{\rho} (3\lambda^7 s^4 - \lambda^9 s^6 \\ & \quad - 3\lambda^5 s^2 + \lambda^3) U_1^2 + \frac{3}{2} \frac{E}{\rho} (3\lambda^7 s^4 - \lambda^9 s^6 - 3\lambda^5 s^2 \end{aligned}$$

$$\begin{aligned}
& + \lambda^3) U_2^2 + \left(\frac{1}{2} g \lambda s^2 + \frac{P_0}{\rho A} \right) U_{2,ss} + \frac{1}{2} \frac{E}{\rho} (\lambda^3 s^2 - \lambda) \\
& \cdot U_{2,s}^2 + \frac{E}{\rho} (\lambda - \lambda^3 s^2) U_{1,s}^3 + 2 \frac{E}{\rho} \lambda^3 s U_2 U_{2,s} + 2 \\
& \cdot \frac{E}{\rho} (3 \lambda^7 s^4 - 3 \lambda^5 s^2 - \lambda^9 s^6 + \lambda^3) U_{1,s} U_1^2 + 2 \frac{E}{\rho} (\lambda^6 s^3 \\
& - \lambda^4 s) U_{2,s} U_2^2 - \frac{E}{\rho} \lambda^3 s U_1 U_{1,s}^2 + \frac{E}{\rho} (\lambda^3 s^2 - \lambda) U_2 U_{2,ss} \\
& + \frac{E}{\rho} (\lambda - \lambda^3 s^2) U_1 U_{1,s} U_{1,ss} + \frac{E}{\rho} \left(\frac{1}{2} \lambda^6 s^4 - \lambda^4 s^2 \right. \\
& + \frac{1}{2} \lambda^2) U_2 U_{2,s}^2 + \frac{3}{2} \frac{E}{\rho} U_{2,s}^2 U_{2,ss} + \frac{E}{\rho} \left(\frac{1}{2} \lambda^2 - \lambda^4 s^2 \right. \\
& + \frac{1}{2} \lambda^6 s^4) U_2^2 U_{2,ss} + \frac{E}{\rho} (\lambda - \lambda^3 s^2) U_1 U_{1,ss} - 2 \frac{E}{\rho} \\
& \cdot \lambda^3 s U_1 U_{1,s} = 0,
\end{aligned} \tag{9}$$

where $\lambda = \rho A g / P_0$. The dynamic tension T is that

$$T = P_0 + E A \epsilon. \tag{10}$$

In the quasi-static fluid, $v_t = 0$ and $v_n = 0$. Then the initial equations may be simplified and rewritten as follows:

$$\begin{aligned}
& A_1 U_{1,ss} - A_2 U_{1,tt} + A_3 + A_4 U_1 + A_5 U_2 + A_6 U_{1,s} \\
& + A_7 U_{2,s} + A_8 U_1^2 + A_9 U_2^2 + A_{10} U_1^3 + A_{11} U_2^3 \\
& + A_{12} U_{1,s} U_{1,ss} + A_{13} U_{2,s} U_{2,ss} + A_{14} U_{1,s}^2 U_{1,ss} \\
& + A_{15} U_2 U_{2,s} + A_{16} U_2^2 U_{2,s} + A_{17} U_1^2 U_{1,s} \\
& + A_{18} U_{1,s}^2 U_1 + A_{19} U_{2,s}^2 U_2 + A_{20} U_{2,s}^3 \\
& + A_{21} U_2 U_{2,s} U_{2,ss} + A_{22} U_1^2 U_{1,ss} \\
& + A_{23} \operatorname{sgn}(U_{1,t}) U_{1,t}^2 = 0 \\
& B_1 U_{2,ss} - B_2 U_{2,tt} + B_3 + B_4 U_1 + B_5 U_2 + B_6 U_{1,s} \\
& + B_7 U_{2,s} + B_8 U_1^2 + B_9 U_2^2 + B_{10} U_1^3 + B_{11} U_2^3 \\
& + B_{12} U_{1,s}^2 + B_{13} U_{2,s}^2 + B_{14} U_{1,s}^3 + B_{15} U_2 U_{2,s} \\
& - B_{16} U_1 U_{1,s} + B_{17} U_{1,s} U_1^2 + B_{18} U_{2,s} U_2^2 - B_{19} U_1 U_{1,s}^2 \\
& + B_{20} U_2 U_{2,s}^2 + B_{21} U_1 U_{1,ss} + B_{22} U_2 U_{2,ss} \\
& + B_{23} U_1 U_{1,s} U_{1,ss} + B_{24} U_{2,s}^2 U_{2,ss} + B_{25} U_2^2 U_{2,ss} \\
& + B_{26} \operatorname{sgn}(U_{2,t}) U_{2,t}^2 = 0,
\end{aligned} \tag{11}$$

where A_i and B_i are coefficients, shown in the Appendix.

4. Results and Discussions

Assuming that U_1, U_2 are the polynomials of T with m and n ranks, respectively, the nonlinear term $A_{14} \kappa^4 (U_1')^2 U_1''$ in (11) is $3m + 4$ -order polynomials of T , and $A_{21} \kappa^3 U_2 U_2' U_2''$ is $3n + 3$ -order polynomials of T , while the highest order linear term is $A_1 \kappa^2 U_1''$, which is $m + 2$ polynomials of T . Similarly, in (12), the nonlinear terms $B_{23} \kappa^3 U_1 U_1' U_1''$ and $B_{24} \kappa^4 (U_2')^2 U_2''$ are $3m + 3$ and $3n + 4$ polynomials of T , respectively, and the highest order linear term $B_1 \kappa^2 U_2''$ is $n + 2$. To look for the traveling wave solution of (11) and (12), the parameters m and n are first determined by balancing the linear term of highest order with the nonlinear term:

$$\begin{aligned}
\max(3m + 4, 3n + 3) &= m + 2 \\
\max(3n + 4, 3m + 3) &= n + 2.
\end{aligned} \tag{13}$$

It is solved that

$$\begin{aligned}
(1) \quad & \begin{cases} m = -1 \\ n = -1 \end{cases} \\
(2) \quad & \begin{cases} m = -1 \\ n = -2 \end{cases} \\
(3) \quad & \begin{cases} m = -2 \\ n = -1 \end{cases} \\
(4) \quad & \begin{cases} m = -\frac{1}{2} \\ n = -\frac{1}{2} \end{cases}.
\end{aligned} \tag{14}$$

For cases (2), (3), and (4), there is no nontrivial solution in terms of tanh function. For case (1), the following transformation is introduced:

$$\begin{aligned}
U_1(\xi) &= \frac{1}{b_0 + b_1 \tanh(\xi)}, \\
U_2(\xi) &= \frac{1}{a_0 + a_1 \tanh(\xi)}, \\
\xi &= \kappa(x - c_1 t),
\end{aligned} \tag{15}$$

where $a_0, a_1, b_0, b_1, \kappa$, and c_1 are undetermined coefficients.

Submitting the transformation into (9), collecting the coefficients of T with the same orders, the nonlinear algebraic equations can be obtained:

$$\begin{aligned}
P_1 &= A_5 b_0 + A_3 + A_9 b_0^2 + A_4 a_0 + A_{15} \kappa b_1 b_0 \\
&+ A_{16} \kappa b_1 b_0^2 + A_8 a_0^2 + A_6 \kappa a_1 + A_{10} a_0^3 + A_{11} b_0^3 \\
&+ A_{18} \kappa^2 a_0 a_1^2 + A_{17} \kappa a_1 a_0^2 + A_{19} \kappa^2 b_0 b_1^2 + A_7 \kappa b_1 \\
&+ A_{20} \kappa^3 b_1^3 = 0
\end{aligned}$$

$$\begin{aligned}
P_2 &= 2A_8a_1a_0 + 2A_9b_1b_0 + 3A_{10}a_1a_0^2 + A_4a_1 + A_5b_1 \\
&\quad + 3A_{11}b_1b_0^2 + A_{15}\kappa b_1^2 + A_{18}\kappa^2a_1^3 + A_{19}\kappa^2b_1^3 \\
&\quad + 2A_{16}\kappa b_0b_1^2 + 2A_{17}\kappa a_0a_1^2 = 0 \\
P_3 &= 3A_{10}a_0a_1^2 + A_8a_1^2 + A_9b_1^2 + 3A_{11}b_0b_1^2 + A_{16}\kappa b_1^3 \\
&\quad + A_{17}\kappa a_1^3 = 0 \\
P_4 &= A_{11}b_1^3 + A_{10}a_1^3 = 0 \\
P_5 &= B_7\kappa b_1 + B_{12}\kappa^2a_1^2 - B_{19}\kappa^2a_0a_1^2 + B_6\kappa a_1 + B_4a_0 \\
&\quad + B_8a_0^2 + B_9b_0^2 + B_{10}a_0^3 + B_3 + B_{11}b_0^3 + B_5b_0 \\
&\quad + B_{13}\kappa^2b_1^2 + B_{15}\kappa b_1b_0 + B_{18}\kappa b_1b_0^2 + B_{14}\kappa^3a_1^3 \\
&\quad - B_{16}\kappa a_0a_1 + B_{17}\kappa a_1a_0^2 + B_{20}\kappa^2b_1^2b_0 = 0 \\
P_6 &= 2B_8a_1a_0 + 2B_9b_1b_0 + 3B_{10}a_1a_0^2 + B_4a_1 + B_5b_1 \\
&\quad + 3B_{11}b_1b_0^2 + B_{15}\kappa b_1^2 + 2B_{18}\kappa b_1^2b_0 + B_{20}\kappa^2b_1^3 \\
&\quad - B_{19}\kappa^2a_1^3 = 0 \\
P_7 &= 3B_{10}a_0a_1^2 + B_8a_1^2 + B_9b_1^2 + 3B_{11}b_0b_1^2 + B_{18}\kappa b_1^3 \\
&\quad + B_{17}\kappa a_1^3 = 0 \\
P_8 &= B_{11}b_1^3 + B_{10}a_1^3 = 0.
\end{aligned} \tag{16}$$

Solving them with RATH package [17, 18], the four solutions are obtained.

Case 1. The parameters are that

$$\begin{aligned}
c_1 &= -\frac{\sqrt{B_1B_2}}{B_2} \\
\kappa &= \frac{\sqrt{(-B_4A_9 + A_4B_9)(-4A_4B_3A_9^2 + 4A_3B_4A_9^2 - A_9B_4A_5^2 + B_9A_4A_5^2)B_9}}{2(-B_4A_9 + A_4B_9)B_7A_9} \\
a_0 &= -\frac{A_5}{2A_9} \\
a_1 &= \frac{\sqrt{(-B_4A_9 + A_4B_9)(-4A_4B_3A_9^2 + 4A_3B_4A_9^2 - A_9B_4A_5^2 + B_9A_4A_5^2)}}{2(-B_4A_9 + A_4B_9)A_9} \\
b_0 &= \frac{-B_4A_9 + A_4B_9}{2(-B_3A_9 + A_3B_9)} \\
b_1 &= \frac{-A_6\sqrt{(-B_4A_9 + A_4B_9)(-4A_4B_3A_9^2 + 4A_3B_4A_9^2 - A_9B_4A_5^2 + B_9A_4A_5^2)}}{2(-B_3A_9 + A_3B_9)B_7A_4A_9}B_9.
\end{aligned} \tag{17}$$

The constraint parameters are that

$$\begin{aligned}
B_8 &= -\frac{B_4(-4A_6^2B_9^2A_9^2A_4B_3 + 4A_6^2B_9^2A_9^2A_3B_4 - A_6^2B_9^2A_5^2B_4A_9 + A_6^2B_9^3A_5^2A_4 + A_4^2A_9^3B_7^2B_4 - A_4^3A_9^2B_7^2B_9)}{4A_4^2A_9^2B_7^2(B_9A_3 - A_9B_3)} \\
A_7 &= \frac{A_9B_7}{B_9}, \\
B_5 &= \frac{A_5B_9}{A_9}, \\
A_8 &= -\frac{-4A_6^2B_9^2A_9^2A_4B_3 + 4A_6^2B_9^2A_9^2A_3B_4 - A_6^2B_9^2A_5^2B_4A_9 + A_6^2B_9^3A_5^2A_4 + A_4^2A_9^3B_7^2B_4 - A_4^3A_9^2B_7^2B_9}{4A_4^2A_9^2B_7^2(B_9A_3 - A_9B_3)},
\end{aligned}$$

$$\begin{aligned}
 B_6 &= \frac{A_6 B_4}{A_4}, \\
 A_1 &= \frac{A_2 B_1}{B_2}.
 \end{aligned}
 \tag{18}$$

And other parameters are arbitrary.

Case 2. The parameters are that

$$\begin{aligned}
 c_1 &= \frac{\sqrt{B_1 B_2}}{B_2}, \\
 a_0 &= -\frac{A_5}{2A_9}, \\
 a_1 &= -\frac{\sqrt{(-B_4 A_9 + A_4 B_9)(-4A_4 B_3 A_9^2 + 4A_3 B_4 A_9^2 - A_9 B_4 A_5^2 + B_9 A_4 A_5^2)}}{2(-B_4 A_9 + A_4 B_9) A_9}, \\
 b_1 &= \frac{A_6 \sqrt{(-B_4 A_9 + A_4 B_9)(-4A_4 B_3 A_9^2 + 4A_3 B_4 A_9^2 - A_9 B_4 A_5^2 + B_9 A_4 A_5^2)}}{2(-B_3 A_9 + A_3 B_9) B_7 A_4 A_9} B_9, \\
 b_0 &= -\frac{-B_4 A_9 + A_4 B_9}{2(-B_3 A_9 + A_3 B_9)}, \\
 \kappa &= -\frac{\sqrt{(-B_4 A_9 + A_4 B_9)(-4A_4 B_3 A_9^2 + 4A_3 B_4 A_9^2 - A_9 B_4 A_5^2 + B_9 A_4 A_5^2)} B_9}{2(-B_4 A_9 + A_4 B_9) B_7 A_9}.
 \end{aligned}
 \tag{19}$$

The constraint parameters are that

$$\begin{aligned}
 B_8 &= -\frac{B_4 (-4A_6^2 B_9^2 A_9^2 A_4 B_3 + 4A_6^2 B_9^2 A_9^2 A_3 B_4 - A_6^2 B_9^2 A_5^2 B_4 A_9 + A_6^2 B_9^3 A_5^2 A_4 + A_4^2 A_9^3 B_7^2 B_4 - A_4^3 A_9^2 B_7^2 B_9)}{4A_4^2 A_9^2 B_7^2 (B_9 A_3 - A_9 B_3)}, \\
 A_7 &= \frac{A_9 B_7}{B_9}, \\
 B_5 &= \frac{A_5 B_9}{A_9}, \\
 A_8 &= -\frac{-4A_6^2 B_9^2 A_9^2 A_4 B_3 + 4A_6^2 B_9^2 A_9^2 A_3 B_4 - A_6^2 B_9^2 A_5^2 B_4 A_9 + A_6^2 B_9^3 A_5^2 A_4 + A_4^2 A_9^3 B_7^2 B_4 - A_4^3 A_9^2 B_7^2 B_9}{4A_4^2 A_9^2 B_7^2 (B_9 A_3 - A_9 B_3)}, \\
 B_6 &= \frac{A_6 B_4}{A_4}, \\
 A_1 &= \frac{A_2 B_1}{B_2}.
 \end{aligned}
 \tag{20}$$

And other parameters are arbitrary.

Case 3. The parameters are that

$$\begin{aligned}
 a_1 &= -\frac{\sqrt{(-B_4 A_9 + A_4 B_9)(-4A_4 B_3 A_9^2 + 4A_3 B_4 A_9^2 - A_9 B_4 A_5^2 + B_9 A_4 A_5^2)}}{2(-B_4 A_9 + A_4 B_9) A_9}, \\
 c_1 &= -\frac{\sqrt{B_1 B_2}}{B_2}
 \end{aligned}$$

$$\begin{aligned}
b_1 &= \frac{A_6 \sqrt{(-B_4 A_9 + A_4 B_9) (-4A_4 B_3 A_9^2 + 4A_3 B_4 A_9^2 - A_9 B_4 A_5^2 + B_9 A_4 A_5^2)}}{2(-B_3 A_9 + A_3 B_9) B_7 A_4 A_9} B_9, \\
a_0 &= -\frac{A_5}{2A_9} \\
b_0 &= -\frac{-B_4 A_9 + A_4 B_9}{2(-B_3 A_9 + A_3 B_9)}, \\
\kappa &= -\frac{\sqrt{(-B_4 A_9 + A_4 B_9) (-4A_4 B_3 A_9^2 + 4A_3 B_4 A_9^2 - A_9 B_4 A_5^2 + B_9 A_4 A_5^2)} B_9}{2(-B_4 A_9 + A_4 B_9) B_7 A_9}.
\end{aligned} \tag{21}$$

The constraint parameters are that

$$\begin{aligned}
B_8 &= -\frac{B_4 (-4A_6^2 B_9^2 A_9^2 A_4 B_3 + 4A_6^2 B_9^2 A_9^2 A_3 B_4 - A_6^2 B_9^2 A_5^2 B_4 A_9 + A_6^2 B_9^3 A_5^2 A_4 + A_4^2 A_9^3 B_7^2 B_4 - A_4^3 A_9^2 B_7^2 B_9)}{4A_4^2 A_9^2 B_7^2 (B_9 A_3 - A_9 B_3)}, \\
B_5 &= \frac{A_5 B_9}{A_9}, \\
B_6 &= \frac{A_6 B_4}{A_4} \\
A_8 &= -\frac{-4A_6^2 B_9^2 A_9^2 A_4 B_3 + 4A_6^2 B_9^2 A_9^2 A_3 B_4 - A_6^2 B_9^2 A_5^2 B_4 A_9 + A_6^2 B_9^3 A_5^2 A_4 + A_4^2 A_9^3 B_7^2 B_4 - A_4^3 A_9^2 B_7^2 B_9}{4A_4^2 A_9^2 B_7^2 (B_9 A_3 - A_9 B_3)}, \\
A_7 &= \frac{A_9 B_7}{B_9}, \\
A_1 &= \frac{A_2 B_1}{B_2}.
\end{aligned} \tag{22}$$

And other parameters are arbitrary.

Case 4. The parameters are that

$$\begin{aligned}
c_1 &= \frac{\sqrt{B_1 B_2}}{B_2}, \\
\kappa &= \frac{\sqrt{(-B_4 A_9 + A_4 B_9) (-4A_4 B_3 A_9^2 + 4A_3 B_4 A_9^2 - A_9 B_4 A_5^2 + B_9 A_4 A_5^2)} B_9}{2(-B_4 A_9 + A_4 B_9) B_7 A_9} \\
b_0 &= -\frac{-B_4 A_9 + A_4 B_9}{2(-B_3 A_9 + A_3 B_9)}, \\
a_1 &= \frac{\sqrt{(-B_4 A_9 + A_4 B_9) (-4A_4 B_3 A_9^2 + 4A_3 B_4 A_9^2 - A_9 B_4 A_5^2 + B_9 A_4 A_5^2)}}{2(-B_4 A_9 + A_4 B_9) A_9} \\
a_0 &= -\frac{A_5}{2A_9}, \\
b_1 &= \frac{-A_6 \sqrt{(-B_4 A_9 + A_4 B_9) (-4A_4 B_3 A_9^2 + 4A_3 B_4 A_9^2 - A_9 B_4 A_5^2 + B_9 A_4 A_5^2)}}{2(-B_3 A_9 + A_3 B_9) B_7 A_4 A_9} B_9.
\end{aligned} \tag{23}$$

FIGURE 2: The displacements and tension surfaces in Case 1.

The constraint parameters are that

$$\begin{aligned}
 B_8 &= -\frac{B_4 \left(-4A_6^2 B_9^2 A_9^2 A_4 B_3 + 4A_6^2 B_9^2 A_9^2 A_3 B_4 - A_6^2 B_9^2 A_5^2 B_4 A_9 + A_6^2 B_9^3 A_5^2 A_4 + A_4^2 A_9^3 B_7^2 B_4 - A_4^3 A_9^2 B_7^2 B_9 \right)}{4A_4^2 A_9^2 B_7^2 (B_9 A_3 - A_9 B_3)}, \\
 B_5 &= \frac{A_5 B_9}{A_9}, \\
 B_6 &= \frac{A_6 B_4}{A_4}, \\
 A_8 &= -\frac{-4A_6^2 B_9^2 A_9^2 A_4 B_3 + 4A_6^2 B_9^2 A_9^2 A_3 B_4 - A_6^2 B_9^2 A_5^2 B_4 A_9 + A_6^2 B_9^3 A_5^2 A_4 + A_4^2 A_9^3 B_7^2 B_4 - A_4^3 A_9^2 B_7^2 B_9}{4A_4^2 A_9^2 B_7^2 (B_9 A_3 - A_9 B_3)}, \\
 A_7 &= \frac{A_9 B_7}{B_9}, \\
 A_1 &= \frac{A_2 B_1}{B_2}.
 \end{aligned} \tag{24}$$

And other parameters are arbitrary.

To express the solutions clearly, the plots of the solutions are drawn, as shown in Figures 2–5, and the parameters used are shown in Table 1. The four solitary solutions are 2 left traveling waves and 2 right traveling waves according to the expressions of c . From the figures, it can be seen that, in the four solitary solutions, two of them introduce solitary tension in the system, and the tension does not change with the time delay. The other solitary solutions in U_1 and U_2 induce solitary wave tension, but with the time, the tension attenuates and disappears, and the energy is absorbed by the line and depleted by the damping from structure and external medium. The solitary tension wave in line may induce the breakage of line structure.

In the problem of taut-slack mooring line, the pretension is an important factor which relates to the line station. At the same exciting loading, when the pretension is altered, the tension curves are different, as shown in Figure 6. It can be seen that when the pretension is 10000 N, the solution is 2-solitary solution, and with the pretension increasing, the tension changes from solitary solution to snap tension, and with the continuous increasing, the curve converts to continuous line, until straight line, which is corresponding to the taut mooring line. In the process of increasing the pretension, the mooring line transfers from slack to taut, accompanied with tension skip.

Given P_0 as 26000 N, the tension at $s = 1900$ m is calculated as shown in Figure 7. The results show that there is a tension skip at about $t = 7$ s, and the tension decreases.

FIGURE 3: The displacements and tension surfaces in Case 2.

FIGURE 4: The displacements and tension surfaces in Case 3.

FIGURE 5: The displacements and tension surfaces in Case 4.

When the parameters are given the same as those in [19], where the length of cable is 30 m, excitation frequency is 1.2 Hz, and amplitude is 8 cm, the calculated result is shown in Figure 8, and the experimental result is shown in

Figure 9. The calculated maximum tension is 945.5 N, and it is 790.8 N in the experiment. This is because the cable is elastic, and the pretension is not so accurate in the experiment. Excluding the effect, the theoretical result is consistent with

(a) $P_0 = 10000$ N (b) $P_0 = 15000$ N (c) $P_0 = 20000$ N (d) $P_0 = 25000$ N (e) $P_0 = 30000$ N (f) $P_0 = 40000$ N

FIGURE 6: Tension curve with different pretension.

FIGURE 7: Tension history curve in mooring line at $s = 1900$ m with the method in this paper.

TABLE 1: Parameters used in the calculation.

Quantity	Symbol	Value
Modulus of elasticity	E	1.0×10^{11} Pa
Density of cable	ρ_c	3750 kg/m ³
Cross-sectional area	A	314.2 mm ²
Drag coefficient in normal direction	C_{Dn}	1.05
Inertia coefficient	C_m	1
Exciting frequency	ω	5 rad/s
Static pretension	P_0	27 kN
Length	L	2000 m
Water density	ρ_w	1025 kg/m ³
Drag coefficient in tangent direction	C_{Dt}	0.01
Added mass coefficients	$C_{at} C_{an}$	1
Exciting amplitude	A_b	10 m

the experimental one. It indicates that the calculating method in this paper may be believable and feasible.

The results show that the skip tension is reduced by the system parameters, and different combination of parameters may introduce different tension in line, and the uncertainty may cause the breakage of mooring system.

5. Conclusions

In this paper, the continuous model is introduced, and the nonlinear partial differential equations of taut-slack mooring line system are transferred to nonlinear algebraic equations through tanh method, and four solitary solutions are obtained further. To express the results clearly, the curve surfaces of displacement U_1 , U_2 and tension T are plotted. The results show that there are four different solutions in the system. With the pretension increasing, the tension changes from solitary solution to snap tension, and when the pretension increased further, the curve converts to continuous line, until straight line, which is corresponding to the taut mooring line. In the process of increasing the pretension, the mooring line transfers from slack to taut, accompanied with tension skip, which is reduced by the system parameters, and different combination of parameters may introduce different tension in line, and the uncertainty may cause the breakage of mooring system. The results have an agreement with

FIGURE 8: Tension history curves in mooring line simulated with the same parameters in [19].

experiment, which shows that the calculating method in this paper may be believable and feasible.

Appendix

$$A_1 = \frac{E}{\rho} + \frac{P_0}{\rho A} + \frac{1}{2} g \lambda s^2$$

$$A_2 = 1 + C_{at} \rho_w \frac{\pi d^2}{4 \rho A}$$

$$A_3 = g \lambda s$$

$$A_4 = g \left(\frac{3}{2} \lambda^3 s^2 - \frac{1}{2} \lambda^7 s^6 - \lambda \right)$$

$$A_5 = \left(g \lambda^2 s + 2 g \lambda^4 s^3 + 2 \frac{E}{\rho} \lambda^3 s \right)$$

$$A_6 = g \lambda s$$

$$A_7 = \left(g \lambda^2 s^2 + \frac{E}{\rho} \lambda^3 s^2 + g \lambda^4 s^4 - \frac{E}{\rho} \lambda - 2g \right)$$

$$A_8 = 2 \frac{E}{\rho} (\lambda^6 s^3 - \lambda^4 s)$$

FIGURE 9: Tension history curves in mooring line end in middle stage at exciting end [19].

$$A_9 = 6 \frac{E}{\rho} (\lambda^6 s^3 - \lambda^4 s)$$

$$A_{10} = \frac{E}{\rho} \left(2\lambda^6 s^2 - 3\lambda^8 s^4 + 2\lambda^{10} s^6 - \frac{1}{2}\lambda^{12} s^8 - \frac{1}{2}\lambda^4 \right)$$

$$A_{11} = 3 \frac{E}{\rho} (\lambda^5 s - 2\lambda^7 s^3 + \lambda^9 s^5)$$

$$A_{12} = 3 \frac{E}{\rho}$$

$$A_{13} = 4 \frac{E}{\rho} (\lambda^2 - 2\lambda^4 s^2 + \lambda^6 s^4)$$

$$A_{14} = \frac{3}{2} \frac{E}{\rho}$$

$$A_{15} = 4 \frac{E}{\rho} (\lambda^2 - 2\lambda^4 s^2 + \lambda^6 s^4)$$

$$A_{16} = 2 \frac{E}{\rho} (3\lambda^5 s^2 - 3\lambda^7 s^4 + \lambda^9 s^6 - \lambda^3)$$

$$A_{17} = 2 \frac{E}{\rho} (\lambda^6 s^3 - \lambda^4 s)$$

$$A_{18} = \frac{E}{\rho} \left(\frac{1}{2}\lambda^6 s^4 + \frac{1}{2}\lambda^2 - \lambda^4 s^2 \right)$$

$$A_{19} = \frac{E}{\rho} \lambda^3 s$$

$$A_{20} = \frac{E}{\rho} (\lambda^3 s^2 - \lambda)$$

$$A_{21} = \frac{E}{\rho} (\lambda^3 s^2 - \lambda)$$

$$A_{22} = \frac{E}{\rho} \left(\frac{1}{2}\lambda^2 - \lambda^4 s^2 + \frac{1}{2}\lambda^6 s^4 \right)$$

$$A_{23} = C_{dt} \frac{\rho_w d}{2\rho A}$$

$$B_1 = \frac{1}{2} g \lambda s^2 + \frac{P_0}{\rho A}$$

$$B_2 = 1 + C_{an} \rho_w \frac{\pi d^2}{4\rho A}$$

$$B_3 = g (1 - \lambda^4 s^4 - \lambda^2 s^2)$$

$$B_4 = g (-\lambda^2 s - 2\lambda^4 s^3)$$

$$B_5 = 2 \frac{E}{\rho} \lambda^4 s^2 - \frac{E}{\rho} \lambda^6 s^4 - g \lambda - \frac{1}{2} g \lambda^7 s^6 + \frac{3}{2} g \lambda^3 s^2 - \frac{E}{\rho} \lambda^2$$

$$B_6 = 2g - \frac{E}{\rho} \lambda^3 s^2 - g \lambda^4 s^4 - g \lambda^2 s^2 + \frac{E}{\rho} \lambda$$

$$B_7 = g \lambda s$$

$$B_8 = \frac{1}{2} \frac{E}{\rho} (3\lambda^7 s^4 - \lambda^9 s^6 - 3\lambda^5 s^2 + \lambda^3)$$

$$B_9 = \frac{3}{2} \frac{E}{\rho} (3\lambda^7 s^4 - \lambda^9 s^6 - 3\lambda^5 s^2 + \lambda^3)$$

$$B_{10} = 3 \frac{E}{\rho} (-\lambda^5 s + 2\lambda^7 s^3 - \lambda^9 s^5)$$

$$B_{11} = \frac{E}{\rho} \left(2\lambda^6 s^2 - 3\lambda^8 s^4 + 2\lambda^{10} s^6 - \frac{1}{2}\lambda^{12} s^8 - \frac{1}{2}\lambda^4 \right)$$

$$B_{12} = \frac{5}{2} \frac{E}{\rho} (\lambda - \lambda^3 s^2)$$

$$B_{13} = \frac{1}{2} \frac{E}{\rho} (\lambda^3 s^2 - \lambda)$$

$$B_{14} = \frac{E}{\rho} (\lambda - \lambda^3 s^2)$$

$$B_{15} = 2 \frac{E}{\rho} \lambda^3 s$$

$$B_{16} = 2 \frac{E}{\rho} \lambda^3 s$$

$$B_{17} = 2 \frac{E}{\rho} (3\lambda^7 s^4 - 3\lambda^5 s^2 - \lambda^9 s^6 + \lambda^3)$$

$$B_{18} = 2 \frac{E}{\rho} (\lambda^6 s^3 - \lambda^4 s)$$

$$B_{19} = \frac{E}{\rho} \lambda^3 s$$

$$\begin{aligned}
B_{20} &= \frac{E}{\rho} \left(\frac{1}{2} \lambda^6 s^4 - \lambda^4 s^2 + \frac{1}{2} \lambda^2 \right) \\
B_{21} &= \frac{E}{\rho} (\lambda - \lambda^3 s^2) \\
B_{22} &= \frac{E}{\rho} (\lambda^3 s^2 - \lambda) \\
B_{23} &= \frac{E}{\rho} (\lambda - \lambda^3 s^2) \\
B_{24} &= \frac{3}{2} \frac{E}{\rho} \\
B_{25} &= \frac{E}{\rho} \left(\frac{1}{2} \lambda^2 - \lambda^4 s^2 + \frac{1}{2} \lambda^6 s^4 \right) \\
B_{26} &= C_{dn} \frac{\rho_w d}{2\rho A}.
\end{aligned}
\tag{A.1}$$

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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